

Let  $S$  be a bounded above subset of  $\mathbb{R}$ , then there exists  $\alpha \in \mathbb{R}$ , denoted by  $\sup(S)$  satisfying one of the following equivalent conditions:

1. if  $\gamma < \alpha$ , then  $\gamma$  is not an upper bound
2. if  $\gamma$  is an upper bound of  $S$ , then  $\gamma \geq \alpha$
3. if  $\delta < \alpha$ , then there exists  $s \in S$ , such that  $\delta < s \leq \alpha$
4. for all  $\epsilon > 0$ , there exists  $s \in S$  such that  $s - \epsilon < s \leq \alpha$

Similar for bounded below and  $\inf(S)$ .