Leaping Limestone

Why can you draw dotted lines on a chalkboard?

Hold a piece of chalk firmly at one end and place the other on the chalkboard. Angle it away from the direction of movement, and while applying slight pressure, start dragging it down. After some practice, you can easily draw dotted lines. If you haven't given it a go, try now! We are going to explore a simple mechanical model of this system, and try to derive the dot spacing (distance between adjacent dots).

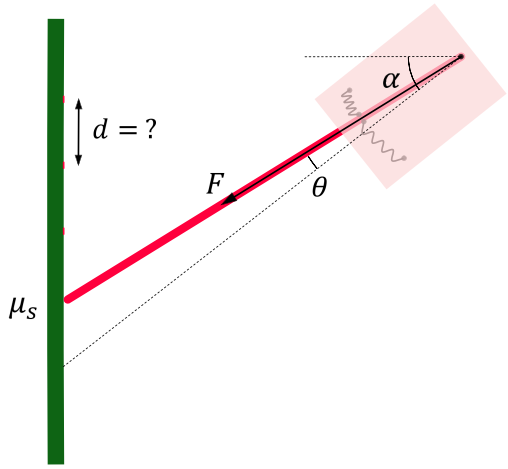
1. Before looking at our proposed model however, you should try to devise your own. This is a good way to practice being a real physicist!

   (a) List the physical parameters that define this problem (e.g. length of chalk).

   (b) Try varying some of these parameters (for instance, setting some to zero) while keeping others unchanged. How is the dot spacing affected?

   (c) Try to come up with a physical model that is consistent with your observations.

2. In our model, a piece of red chalk, length ℓ, is held in your hand, represented below by the beige box. Initially, the chalk is held straight, such that the angle $\theta = 0$ and the chalk makes an angle α with the horizontal. You place the chalk on the board, exerting force F along its length, and start moving your hand down without changing its orientation. The coefficient of static friction between the two is μ_s.



   We model your grab as a torque τ applied to the chalk about its held end:

$$\tau = M\theta$$

   where M is a constant with units of torque. Assuming $\theta \ll \alpha$ and that you move your hand slowly, find the angle θ_0 at which the chalk slips and hence the dot spacing d.

— Pedram Amani