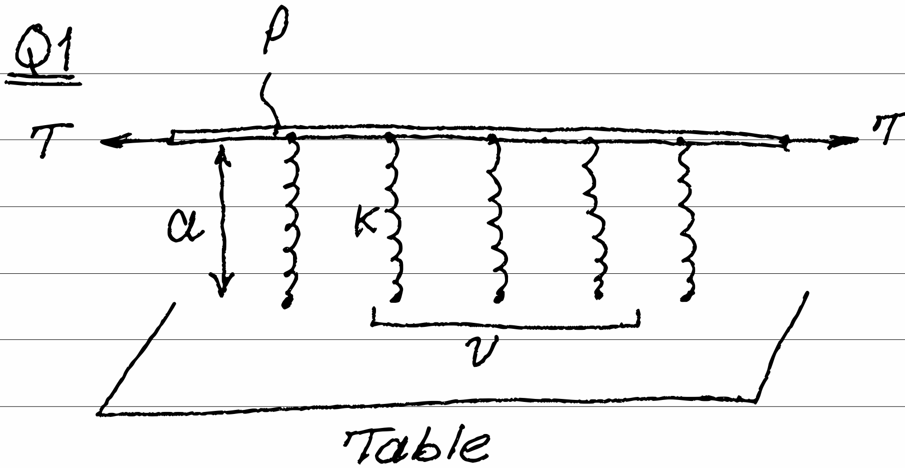
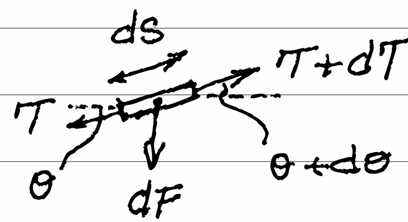
Q1



consider element of string at position (x,y):



, $\theta \ll 0$

$$\begin{cases} d(T\cos(\theta)) \approx dT = 0 \\ d(T\sin(\theta)) - \underbrace{dF}_{K\nu dx \cdot y} = \underbrace{dm}_{\rho dx}\ \ddot{y} \end{cases}, \quad \tan(\theta) = \frac{dy}{dx}$$

$$\Rightarrow \underbrace{d\left(T\frac{dy}{dx}\right)}_{\cancel{dT}\cdot\frac{dy}{dx} + T\frac{d^2y}{dx}} - K\nu y\,dx = \rho\ddot{y}\,dx$$

$\cancel{dT} \to 0$

Q1

$$\Rightarrow \boxed{T \frac{d^2y}{dx^2} - K\nu y = \rho \frac{d^2y}{dt^2}}$$

$$y = e^{i\omega t}\, Y(x):$$

$$T \cdot Y'' - K\nu Y = -\rho\omega^2 Y$$

$$Y'' + \left(\frac{\rho\omega^2 - K\nu}{T}\right) Y = 0$$

$$Y = e^{ikx}:$$

$$-k^2 + \left(\frac{\rho\omega^2 - K\nu}{T}\right) = 0$$

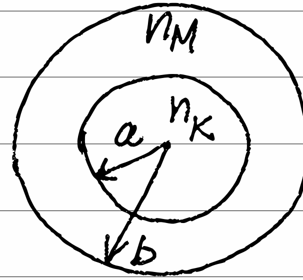
$$K(\omega) = \sqrt{\frac{\rho\omega^2 - K\nu}{T}}$$

$$\boxed{y = \sum_{\omega} A_\omega e^{i(\omega t + kx)} + \sum_{\omega} B_\omega e^{i(\omega t - kx)}}$$

Q1

$$K(\omega_0) = 0 \Rightarrow \boxed{\omega_0 = \sqrt{\frac{k\nu}{\rho}}}$$

Q2



a) $n_K \sin(\theta_0) = n_M$

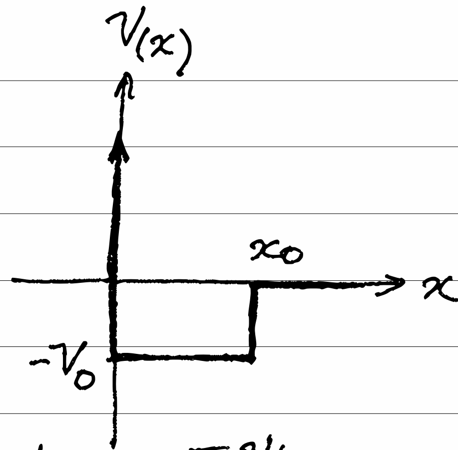
$\Rightarrow \theta_0 = \sin^{-1}\left(\frac{n_M}{n_K}\right) \approx \boxed{83.6^\circ}$

b) $\Delta t_{max} = \frac{L}{v \sin(\theta_0)} - \frac{L}{v}$, $v = \frac{c}{n_K}$

$= \frac{L n_K}{c}\left[\frac{1}{\sin(\theta_0)} - 1\right] \approx \boxed{30.2\,L \text{ ps}}$

To minimize time difference, ensure $\frac{n_K}{n_M}$ is as low as possible. So that internal reflection angle is closer to 90°.

Q3



$$\Psi_{(x,t)} = e^{-\frac{iEt}{\hbar}} \psi_{(x)}$$

a) $$-\frac{\hbar^2}{2m}\psi''_{(x)} + V_{(x)}\psi_{(x)} = E\psi_{(x)}$$

I) $\psi_{I} = 0 \implies \boxed{\Psi_{I} = 0}$

II) $$-\frac{\hbar^2}{2m}\psi''_{II} = (E+V_0)\psi_{II}$$

$$\psi_{II} = e^{iK_E x}, \quad K_E = \sqrt{\frac{2m(E+V_0)}{\hbar^2}}$$

$$\boxed{\Psi_{I} = \sum_E A_E e^{i(K_E x - Et/\hbar)}}$$

NOTE: $+B_E$ term for $-K_E$

III) $$-\frac{\hbar^2}{2m}\psi''_{II} = E\psi_{III}$$

$$\psi_{III} = e^{K'_E x}, \quad K'_E = \sqrt{\frac{-2mE}{\hbar^2}}$$

$$\boxed{\Psi_{III} = \sum_E A'_E e^{(K'_E x - iEt/\hbar)}}$$

Q3

b) $\psi_{I}(0) = \psi_{II}(0) = 0$

$\psi_{II}(x_0) = \psi_{III}(x_0)\ , \ \psi'_{II}(x_0) = \psi'_{III}(x_0)$

$\psi_{III}(\infty) = 0$

c) $\psi_{II} = A\sin(k_E x) + \overbrace{B\cos(k_E x)}^{0 \text{ as } \psi_{II}(0)=0}$

$\psi_{III} = \underbrace{A e^{k'_E x}}_{0 \text{ as } \psi_{III}(\infty)=0} + B e^{-k'_E x}$

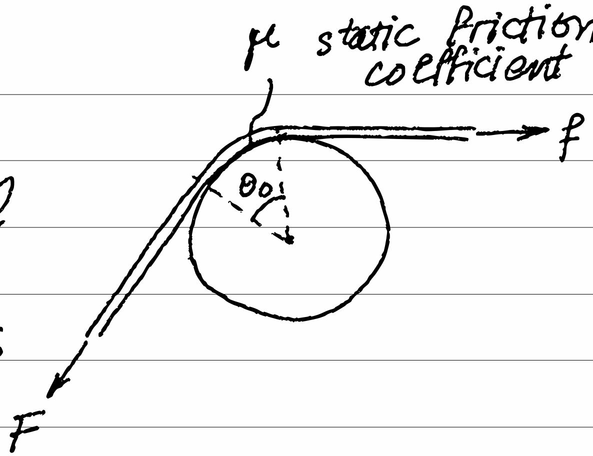
$$A\sin(k_E x_0) = B e^{-k'_E x_0}$$
$$A k_E \cos(k_E x_0) = -B k'_E e^{-k'_E x_0}$$

$\Rightarrow \cot(k_E x_0) = -k'_E$

$$\boxed{\cot\left(\sqrt{\frac{2m(E+V_0)}{\hbar^2}}\, x_0\right) = -\sqrt{\frac{-2mE}{\hbar^2}}}$$

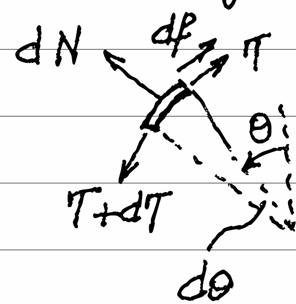
d) $\boxed{\dfrac{\sqrt{2mV_0}\,x_0}{\hbar} > \pi/2}$

Q5



consider general case where θ_0 of rope contacts the piling.

Consider rope element at angle θ:



$$dN = T\,d\theta$$
$$df = \mu\,dN = \mu T\,d\theta$$

$$\Rightarrow dT = \mu T\,d\theta$$

$$\int_f^F \frac{dT}{T} = \int_0^{\theta_0} \mu\,d\theta \Rightarrow \ln(F/f) = \mu\theta_0$$

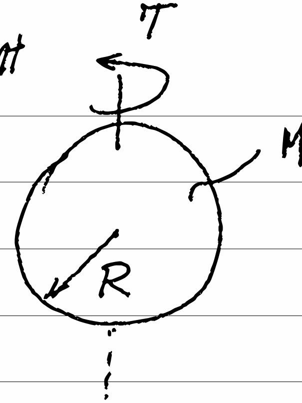
Now consider the problem: $\ln(F/f) = 2\pi\mu$

$\ln(F'/f) = 4\pi\mu$

$$\therefore \boxed{F' = \frac{F^2}{f}}$$

Q6

disk slice moment of inertia



a) $$I = 2\int_{z=0}^{R} \frac{\rho \pi r^2 dz \cdot r^2}{2}$$

$$r_{(z)} = \sqrt{R^2 - z^2}, \quad \rho = \frac{M}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow I = \pi \rho \int_0^R (R^2 - z^2)^2 dz$$

$$= \pi \rho R^5 \big(\underbrace{1 - 2/3 + 1/5}_{8/15}\big) = \frac{2}{5} M R^2$$

$$\boxed{\approx 7.68 \times 10^{47} \ kg\, m^2}$$

$$\boxed{L = I\omega = \frac{2\pi I}{T} \approx 2.54 \times 10^{42} \ \frac{kg\, m^2}{s}}$$

$\omega = 2\pi / T$

$$\boxed{E = I\omega^2/2 \approx 4.20 \times 10^{36} \ J}$$

b) Angular moment is conserved as there is no external torque on the system of star and neutron star
Mechanical energy is not conserved as some of it is dissipated (heat, etc.) during collision.

c) $\omega' = \frac{2\pi}{T} = \boxed{1.57 \times 10^3 \ 1/s}$

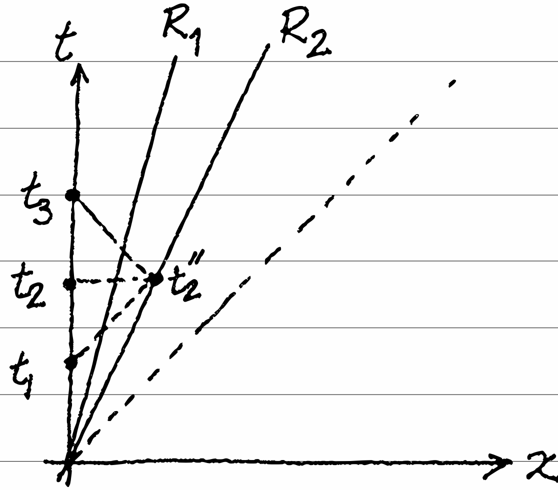
$E' = \frac{I'\omega'^2}{2}, \quad I\omega = I'\omega'$

$\Rightarrow E' = \frac{I\omega\omega'}{2} \approx \boxed{1.99 \times 10^{45} \ J}$

d) Some of the linear kinetic energy of star/neutron star is converted to rotational energy.

Q7

a)



b) $$v_{2/1} = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} = \frac{0.2c}{0.52} = \boxed{\frac{5}{13}c}$$

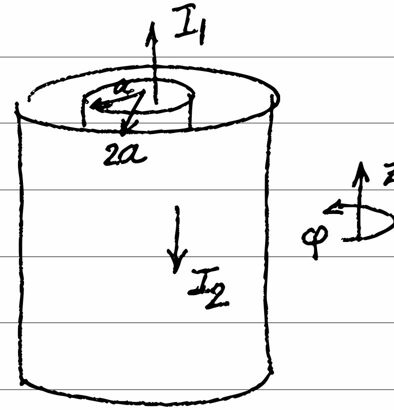
c) Solve geometrically. Let $t_2 = \frac{t_1 + t_3}{2}$.

$$t_3 = t_1 + 2(t_2 - t_1)$$
$$t_1 + t_2 = \underbrace{\frac{1}{0.8}}_{\text{slope of } R_2 \text{ worldline}} \cdot t_2$$

$$t_2 = 4t_1$$
$$t_3 = 7t_1 = \boxed{7h}$$

Q8

Let $\vec{B}^{bw}$ be the magnetic field between and $\vec{B}^{out}$ magnetic field outside.



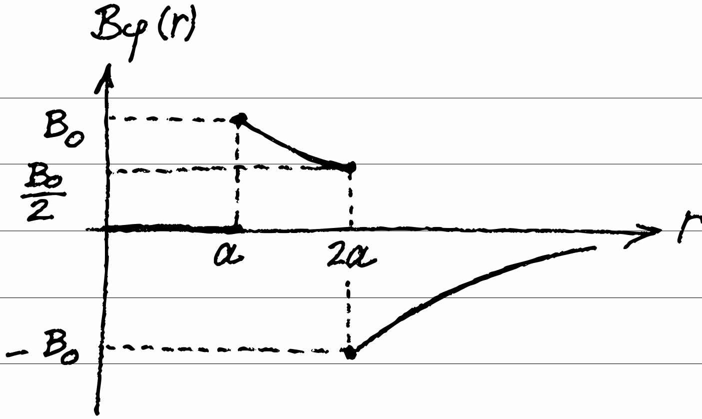
B_ρ, B_z are zero by symmetry.

$$2\pi r B_\varphi^{bw} = \mu_0 I_1 \Rightarrow B_\varphi^{bw} = \frac{\mu_0 I_1}{2\pi r}, \quad a < r < 2a$$

$$\left|B_\varphi^{bw}\right|_{max} = \left|B_\varphi^{bw}(r=a)\right| = \frac{\mu_0 I_1}{2\pi a}$$

$$B_\varphi^{out} = \frac{\mu_0 \overbrace{(I_1 - I_2)}^{-2I_1}}{2\pi r}, \quad 2a < r$$

$$\left|B_\varphi^{out}\right|_{max} = \left|B_\varphi^{out}(r=2a)\right| = \frac{\mu_0 I_1}{2\pi a}$$



$$B_0 \equiv \frac{\mu_0 I_1}{2\pi a}$$

The question was very ambiguous. I don't understand what it is asking for!
Best I could do is plot $B_\varphi(r)$ above.

Q9

a) $N = N_0 \cdot 2^{-t/T}$ half-life

$\Rightarrow \left. N/N_0 \right|_{t=T/2} \approx 0.71\, N_0$

$\therefore$ $\boxed{\text{71\% is present}}$

b) Fasty's misconception is that number of parent nuclei (N) varies linearly with time. While N varies exponentially.

Holda is correct. But I don't like the "just use the formula" reasoning.

Marcela did not rearrange the formula correctly. It should be $N/N_0 = e^{-\lambda t}$. Furthermore N is number of remaining parent nuclei, not decayed.

Q10

a) $P = \overbrace{2\pi r l}^{\text{area}} \cdot \sigma T^4 \approx \boxed{223\ W}$

b) $P_{eye} = P \cdot \dfrac{\pi \rho^2}{4\pi D^2} \approx \boxed{5.01 \times 10^{-12}\ W}$

c) $P_{eye} = n \underbrace{hf}_{\text{photon energy}} = n \cdot \dfrac{hc}{\lambda}$

$n = \dfrac{\lambda P_{eye}}{hc} \approx \boxed{1.51 \times 10^{7}\ 1/s}$

d) $P_{total} = P_S - P_E = 0$

S : sun
E : earth
AU : astronom. unit

$P_S = \dfrac{\pi R_E^2}{4\pi D^2} \cdot \sigma T_S^4 \cdot 4\pi R_S^2$

$P_E = \sigma T_E^4 \cdot 4\pi R_E^2$

Solving for T_E : $\boxed{T_E \approx 246\ K}$