Sejam  $u=f(x),\,v=g(x),\,u'=\frac{df(x)}{dx}$  e  $v'=\frac{dg(x)}{dx}$ . Sejam também  $a,\,c$  e n constantes.

#### **Derivadas**

#### Regras Fundamentais

$$1. \ (au)' = au'$$

2. 
$$(u+v)' = u' + v'$$

3. 
$$(uv)' = u'v + uv'$$

$$4. \left(\frac{u}{v}\right)' = \frac{u'v - uv}{v^2}$$

5. 
$$(u(v))' = u'(v) v'$$
 (Regra da Cadeia)

## Funções Básicas

| Função        | Derivada         | Restrições          |
|---------------|------------------|---------------------|
| a             | 0                |                     |
| ax            | a                |                     |
| $x^n$         | $n x^{n-1}$      |                     |
| $\frac{1}{x}$ | $-\frac{1}{x^2}$ |                     |
| $a^x$         | $a^x \ln a$      | $(a > 0, a \neq 1)$ |
| $e^x$         | $e^x$            | _                   |
| $\ln x$       | $\frac{1}{x}$    |                     |

## Funções Trigonométricas

| Função   | Derivada                |
|----------|-------------------------|
| $\sin u$ | $u'\cos u$              |
| $\cos u$ | $-u'\sin u$             |
| $\tan u$ | $u' \sec^2 u$           |
| $\cot u$ | $-u'\csc^2 u$           |
| $\sec u$ | $u'\sec u \cdot \tan u$ |
| $\csc u$ | $-u'\csc u\cdot\cot u$  |

# Funções Trigonométricas Inversas

| Função      | Derivada                      | Restrições |
|-------------|-------------------------------|------------|
| $\arcsin u$ | $\frac{u'}{\sqrt{1-u^2}}$     |            |
| $\arccos u$ | $\frac{-u'}{\sqrt{1-u^2}}$    |            |
| $\arctan u$ | $\frac{u'}{\sqrt{1+u^2}}$     |            |
| arccotan u  | $\frac{-u'}{\sqrt{1+u^2}}$    |            |
| arcsec u    | $\frac{u'}{ u \sqrt{u^2-1}}$  | u  > 1     |
| arccsc u    | $\frac{-u'}{ u \sqrt{u^2-1}}$ | u  > 1     |

## Funções Genéricas

| Função        | Derivada   | Restrições          |
|---------------|--|---------------------|
| $u^n$         | $nu^{n-1}u'$   |                     |
| uv            | u'v + v'u  |                     |
| $\frac{u}{v}$ | $\frac{u'v - v'u}{v^2}$                                      |                     |
| $a^u$         | $a^u \ln a u'$   | $(a > 0, a \neq 1)$ |
| $e^u$         | $e^u u'$   |                     |
| $\log_a u$    | $\frac{u'}{u} \log_a e = \frac{u'}{u} \cdot \frac{1}{\ln a}$ |                     |
| $\ln u$       | $\frac{1}{u} u'$   |                     |
| $u^v$         | $vu^{v-1}u' + u^v \ln u \ v'$                                |                     |

# Integrais

#### Regras Fundamentais

- 1.  $\int au \ dx = a \int u \ dx$
- 2.  $\int (u+v) dx = \int u dx + \int v dx$
- 3.  $\int u'v \ dx = uv \int uv' \ dx$

# Funções Genéricas

| Função                | Integral                                  | Restrições        |
|-----------------------|---|-------------------|
| $\int du$             | u+c                                       |                   |
| $\int u^n du$         | $\frac{u^{n+1}}{n+1} + c$                 | $n \neq -1$       |
| $\int \frac{1}{u} du$ | $\ln u  + c$                              |                   |
| $\int a^u du$         | $\frac{a^u}{\ln a} + c$                   | $a > 0, a \neq 1$ |
| $\int \ln x  dx$      | $x \ln x - x + c$                         |                   |
| $\int \log_a x  dx$   | $c \frac{x}{\ln a} + c - \frac{1}{\ln a}$ |                   |
|                       | $e^u + c$                                 |                   |

#### Funções Racionais

| Função                        | Integral  | Restrições  |
|-------------------------------|---|-------------|
| $\int \frac{1}{u^2 + a^2} du$ | $\frac{1}{a}\arctan\frac{u}{a} + c$               |             |
| $\int \frac{1}{u^2 - a^2} du$ | $\frac{1}{2a}\ln\left \frac{u-a}{u+a}\right  + c$ | $u^2 > a^2$ |
| $\int \frac{1}{a^2 - u^2} du$ | $\frac{1}{2a}\ln\left \frac{a+u}{a-u}\right  + c$ | $u^2 < a^2$ |

# Funções Irracionais

| Função                                | Integral  | Restrições  |
|---------------------------------------|---|-------------|
| $\int \frac{1}{\sqrt{u^2 + a^2}} du$  | $ \ln u + \sqrt{u^2 + a^2}  + c $                   |             |
| $\int \frac{1}{\sqrt{u^2 - a^2}} du$  | $ \ln u + \sqrt{u^2 - a^2}  + c $                   | $u^2 > a^2$ |
| $\int \frac{1}{\sqrt{a^2 - u^2}} du$  | $\arcsin\frac{u}{a} + c$                            | $u^2 < a^2$ |
| $\int \frac{1}{u\sqrt{u^2 - a^2}} du$ | $\frac{1}{a}$ arcsec $\left \frac{u}{a}\right  + c$ | $u^2 > a^2$ |

## Funções Trigonométricas

| Função                          | Integral   | Restrições |
|---------------------------------|--|------------|
| $\int \sin u \ du$              | $-\cos u + c$                                      |            |
| $\int \cos u \ du$              | $\sin u + c$                                       |            |
| $\int \tan u \ du$              | $\ln \sec u  + c$                                  |            |
| $\int \cot u \ du$              | $     \ln \sin u  + c $                            |            |
| $\int \sec u \ du$              | $     \ln \sec u + \tan u  + c $                   |            |
| $\int \csc u \ du$              | $\ln \csc u - \cot u  + c$                         |            |
| $\int \sec u \cdot \tan u \ du$ | $\sec u + c$                                       |            |
| $\int \csc u \cdot \cot u \ du$ | $-\csc u + c$                                      |            |
| $\int \sin^2 u \ du$            | $\frac{1}{2} \left( u - \sin u \cos u \right) + c$ |            |
| $\int \cos^2 u \ du$            | $\frac{1}{2}\left(u+\sin u \cos u\right)+c$        |            |
| $\int \sec^2 u \ du$            | $\tan u + c$                                       |            |
| $\int \csc^2 u \ du$            | $-\cot u + c$                                      |            |

Última revisão: 14/04/2019

#### Definições e Identidades trigonométricas

1. 
$$\tan(x) = \frac{\sin x}{\cos x}$$

2. 
$$\sin(-x) = -\sin(x)$$

$$3. \cos(-x) = \cos(x)$$

$$4. \tan(-x) = -\tan(x)$$

5. 
$$\sin x = \cos(\frac{\pi}{2} - x)$$

6. 
$$\cos x = \sin(\frac{\pi}{2} - x)$$

7. 
$$\tan x = \cot(\frac{\pi}{2} - x)$$

8. 
$$\cot x = \tan(\frac{\pi}{2} - x)$$

9. 
$$\sec x = \csc(\frac{\pi}{2} - x)$$

10. 
$$\csc x = \sec(\frac{\pi}{2} - x)$$

11. 
$$\sec x = \frac{1}{\cos x}$$

12. 
$$\cos x = \frac{1}{\sin x}$$

13. 
$$\cot x = \frac{1}{\tan x}$$

14. 
$$\sin^2 x + \cos^2 x = 1$$

15. 
$$\sec^2 x - \tan^2 x = 1$$

16. 
$$\csc^2 x - \cot^2 x = 1$$

17. 
$$\sin 2x = 2\sin x \cdot \cos x$$

18. 
$$\cos 2x = \cos^2 x - \sin^2 x$$

19. 
$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

20. 
$$\tan \frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$$

21. 
$$\sin(x \pm y) = \sin(x) \cos(y) \pm \sin(y) \cos(x)$$

22. 
$$\cos(x \pm y) = \cos(x) \cos(y) \mp \sin(x) \sin(y)$$

23. 
$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x + \tan y}$$

24. 
$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x + \tan y}$$

25. 
$$\cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x}$$

26. 
$$2\sin x \cos y = \sin(x - y) + \sin(x + y)$$

27. 
$$2\cos x \sin y = \sin(x+y) - \sin(x-y)$$

28. 
$$2\sin x \sin y = \cos(x - y) - \cos(x + y)$$

29. 
$$2\cos x \cos y = \cos(x - y) + \cos(x + y)$$

30. 
$$1 + \cos x = 2 \cos^2 \frac{x}{2}$$

31. 
$$1 - \cos x = 2 \sin^2 \frac{x}{2}$$

32. 
$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

33. 
$$\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

34. 
$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

35. 
$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

36. 
$$\tan x + \tan y = \frac{\sin(x+y)}{\cos x \cos y}$$