Assignment 1

Precision Loss Examples - 5 points

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Introduction

In numerical computations, precision loss can occur when operations involve subtraction of nearly equal numbers or division by small values. These errors can significantly affect the accuracy of results in scientific and engineering problems. In this assignment, you will explore two examples that highlight precision loss and implement solutions to minimize it using C programming.

Example 1: Arithmetic Operations (1.5 points)

The following subtraction:

```c
int main(){
 double a = 2.0;
 for (int i = 0; i < 10; i++)
 {
  a -= 1e-16;
 }
 printf("result = %0.15lf\n",a);
}
```

should result in 1.999999999999999. However, it is returning 2.000000000000000. The same error happens when `a += 1e-16`.

1. Explain why this is happening.

2. When `a=1.0` with `a += 1e-16`, we find the same error. However, when `a=1.0` and `a -= 1e-16`, we don't have any errors. Why?

3. How can we fix the problem only using addition operator (only + is allowed, NOT ×)?

Example 2: Quadratic Equation (1.5 points)

The quadratic equation is given by:

$$ax^2 + bx + c = 0$$

The roots of this equation can be computed using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Explain why the solution(s) are not correct for the given values $a = 1$, $b = 10^8$, and $c = 1$.

2. Write a C program (`main1.c`) to compute the roots of the quadratic equation using a numerically stable method that reduces the precision loss for the given $a = 1$, $b = 10^8$, and $c = 1$.

3. Include the computed root values in your report.

Example 3: Precision Loss in a Mathematical Function (1.5 points)

Consider the following mathematical expression:

$$f(x) = \sqrt{\left(1 - \sqrt{1 - x^2}\right)}$$

1. Explain the cause of precision loss in this formula.

2. Write a C program (`main2.c`) to compute $f(x)$ for x values close to zero using a numerically stable alternative. Your code must compute and print the result for original and alternative functions when `x=1e-8`.

3. Include the computed results from both functions when `x=1e-8` in your report.

Submission On Avenue to Learn

Provide your explanations for each example, including why precision loss occurs and the steps you took to minimize it. Include the C code for each example, ensuring it is well-documented and easy to understand. Include the computed results for the given scenarios in your report as well as how to compile and run the code. Submit your assignment as:

1. `main1.c`,

2. `main2.c`,

3. a PDF file named `Report.pdf` (0.5 points) containing your explanations, code, and results.

Create a section called Appendix in your report and include all your codes in this section as a text based format (not screenshots). Please do **NOT** submit any zip files.