Hoja de formulas

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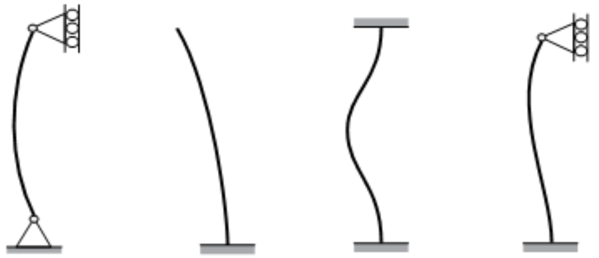


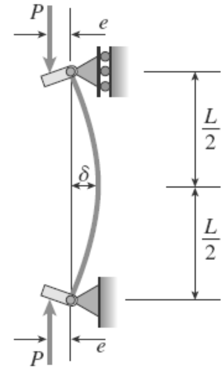
$L_e = L$ $\quad$ $L_e = 2\,L$ $\quad$ $L_e = 0{,}5\,L$ $\quad$ $L_e = 0{,}7\,L$

n^2 ← modos $\qquad$ $\lambda_{lim} = \sqrt{\frac{E}{\sigma_p}}$

$$P_{crit} = \frac{\pi^2\,E\,I}{L_e^2} \qquad r_g^2 = I/A \qquad \lambda = L_e/r_g$$

$$\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right]$$

$$\sigma_{\text{máx}} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$



$$P_{\text{perm}} = \sigma_{\text{perm}}\,A \qquad \frac{KL}{r} \geq \left(\frac{KL}{r}\right)_c \quad n_2 = \frac{23}{12} \approx 1.92 \quad \frac{\sigma_{\text{perm}}}{\sigma_Y} = \frac{(KL/r)_c^2}{2n_2(KL/r)^2}$$

$$\left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} \qquad \frac{KL}{r} \leq \left(\frac{KL}{r}\right)_c \quad n_1 = \frac{5}{3} + \frac{3(KL/r)}{8(KL/r)_c} - \frac{(KL/r)^3}{8(KL/r)_c^3} \quad \frac{\sigma_{\text{perm}}}{\sigma_Y} = \frac{1}{n_1}\left[1 - \frac{(KL/r)^2}{2(KL/r)_c^2}\right]$$

$$\underline{\underline{S}} = s_{ij}\underline{e}_i\underline{e}_j = \left(\sigma_{ij} - \delta_{ij}\sigma_h\right)\underline{e}_i\underline{e}_j\ , \quad s_{ii} = 0 \qquad \sigma'_{ij} = Q_{ip}Q_{jq}\sigma_{pq} \qquad Q_{ij} = \cos(x'_i, x_j)$$

$$\sigma_h = p = \frac{E}{3(1-2\upsilon)}\varepsilon_{pp} = K\,\varepsilon_{pp} \qquad \sigma_{ij} = C_{ijkl}(\epsilon_{kl} - \epsilon_{kl}^{\theta}) \qquad \epsilon_{ij}^{\theta} = \alpha\Delta\theta\delta_{ij}$$

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_m}{\partial x_i}\frac{\partial u_m}{\partial x_j}\right) \quad e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad \hat{e}_{ij} = e_{ij} - \frac{1}{3}e_{kk}\delta_{ij} \quad \tilde{e}_{ij} = \frac{1}{3}e_{kk}\delta_{ij} = \frac{1}{3}\vartheta\delta_{ij}$$

$$e_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} \qquad \sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + \mu(\varepsilon_{ij} + \varepsilon_{ji})$$

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{12}\\ 2\varepsilon_{23}\\ 2\varepsilon_{31}\end{bmatrix} = \frac{1}{E}\begin{bmatrix}1 & -\nu & -\nu & 0 & 0 & 0\\ & 1 & -\nu & 0 & 0 & 0\\ & & 1 & 0 & 0 & 0\\ & & & 2(1+\nu) & 0 & 0\\ & Symm. & & & 2(1+\nu) & 0\\ & & & & & 2(1+\nu)\end{bmatrix}\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{bmatrix}$$

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\ & 1-\nu & \nu & 0 & 0 & 0\\ & & 1-\nu & 0 & 0 & 0\\ & & & \frac{1-2\nu}{2} & 0 & 0\\ & & & & \frac{1-2\nu}{2} & 0\\ & Symm. & & & & \frac{1-2\nu}{2}\end{bmatrix}\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ 2\varepsilon_{12}\\ 2\varepsilon_{23}\\ 2\varepsilon_{31}\end{bmatrix}$$

	E	v	k	μ	λ
E, v	E	v	$\frac{E}{3(1-2v)}$	$\frac{E}{2(1+v)}$	$\frac{Ev}{(1+v)(1-2v)}$
E, k	E	$\frac{3k-E}{6k}$	k	$\frac{3kE}{9k-E}$	$\frac{3k(3k-E)}{9k-E}$
E, μ	E	$\frac{E-2\mu}{2\mu}$	$\frac{\mu E}{3(3\mu-E)}$	μ	$\frac{\mu(E-2\mu)}{3\mu-E}$
E, λ	E	$\frac{2\lambda}{E+\lambda+R}$	$\frac{E+3\lambda+R}{6}$	$\frac{E-3\lambda+R}{4}$	λ
v, k	$3k(1-2v)$	v	k	$\frac{3k(1-2v)}{2(1+v)}$	$\frac{3kv}{1+v}$
v, μ	$2\mu(1+v)$	v	$\frac{2\mu(1+v)}{3(1-2v)}$	μ	$\frac{2\mu v}{1-2v}$
v, λ	$\frac{\lambda(1+v)(1-2v)}{v}$	v	$\frac{\lambda(1+v)}{3v}$	$\frac{\lambda(1-2v)}{2v}$	λ
k, μ	$\frac{9k\mu}{6k+\mu}$	$\frac{3k-2\mu}{6k+2\mu}$	k	μ	$k-\frac{2}{3}\mu$
k, λ	$\frac{9k(k-\lambda)}{3k-\lambda}$	$\frac{\lambda}{3k-\lambda}$	k	$\frac{3}{2}(k-\lambda)$	λ
μ, λ	$\frac{\mu(3\lambda+2\mu)}{\lambda+\mu}$	$\frac{\lambda}{2(\lambda+\mu)}$	$\frac{3\lambda+2\mu}{3}$	μ	λ

$R = \sqrt{E^2 + 9\lambda^2 + 2E\lambda}$

Hoja de formulas

Solution	To Convert to:	E is Replaced by:	ν is Replaced by:
Plane stress	Plane strain	$\dfrac{E}{1-\nu^2}$	$\dfrac{\nu}{1-\nu}$
Plane strain	Plane stress	$\dfrac{1+2\nu}{(1+\nu)^2}E$	$\dfrac{\nu}{1+\nu}$

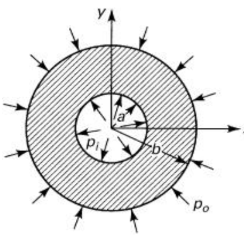
Plane Strain
$$\begin{Bmatrix}\sigma_{rr}\\ \sigma_{\theta\theta}\\ \sigma_{zz}\end{Bmatrix}=\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu\\ \nu & 1-\nu\\ \nu & \nu\end{bmatrix}\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\end{bmatrix}$$

$$\varepsilon_{rr}=u_{r,r},\ \varepsilon_{\theta\theta}=\tfrac{1}{r}\left(u_{\theta,\theta}+u_r\right),\ \varepsilon_{zz}=u_{z,z}$$

$$2\varepsilon_{\theta r}=\tfrac{1}{r}\left(u_{r,\theta}-u_\theta\right)+u_{\theta,r},\quad 2\varepsilon_{rz}=u_{z,r}+u_{r,z},\quad 2\varepsilon_{z\theta}=u_{\theta,z}+\tfrac{1}{r}u_{z,\theta}$$

Plane Stress
$$\begin{Bmatrix}\sigma_{rr}\\ \sigma_{\theta\theta}\end{Bmatrix}=\frac{E}{(1-\nu^2)}\begin{bmatrix}1 & \nu\\ \nu & 1\end{bmatrix}\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{\theta\theta}\end{bmatrix}\ ;\quad \varepsilon_{zz}=-\frac{\nu}{1-\nu}\left(\varepsilon_{rr}+\varepsilon_{\theta\theta}\right)$$

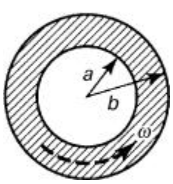
Casos axisimétricos seleccionados (Plane Stress):



$$\sigma_r=\frac{a^2p_i-b^2p_o}{b^2-a^2}-\frac{(p_i-p_o)a^2b^2}{(b^2-a^2)r^2}$$

$$\sigma_\theta=\frac{a^2p_i-b^2p_o}{b^2-a^2}+\frac{(p_i-p_o)a^2b^2}{(b^2-a^2)r^2}$$

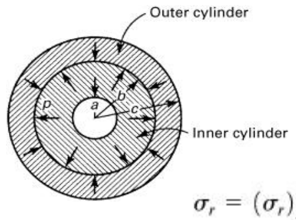
$$u=\frac{1-\nu}{E}\frac{(a^2p_i-b^2p_o)r}{b^2-a^2}+\frac{1+\nu}{E}\frac{(p_i-p_o)a^2b^2}{(b^2-a^2)r}$$



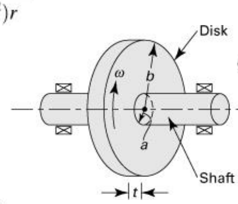
$$\sigma_r=\frac{3+\nu}{8}\left(a^2+b^2-r^2-\frac{a^2b^2}{r^2}\right)\rho\omega^2$$

$$\sigma_\theta=\frac{3+\nu}{8}\left(a^2+b^2-\frac{1+3\nu}{3+\nu}r^2+\frac{a^2b^2}{r^2}\right)\rho\omega^2$$

$$u=\frac{(3+\nu)(1-\nu)}{8E}\left(a^2+b^2-\frac{1+\nu}{3+\nu}r^2+\frac{1+\nu}{1-\nu}\frac{a^2b^2}{r^2}\right)\rho\omega^2r$$



$$p=\frac{E\delta}{b}\frac{(b^2-a^2)(c^2-b^2)}{2b^2(c^2-a^2)}$$



$$\sigma_r=(\sigma_r)_p+\frac{3+\nu}{8}\left(a^2+b^2-\frac{a^2b^2}{r^2}-r^2\right)\rho\omega^2$$

$$\sigma_\theta=(\sigma_\theta)_p+\frac{3+\nu}{8}\left(a^2+b^2-\frac{1+3\nu}{3+\nu}r^2+\frac{a^2b^2}{r^2}\right)\rho\omega^2$$

$$u=(u)_p+\frac{(3+\nu)(1-\nu)}{8E}\left(a^2+b^2-\frac{1+\nu}{3+\nu}r^2+\frac{1+\nu}{1-\nu}\frac{a^2b^2}{r^2}\right)\rho\omega^2r$$

$$\sigma_r=\sigma_\theta=-p$$

$$u=-\frac{1-\nu}{E_s}pr\qquad T=Fa=2\pi a^2fpt$$

$$\delta=\frac{ap}{E_d}\left(\frac{a^2+b^2}{b^2-a^2}+\nu\right)+\frac{ap}{E_s}(1-\nu)+\frac{a\rho\omega^2}{4E_d}\left[(1-\nu)a^2+(3+\nu)b^2\right]$$

$T=\dfrac{\sigma_0}{2}$ (en t. uniaxial) ← sale del c. de MOHR

criterios de fallo

$$\sigma_{VM}=\tfrac{1}{\sqrt{2}}\sqrt{(\sigma_I-\sigma_{II})^2+(\sigma_{II}-\sigma_{III})^2+(\sigma_{III}-\sigma_I)^2}=\sqrt{I_1^2-3I_2}$$

$$\sigma_{VM}=\sqrt{\sigma_{11}^2+\sigma_{22}^2+\sigma_{33}^2-\sigma_{22}\sigma_{33}-\sigma_{33}\sigma_{11}-\sigma_{11}\sigma_{22}+3(\sigma_{23}^2+\sigma_{13}^2+\sigma_{12}^2)}$$

$$\sigma_{VM}=\tfrac{1}{\sqrt{2}}\sqrt{(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+6(\sigma_{23}^2+\sigma_{13}^2+\sigma_{12}^2)}$$

fragil

$$\sigma_u=\text{MAX}(|\sigma_1|,|\sigma_2|,|\sigma_3|)\qquad \text{(at fracture)}$$

$$\frac{\sigma_o}{2}=\text{MAX}\left(\frac{|\sigma_1-\sigma_2|}{2},\frac{|\sigma_2-\sigma_3|}{2},\frac{|\sigma_3-\sigma_1|}{2}\right)\qquad \text{(at yielding)}$$

$$\sigma_u=\text{MAX}(|\sigma_1|,|\sigma_2|,|\sigma_3|)\qquad \text{(at fracture)}$$

$$\tau_h=\frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}\qquad \tau_o=\text{MAX}\left(\frac{|\sigma_1-\sigma_2|}{2},\frac{|\sigma_2-\sigma_3|}{2},\frac{|\sigma_3-\sigma_1|}{2}\right)\quad \text{(at yielding)}\qquad \tau_{ho}=\frac{\sqrt{2}}{3}\sigma_o$$

$$|\tau|+\underbrace{\mu\sigma}_{\text{fragil}}=\tau_i\qquad m=\sin\phi=\frac{\mu}{\sqrt{1+\mu^2}}\qquad \sigma'_{uc}=-2\tau_i\sqrt{\frac{1+m}{1-m}}\qquad \sigma_i=-|\sigma'_{uc}|+\sigma_{ut}\frac{1+m}{1-m}$$

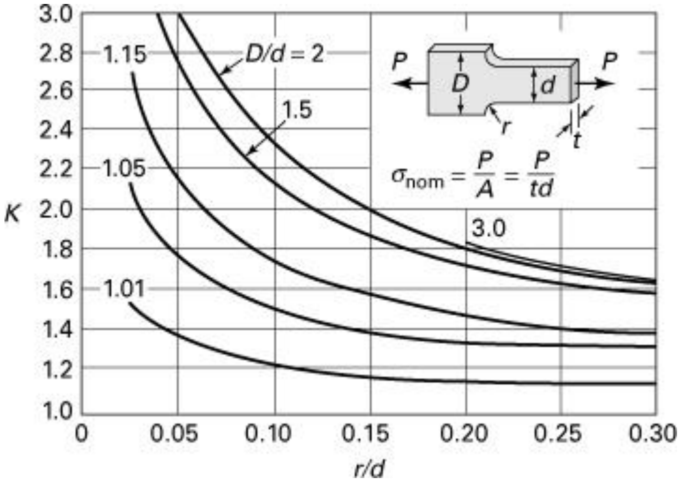
$$C_{12}=\frac{1}{1-m}\left[|\sigma_1-\sigma_2|+m(\sigma_1+\sigma_2)\right]\qquad \bar{\sigma}_{CM}=\text{MAX}(C_{12},C_{23},C_{31}),\qquad X_{CM}=\frac{|\sigma'_{uc}|}{\bar{\sigma}_{CM}}$$

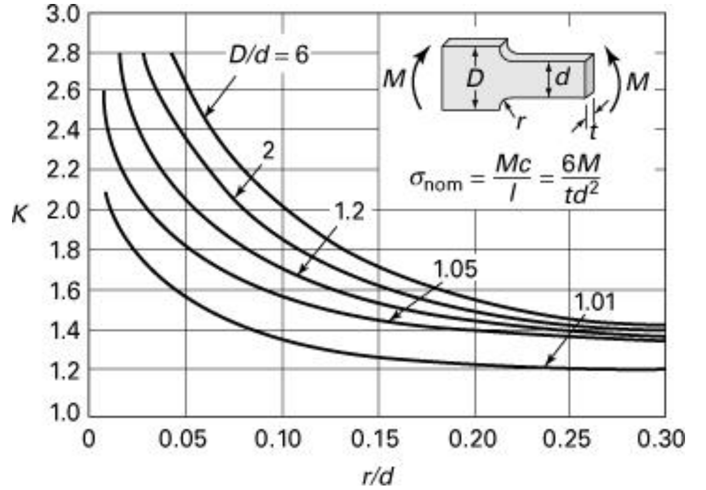
$$\bar{\sigma}_{CM}=0,\quad X_{CM}=\infty,\quad \text{if MAX}\le 0$$

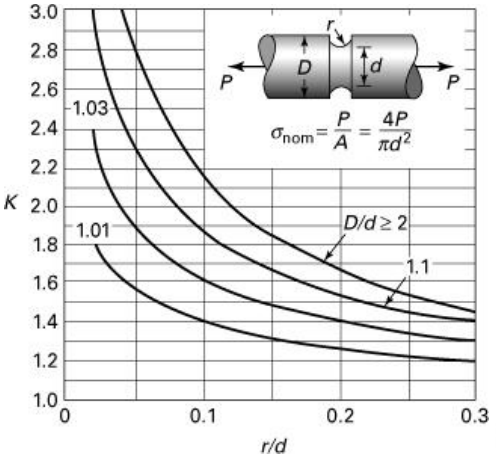
$$C_{23}=\frac{1}{1-m}\left[|\sigma_2-\sigma_3|+m(\sigma_2+\sigma_3)\right]\qquad \bar{\sigma}_{NP}=\text{MAX}(\sigma_1,\sigma_2,\sigma_3),\qquad X_{NP}=\frac{\sigma_{ut}}{\bar{\sigma}_{NP}}$$

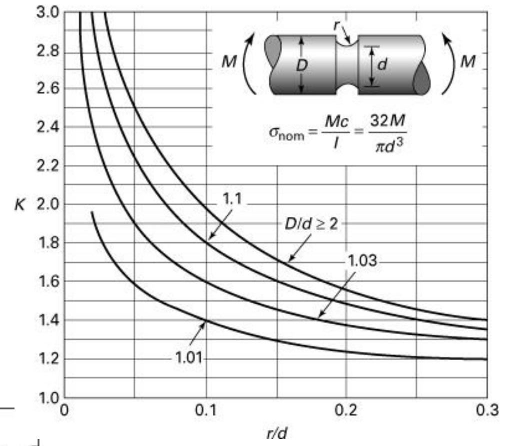
$$C_{31}=\frac{1}{1-m}\left[|\sigma_3-\sigma_1|+m(\sigma_3+\sigma_1)\right]\qquad \bar{\sigma}_{NP}=0,\quad X_{NP}=\infty,\quad \text{if MAX}\le 0$$

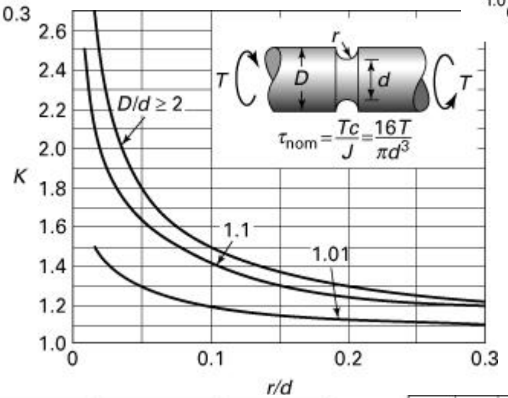
$$X_{MM}=\text{MIN}(X_{CM},X_{NP})$$

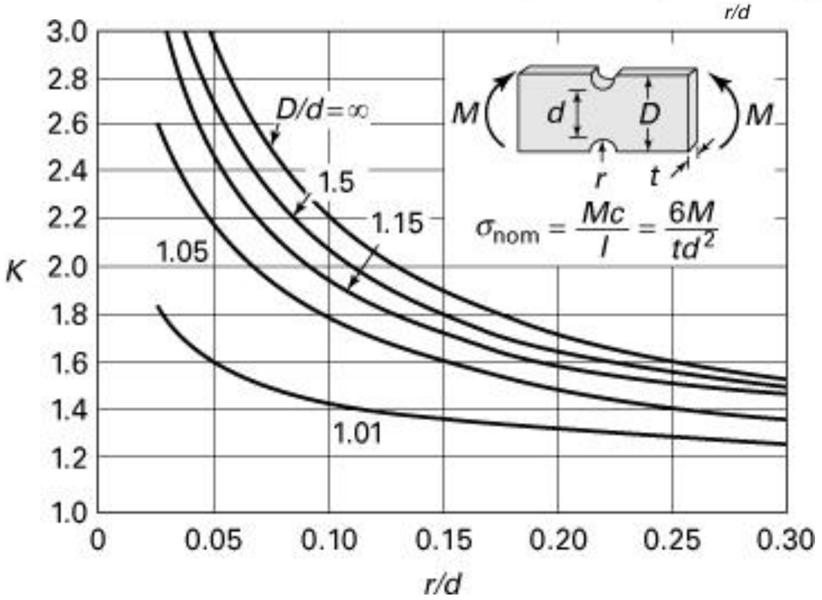

K
r/d
D/d = 2
1.15
1.5
1.05
3.0
1.01
P
D
d
r
t
$\sigma_{nom} = \frac{P}{A} = \frac{P}{td}$

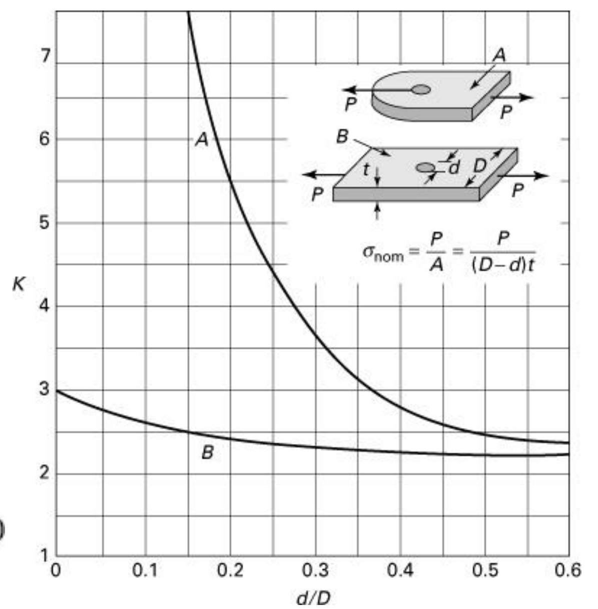

K
r/d
D/d = 6
2
1.2
1.05
1.01
M
D
d
r
t
$\sigma_{nom} = \frac{Mc}{I} = \frac{6M}{td^2}$


K
r/d
1.03
1.01
D/d ≥ 2
1.1
P
D
d
r
$\sigma_{nom} = \frac{P}{A} = \frac{4P}{\pi d^2}$

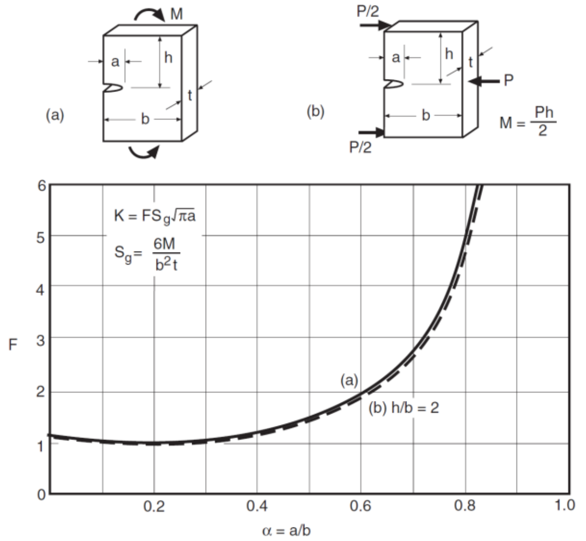

K
r/d
1.1
D/d ≥ 2
1.03
1.01
M
D
d
r
$\sigma_{nom} = \frac{Mc}{I} = \frac{32M}{\pi d^3}$

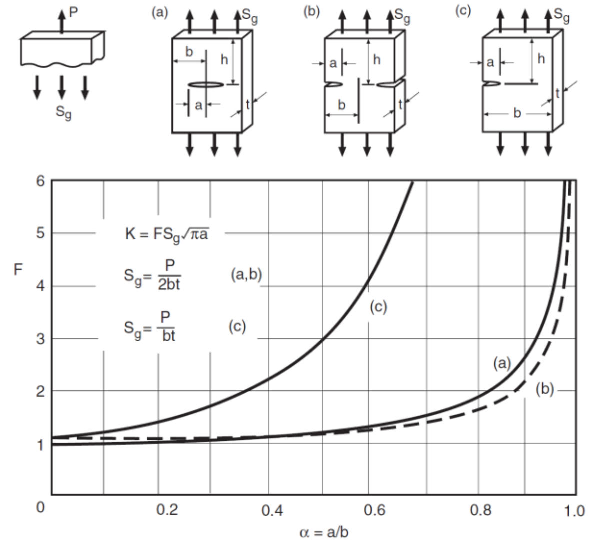

K
r/d
D/d ≥ 2
1.1
1.01
T
D
d
r
$\tau_{nom} = \frac{Tc}{J} = \frac{16T}{\pi d^3}$


K
r/d
D/d = ∞
1.5
1.15
1.05
1.01
M
d
D
r
t
$\sigma_{nom} = \frac{Mc}{I} = \frac{6M}{td^2}$


K
d/D
A
B
P
t
d
D
$\sigma_{nom} = \frac{P}{A} = \frac{P}{(D-d)t}$

$$X_a = \frac{a_c}{a} = \left(\frac{F}{F_c} X_K\right)^2 \qquad X_K = \frac{K_{Ic}}{K} = \frac{K_{Ic}}{F S_g \sqrt{\pi a}}$$





Values for small a/b and limits for 10% accuracy:

(a, b) $K = 1.12S_g\sqrt{\pi a}$ $\quad (a/b \le 0.4)$

Expressions for any $\alpha = a/b$:

$$\text{(a)}\quad F = \sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}}\left[\frac{0.923 + 0.199\left(1 - \sin\frac{\pi\alpha}{2}\right)^4}{\cos\frac{\pi\alpha}{2}}\right] \qquad (\text{large } h/b)$$

(b) F is within 3% of (a) for $h/b = 4$, and within 6% for $h/b = 2$, at any a/l

$$F = \frac{1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.7\alpha^2)}{\sqrt{\pi}(1+2\alpha)(1-\alpha)^{3/2}} \qquad (h/b = 2)$$

Values for small a/b and limits for 10% accuracy:

(a) $K = S_g\sqrt{\pi a}$ $\quad (a/b \le 0.4)$

(b) $K = 1.12S_g\sqrt{\pi a}$ $\quad (a/b \le 0.6)$

(c) $K = 1.12S_g\sqrt{\pi}$ $\quad (a/b \le 0.13)$

Expressions for any $\alpha = a/b$:

$$\text{(a)}\quad F = \frac{1 - 0.5\alpha + 0.326\alpha^2}{\sqrt{1-\alpha}} \qquad (h/b \ge 1.5)$$

$$\text{(b)}\quad F = \left(1 + 0.122\cos^4\frac{\pi\alpha}{2}\right)\sqrt{\frac{2}{\pi\alpha}\tan\frac{\pi\alpha}{2}} \qquad (h/b \ge 2)$$

$$\text{(c)}\quad F = 0.265(1-\alpha)^4 + \frac{0.857 + 0.265\alpha}{(1-\alpha)^{3/2}} \qquad (h/b \ge 1)$$

Table 8.1 Fracture Toughness and Corresponding Tensile Properties for Representative Metals at Room Temperature

Material	Toughness K_{Ic}	Yield σ_o	Ultimate σ_u	Elong. $100\varepsilon_f$	Red. Area $\%RA$
	MPa$\sqrt{m}$ (ksi$\sqrt{in}$)	MPa (ksi)	MPa (ksi)	%	%
(a) Steels					
AISI 1144	66 (60)	540 (78)	840 (122)	5	7
ASTM A470-8 (Cr-Mo-V)	60 (55)	620 (90)	780 (113)	17	45
ASTM A517-F	187 (170)	760 (110)	830 (121)	20	66
AISI 4130	110 (100)	1090 (158)	1150 (167)	14	49
18-Ni maraging air melted	123 (112)	1310 (190)	1350 (196)	12	54
18-Ni maraging vacuum melted	176 (160)	1290 (187)	1345 (195)	15	66
300-M 650°C temper	152 (138)	1070 (156)	1190 (172)	18	56
300-M 300°C temper	65 (59)	1740 (252)	2010 (291)	12	48
(b) Aluminum and Titanium Alloys (L-T Orientation)					
2014-T651	24 (22)	415 (60)	485 (70)	13	—
2024-T351	34 (31)	325 (47)	470 (68)	20	—
2219-T851	36 (33)	350 (51)	455 (66)	10	—
7075-T651	29 (26)	505 (73)	570 (83)	11	—
7475-T7351	52 (47)	435 (63)	505 (73)	14	—
Ti-6Al-4V annealed	66 (60)	925 (134)	1000 (145)	16	34

Sources: Data in [Barsom 87] p. 172, [Boyer 85] pp. 6.34, 6.35, and 9.8, [MILHDBK 94] pp. 3.10–3.12 and 5.3, and [Ritchie 77].

Table 8.2 Fracture Toughness of Some Polymers and Ceramics at Room Temperature

Material Polymers[1]	K_{Ic} MPa$\sqrt{m}$	(ksi$\sqrt{in}$)	Material Ceramics[2]	K_{Ic} MPa$\sqrt{m}$	(ksi$\sqrt{in}$)
ABS	3.0	(2.7)	Soda-lime glass	0.76	(0.69)
Acrylic	1.8	(1.6)	Magnesia, MgO	2.9	(2.6)
Epoxy	0.6	(0.55)	Alumina, Al_2O_3	4.0	(3.6)
PC	2.2	(2.0)	Al_2O_3, 15% ZrO_2	10	(9.1)
PET	5.0	(4.6)	Silicon carbide SiC	3.7	(3.4)
Polyester	0.6	(0.55)			
PS	1.15	(1.05)	Silicon nitride Si_3N_4	5.6	(5.1)
PVC	2.4	(2.2)			
PVC rubber mod.	3.35	(3.05)	Dolomitic limestone	1.30	(1.18)
			Westerly granite	0.89	(0.81)
			Concrete	1.19	(1.08)

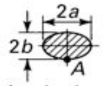
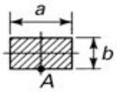
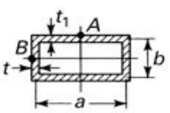
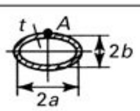
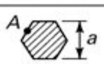
Notes: [1,2]See Tables 4.3 and 3.10, respectively, for additional properties of similar materials.
Sources: Data in [ASM 88] p. 739, [Karfakis 90], [Kelly 86] p. 376, [Shah 95] p. 176, and [Williams 87] p. 243.

Hoja de formulas

$$\theta = \frac{h}{2AG}\oint \frac{ds}{t} = \frac{1}{2AG}\oint \tau\, ds \qquad \theta = \frac{3T}{bt^3G}$$

$$\tau = \frac{T}{2At} \qquad \tau = \frac{h}{t} \qquad T = 2Ah \qquad \tau_r = \frac{M_t}{I_P}\, r$$

$$C = \frac{T}{\theta} = \tfrac{1}{3}bt^3G = J_eG \qquad J_e = \sum \tfrac{1}{3}bt^3 \qquad \tau_i = \frac{T.b_i}{J_e}$$

<table>
<tr><th>Cross section</th><th colspan="2">Maximum shearing stress</th><th colspan="2">Angle of twist per unit length</th></tr>
<tr><td>For circular bar: $a = b$</td><td colspan="2">$\tau_A = \frac{2T}{\pi ab^2}$</td><td colspan="2">$\theta = \frac{(a^2 + b^2)T}{\pi a^3b^3G}$</td></tr>
<tr><td>Equilateral triangle</td><td colspan="2">$\tau_A = \frac{20T}{a^3}$</td><td colspan="2">$\theta = \frac{46.2T}{a^4G}$</td></tr>
<tr><td rowspan="3"></td><td colspan="2">$\tau_A = \frac{T}{\alpha ab^2}$</td><td colspan="2">$\theta = \frac{T}{\beta ab^3G}$</td></tr>
<tr><td>a/b</td><td colspan="2">β</td><td colspan="2">α</td></tr>
<tr><td>1.0
1.5
2.0
2.5
3.0
4.0
5.0
10.0
∞</td><td colspan="2">0.141
0.196
0.229
0.249
0.263
0.281
0.291
0.312
0.333</td><td colspan="2">0.208
0.231
0.246
0.256
0.267
0.282
0.292
0.312
0.333</td></tr>
<tr><td></td><td colspan="2">$\tau_A = \frac{T}{2abt_1}$
$\tau_B = \frac{T}{2abt}$</td><td colspan="2">$\theta = \frac{(at + bt_1)T}{2tt_1a^2b^2G}$</td></tr>
<tr><td>For circular tube: $a = b$</td><td colspan="2">$\tau_A = \frac{T}{2\pi abt}$</td><td colspan="2">$\theta = \frac{\sqrt{2(a^2 + b^2)}T}{4\pi a^2b^2tG}$</td></tr>
<tr><td>Hexagon</td><td colspan="2">$\tau_A = \frac{5.7T}{a^3}$</td><td colspan="2">$\theta = \frac{8.8T}{a^4G}$</td></tr>
</table>

Hoja de formulas

$$\nabla^4 w = \frac{p}{D} \qquad D = \frac{Et^3}{12(1-\nu^2)} \qquad \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right)$$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \qquad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x) = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right)$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \qquad \tau_{xy} = \frac{E}{2(1+\nu)}\gamma_{xy} = -\frac{Ez}{1+\nu}\frac{\partial^2 w}{\partial x\,\partial y}$$

$$M_{xy} = -D(1-\nu)\frac{\partial^2 w}{\partial x\,\partial y} \qquad \sigma_{x,\max} = \frac{6M_x}{t^2}, \qquad \sigma_{y,\max} = \frac{6M_y}{t^2}, \qquad \tau_{xy,\max} = \frac{6M_{xy}}{t^2}$$

$$Q_x = -D\frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right), \qquad Q_y = -D\frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \qquad \tau_{xz,\max} = \frac{3}{2}\frac{Q_x}{t}, \qquad \tau_{yz,\max} = \frac{3}{2}\frac{Q_y}{t}$$

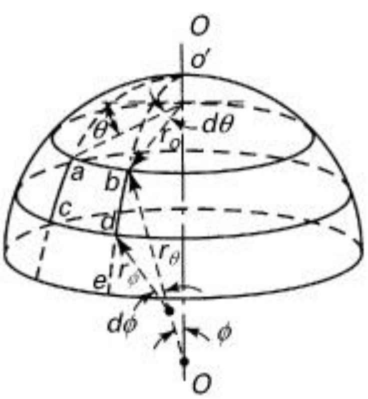
$$\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{p}{D}$$

$$\frac{D}{p_o}w = \int\frac{1}{r}\int r\int\left[\frac{1}{r}\left(\frac{r^2}{2} + c_1\right)\right]dr\,dr\,dr = \frac{r^4}{64} + \frac{c_1 r^2}{4}(\ln r - 1) + \frac{c_2 r^2}{4} + c_3\ln r + c_4$$

$$M_r = -D\left(\frac{d^2 w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right)$$

$$M_\theta = -D\left(\frac{1}{r}\frac{dw}{dr} + \nu\frac{d^2 w}{dr^2}\right)$$

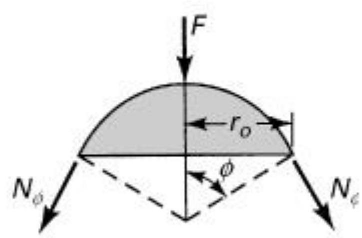
$$Q_r = -D\frac{d}{dr}\left(\frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right)$$



$$\frac{N_\phi}{r_\phi} + \frac{N_\theta}{r_\theta} = -p_z$$

$$2\pi r_0 N_\phi \sin\phi + F = 0$$

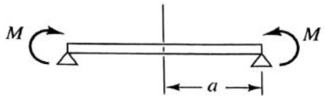
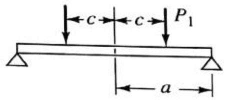
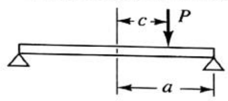
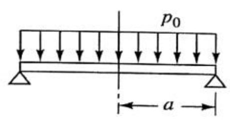
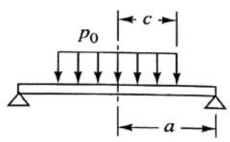
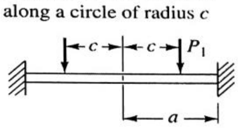
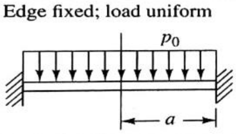
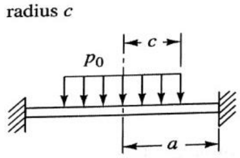
$$r_0 = r_\theta\, sen\, \emptyset$$



$$\varepsilon_\varphi = \frac{1}{r_\varphi}\frac{dv}{d\varphi} - \frac{w}{r_\varphi} \qquad \varepsilon_\theta = \frac{1}{r_0}(\mathrm{v}\cos\varphi - w\,\mathrm{sen}\,\varphi)$$

$$\frac{dv}{d\varphi} - v\cot\varphi = \frac{1}{E}\left[\sigma_\varphi(r_\varphi + \nu r_\theta) - \sigma_\theta(r_\theta + \nu r_\varphi)\right] = f(\varphi) \qquad v = \left[\int\frac{f(\phi)}{\sin\phi}d\phi + c\right]\sin\phi$$

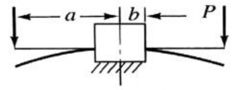
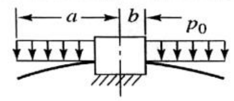
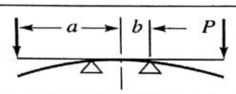
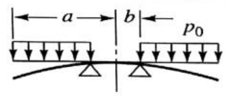
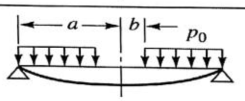
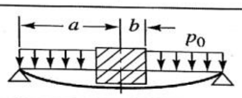
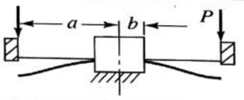
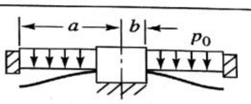
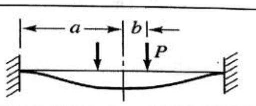
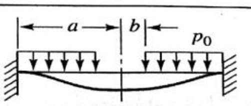
$$U = \int\frac{N^2 dx}{2AE} + \int\frac{M^2 dx}{2EI} + \int\frac{\alpha V^2 dx}{2AG} + \int\frac{T^2 dx}{2JG} \qquad \frac{\partial U}{\partial P_i} = \delta_i \qquad \frac{\partial U}{\partial C_i} = \theta_i$$

Case No.	Support and Loading	Maximum Bending Stress σ, Deflection w at Center, and Slope θ at Edge
1.	Edge simply supported (or no support); load uniform along edge 	$\sigma = 6\frac{M}{t^2}$ (uniform) $w = 6(1-\nu)\frac{Ma^2}{Et^3}$ $\theta = 12(1-\nu)\frac{Ma}{Et^3}$
2.	Edge simply supported; load uniform along a circle of radius c 	$\sigma = \frac{3}{2}\frac{P_1 c}{t^2}\left[(1-\nu)\left(1-\frac{c^2}{a^2}\right)+2(1+\nu)\ln\frac{a}{c}\right]$ (at center) $w = \frac{3(1-\nu)}{2}\frac{P_1 c}{Et^3} \times\left[(3+\nu)(a^2-c^2)-2(1+\nu)c^2\ln\frac{a}{c}\right]$ $\theta = 6(1-\nu)\frac{P_1 ac}{Et^3}\left(1-\frac{c^2}{a^2}\right)$
3.	Concentrated load at a distance c from the center 	Deflection w at center approximately same as Case 2 for edge simply supported, and same as Case 6 for edge fixed.
4.	Edge simply supported; load uniform 	$\sigma = \frac{3(3+\nu)}{8}\frac{p_0 a^2}{t^2}$ (at center) $w = \frac{3(1-\nu)(5+\nu)}{16}\frac{p_0 a^4}{Et^3}$ $\theta = \frac{3(1-\nu^2)}{2}\frac{p_0 a^3}{Et^3}$
5.	Edge simply supported; uniform load on circular area of radius c 	$\sigma = \frac{3}{8}\frac{p_0 c^2}{t^2}\left[4-(1-\nu)\frac{c^2}{a^2}+4(1+\nu)\ln\frac{a}{c}\right]$ (at center) $w = \frac{3(1-\nu)}{16}\frac{p_0 c^2}{Et^3} \times\left[4(3+\nu)a^2-(7+3\nu)c^2-4(1+\nu)c^2\ln\frac{a}{c}\right]$ $\theta = \frac{3(1-\nu)}{2}\frac{p_0 ac^2}{Et^3}\left(2-\frac{c^2}{a^2}\right)$
6.	Edge fixed; load uniform along a circle of radius c 	$\sigma = \frac{3(1+\nu)}{2}\frac{P_1 c}{t^2}\left(\frac{c^2}{a^2}+2\ln\frac{a}{c}-1\right)$ (at center) $\sigma = 3\frac{P_1 c}{t^2}\left(1-\frac{c^2}{a^2}\right)$ (at edge) $w = \frac{3(1-\nu^2)}{2}\frac{P_1 c}{Et^3}\left(b^2-c^2-2c^2\ln\frac{a}{c}\right)$
7.	Edge fixed; load uniform 	$\sigma = \frac{3}{4}\frac{p_0 a^2}{t^2}$ (at edge) $w = \frac{3(1-\nu^2)}{16}\frac{p_0 a^{4\dagger}}{Et^3}$
8.	Edge fixed; load uniform over a circular area of radius c 	$\sigma = \frac{3(1+\nu)}{8}\frac{p_0 c^2}{t^2}\left(\frac{c^2}{a^2}+4\ln\frac{a}{c}\right)$ (at center) $\sigma = \frac{3}{4}\frac{p_0 c^2}{t^2}\left(2-\frac{c^2}{a^2}\right)$ (at edge) $w = \frac{3(1-\nu^2)}{4}\frac{p_0 c^2}{Et^3}\left(a^2-\frac{3}{4}c^2-c^2\ln\frac{a}{c}\right)$

NOTATION: a = radius of plate; p_0 = uniform load per unit area; P_1 = load per unit length, uniformly distributed along a centric circle of radius c; P = concentrated load; M = moments per unit length uniformly distributed along edge; ν = Poisson's ratio; E = modulus of elasticity.

†For thicker plates ($a > 10t$), a modified bending theory gives $w = \frac{3}{16}\alpha(1-\nu^2)(p_0 a^4/Et^3)$, where the constant $\alpha = 1 + 5.72(t/a)^2$.

Table 4.3 Variously supported and uniformly loaded annular plates

Case No.	Support	Load Diagram	Maximum Stress s_{max}	Deflection between Edges w_{max}
1.	Inner edge fixed		$k_1 \dfrac{P}{t^2}$	$h_1 \dfrac{Pa^2}{Et^3}$
2.	Inner edge fixed		$k_2 \dfrac{p_0 a^2}{t^2}$	$h_2 \dfrac{p_0 a^4}{Et^3}$
3.	Inner edge simply supported		$k_3 \dfrac{P}{t^2}$	$h_3 \dfrac{Pa^2}{Et^3}$
4.	Inner edge simply supported		$k_4 \dfrac{p_0 a^2}{t^2}$	$h_4 \dfrac{p_0 a^4}{Et^3}$
5.	Outer edge simply supported		$k_5 \dfrac{p_0 a^2}{t^2}$	$h_5 \dfrac{p_0 a^4}{Et^3}$
6.	Outer edge simply supported, inner-edge rotation prevented		$k_6 \dfrac{p_0 a^2}{t^2}$	$h_6 \dfrac{p_0 a^4}{Et^3}$
7.	Inner edge fixed, outer-edge rotation prevented		$k_7 \dfrac{P}{t^2}$	$h_7 \dfrac{Pa^2}{Et^3}$
8.	Inner edge fixed, outer-edge rotation prevented		$k_8 \dfrac{p_0 a^2}{t^2}$	$h_8 \dfrac{p_0 a^4}{Et^3}$
9.	Outer edge fixed		$k_9 \dfrac{P}{t^2}$	$h_9 \dfrac{Pa^2}{Et^3}$
10.	Outer edge fixed		$k_{10} \dfrac{p_0 a^2}{t^2}$	$h_{10} \dfrac{p_0 a^4}{Et^3}$

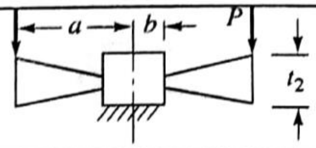
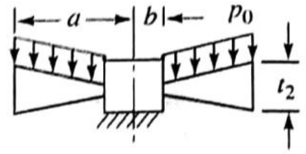
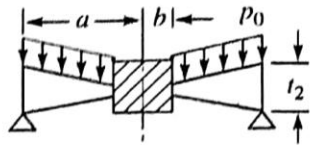
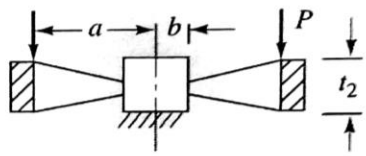
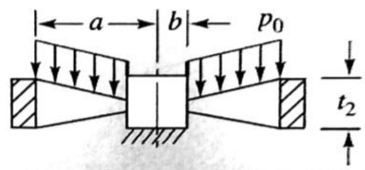
NOTATION: a = outer radius of plate; b = inner radius of plate; t = thickness of plate; E = modulus of elasticity; P = total load uniformly distributed along outer or inner edge; p_0 = uniform load per unit surface area of plate.

Table 4.4 Coefficients k and h for cases 1 through 10 in Table 4.3

a/b	1.25	1.50	2.00	3.00	4.00	5.00
k_1	0.227	0.428	0.753	1.205	1.514	1.745
h_1	0.0051	0.0249	0.0877	0.209	0.293	0.350
k_2	0.135	0.410	1.04	2.15	2.99	3.69
h_2	0.0023	0.0183	0.0938	0.2925	0.448	0.564
k_3	1.10	1.26	1.48	1.88	2.17	2.34
h_3	0.341	0.519	0.672	0.734	0.724	0.704
k_4	0.66	1.19	2.04	3.34	4.30	5.10
h_4	0.202	0.491	0.902	1.220	1.300	1.310
k_5	0.592	0.976	1.440	1.880	2.080	2.19
h_5	0.1841	0.4139	0.6640	0.8237	0.8296	0.813
k_6	0.122	0.336	0.74	1.21	1.45	1.59
h_6	0.0034	0.0313	0.1250	0.291	0.417	0.492
k_7	0.115	0.220	0.405	0.703	0.933	1.13
h_7	0.0013	0.0064	0.0237	0.0619	0.0923	0.114
k_8	0.090	0.273	0.71	1.54	2.23	2.80
h_8	0.0008	0.0062	0.0329	0.1096	0.1792	0.2338
k_9	0.194	0.320	0.454	0.673	1.021	1.305
h_9	0.00504	0.0242	0.0810	0.172	0.217	0.288
k_{10}	0.105	0.259	0.480	0.657	0.710	0.730
h_{10}	0.00199	0.0139	0.0575	0.130	0.162	0.175

SOURCE: Data rearranged from Wahl and Lobo [4.10].

Table 4.5 Variously supported and uniformly loaded annular plates with linearly varying thickness

Case No. (Corresponding to Table 4.3)	Support	Load Diagram	Maximum Stress σ_{max}	Deflection between Edges w_{max}
1.	Inner edge fixed		$k_1 \frac{P}{a^2}$	$h_1 \frac{Pa^2}{Et_2^3}$
2.	Inner edge fixed		$k_2 \frac{p_0 a^2}{t_2^2}$	$h_2 \frac{p_0 a^4}{Et_2^3}$
6.	Outer edge simply supported, inner-edge rotation prevented		$k_6 \frac{p_0 a^2}{t_2^2}$	$h_6 \frac{p_0 a^4}{Et_2^3}$
7.	Inner edge fixed, outer-edge rotation prevented		$k_7 \frac{P}{t_2^2}$	$h_7 \frac{Pa^2}{Et_2^3}$
8.	Inner edge fixed, outer-edge rotation prevented		$k_8 \frac{p_0 a^2}{t_2^2}$	$h_8 \frac{p_0 a^4}{Et_2^3}$

NOTATION: a = outer radius of plate; b = inner radius of plate; t = outer edge thickness of plate; E = modulus of elasticity; P = total load uniformly distributed along outer edge; p_0 = uniform load per unit surface area of plate.

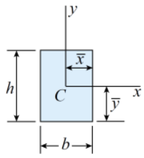
Table 4.6 Coefficients k and h for cases 1, 2, 6, 7, and 8 in Table 4.5. Poisson's ratio $\nu = 1/3$

a/b	1.25	1.50	2.00	3.00	4.00	5.00
k_1	0.353	0.933	2.63	6.88	11.47	16.51
h_1	0.0082	0.0583	0.345	1.358	2.39	3.27
k_2	0.249	0.638	3.96	13.64	26.0	40.6
h_2	0.0037	0.0453	0.401	2.12	4.25	6.28
k_6	0.149	0.991	2.23	5.57	7.78	9.16
h_6	0.0055	0.0564	0.412	1.673	2.79	3.57
k_7	0.159	0.396	0.091	3.31	6.55	10.78
h_7	0.0017	0.0112	0.0606	0.261	0.546	0.876
k_8	0.1275	0.515	2.05	7.97	17.35	30.0
h_8	0.0011	0.0115	0.0934	0.537	1.261	2.16

Hoja de formulas

Notation: A = area
$\bar{x}, \bar{y}$ = distances to centroid C
I_x, I_y = moments of inertia with respect to the x and y axes, respectively
I_{xy} = product of inertia with respect to the x and y axes
$I_P = I_x + I_y$ = polar moment of inertia with respect to the origin of the x and y axes
I_{BB} = moment of inertia with respect to axis B-B

1

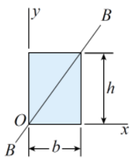


Rectangle (Origin of axes at centroid)

$A = bh \quad \bar{x} = \frac{b}{2} \quad \bar{y} = \frac{h}{2}$

$I_x = \frac{bh^3}{12} \quad I_y = \frac{hb^3}{12} \quad I_{xy} = 0 \quad I_P = \frac{bh}{12}(h^2 + b^2)$

2

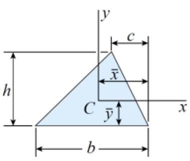


Rectangle (Origin of axes at corner)

$I_x = \frac{bh^3}{3} \quad I_y = \frac{hb^3}{3} \quad I_{xy} = \frac{b^2h^2}{4} \quad I_P = \frac{bh}{3}(h^2 + b^2)$

$I_{BB} = \frac{b^3h^3}{6(b^2 + h^2)}$

3



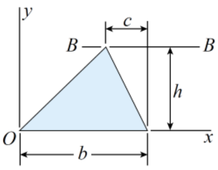
Triangle (Origin of axes at centroid)

$A = \frac{bh}{2} \quad \bar{x} = \frac{b + c}{3} \quad \bar{y} = \frac{h}{3}$

$I_x = \frac{bh^3}{36} \quad I_y = \frac{bh}{36}(b^2 - bc + c^2)$

$I_{xy} = \frac{bh^2}{72}(b - 2c) \quad I_P = \frac{bh}{36}(h^2 + b^2 - bc + c^2)$

4

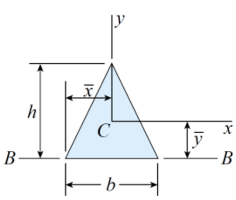


Triangle (Origin of axes at vertex)

$I_x = \frac{bh^3}{12} \quad I_y = \frac{bh}{12}(3b^2 - 3bc + c^2)$

$I_{xy} = \frac{bh^2}{24}(3b - 2c) \quad I_{BB} = \frac{bh^3}{4}$

5



Isosceles triangle (Origin of axes at centroid)

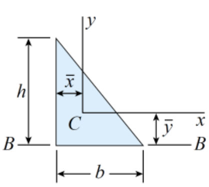
$A = \frac{bh}{2} \quad \bar{x} = \frac{b}{2} \quad \bar{y} = \frac{h}{3}$

$I_x = \frac{bh^3}{36} \quad I_y = \frac{hb^3}{48} \quad I_{xy} = 0$

$I_P = \frac{bh}{144}(4h^2 + 3b^2) \quad I_{BB} = \frac{bh^3}{12}$

(*Note:* For an equilateral triangle, $h = \sqrt{3}\, b/2$.)

6



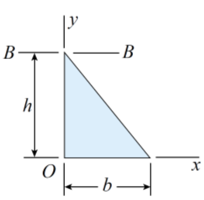
Right triangle (Origin of axes at centroid)

$A = \frac{bh}{2} \quad \bar{x} = \frac{b}{3} \quad \bar{y} = \frac{h}{3}$

$I_x = \frac{bh^3}{36} \quad I_y = \frac{hb^3}{36} \quad I_{xy} = -\frac{b^2h^2}{72}$

$I_P = \frac{bh}{36}(h^2 + b^2) \quad I_{BB} = \frac{bh^3}{12}$

7

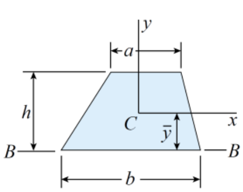


Right triangle (Origin of axes at vertex)

$I_x = \frac{bh^3}{12} \quad I_y = \frac{hb^3}{12} \quad I_{xy} = \frac{b^2h^2}{24}$

$I_P = \frac{bh}{12}(h^2 + b^2) \quad I_{BB} = \frac{bh^3}{4}$

8

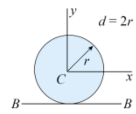


Trapezoid (Origin of axes at centroid)

$A = \frac{h(a + b)}{2} \quad \bar{y} = \frac{h(2a + b)}{3(a + b)}$

$I_x = \frac{h^3(a^2 + 4ab + b^2)}{36(a + b)} \quad I_{BB} = \frac{h^3(3a + b)}{12}$

9

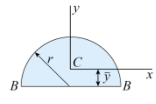


Circle (Origin of axes at center)

$A = \pi r^2 = \frac{\pi d^2}{4} \quad I_x = I_y = \frac{\pi r^4}{4} = \frac{\pi d^4}{64}$

$I_{xy} = 0 \quad I_P = \frac{\pi r^4}{2} = \frac{\pi d^4}{32} \quad I_{BB} = \frac{5\pi r^4}{4} = \frac{5\pi d^4}{64}$

10

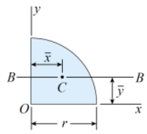


Semicircle (Origin of axes at centroid)

$A = \frac{\pi r^2}{2} \quad \bar{y} = \frac{4r}{3\pi}$

$I_x = \frac{(9\pi^2 - 64)r^4}{72\pi} \approx 0.1098r^4 \quad I_y = \frac{\pi r^4}{8} \quad I_{xy} = 0 \quad I_{BB} = \frac{\pi r^4}{8}$

11

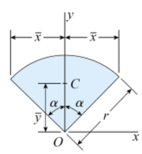


Quarter circle (Origin of axes at center of circle)

$A = \frac{\pi r^2}{4} \quad \bar{x} = \bar{y} = \frac{4r}{3\pi}$

$I_x = I_y = \frac{\pi r^4}{16} \quad I_{xy} = \frac{r^4}{8} \quad I_{BB} = \frac{(9\pi^2 - 64)r^4}{144\pi} \approx 0.05488r^4$

13



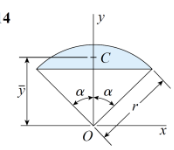
Circular sector (Origin of axes at center of circle)

α = angle in radians $(\alpha \leq \pi/2)$

$A = \alpha r^2 \quad \bar{x} = r \sin \alpha \quad \bar{y} = \frac{2r \sin \alpha}{3\alpha}$

$I_x = \frac{r^4}{4}(\alpha + \sin \alpha \cos \alpha) \quad I_y = \frac{r^4}{4}(\alpha - \sin \alpha \cos \alpha) \quad I_{xy} = 0 \quad I_P = \frac{\alpha r^4}{2}$

14



Circular segment (Origin of axes at center of circle)

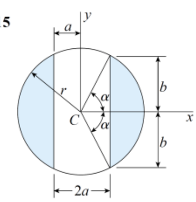
α = angle in radians $(\alpha \leq \pi/2)$

$A = r^2(\alpha - \sin \alpha \cos \alpha) \quad \bar{y} = \frac{2r}{3}\left(\frac{\sin^3 \alpha}{\alpha - \sin \alpha \cos \alpha}\right)$

$I_x = \frac{r^4}{4}(\alpha - \sin \alpha \cos \alpha + 2 \sin^3 \alpha \cos \alpha) \quad I_{xy} = 0$

$I_y = \frac{r^4}{12}(3\alpha - 3 \sin \alpha \cos \alpha - 2 \sin^3 \alpha \cos \alpha)$

15



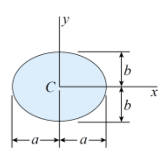
Circle with core removed (Origin of axes at center of circle)

α = angle in radians $(\alpha \leq \pi/2)$

$\alpha = \arccos \frac{a}{r} \quad b = \sqrt{r^2 - a^2} \quad A = 2r^2\left(\alpha - \frac{ab}{r^2}\right)$

$I_x = \frac{r^4}{6}\left(3\alpha - \frac{3ab}{r^2} - \frac{2ab^3}{r^4}\right) \quad I_y = \frac{r^4}{2}\left(\alpha - \frac{ab}{r^2} + \frac{2ab^3}{r^4}\right) \quad I_{xy} = 0$

16



Ellipse (Origin of axes at centroid)

$A = \pi ab \quad I_x = \frac{\pi ab^3}{4} \quad I_y = \frac{\pi ba^3}{4}$

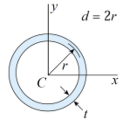
$I_{xy} = 0 \quad I_P = \frac{\pi ab}{4}(b^2 + a^2)$

Circumference $\approx \pi[1.5(a + b) - \sqrt{ab}\,] \quad (a/3 \leq b \leq a)$

$\approx 4.17b^2/a + 4a \quad (0 \leq b \leq a/3)$

Notation: A = area
$\bar{x}, \bar{y}$ = distances to centroid C
I_x, I_y = moments of inertia with respect to the x and y axes, respectively
I_{xy} = product of inertia with respect to the x and y axes
$I_P = I_x + I_y$ = polar moment of inertia with respect to the origin of the x and y axes
I_{BB} = moment of inertia with respect to axis B-B

22

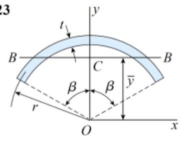


Thin circular ring (Origin of axes at center)
Approximate formulas for case when t is small

$$A = 2\pi r t = \pi d t \qquad I_x = I_y = \pi r^3 t = \frac{\pi d^3 t}{8}$$

$$I_{xy} = 0 \qquad I_P = 2\pi r^3 t = \frac{\pi d^3 t}{4}$$

23



Thin circular arc (Origin of axes at center of circle)
Approximate formulas for case when t is small

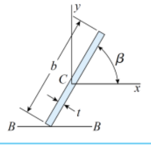
β = angle in radians (*Note:* For a semicircular arc, $\beta = \pi/2$.)

$$A = 2\beta r t \qquad \bar{y} = \frac{r \sin\beta}{\beta}$$

$$I_x = r^3 t(\beta + \sin\beta\cos\beta) \qquad I_y = r^3 t(\beta - \sin\beta\cos\beta)$$

$$I_{xy} = 0 \qquad I_{BB} = r^3 t\left(\frac{2\beta + \sin 2\beta}{2} - \frac{1 - \cos 2\beta}{\beta}\right)$$

24



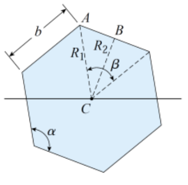
Thin rectangle (Origin of axes at centroid)
Approximate formulas for case when t is small

$$A = bt$$

$$I_x = \frac{tb^3}{12}\sin^2\beta \qquad I_y = \frac{tb^3}{12}\cos^2\beta \qquad I_{BB} = \frac{tb^3}{3}\sin^2\beta$$

25

Regular polygon with *n* sides (Origin of axes at centroid)



C = centroid (at center of polygon)

n = number of sides ($n \geq 3$) $\qquad b$ = length of a side

β = central angle for a side $\qquad \alpha$ = interior angle (or vertex angle)

$$\beta = \frac{360°}{n} \qquad \alpha = \left(\frac{n-2}{n}\right)180° \qquad \alpha + \beta = 180°$$

R_1 = radius of circumscribed circle (line CA) $\qquad R_2$ = radius of inscribed circle (line CB)

$$R_1 = \frac{b}{2}\csc\frac{\beta}{2} \qquad R_2 = \frac{b}{2}\cot\frac{\beta}{2} \qquad A = \frac{nb^2}{4}\cot\frac{\beta}{2}$$

I_c = moment of inertia about any axis through C (the centroid C is a principal point and every axis through C is a principal axis)

$$I_c = \frac{nb^4}{192}\left(\cot\frac{\beta}{2}\right)\left(3\cot^2\frac{\beta}{2} + 1\right) \qquad I_P = 2I_c$$

Hoja de formulas

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31,34 ESTÁTICA:

Rotación de tensiones

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} cos(2\theta) + \tau_{xy} sen(2\theta)$$

$$\tau_{x_1 y_1} = -\frac{\sigma_x - \sigma_y}{2} sen(2\theta) + \tau_{xy} cos(2\theta)$$

$$\sigma_{prom} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$tan(2\theta_P) = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Teorema de Castigliano

$$\delta_i = \frac{\partial U}{\partial P_i}$$

Equilibrio:

$\sum \bar{F} = m\bar{a}$

$$\sum \bar{M} = I \propto$$
$$\bar{M} = \bar{r} \times \bar{F}$$

Potencia: $P = F.V = T.\omega$

Tensiones y Deformaciones

$\sigma_{prom} = \frac{F_n}{A}$ $\qquad \tau_{prom} = \frac{F_t}{A}$

$\sigma = E.\varepsilon$ $\qquad \tau = G.\gamma$

$\int d\delta = \int \epsilon \, dx$ $\qquad \varepsilon_{total} = \varepsilon_{mec} + \varepsilon_{ter}$

$\varepsilon_{ter} = \propto . \Delta T$

$U_e = \int F_e(x)\, dx$

Torsión:

$$\tau = \frac{M_x\, r}{J} \qquad \varphi_{B-A} = \int_A^B \frac{M_x(x)}{J(x)G(x)} dx \qquad \tau_{prom} = \frac{M_x}{2\, t\, A_m}$$

Áreas:

$$\bar{x} = \frac{\int_A x\, dA}{\int_A dA} \qquad \bar{y} = \frac{\int_A y\, dA}{\int_A dA}$$

$$I_y = \int_A z^2 dA \qquad I_z = \int_A y^2 dA \qquad I_{yz} = \int_A yz\, dA$$

$$I_p = \int_A r^2 dA = I_y + I_z$$

$$I_{y'} = I_y + A{d_z}^2 \qquad I_{z'} = I_z + A{d_y}^2 \qquad I_{y'z'} = I_{ysz} + Ad_y d_z$$

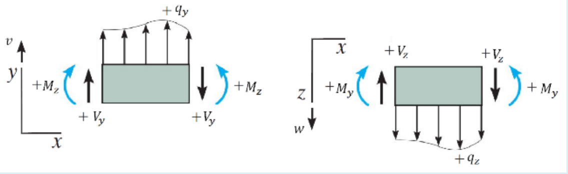
$$I_{y'} = \frac{I_y + I_z}{2} + \frac{I_y - I_z}{2} \cos 2\theta - I_{yz} \sin 2\theta$$

$$I_{z'} = \frac{I_y + I_z}{2} - \frac{I_y - I_z}{2} \cos 2\theta + I_{yz} \sin 2\theta$$

$$I_{y'z'} = \frac{I_y - I_z}{2} \sin 2\theta - I_{yz} \cos 2\theta$$

$$\tan 2\theta_p = -\frac{2\, I_{yz}}{I_y - I_z}$$

Flexión:



$$q_y(x) = \frac{dV_y}{dx} \qquad V_y(x) = \frac{dM_z}{dx} \qquad M_z(x) = EI\frac{d^2v}{dx^2}$$

$$q_z(x) = -\frac{dV_z}{dx} \qquad V_z(x) = \frac{dM_y}{dx} \qquad M_y(x) = -EI\frac{d^2w}{dx^2}$$

$$\sigma = \frac{N_x}{A} - \frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y} \qquad \tau = \frac{VQ}{I\, b} \qquad f = \frac{VQ}{I}$$

$$Q = \int_{A'} y\, dA' = \bar{y}' A'$$

Constantes: $g = 9.81\ m/s2$