

# Inf2C - Computer Systems

## Lecture 13

### Memory Hierarchy and Caches

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# Previous lecture: multi-cycle processor

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- Execution time:  
 $(\text{inst count}) \times (\text{cycles/inst}) \times (\text{cycle time})$
- Multi-cycle processor: reduce the cycle time by breaking up an instruction's execution into multiple simple operations (1 or 2 per cycle)
  - Control FSM sequences the cycles
  - Reuse components (memory, ALU) to reduce “cost”

# Memory requirements

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- Programmers wish for memory to be
  - Large
  - Fast
  - Random access
- Wish not achievable with 1 kind of memory
  - Issues of cost and technical feasibility
- Idea of a **memory hierarchy**: approximate the “ideal” large+fast memory through a combination of different kinds of memories



# Memory examples

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Technology	Typical access time	Price per GB
SRAM	1-10 ns	£1000
DRAM	~100 ns	£10
Flash SSD	~100 $\mu$ s	£1
Magnetic disk	~10 ms	£0.1

Which of these is “main memory”? **DRAM**



# Memory hierarchy overview

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- Use a combination of memory kinds
  - Smaller amounts of expensive but fast memory closer to the processor
  - Larger amounts of cheaper but slower memory farther from the processor

- Idea is not new:

**“Ideally one would desire an indefinitely large memory capacity such that any particular ... word would be immediately available... we are ... forced to recognize the possibility of constructing a hierarchy of memories, each of which has greater capacity than the preceding but which is less quickly accessible.”**

A. W. Burks, H. H. Goldstine, and J. von Neumann - 1946



# Why is a memory hierarchy effective?

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- Temporal Locality:
  - A recently accessed memory location (instruction or data) is likely to be accessed again in the near future
- Spatial Locality:
  - Memory locations (instructions or data) close to a recently accessed location are likely to be accessed in the near future
- Why does locality exist in programs?
  - Instruction reuse: loops, functions
  - Data working sets: arrays, temporary variables, objects



# Example of Temporal & Spatial Locality

Matrix – matrix multiplication:

Spatial locality

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & \dots \end{bmatrix}$$

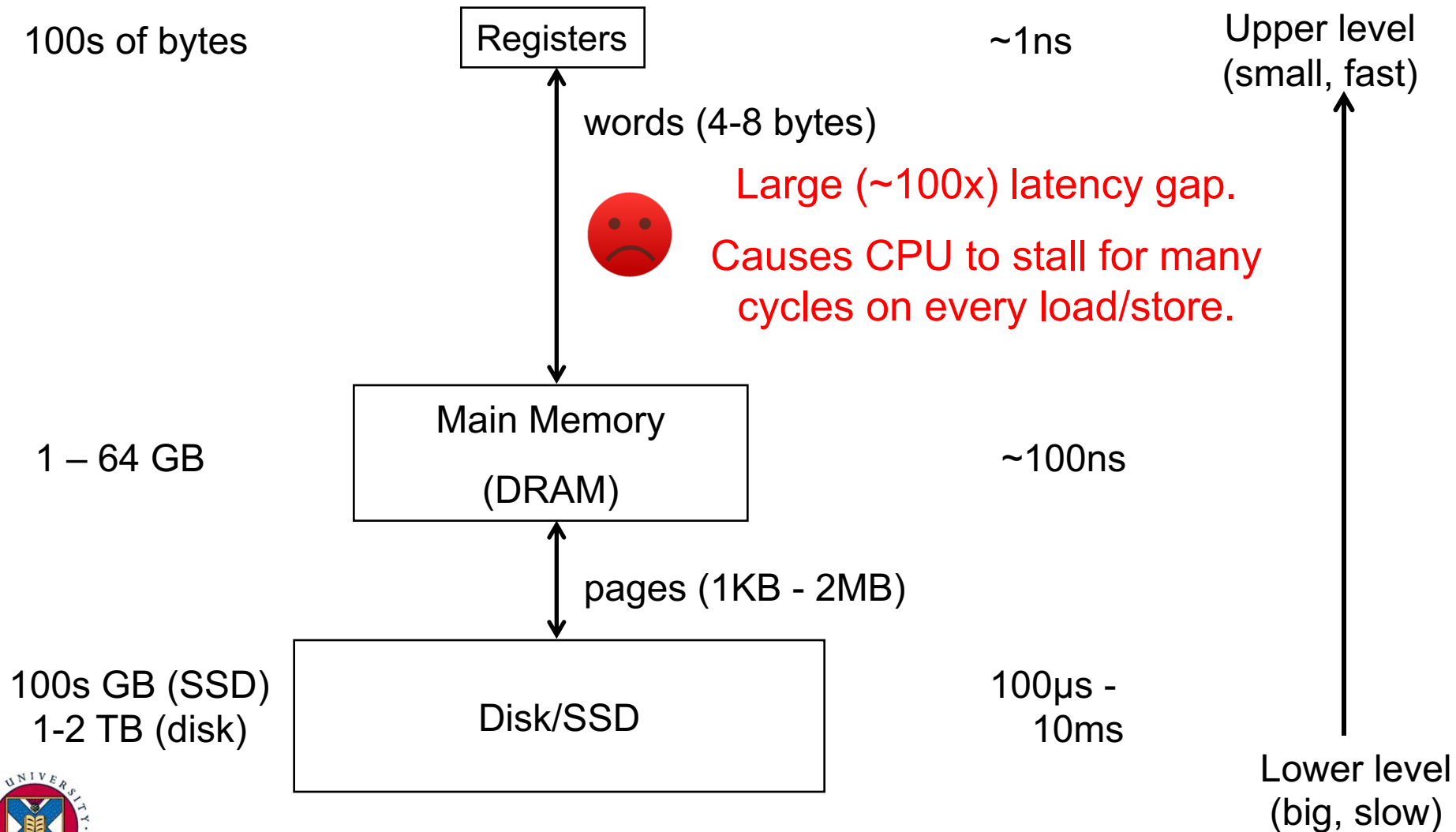
Temporal locality

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \end{bmatrix}$$

```
for i = 1 to M
  for j = 1 to N
    for k = 1 to P
      c[i,j] = c[i,j] + a[i,k] * b[k,j]
```

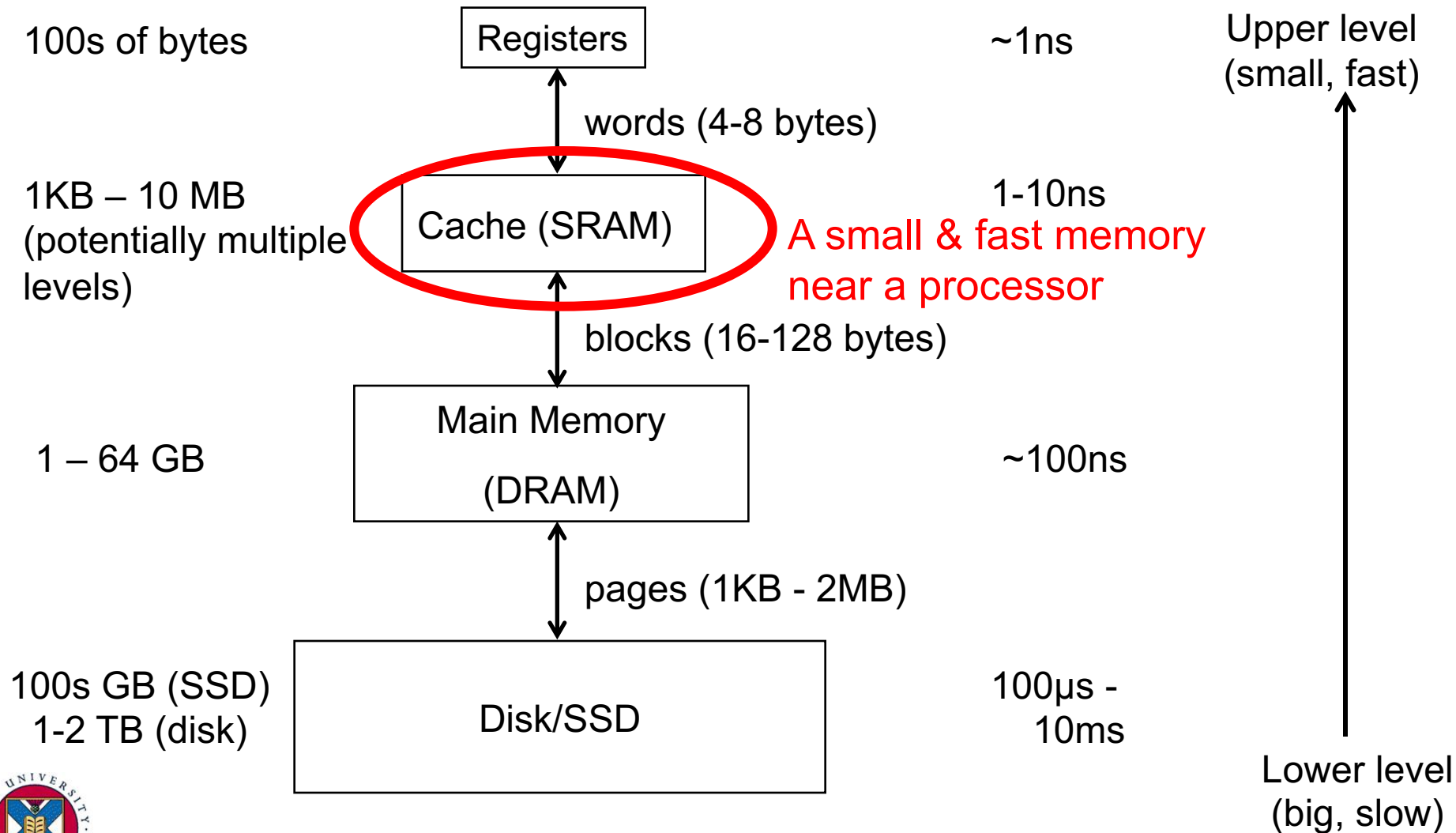
Temporal & spatial  
locality in the code  
itself

# Levels of the memory hierarchy





# Levels of the memory hierarchy



# Memory hierarchy in a modern processor

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- Small, fast **cache** next to a processor backed up by larger & slower cache(s) and main memory give the impression of a single, large, fast memory
- Take advantage of temporal locality
  - If access data from slower memory, move it to faster memory
  - If data in faster memory unused recently, move it to slower memory
- Take advantage of spatial locality
  - If need to move a word from slower to faster memory, move adjacent words at same time
  - Gives rise to **block** & **pages**: units of storage within the memory hierarchy composed of multiple contiguous words

# Control of data transfers in hierarchy

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- Q. Should the SW or HW be responsible for moving data between levels of the memory hierarchy?
- A. It depends: there is a trade-off between ease of programming, hardware complexity, and performance.
  - *SW (compiler)*: between registers and cache/main memory
  - *HW*: between caches and main memory (SW is usually unaware of caches)
  - *SW (Operating System)*: between main memory and disk

# HW-managed transfers between levels

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- Occurs between cache memory and main memory levels
  - Programmer & processor both oblivious to where data resides
    - Just issue loads & stores to “memory”
  - Cache Hardware manages transfers between levels
    - Data moved or copied between levels automatically in response to the program’s memory accesses
    - Memory always has a copy of cached data, but data in the cache may be more recent
      - This creates interesting problems.
- Discussed in Computer Architecture and Parallel Architectures ☺



# Cache terminology

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- **Block** (or **line**): the unit of data stored in the cache
  - Typically in the range of 32-128 bytes
- **Hit**: data is found (this is what we want to happen)
  - Memory access completes quickly
- **Miss**: data not found
  - Must continue the search at the next level of the memory hierarchy (could be another cache or main memory)
  - After data is eventually located, it is copied to the memory level where the miss happened

# More cache terminology

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- **Hit rate (hit ratio)**: fraction of accesses that are hits at a given level of the memory hierarchy
- **Miss rate (miss ratio)**: fraction of accesses that are misses at a given level ( $= 1 - \text{hit rate}$ )
- **Allocation**: placement of a new block into the cache, which typically results in an eviction of another block.
- **Eviction**: displacement of a block from the cache, which commonly happens when a new block is allocated in its place.

# Cache basics

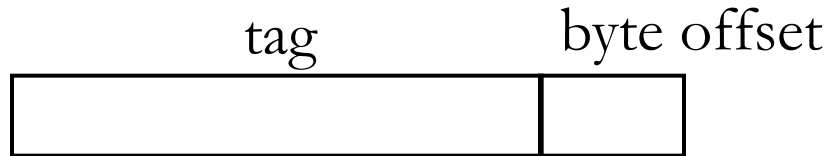
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- Data are identified in (main) memory by their full 32-bit address
- Problem: how to map a 32-bit address to a much smaller memory, such as a cache?
- Answer: associate with each data block in cache:
  - a **tag** word, indicating the address of the main memory block it holds
  - a **valid bit**, indicating the block is in use

# Fully-associative cache

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requested address:

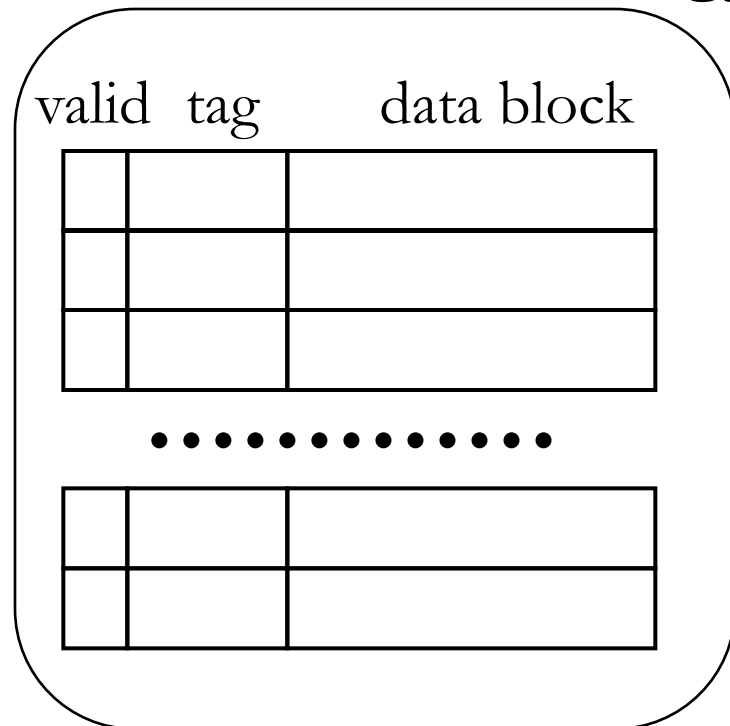


Correct cache block identified  
by matching tags

Byte offset selects word/byte  
within block

Address tag can potentially  
match tag of *any* cache block

Cache





# Cache Replacement

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- Least Recently Used (LRU)
  - Evict the cache block that hasn't been accessed longest
  - Relies on past behaviour as a predictor of the future
- FIFO – replace in same order as filled
  - Simple to implement
- Example:
  - Cache with 4 blocks
  - Access addresses: 0 2 6 0 7 8

LRU

0
8
6
7

FIFO

8
2
6
7

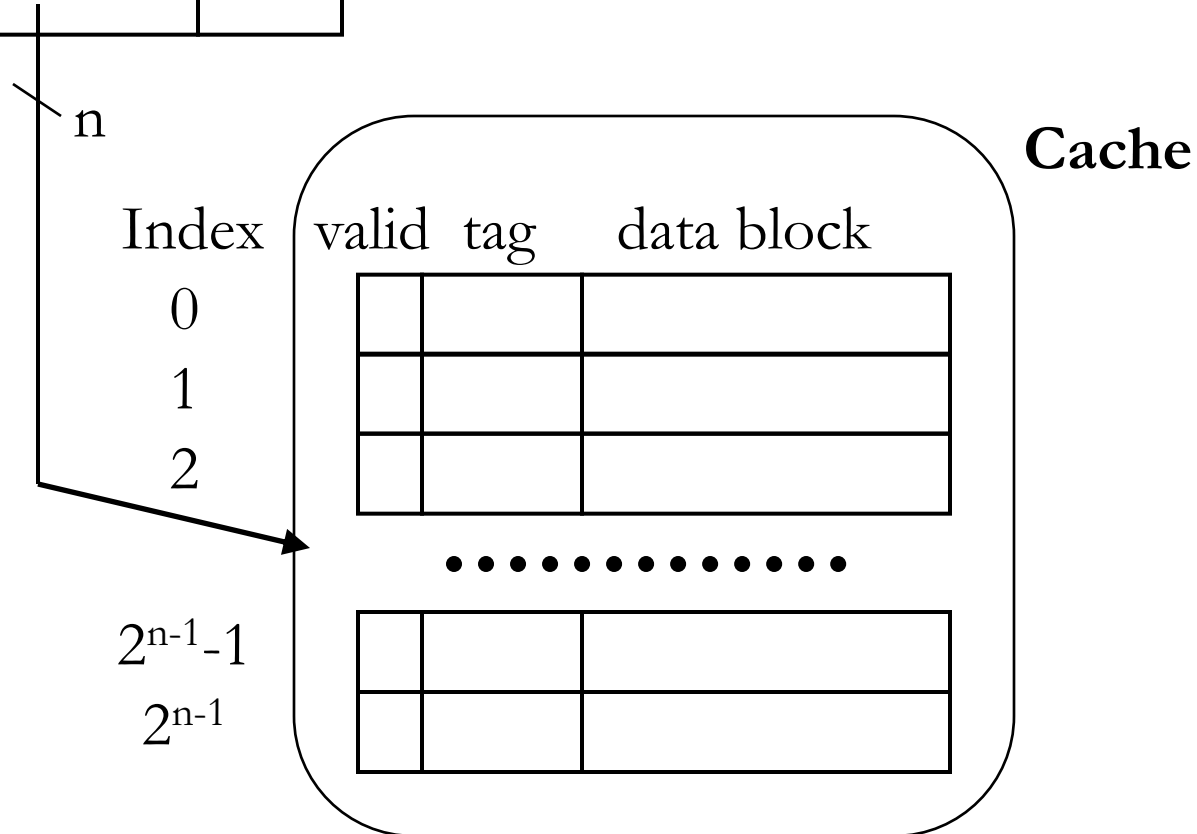
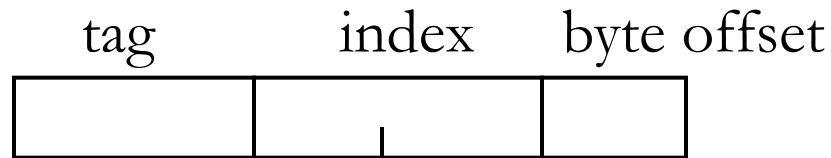
# Direct-mapped cache

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- In a fully-associative cache, search for matching tags is either very slow, or requires a very expensive memory type called Content Addressable Memory (CAM)
- By restricting the cache location where a data item can be stored, we can simplify the cache
- In a **direct-mapped** cache, a data item can be stored in one location only, determined by its address
  - Use some of the address bits as index to the cache array

# Address mapping for direct-mapped cache

requested address:



# Example problem

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Given a 4 KB direct-mapped cache with 4-byte blocks and 32-bit addresses.

Question: How many tag, index, and offset bits does the address decompose into?

# Example problem

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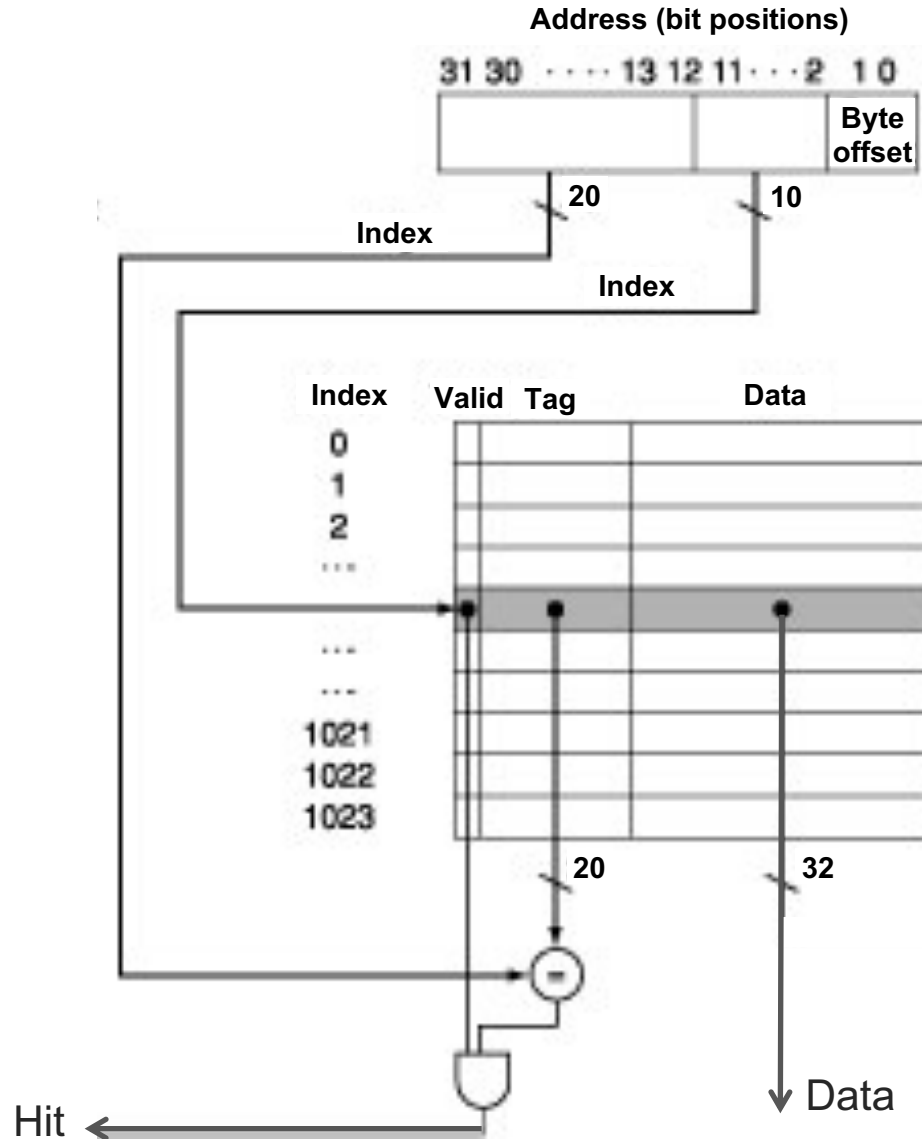
Given a 4 KB direct-mapped cache with 4-byte blocks and 32-bit addresses.

Question: How many tag, index, and offset bits does the address decompose into?

Answer:

- $4 \text{ KB} / 4 \text{ bytes per block} = 1\text{K blocks}$ 
  - Requires a 10-bit index
- 4-byte block: requires a 2-bit offset
- Tag:  $32 - 10 - 2 = 20 \text{ bits}$

# Direct-mapped cache in detail



# Cache Associativity Options

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## ■ Fully Associative

- The block can go into any location in the cache
- Good: Most flexible approach → lowest miss rate
- Bad: Must search the whole cache to find the block (bad for speed and power)

## ■ Direct mapped

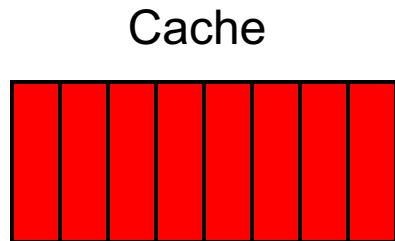
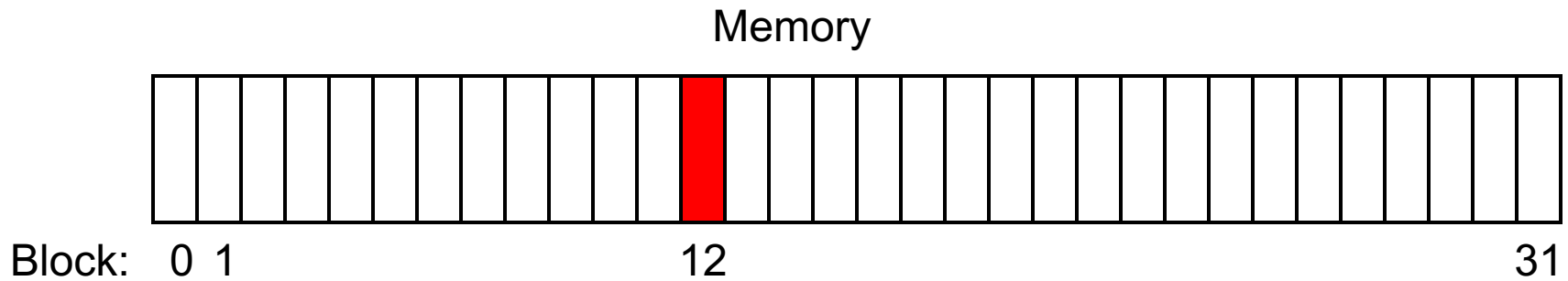
- The block can only go into one location in the cache
- Good: very simple hardware (fast and low power)
- Bad: Blocks mapping to the same location (**thrashing**) → increased miss rates

## ■ Set Associative

- Split the cache into groups (**sets**) of  $m$  blocks each → **m-way set-associative**
- A given block can only go into one set (based on block address), but within that set it can go anywhere
- Typical degree of associativity is 2 – 16



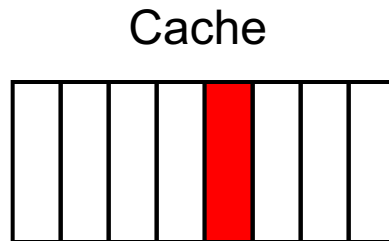
# Cache Block Placement



Block: 0 1 2 3 4 5 6 7

Fully associative:

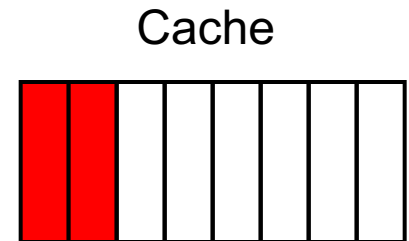
block 12 can go anywhere  
in the cache



Block: 0 1 2 3 4 5 6 7

Direct mapped:

block 12 can only go  
into block 4  
( $12 \bmod 8$ )



Block: 0 1 2 3 4 5 6 7  
Set: 0 1 2 3

Set associative:

block 12 can go anywhere  
in set 0  
( $12 \bmod 4$ )





# Example problem

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Given a 4 KB, 4-way set-associative cache with 4-byte blocks and 32-bit addresses.

Question: How many tag, index, and offset bits does the address decompose into?

Answer:

- $4 \text{ KB} / 4 \text{ bytes per block} = 1\text{K blocks}$
- But.. there are 4 ways per set  $\rightarrow 256 \text{ sets}$ 
  - Requires an 8-bit index to select the set
- 4-byte block: requires a 2-bit offset
- Tag:  $32 - 8 - 2 = 22 \text{ bits}$

# Writing to caches: on a hit

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- **Write through** – write to both cache and memory
  - Good: memory and cache always synchronized
  - Bad: writes are slow and require memory bandwidth
- **Write back** – write to cache only
  - Each cache block has a **dirty bit**, set if the block has been written to
  - When a **dirty** cache block is replaced, it is written to memory
  - Good: writes are fast and generate little memory traffic
  - Bad: memory can have stale data for some time



# Writing to caches: on a miss

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- **Write allocate** – bring the block into the cache and write to it
  - Useful if locality exists
- **Write no-allocate** – do not bring the block into the cache; modify data only in memory
  - Useful if no locality
  - Guarantees that cache and memory are synchronized (have the same value for an address)

# Coursework 2: cache simulator

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Goals: understand caches, get practice with C

Your job: using C, write a configurable cache simulator that supports the following parameters:

- Number of blocks
- Block size
- Associativity
- Replacement policy

Input: list of addresses (in a file)

Output: cache configuration details and hit/miss stats

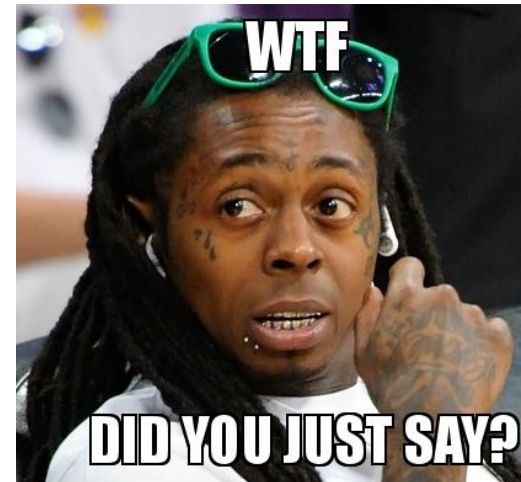


**Due: Wed, Nov 28, 4pm**

# It's only a simulator!

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- **Simulator:** Not the same as real hardware!
  - Don't think bits & bytes; think data types!
  - E.g., is `bit` the best data type for indicating that a cache block is valid?
- The cache will not store data
  - Cache size refers only to the size of the data portion



# Coursework 2 Tips

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- Start the coursework by understanding lecture slides, and Tutorial 4 (questions 2 & 3)
  - Don't ignore the text book – it's very useful
  - Read others' questions on Piazza
    - Read previous questions to avoid double posting
- You must allocate memory dynamically
- You must have a clean, modular simulator design
- Your code must compile & run on DICE with the specified flags (and only those flags)

