Technical Report

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Contents

Chapter 1

Introduction

This document provides an overview of various algorithms and their significance in computational problem-solving. It aims to set the foundation for understanding the design, analysis, and application of different algorithms across various domains.

Chapter 2

Algorithms

This chapter presents detailed analyses of different algorithms. It explores their mechanisms, use cases, and the theoretical concepts that underlie their effectiveness. Each algorithm is discussed with practical insights to highlight its strengths and potential limitations.

2.1 Optimization of Spacecraft Trajectory

2.1.1 Introduction

In this section, we present an algorithm designed to optimize a spacecraft's trajectory by maximizing fuel efficiency and utilizing gravitational assists. The algorithm calculates the optimal time and thrust parameters to minimize the cost function, which incorporates several penalties and efficiencies.

2.1.2 Variables

- m_{fuel}: Total mass of fuel onboard in kilograms (kg).
- T_{initial}: Initial thrust in Newtons (N).
- d_{closest}: Closest approach distance to the celestial body in meters (m).
- t: Time for which the thrust is applied in seconds (s).
- T: Thrust applied in Newtons (N).
- Δv: Change in velocity (delta-v) in meters per second (m/s).
- E_{grav}: Effect of gravitational assist.
- d_{traveled}: Distance traveled by the spacecraft in meters (m).
- m_{used}: Fuel mass used in kilograms (kg).

- P_{time}: Penalty for the elapsed time.
- $P_{\text{efficiency}}$: Penalty based on efficiency related to the closest approach.
- C: Total cost function value.

2.1.3 Formulas

$$
\begin{aligned}
\Delta v &= \frac{T \cdot t}{m_{\text{fuel}}} \\
E_{\text{grav}} &= \begin{cases} \ln(d_{\text{closest}}) & \text{if } d_{\text{closest}} > 1 \\ 0 & \text{otherwise} \end{cases} \\
d_{\text{traveled}} &= \frac{\Delta v \cdot t \cdot E_{\text{grav}}}{2} \\
m_{\text{used}} &= m_{\text{fuel}} - \frac{T \cdot t}{\Delta v} \\
P_{\text{time}} &= 0.05 \cdot t^{1.2} \\
P_{\text{efficiency}} &= 0.01 \cdot (d_{\text{closest}} - 7 \times 10^6)^2 \\
C &= m_{\text{used}} + P_{\text{time}} + P_{\text{efficiency}}
\end{aligned}
$$

2.1.4 Code Listing

```
import numpy as np
from scipy.optimize import minimize


# Function to optimize the spacecraft's trajectory based on fuel efficiency and gravitational assist
def optimize_trajectory(fuel_mass, initial_thrust, closest_approach):
    """
    Objective function for trajectory optimization using gravitational assist.

    Parameters:
    - fuel_mass (float): Total mass of fuel onboard in kg.
    - initial_thrust (float): Initial thrust in Newtons.
    - closest_approach (float): Closest approach distance to the celestial body in meters.

    Returns:
    - Optimal parameters (list): [time, thrust] values that minimize the cost function.
    - Cost (float): Minimized cost function value.
    """

    # Define the cost function for optimization
    def objective(params):
        time, thrust = params  # Unpack parameters: time and thrust

```

```
        # Calculate delta-v (velocity change) based on thrust and time
        delta_v = thrust * time / fuel_mass

        # Calculate the effect of gravitational assist based on closest approach distance
        gravitational_assist_effect = np.log(closest_approach) if closest_approach > 1 else 0

        # Simulate distance traveled using delta-v and gravitational effect
        distance_traveled = delta_v * time * gravitational_assist_effect / 2

        # Calculate the cost considering fuel usage, time penalty, and gravitational assist efficiency
        fuel_used = fuel_mass - (thrust * time) / delta_v
        time_penalty = 0.05 * time**1.2  # Small non-linear penalty for extended time
        efficiency_penalty = 0.01 * (closest_approach - 7e6) ** 2  # Optimal closest approach is around 7 million meters

        # Total cost combines all penalties and fuel usage
        cost = fuel_used + time_penalty + efficiency_penalty
        return cost

    # Initial guess for time and thrust
    initial_guess = [1e6, initial_thrust]

    # Define bounds for time and thrust to avoid unrealistic values
    bounds = [(1e5, 1e7), (5000, 3e5)]

    # Minimize the objective function to find the optimal time and thrust
    result = minimize(objective, initial_guess, bounds=bounds)
    return result.x, result.fun  # Return optimal parameters and the minimized cost
```

2.2 Predator-Prey System

2.2.1 Introduction

The predator-prey system is a set of differential equations known as the Lotka-Volterra equations, which describe the dynamics of biological systems in which two species interact: a prey and a predator.

2.2.2 Variables

- p_{prey}: Population of prey at time t.
- p_{predator}: Population of predators at time t.
- t: Time variable.
- α: Growth rate of the prey.

- β: Rate at which predators consume prey.
- δ: Growth rate of predators due to consumption of prey.
- γ: Natural death rate of predators.

2.2.3 Formulas

$$\frac{dp_{\text{prey}}}{dt} = \alpha p_{\text{prey}} - \beta p_{\text{prey}} p_{\text{predator}}$$
$$\frac{dp_{\text{predator}}}{dt} = \delta p_{\text{prey}} p_{\text{predator}} - \gamma p_{\text{predator}}$$

2.2.4 Code Listing

```python
# Define the predator-prey system as a system of differential equations
def predator_prey_system(populations, t, alpha, beta, delta, gamma):
    """
    Calculate the rate of change for prey and predator populations based on Lotka-Volterra equations.

    Parameters:
    - populations (list): A list with two elements: [prey, predator] populations at time t.
    - t (float): Time variable, required for odeint even though it's unused here.
    - alpha (float): Prey growth rate.
    - beta (float): Rate at which predators consume prey.
    - delta (float): Predator growth rate from consuming prey.
    - gamma (float): Natural predator death rate.

    Returns:
    - list: Contains the rate of change [dprey_dt, dpredator_dt] for prey and predator populations.
    """
    prey, predator = (
        populations  # Unpack the populations array into prey and predator variables
    )
    dprey_dt = alpha * prey - beta * prey * predator  # Rate of change for prey
    dpredator_dt = (
        delta * prey * predator - gamma * predator
    )  # Rate of change for predator
    return [dprey_dt, dpredator_dt]
```

2.3 Gravitational Assist Algorithm

2.3.1 Introduction

This section outlines an algorithm for calculating the final velocity of a spacecraft after a gravitational assist with a planet. The calculation considers the

initial velocities, masses, and distances involved.

2.3.2 Variables

- v_{sc}: Initial velocity of the spacecraft in m/s.
- v_{p}: Velocity of the planet in m/s.
- m_{p}: Mass of the planet in kg.
- d_{closest}: Distance of closest approach in meters.
- $v_{\text{rel, init}}$: Relative initial velocity between spacecraft and planet.
- v_{∞}: Velocity at infinity (far from the planet) after gravitational interaction.
- v_{final}: Spacecraft's final velocity after the gravitational assist.
- G: Gravitational constant, $6.67430 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$.

2.3.3 Formulas

$$
\begin{aligned}
v_{\text{rel, init}} &= |v_{\text{sc}} - v_{\text{p}}| \\
v_{\infty} &= \sqrt{v_{\text{rel, init}}^2 + \frac{2 \cdot G \cdot m_{\text{p}}}{d_{\text{closest}}}} \\
v_{\text{final}} &= \sqrt{v_{\text{sc}}^2 + 2 \cdot (v_{\infty}^2 - v_{\text{sc}} \cdot v_{\infty} \cdot \cos(\pi))}
\end{aligned}
$$

2.3.4 Code Listing

```
import numpy as np

# Function to calculate the final velocity of a spacecraft after gravitational assist
def gravitational_assist(spacecraft_velocity, planet_velocity, planet_mass, closest_approach_distance):
    """
    Calculate the final velocity of a spacecraft after a gravitational assist with a planet.

    Parameters:
    - spacecraft_velocity (float): Initial velocity of the spacecraft in m/s.
    - planet_velocity (float): Velocity of the planet in m/s.
    - planet_mass (float): Mass of the planet in kg.
    - closest_approach_distance (float): Distance of closest approach in meters.

    Returns:
    - v_final (float): The spacecraft's final velocity after the gravitational assist.
```

```
    """
    G = 6.67430e-11  # Gravitational constant in m^3 kg^-1 s^-2

    # Relative initial velocity between the spacecraft and the
    planet
    v_rel_initial = abs(spacecraft_velocity - planet_velocity)

    # Calculate velocity at infinity (far from the planet) after
    gravitational interaction
    v_inf = np.sqrt(v_rel_initial**2 + 2 * G * planet_mass /
    closest_approach_distance)

    # Calculate final velocity considering energy changes from the
    approach and assist
    v_final = np.sqrt(spacecraft_velocity**2 + 2 * (v_inf**2 -
    spacecraft_velocity * v_inf * np.cos(np.pi)))

    return v_final
```

2.4 Hohmann Transfer Orbit Calculation

2.4.1 Introduction

The Hohmann transfer algorithm calculates the velocity changes (delta-v) required to transfer a spacecraft from Earth's orbit to Mars's orbit. This is achieved through the use of gravitational interactions, specifically the Sun's gravitational influence.

2.4.2 Variables

- r_{earth}: Radius of Earth's orbit around the Sun in meters.
- r_{mars}: Radius of Mars's orbit around the Sun in meters.
- μ_{sun}: Gravitational constant for the Sun in m^3/s^2.
- v_{earth}: Velocity of Earth in its orbit.
- v_{mars}: Velocity of Mars in its orbit.
- a_{transfer}: Semi-major axis of the Hohmann transfer orbit.
- $v_{\text{transfer\_earth}}$: Velocity required at Earth for the transfer orbit.
- $v_{\text{transfer\_mars}}$: Velocity required at Mars for the transfer orbit.
- Δv_{earth}: Delta-v required to escape Earth's orbit and enter the transfer orbit.
- Δv_{mars}: Delta-v required to enter Mars's orbit from the transfer orbit.
- Δv_{total}: Total delta-v for the entire transfer.

2.4.3 Formulas

$$\mu_{\text{sun}} = 1.32712440018 \times 10^{20}\,\text{m}^3/\text{s}^2$$

$$v_{\text{earth}} = \sqrt{\frac{\mu_{\text{sun}}}{r_{\text{earth}}}}$$

$$v_{\text{mars}} = \sqrt{\frac{\mu_{\text{sun}}}{r_{\text{mars}}}}$$

$$a_{\text{transfer}} = \frac{r_{\text{earth}} + r_{\text{mars}}}{2}$$

$$v_{\text{transfer\_earth}} = \sqrt{\frac{2\mu_{\text{sun}}}{r_{\text{earth}}} - \frac{\mu_{\text{sun}}}{a_{\text{transfer}}}}$$

$$v_{\text{transfer\_mars}} = \sqrt{\frac{2\mu_{\text{sun}}}{r_{\text{mars}}} - \frac{\mu_{\text{sun}}}{a_{\text{transfer}}}}$$

$$\Delta v_{\text{earth}} = v_{\text{transfer\_earth}} - v_{\text{earth}}$$

$$\Delta v_{\text{mars}} = v_{\text{mars}} - v_{\text{transfer\_mars}}$$

$$\Delta v_{\text{total}} = |\Delta v_{\text{earth}}| + |\Delta v_{\text{mars}}|$$

2.4.4 Code Listing

```
import numpy as np


def calculate_hohmann_transfer(earth_orbit_radius, mars_orbit_radius):
    """
    Calculate the delta-v (velocity change) required for a Hohmann transfer orbit from Earth to Mars.

    Parameters:
    - earth_orbit_radius (float): Radius of Earth's orbit around the Sun in meters.
    - mars_orbit_radius (float): Radius of Mars's orbit around the Sun in meters.

    Returns:
    - delta_v_earth (float): Delta-v required to escape Earth's orbit and enter the transfer orbit.
    - delta_v_mars (float): Delta-v required to enter Mars's orbit from the transfer orbit.
    - total_delta_v (float): Total delta-v for the entire transfer.
    """

    # Gravitational constant for the Sun in m^3/s^2
    mu_sun = 1.32712440018e20

    # Calculate orbital velocities of Earth and Mars using circular orbit approximation
    v_earth = np.sqrt(mu_sun / earth_orbit_radius)  # Velocity of Earth in its orbit
```

```
    v_mars = np.sqrt(mu_sun / mars_orbit_radius)  # Velocity of
    Mars in its orbit

    # Calculate the semi-major axis of the Hohmann transfer orbit (
    average distance between Earth and Mars)
    transfer_orbit_semi_major_axis = (earth_orbit_radius +
    mars_orbit_radius) / 2

    # Calculate the velocity required at Earth's position for the
    transfer orbit
    v_transfer_at_earth = np.sqrt(2 * mu_sun / earth_orbit_radius -
     mu_sun / transfer_orbit_semi_major_axis)

    # Calculate the velocity required at Mars's position for the
    transfer orbit
    v_transfer_at_mars = np.sqrt(2 * mu_sun / mars_orbit_radius -
    mu_sun / transfer_orbit_semi_major_axis)

    # Delta-v at Earth: difference between the transfer orbit
    velocity and Earth's orbital velocity
    delta_v_earth = v_transfer_at_earth - v_earth

    # Delta-v at Mars: difference between Mars's orbital velocity
    and the transfer orbit velocity at Mars
    delta_v_mars = v_mars - v_transfer_at_mars

    # Total delta-v required for the Hohmann transfer (sum of
    absolute delta-v values at Earth and Mars)
    total_delta_v = abs(delta_v_earth) + abs(delta_v_mars)

    return delta_v_earth, delta_v_mars, total_delta_v
```