# FYS4150 Project 4: Numerical integration

**Peder Forfang, Andreas Ellewsen**

## 1. Introduction

In this project we study the Ising model in two dimensions, without an external magnetic field. In its simplest form the energy is expressed as

$$E = -J \sum_{<kl>}^{N} s_k s_l \tag{1}$$

with $s_k = \pm 1$, $N$ is the total number of spins and $J$ is a coupling constant expressing the strength of the interaction between neighbouring spins. The symbol $< kl >$ indicates that we sum over nearest neighbours only. We will assume that we have a ferromagnetic ordering, viz $j > 0$. We also assume periodic boundary conditions. This project will be solved using the Metropolis algorithm.

## 2. Analytical solution

To start with we assume that we only have two spins in each direction, such that $L = 2$, giving us a total of 4 spins. This gives a partition function

$$Z = 12 + 4cosh(8J\beta) \tag{2}$$

where $\beta = 1/kT$. An expectation value for energy

$$< E >= \frac{-8Jsinh(8J\beta)}{3 + cosh(8J\beta)} \approx -8Jtanh(8J\beta). \tag{3}$$

A variance for the the energy

$$\sigma_E = 768J^2 \frac{cosh(8J\beta) + 1/4}{cosh(8J\beta) + 3}. \tag{4}$$

Expectation value for the magnetization

$$< M >= 0. \tag{5}$$

Expectation value for the absolute value of the magnetization

$$< |M| >= \frac{2(e^{8J\beta} + 2)}{cosh(8J\beta) + 3}. \tag{6}$$

Variance of the magnetization

$$\sigma_M = \frac{2(e^{8J\beta} + 2)}{3 + cosh(8J\beta)}. \tag{7}$$

Which gives us equations for specific heat capacity

$$C_V = \frac{768J^2}{kT^2} \frac{cosh(8J\beta) + 1/4}{cosh(8J\beta) + 3} \tag{8}$$

and magnetic susceptibility

$$\chi = \frac{1}{kT} \frac{2(e^{8J\beta} + 2)}{3 + cosh(8J\beta)} \tag{9}$$

## 3. Monte Carlo

## 4. Monte Carlo with important sampling

## 5. Conclusions

## 6. Source code

The source code for this document, the c++ project, and the python program for plotting can be found at https://github.com/tellewsen/Project3.

## 7. References

Computational Physics, Lecture notes Fall 2015, Morten Hjorth-Jensen