Tarea 3

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1. Corte meridional

Realizar un programa que realice y grafique un corte meridional de U en donde se observe el paso del huracán y guardar las gráficas de cuatro tiempos diferentes en formato jpg.

```
% author Pedraza-Espitia S.
2 % corte meridional
3 close all
4 clear all
5 %Ruta = ['/media/salva/exfat/'];
6 Ruta = ['/media/sal/exfat/'];
7 Arch = [Ruta, 'wrfout_d02_2012 -08-08_00_UVW.nc'];
8 % atributos, variables y dimensiones
9 Tiempo = ncread(Arch, 'Times')';
Times_l = length (Tiempo);
12 XLAT = ncread(Arch, 'XLAT', [1 1 1], [Inf Inf 1], [1 1 1]);
13 XLAT = double(XLAT);
14 XLONG = ncread(Arch, 'XLONG', [1 1 1], [Inf Inf 1], [1 1 1]);
15 XLONG = double (XLONG);
Niveles = 1:27;
Xlatitud = XLAT(1,:);
[Niveles, Xlatitud] = meshgrid(Niveles, Xlatitud);
19 \text{ figst} = [24 \ 29 \ 34 \ 39];
20 num = 1;
Un94 = ncread(Arch, 'U', [200 1 1 1], [1 inf inf Inf], [1 1 1 1]);
22 Un94 = double(squeeze(Un94));
23 figure
24 for tt = 1:46
25
      clf;
      year = Tiempo(1, 1:4);
26
      mes = Tiempo (1,6:7)
27
      dia = Tiempo(tt, 9:10);
28
      hora = Tiempo(tt, 12:13);
      mins = Tiempo(tt, 15:16);
30
31
      pcolor(Xlatitud, Niveles, Un94(:,:,tt)), shading interp
      ylabel ('Altura (Niveles)')
32
      xlabel ('Latitud')
33
      title (['Corte meridional componente zonal U',...
34
          dia, '-', mes, '-', year, 'a las', hora, ':', mins, 'GMT'])
35
      caxis([-20 20])
36
      colorbar
37
      %title(tt) % codigo para obtener 4 imagenes jpeg:
      if figst (num) == tt
```

```
# % % % fig . PaperUnits = 'inches';
# nombreimg = ['t3fig', num2str(tt),'.jpeg'];
# print(nombreimg, '-djpeg', '-r0')
# if num < 4
# num = num+1;
# else
# continue
# end
# pause(0.2)
# ond</pre>
```

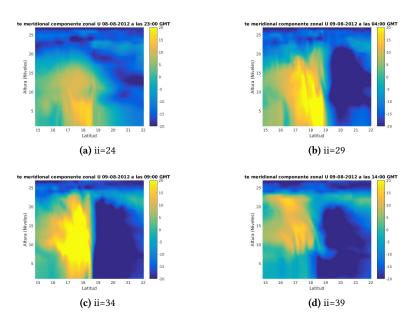


Figura 1: Evolución de la componente U en un corte meridional (longitud fija \approx -94°)

2. Divergencia

Hacer un programa que calcule la divergencia del viento a 10 metros, utilizando diferencias finitas centradas, sin emplear funciones de matlab (DIV), y guardar las gráficas resultantes de cuatro tiempos diferentes. Tomar en cuenta que la latitud y longitud que proporciona el modelo están en grados y deberán convertirse a metros.

En la siguiente Sección 3 se fusionan el ejercicio de obtener divergencia y rotacional en un sólo script. Las gráficas que se piden se muestran en la Figura 2.

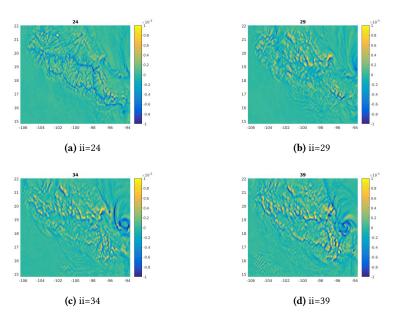


Figura 2: Divergencia en 4 tiempos distintos, una divergencia positiva implica una fuente y una negativa indica hundimiento o sumidero.

3. Rotacional

Hacer un programa que calcule el rotacional del viento en el nivel 8, utilizando diferencias finitas centradas, sin emplear funciones de matlab (CURL), y guardar las gráficas resultantes de cuatro tiempos diferentes. Tomar en cuenta que la latitud y longitud que proporciona el modelo están en grados y deberán convertirse a metros.

```
% author Pedraza-Espitia S.
  % divergencia y rotacional
  close all
  clear all
  %Ruta = ['/media/salva/exfat/'];
  %Ruta = ['/media/sf_salida_WRF/'];
  Ruta = ['/media/sal/exfat/'];
  Arch = [Ruta, 'wrfout_d02_2012 -08-08_00_UVW.nc'];
  %ncdisp(Arch)
11
  Tiempo = ncread(Arch, 'Times')';
12
13 Times_l = length (Tiempo);
14 % [DesdeX , Y, T] [HastaX , Y, T] [cUAL]
15 XLAT = ncread(Arch, 'XLAT', [1 1 1], [Inf Inf 1], [1 1 1]);
16 XLAT = double(XLAT);
17 XLONG = ncread (Arch, 'XLONG', [1 1 1], [Inf Inf 1], [1 1 1]);
18 XLONG = double(XLONG);
20 U10 = ncread(Arch, 'U10');
```

```
V10 = ncread (Arch, 'V10');
22
23
      for ii = 1: Times_l(1)
24
                   Uii = double(U10(:,:,ii));
25
                   Vii = double(V10(:,:,ii));
Speed = sqrt(Uii.^2+Vii.^2);
26
27
                   pcolor(XLONG, XLAT, Speed), shading flat, caxis([0 30]), colorbar
28
                   pause (.1)
29
30 end
31
32 %% convertir diferenciales Dx Dy
[xx, yy, tt] = size(U10);
R = 6370e3;
[Nx, Ny] = size(XLAT);
Del2X = zeros(Nx, Ny);
Del2Y = zeros(Nx, Ny);
39 for ii = 2:Nx-1;
                   for jj = 2:Ny-1
40
                               Del2X(ii,jj) = R*\cos((pi/180)*XLAT(ii,jj))*(XLONG(ii+1,jj)-
41
                   XLONG(ii -1, jj))*pi/180;
                               \label{eq:delay} Del2Y(\,i\,i\,\,,\,j\,j\,\,) \,\,=\,\, R \, \star \, \big(\,XLAT\big(\,i\,i\,\,,\,j\,j\,\,+\,1\big) - XLAT\big(\,i\,i\,\,,\,j\,j\,\,-\,1\big)\,\big) \, \star \, \frac{p\,i}{1} \, / \,1\,8\,0\,;
42
43
44 end
45
Div = zeros(Nx, Ny, tt);
Rot = zeros(Nx, Ny, tt);
48 %% calculo divergencia
49 for kk = 1: tt
                   U=double(U10(:,:,kk));
                  V=double(V10(:,:,kk));
51
                   for ii = 2:Nx-1;
52
53
                               for jj = 2:Ny-1;
                                          Div(ii,jj,kk) = (U(ii+1,jj)-U(ii-1,jj))/Del2X(ii,jj) + (V(ii+1,jj)-U(ii-1,jj))/Del2X(ii,jj) + (V(ii+1,jj)-U(ii-1,jj)-U(ii-1,jj))/Del2X(ii,jj) + (V(ii+1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj))/Del2X(ii-ij-1,jj) + (V(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj)-U(ii-1,jj
54
                    ii , jj +1)-V(ii , jj -1))/Del2Y(ii , jj);
                              end
55
56
57 end
58
figst = [24 \ 29 \ 34 \ 39];
60 \text{ num} = 1;
for kk = 1: Times_l(1)
                 pcolor\left(XLONG,XLAT,Div\left(:\,,:\,,kk\right)\right), shading \ flat \ , \ caxis\left([\,-1e-3\ 1e-3\,]\right), 
62
                   colorbar
                title (int2str(kk))
63
                if figst(num) == kk
64
                               nombreimg = ['t3div', num2str(num),'.jpeg'];
                               print (nombreimg, '-djpeg', '-r0')
66
67
                                if num < 4
                                           num = num + 1;
68
69
70
                                            continue
                               end
71
72
                end
                pause (.2)
73
```

```
75
  76 %% calculo Rotacional
  Uu = ncread (Arch, 'U');
  Vv = ncread(Arch, 'V');
  79 for kk = 1: tt
                            U=double\left(Uu\left(:\;,:\;,8\;,kk\right)\right);
  80
                             V=double(Vv(:,:,8,kk));
  81
                             for ii = 2:Nx-1;
  82
                                              for jj = 2:Ny-1;
  83
                                                               Rot(ii,jj,kk) = (V(ii+1,jj)-V(ii-1,jj))/Del2X(ii,jj) - (U(ii+1,jj)-V(ii-1,jj))/Del2X(ii,jj) - (U(ii+1,jj)-V(ii-1,jj)-V(ii-1,jj))/Del2X(ii,jj) - (U(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii-1,jj)-V(ii
  84
                              ii , jj +1)-U( ii , jj -1) ) / Del2Y( ii , jj );
   85
                                              end
                             end
  86
  87 end
  88
  figst = [24 \ 29 \ 34 \ 39];
  90 num = 1;
  91 for kk = 1: Times_l(1)
                         colorbar
   93
                          title (int2str(kk))
                         pause (.2)
  94
                          if figst(num) == kk
   95
                                              nombreimg = ['t3rot', num2str(num),'.jpeg'];
                                              print (nombreimg, '-djpeg', '-r0')
   97
                                                if num < 4
                                                              num = num + 1;
  99
100
                                                                continue
101
                                              end
102
103
104 end
```

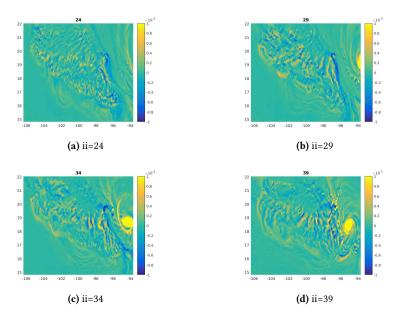


Figura 3: Rotacional en 4 tiempos distintos, se calcula sobre una superfice (nivel 8) y esto implica que sólo resulte una componente que es perpendicular a la superficie, el rotacional sigue la regla de la mano derecha y es proporcional a la velocidad angular.

4. Resolver y' = cos(t) + sin(t) usando RK4

Considera la ecuación diferencial y' = cos(t) + sin(t) con y(t = 0) = 1 y h = 0.1. Escribe un código para calcular la solución con el método de Runge-Kutta de orden 4. Recuerda que:

$$k_1 = h * f(t_i, y_i)$$

$$k_2 = h * f(t_i + h/2, y_i + k_1/2)$$

$$k_3 = h * f(t_i + h/2, y_i + k_2/2)$$

$$k_4 = h * f(t_i + h, y_i + k_3)$$

$$y_{i+1} = y_i + 1/6(k_1 + 2k_2 + 2k_3 + k_4)$$

```
clear all
h = 0.1;
t = 0;
y = 1;

fprintf('Paso 0: t = %6.3f, y = %18.15f\n',t,y);
for ii = 1:126
k1 = h*(cos(t) + sin(t));
k2 = h*(cos(t+h/2) + sin(t+h/2));
k3 = h*(cos(t+h/2) + sin(t+h/2));
k4 = h*(cos(t+h/2) + sin(t+h/2));
```

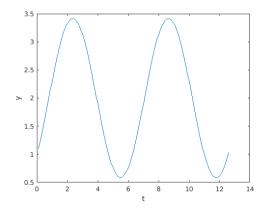


Figura 4: Solución de $y' = \cos t + \sin t$.

5. Resolver $y' = y - 2 * t^3 + 2 \text{ con RK4}$

Considera la ecuación diferencial $y' = y - 2 * t^3 + 2$ con y(t = 0) = 0.5. Escribe un código para calcular la solución con el método de Runge-Kutta de orden 4.

```
clear T Y
_{2} h = 0.01;
t = 0;
y = .5;
RK_f = @(t,y) y - 2*t^3 + 2;
7 fprintf('Paso 0: t = \%6.3 f, y = \%18.15 f \ n', t, y);
  for ii = 1:400
       k1 = h \cdot RK_f(t, y);
       k2 = h * RK_f(t+h/2,y+k1/2);
10
       k3 = h * RK_-f(t+h/2,y+k2/2);
       k4 = h* RK_{-}f(t+h,y+k3);
12
       y = y + (k1 + 2*k2 + 2*k3 + k4) / 6;

t = t + h;
13
14
       T(ii) = t;
15
       Y(ii) = y;
16
       fprintf('Paso %d: t = \%6.3 f, y = \%18.15 f n', ii, t, y);
17
plot(T,Y, 'c');
```

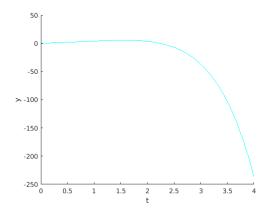


Figura 5: Solución de $y' = y - 2t^3 + 2$.

6. Aguas soméras

$$\frac{\partial u}{\partial t} - fv = -g \frac{\partial h}{\partial x} \tag{1}$$

$$\frac{\partial v}{\partial t} + fu = -g \frac{\partial h}{\partial u} \tag{2}$$

$$\frac{\partial h}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{3}$$

$$\frac{\partial [(1)]}{\partial x} \Rightarrow \frac{\partial^2 u}{\partial x \partial t} - f \frac{\partial v}{\partial x} = -g \frac{\partial^2 h}{\partial x^2} \tag{4}$$

$$\frac{\partial[(2)]}{\partial y} \Rightarrow \frac{\partial^2 v}{\partial y \partial t} + f \frac{\partial v}{\partial y} = -g \frac{\partial^2 h}{\partial y^2}$$
 (5)

$$[(4) + (5)] \Rightarrow \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + f \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) = -g \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) \tag{6}$$

$$\frac{\partial [(3)]}{\partial t} \Rightarrow \frac{\partial^2 h}{\partial t^2} + H \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0 \tag{7}$$

$$[(7) - H * (6)] \Rightarrow \frac{\partial^{2} h}{\partial t^{2}} + H \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - H \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$
$$- H f \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) = g \left(\frac{\partial^{2} h}{\partial x^{2}} + \frac{\partial^{2} h}{\partial y^{2}} \right)$$
$$\Rightarrow \frac{\partial^{2} h}{\partial t^{2}} - H f \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) - g H \left(\frac{\partial^{2} h}{\partial x^{2}} + \frac{\partial^{2} h}{\partial y^{2}} \right) = 0$$
(8)

$$\frac{\partial[(1)]}{\partial y} \Rightarrow \frac{\partial^2 u}{\partial y \partial t} - f \frac{\partial v}{\partial y} = -g \frac{\partial^2 h}{\partial y \partial x} \tag{9}$$

$$\frac{\partial[(2)]}{\partial x} \Rightarrow \frac{\partial^2 v}{\partial x \partial t} + f \frac{\partial u}{\partial x} = -g \frac{\partial^2 h}{\partial x \partial y}$$
 (10)

$$[(9) + (10)] \Rightarrow \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) = f \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \tag{11}$$

$$[(3)] \Rightarrow \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -\frac{1}{H}\frac{\partial h}{\partial t} \tag{12}$$

$$[(12) \text{ y } (11)] \Rightarrow \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) = -f \frac{1}{H} \frac{\partial h}{\partial t}$$
 (13)

$$\frac{\partial [(8)]}{\partial t} \Rightarrow \frac{\partial}{\partial t} \frac{\partial^2 h}{\partial t^2} - Hf \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) - gH \frac{\partial}{\partial t} \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) = 0 \tag{14}$$

$$[(11) \text{ y } (14)] \Rightarrow \frac{\partial}{\partial t} \frac{\partial^2 h}{\partial t^2} - Hf^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - gH \frac{\partial}{\partial t} \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) = 0$$
 (15)

$$[(12) \text{ y } (15)] \Rightarrow \frac{\partial}{\partial t} \frac{\partial^2 h}{\partial t^2} - H f^2 \left(-\frac{1}{H} \frac{\partial h}{\partial t} \right) - g H \frac{\partial}{\partial t} \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) = 0 \tag{16}$$

$$\frac{\partial}{\partial t} \frac{\partial^2 h}{\partial t^2} + f^2 \left(\frac{\partial h}{\partial t} \right) - gH \frac{\partial}{\partial t} \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) = 0 \tag{17}$$

$$\frac{\partial}{\partial t} \left[\frac{\partial^2}{\partial t^2} + f^2 - gH \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \right] h = 0$$
 (18)