UBC Virtual Physics Circle

The Hacker’s Guide to Fermi Estimates

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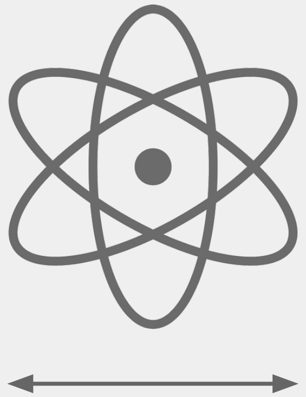
Why estimate?

Physics is fundamentally empirical

Use estimation techniques to

- Apply simplified models quickly
- Check a more rigorous calculation
- Fulfill your curiosity



$$\sim 10^{-10}\ m = 1\ \text{Å}$$

A Fermi estimate

What is the height of this building?

Students, enter a number!

A Fermi estimate

What is the height of this building?

Reasonable estimate: $56\ m$

But unnecessary precision! Perhaps keep one significant figure: $60\ m$

Even further: $\sim 100 = 10^2\ m$

The power of 10

To the nearest order of magnitude:

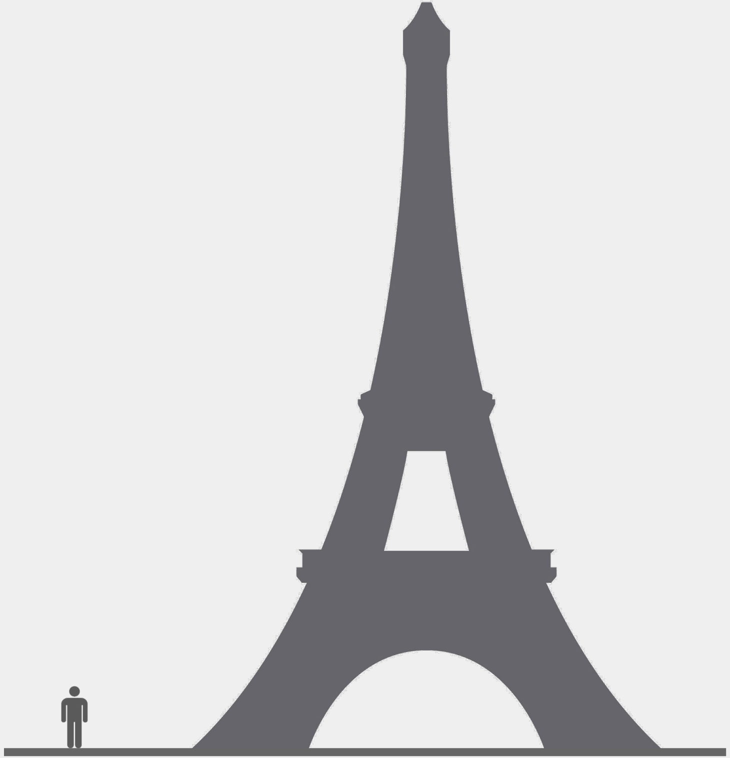
Earth's population $\sim 10 \text{ billion} = 10^{10}$

My height, and yours $\sim 1 = 10^{0}\ m$

Neuron resting potential $\sim -100\, mV = -10^{-1}\ V$

A comparison

Consider the building height in previous example. Is the value closer to my height or the height of the Eiffel tower?



Students choose an option

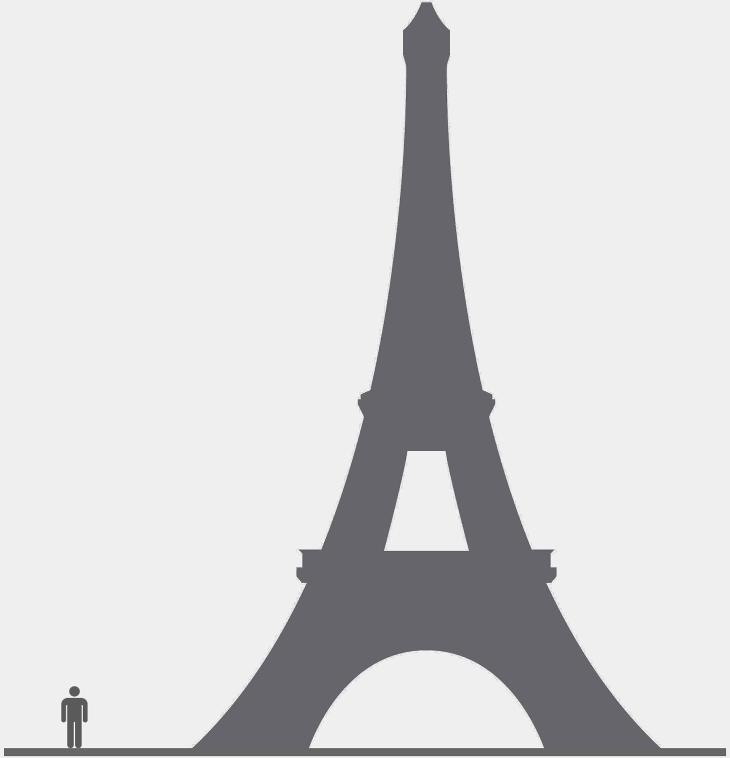
A comparison

Consider the building height in previous example.
Is the value closer to my height or the height of the Eiffel tower?

My height $\approx 2\ m$

Building height $\approx 60\ m$

Eiffel tower height $\approx 300\ m$

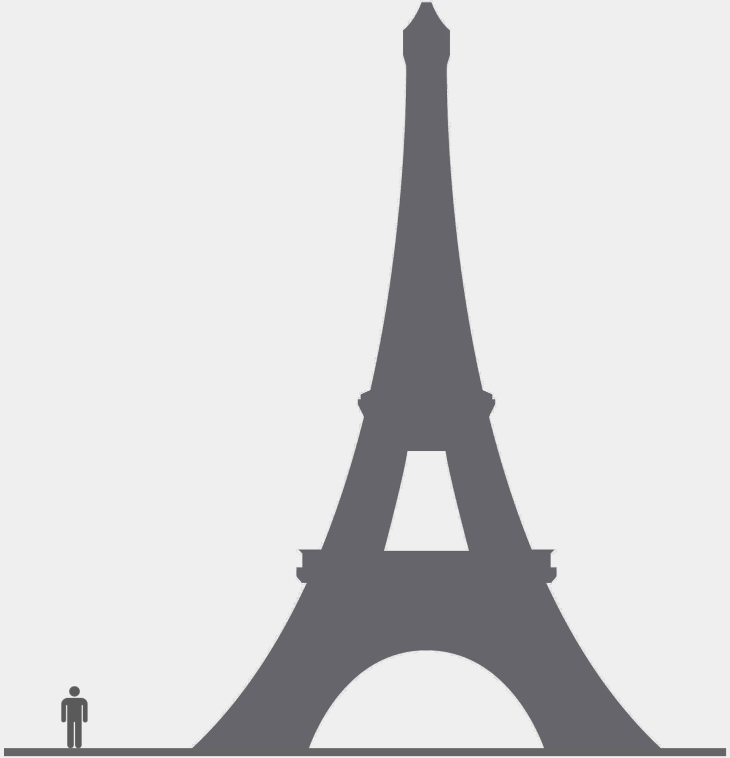


A comparison

Dividing the values:

$$\frac{\text{Building height}}{\text{My height}} = 30$$

$$\frac{\text{Eiffel tower height}}{\text{Building height}} = 5$$



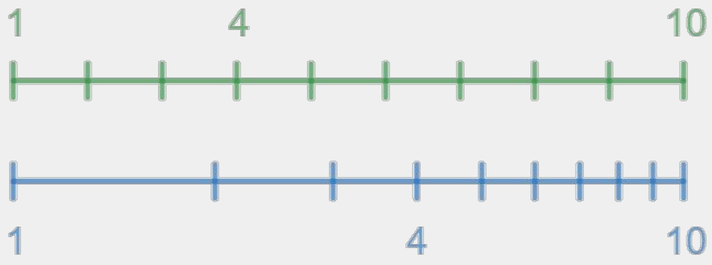
The logarithmic scale

Subtracting on the logarithmic scale is dividing on the linear scale.

$$\log(60) - \log(2) = \log(\tfrac{60}{2}) \approx 1.5$$

$$\log(300) - \log(60) = \log(\tfrac{300}{60}) \approx 0.7$$

Another interesting result: $4 \approx 10^{0.6} \approx 10$



Moon people

Without looking it up, estimate the number of people that have walked on the moon.

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A good general approach is to average an underestimate and overestimate.

- Underestimate: 1 person
- Overestimate: 100 people

Geometric mean

Averaging two estimates:

$$\frac{\log(1) + \log(100)}{2} = 1 = \log(10)$$

$$\sqrt{1 \times 100} = 10 \text{ people}$$

More generally: $\bar{a} = \sqrt{a_1 a_2 \cdots a_n}$

Short break

Meanwhile, feel free to ask questions!

Power for all

Fermi estimate the annual electricity usage of an average household in Vancouver. Answer in GJ units.

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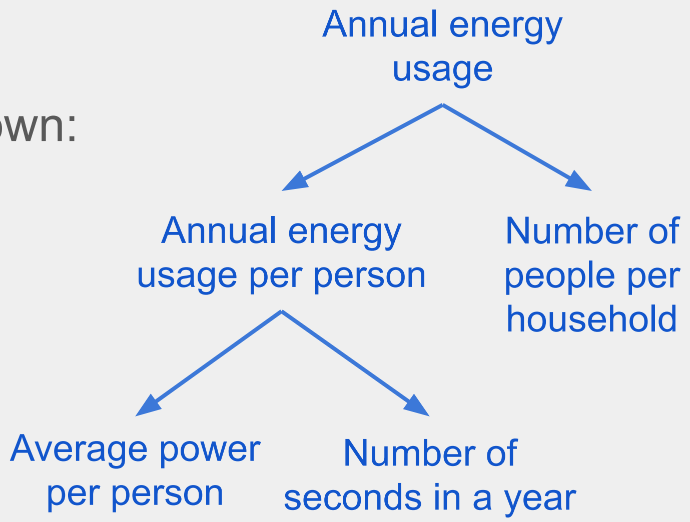
Instead of estimating right away, we can break it down:

- What is the average power usage per person?
- How many seconds in a year?
- How many people per household?



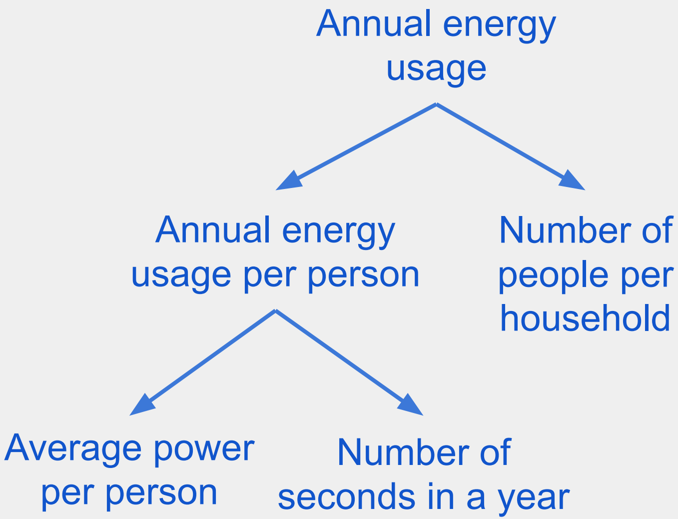
Sub-estimates

Using this divide and conquer strategy:

$$\text{Annual energy usage} = \frac{10 \cdot 60\ J}{1\ s \cdot 1\ \text{person}}$$

$$\times \frac{3600 \cdot 24 \cdot 365\ s}{1\ \text{year}} \times \frac{4\ \text{person}}{1\ \text{household}}$$

$$\approx 80\ \frac{GJ}{1\ \text{household} \cdot 1\ \text{year}}$$



Brilliant idea

Can we supply a significant portion of this energy by harvesting potential energy from rainwater? We get a lot of rain afterall.

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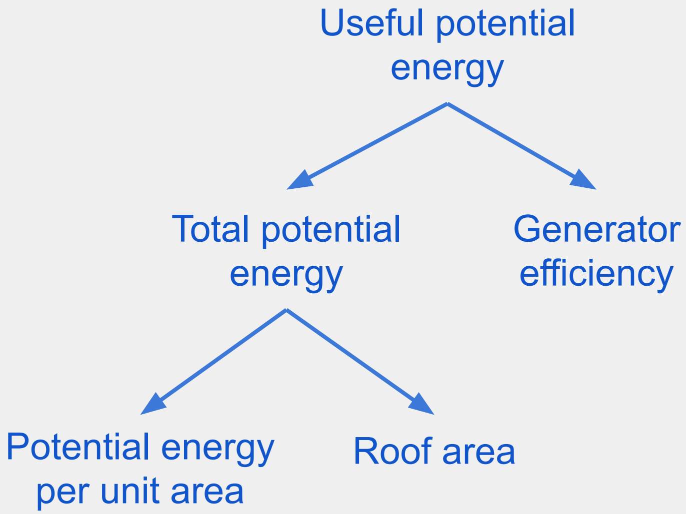
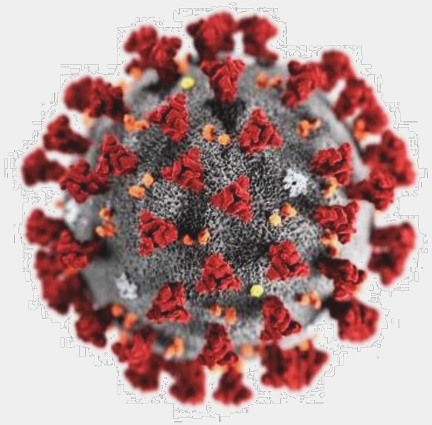
$$U = mgh = \rho V gh$$

$$\text{Useful potential energy} = \frac{10^3 \cdot 1 \cdot 10 \cdot 3\ J}{1\ m^2} \times \frac{5 \cdot 10\ m^2}{1\ \text{roof}} \times 0.5 \approx 0.8\ \frac{MJ}{1\ \text{roof}}$$



And of course

How likely is it that you contract Covid-19 in the next 2 weeks?



Students, write your response!

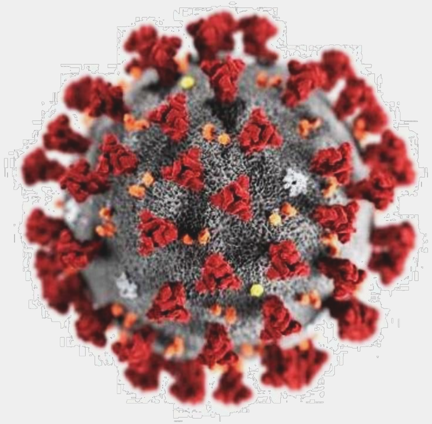
And of course

How likely is it that you contract Covid-19 in the next 2 weeks?

$$\text{Probability of asymptomatic} = \frac{100 \text{ asymptomatic cases}}{5 \text{ million people}} = 2 \cdot 10^{-5}$$

$$\text{Probability of contraction} = 2 \text{ weeks}$$

$$\times \frac{1 \text{ close contact}}{1 \text{ week}} \times \frac{2 \cdot 10^{-5} \text{ contraction probability}}{1 \text{ close contact}} \approx 4 \cdot 10^{-5}$$



$$\text{New cases per 2 weeks} = 200$$

Thank you.

Next time: Random walks!