

Definitions

$$V^\pi(s_t) \triangleq \mathbb{E}_{s_{t+1:\infty}, a_{t:\infty}} \left[\sum_{\ell=0}^{\infty} r_{t+\ell} \right]$$

$$Q^\pi(s_t, a_t) \triangleq \mathbb{E}_{s_{t+1:\infty}, a_{t+1:\infty}} \left[\sum_{\ell=0}^{\infty} r_{t+\ell} \right]$$

$$V^{\pi, \gamma}(s_t) \triangleq \mathbb{E}_{s_{t+1:\infty}, a_{t:\infty}} \left[\sum_{\ell=0}^{\infty} \gamma^\ell r_{t+\ell} \right]$$

$$Q^{\pi, \gamma}(s_t, a_t) \triangleq \mathbb{E}_{s_{t+1:\infty}, a_{t+1:\infty}} \left[\sum_{\ell=0}^{\infty} \gamma^\ell r_{t+\ell} \right]$$

$$\delta_t \triangleq r_t + \gamma V^{\pi, \gamma}(s_{t+1}) - V^{\pi, \gamma}(s_t)$$

$$\delta_t^V \triangleq r_t + \gamma V_\phi^{\pi, \gamma}(s_{t+1}) - V_\phi^{\pi, \gamma}(s_t)$$

Advantage Function

$$A^\pi(s_t, a_t) \triangleq Q^\pi(s_t, a_t) - V^\pi(s_t)$$

γ : introduces bias, lowers variance

$$A^{\pi, \gamma}(s_t, a_t) \triangleq Q^{\pi, \gamma}(s_t, a_t) - V^{\pi, \gamma}(s_t)$$

λ : tunes the bias-variance trade-off
for the approximation of $A^{\pi, \gamma}$

$$\hat{A}_t^{\text{GAE}(\gamma, \lambda)} \triangleq \sum_{\ell=0}^{\infty} (\gamma \lambda)^\ell \delta_{t+\ell}^V$$

Policy Gradient

$$g \triangleq \mathbb{E}_{s_{0:\infty}, a_{0:\infty}} \left[\sum_{t=0}^{\infty} A^\pi(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t | s_t) \right]$$

$$g^\gamma \triangleq \mathbb{E}_{s_{0:\infty}, a_{0:\infty}} \left[\sum_{t=0}^{\infty} A^{\pi, \gamma}(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t | s_t) \right]$$

$$\hat{g} \triangleq \mathbb{E}_{s_{0:\infty}, a_{0:\infty}} \left[\sum_{t=0}^{\infty} \hat{A}_t^{\text{GAE}(\gamma, \lambda)} \nabla_\theta \log \pi_\theta(a_t | s_t) \right]$$