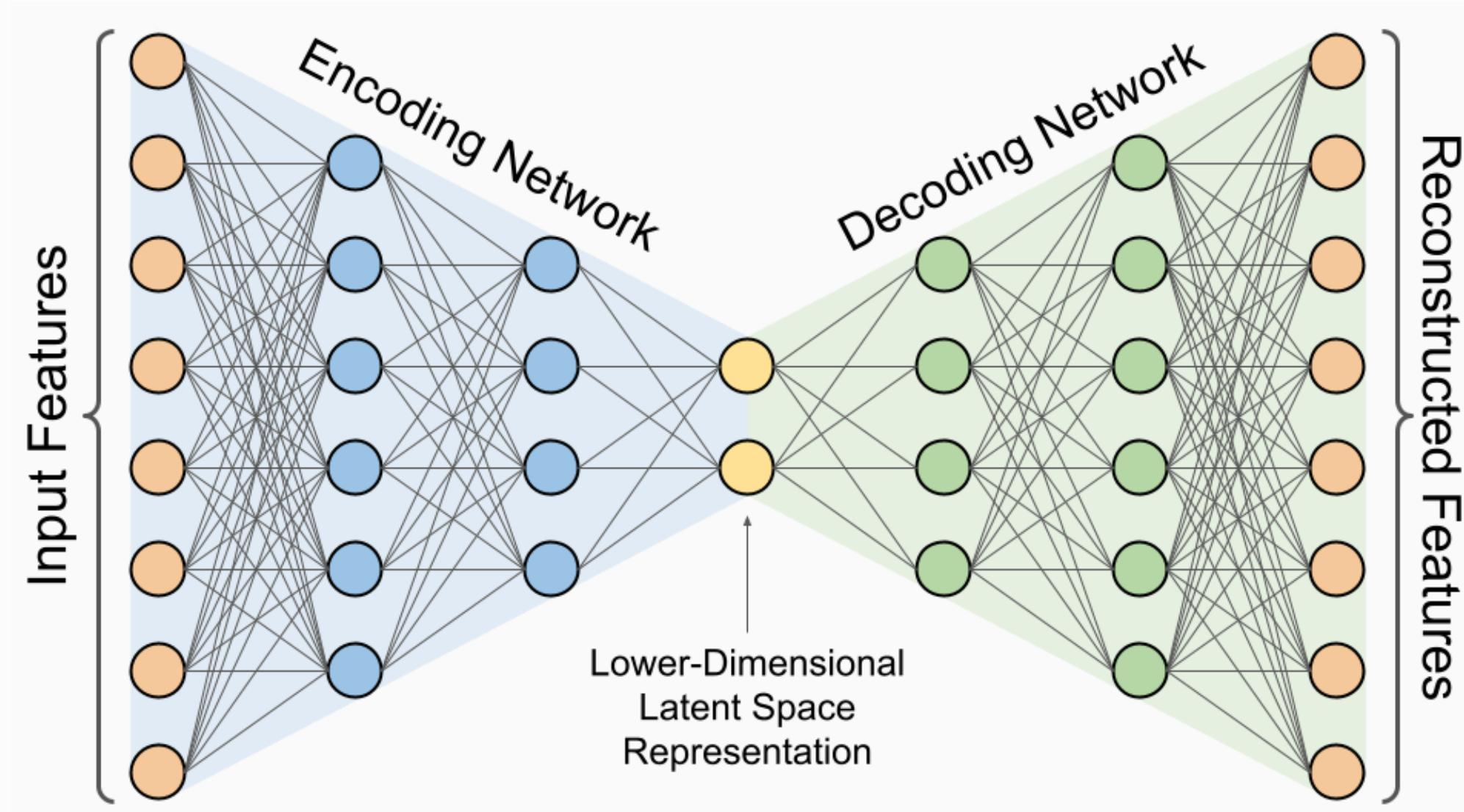
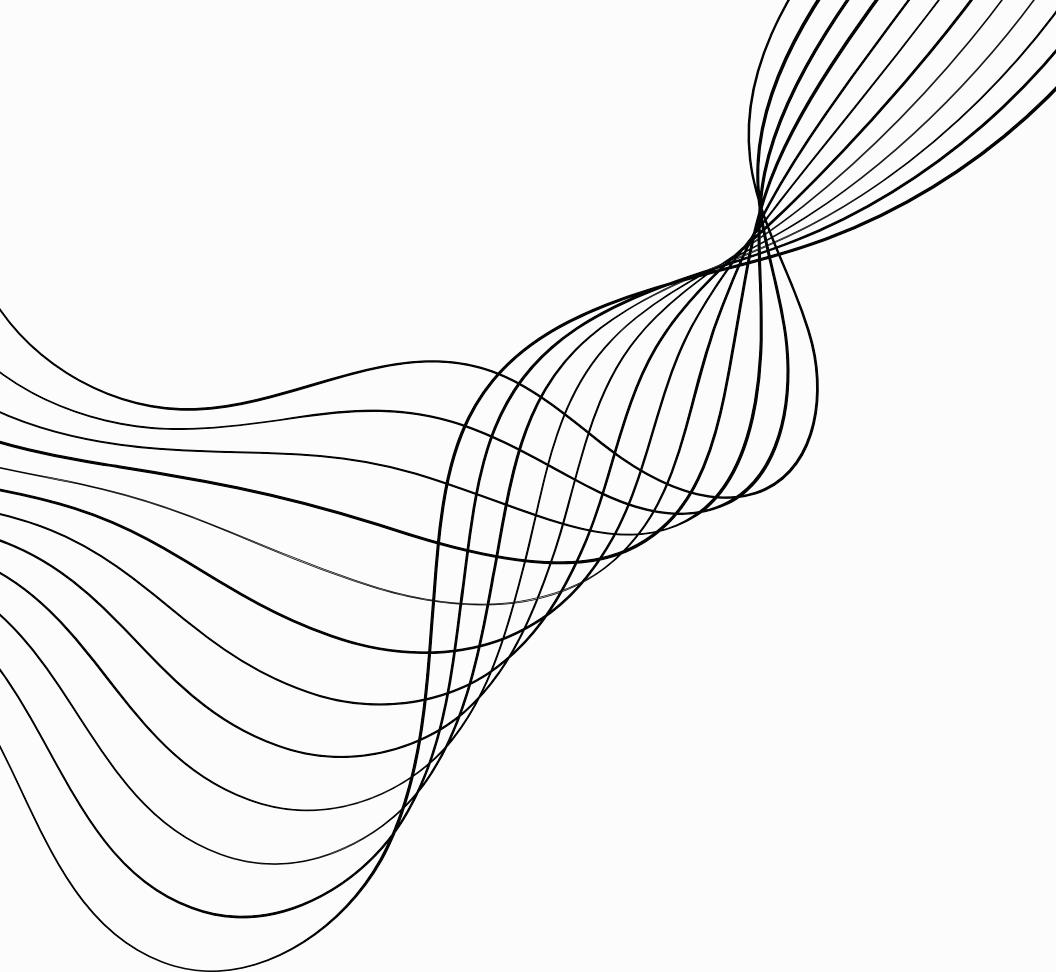


# AUTOENCODER



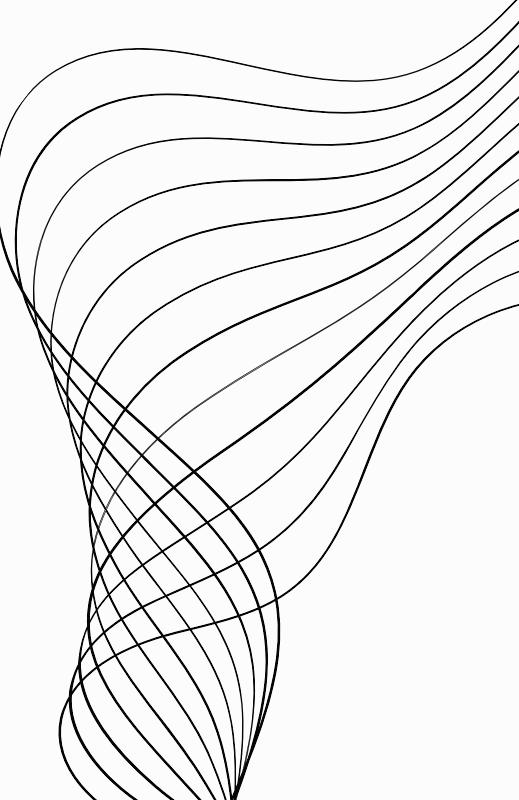


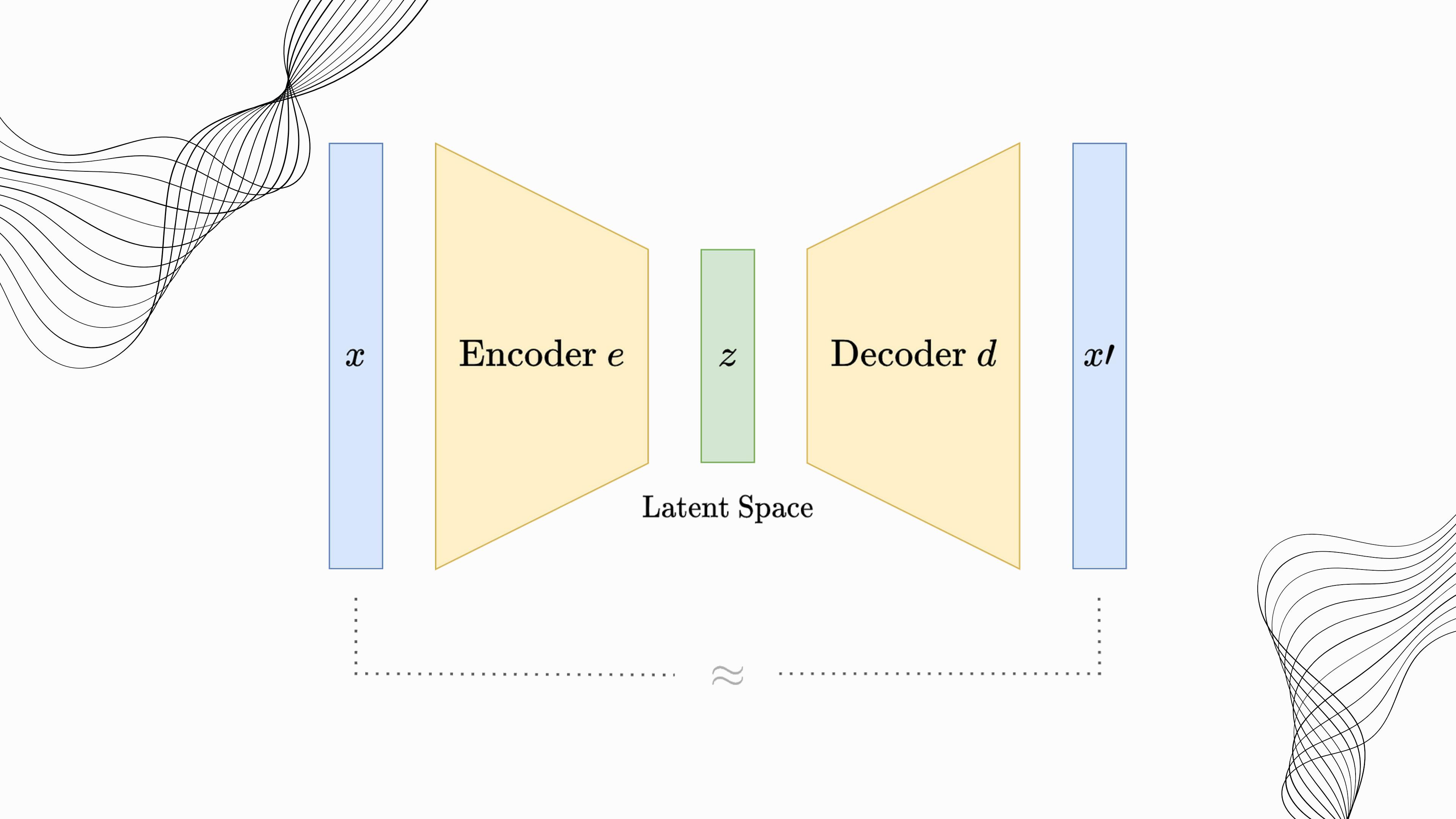
# ESPECIFICAÇÃO

Entrada:  $x$

Saída:  $x'$

Tal que

$$x' \approx x$$




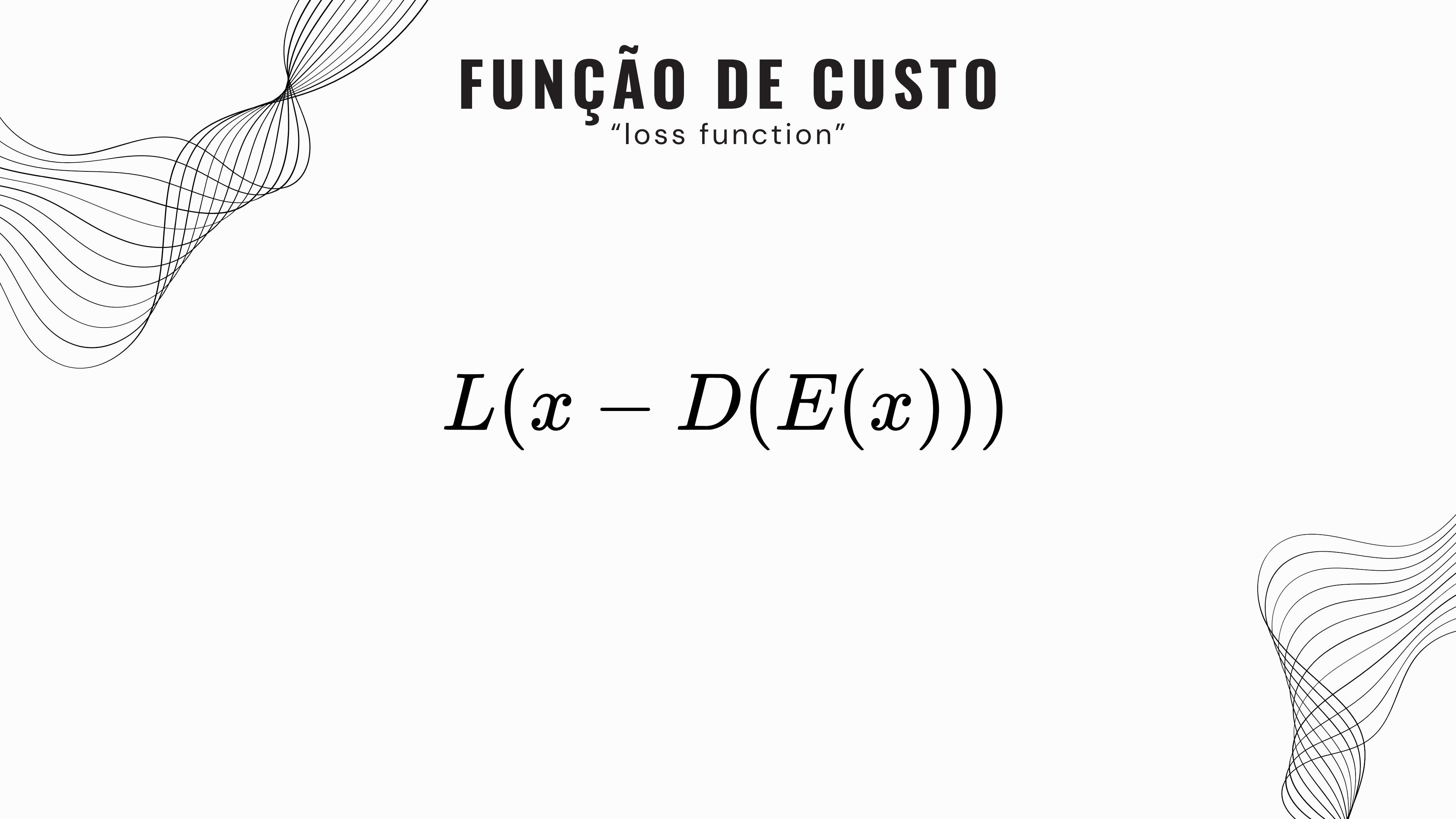
# ESPECIFICAÇÃO

Entrada:  $x$

Saída:  $x' = D(E(x))$

Tal que

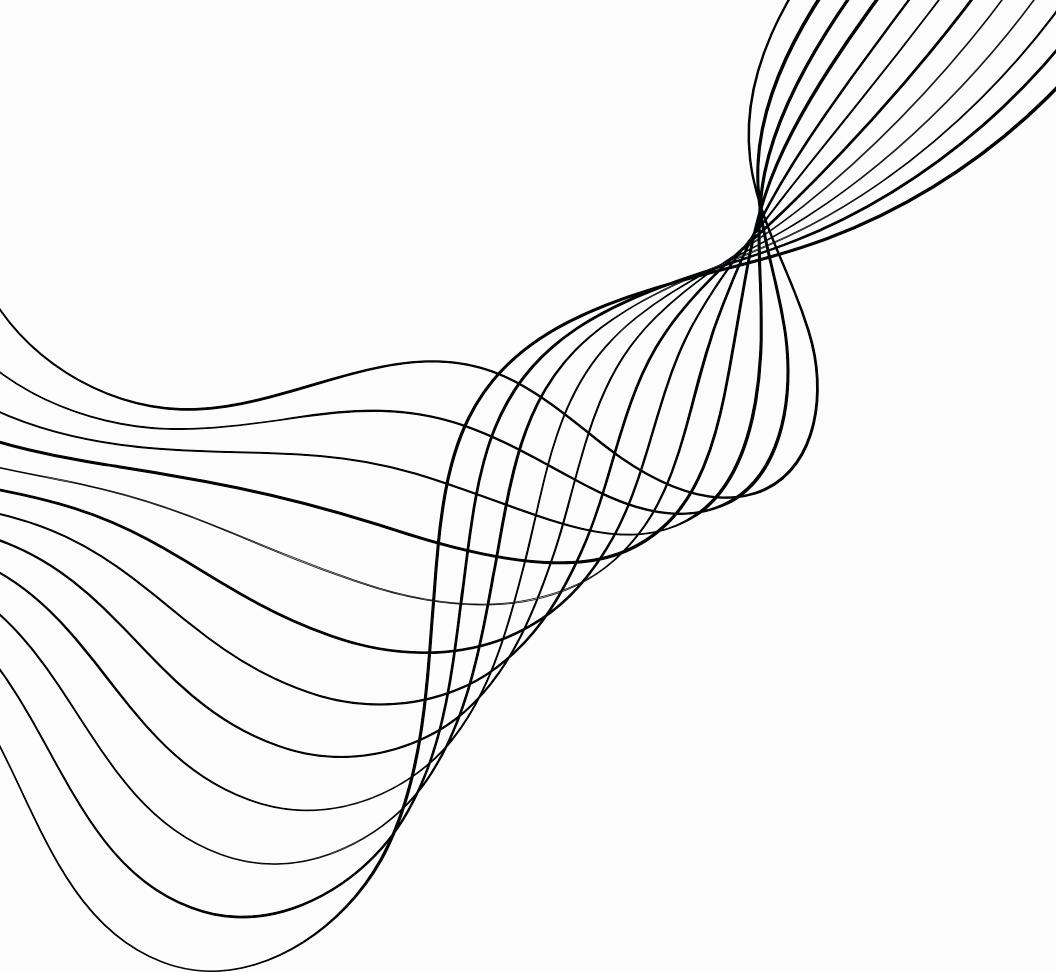
$x' = D(E(x)) \approx x$



# **FUNÇÃO DE CUSTO**

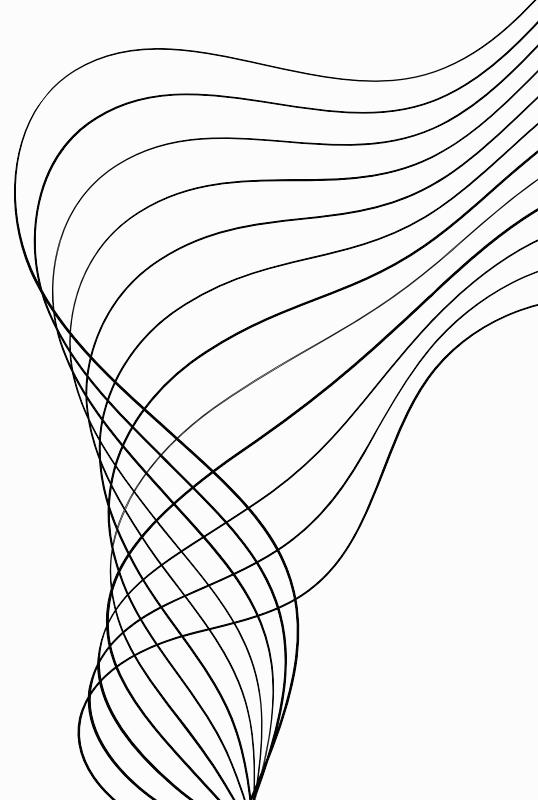
"loss function"

$$L(x - D(E(x)))$$



# **FUNÇÃO DE CUSTO**

"loss function"

$$\min_{D, E} L(x - D(E(x)))$$


# AUTOENCODER LINEAR

$$E = W$$

$$D = W^T$$

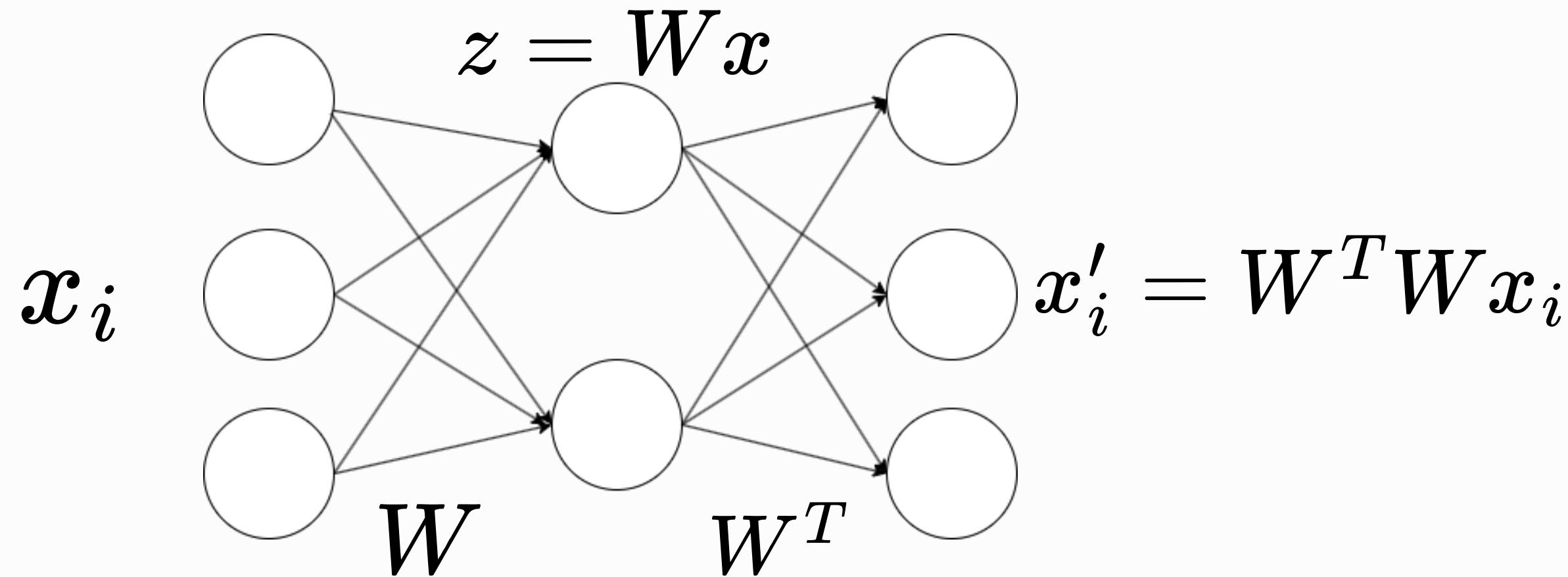
$$D(E(x)) = W^T W x$$

$$L(x - D(E(x))) = ||x - W^T W x||^2$$

# AUTOENCODER LINEAR

$x_i$  vetores de  $X$

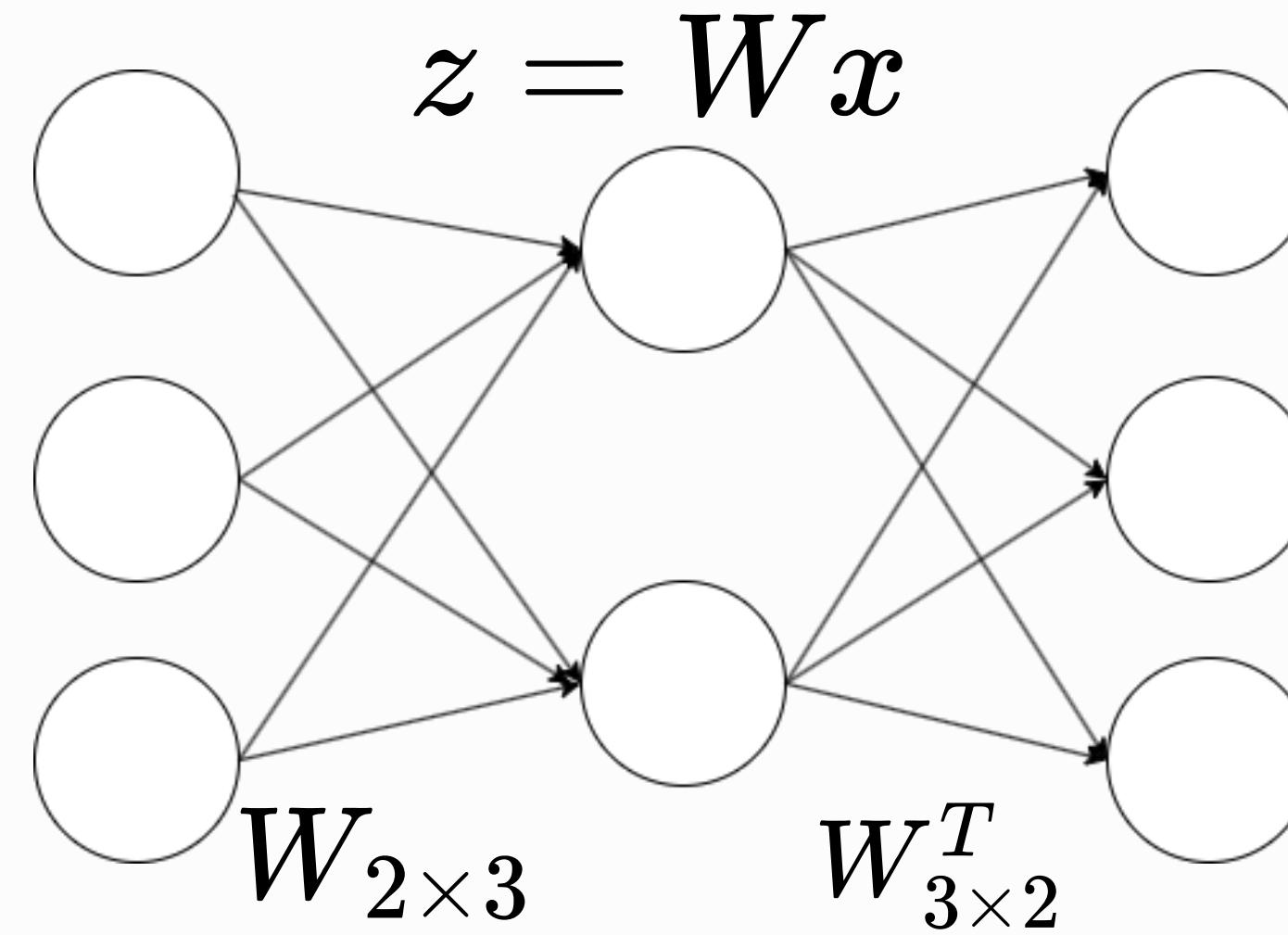
$$\min_W \sum_{i=1}^n \|x_i - W^T W x_i\|^2$$



# AUTOENCODER LINEAR

$X_{3 \times 3}$   
posto 3

$x_i$

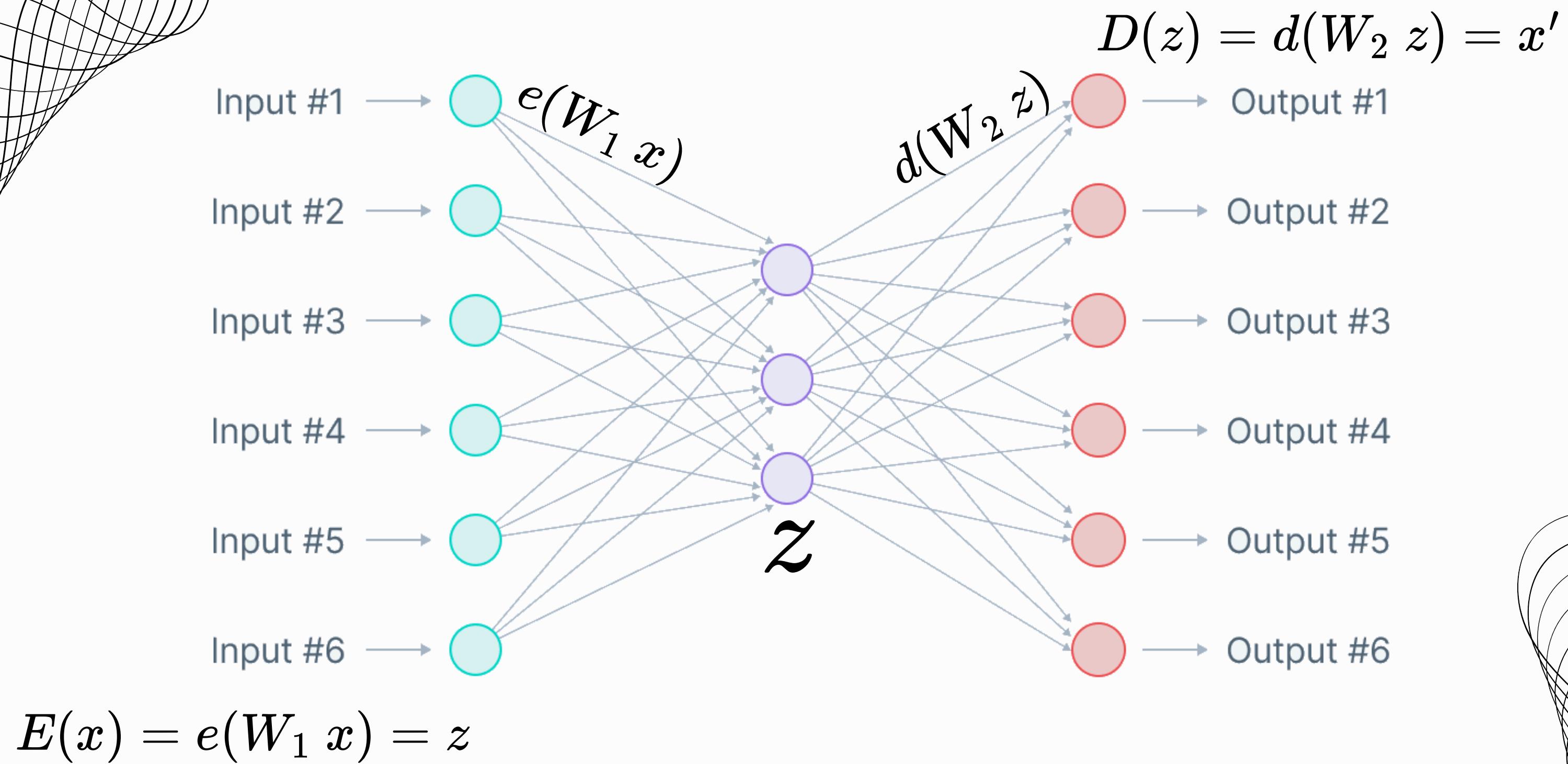


$X'_{3 \times 3}$   
posto 2

$W$  aprende os 2 primeiros componentes principais de  $X$

# AUTOENCODER

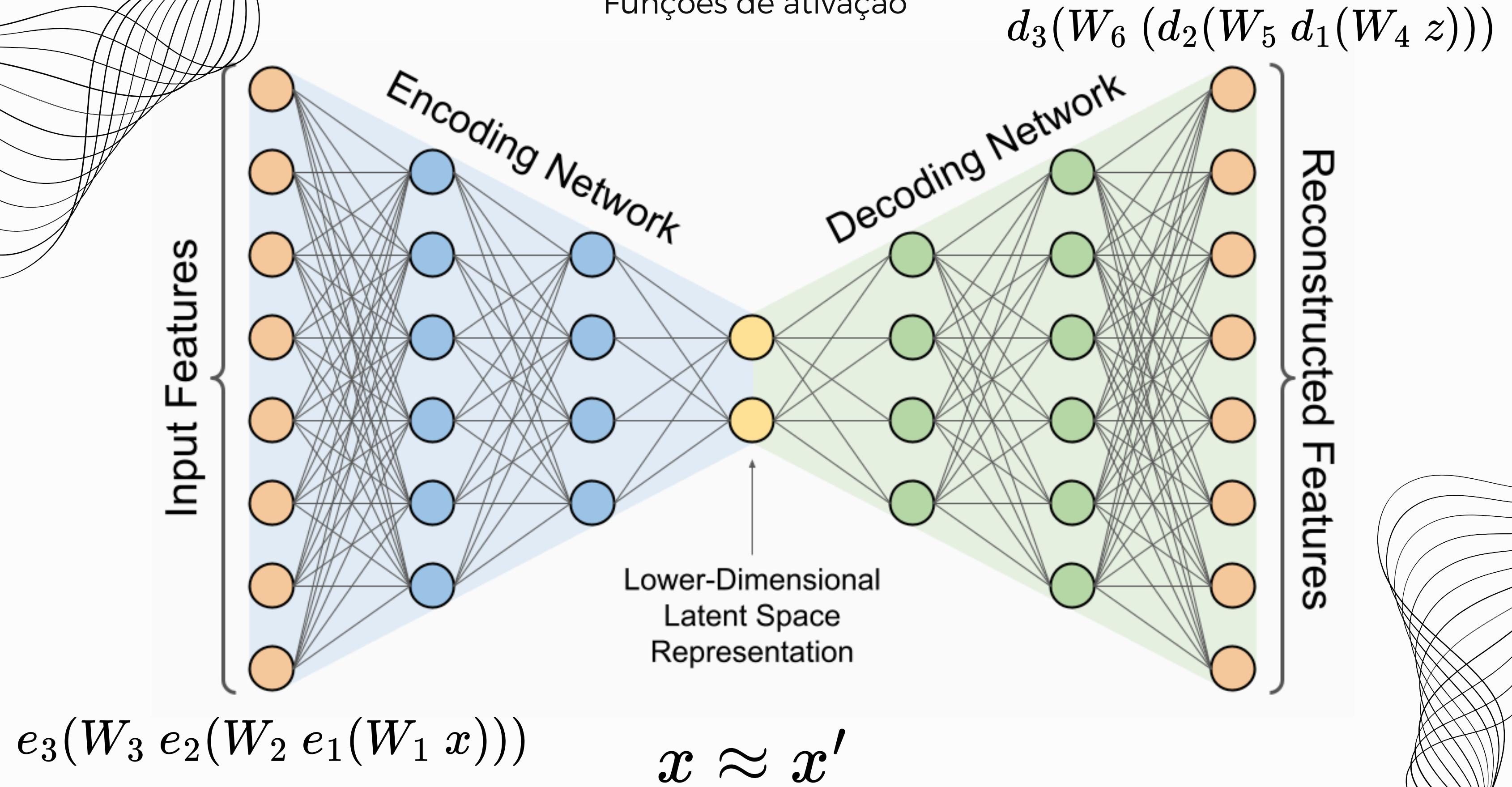
Funções de ativação



$$x \approx x'$$

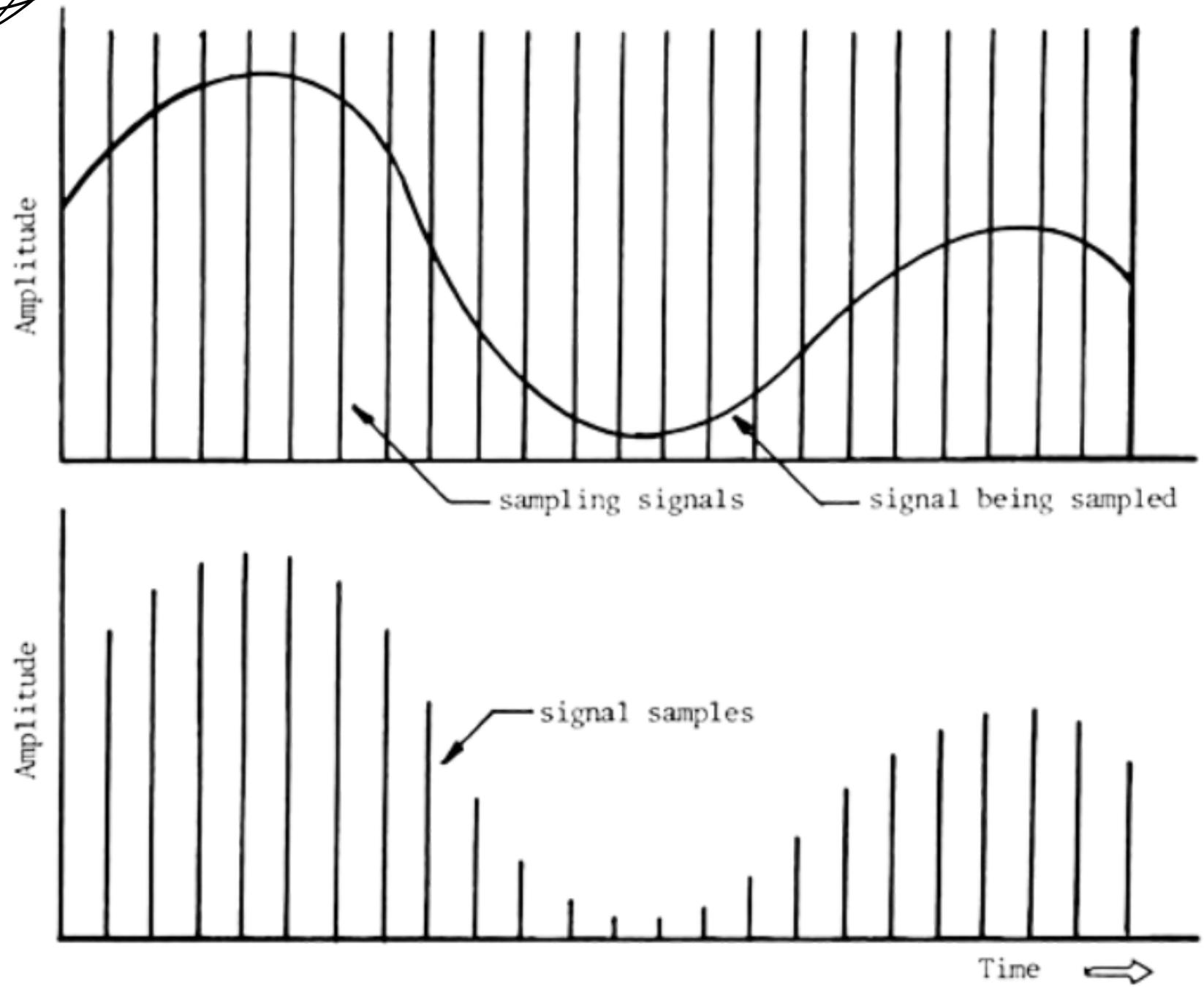
# AUTOENCODER

Funções de ativação



# **EXPERIMENTOS**

# SINAIS



$x =$

$$\begin{bmatrix} f(t_1) \\ f(t_2) \\ \vdots \\ f(t_{100}) \end{bmatrix} \leftarrow \begin{array}{l} t_1 \\ t_2 \\ \vdots \\ t_{100} \end{array}$$

# SINAIS

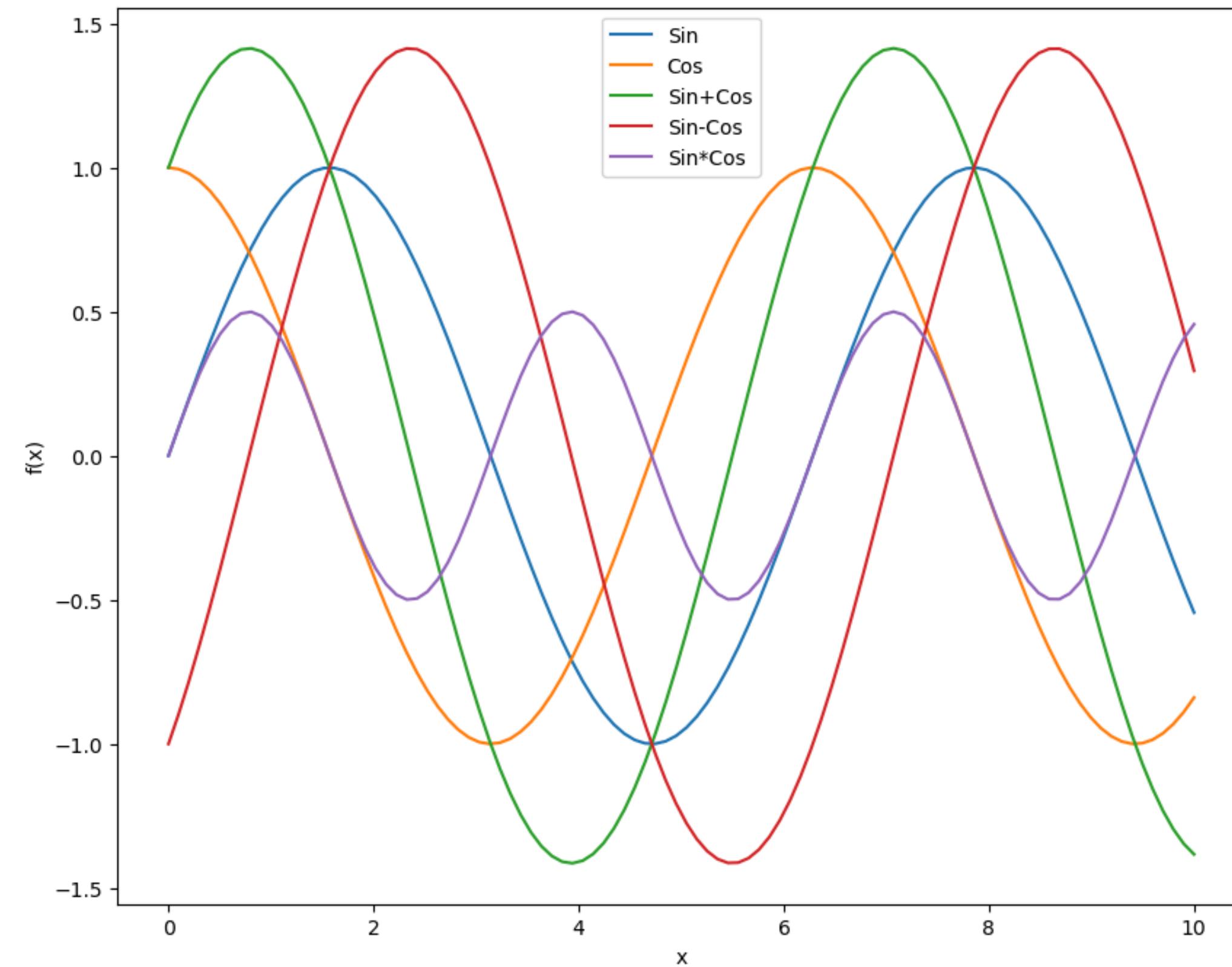
$$X = \begin{bmatrix} f_1(t_1) & f_2(t_1) & \dots & f_{10^5}(t_1) \\ f_1(t_2) & f_2(t_2) & \dots & f_{10^5}(t_2) \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ f_1(t_n) & f_2(t_n) & \dots & f_{10^5}(t_{100}) \end{bmatrix} \leftarrow \begin{array}{l} t_1 \\ t_2 \\ \vdots \\ t_{100} \end{array}$$

# SINAIS

$$f_i(t) \in \left\{ \begin{array}{l} \sin(rt), \cos(rt), \\ \sin(rt) + \cos(rt), \\ \sin(rt) - \cos(rt), \\ \sin(rt) \cdot \cos(rt) \end{array} \right\}$$

$$r \sim \mathcal{N}(0, 2.25)$$

# SINAIS



# CONSTRUÇÃO DO AUTOENCODER

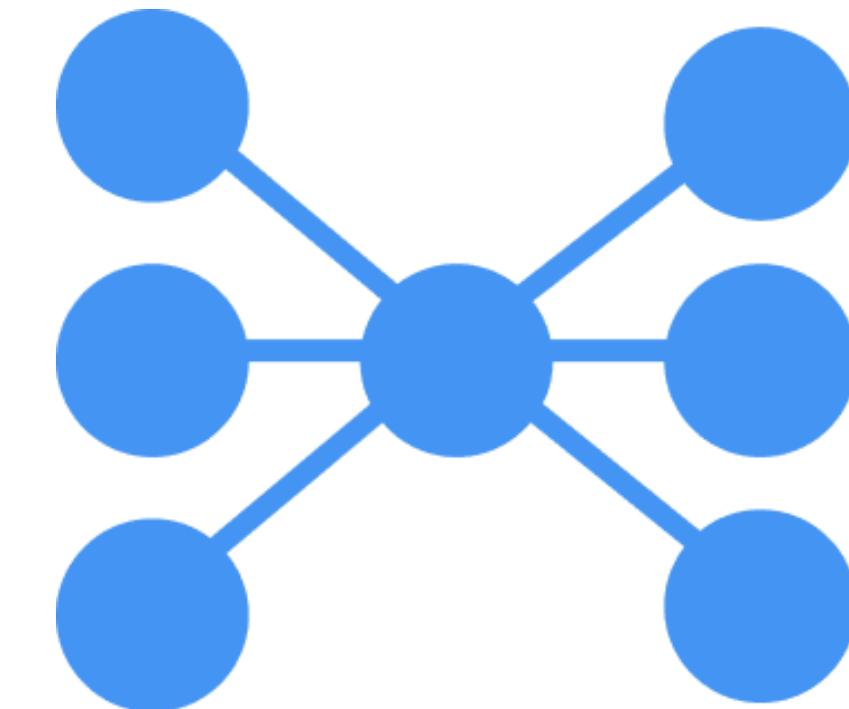
Entrada:  $x \in \mathbb{R}^{100}$

Espaço latente:  $z \in \mathbb{R}$

Saída:  $x' \in \mathbb{R}^{100}$

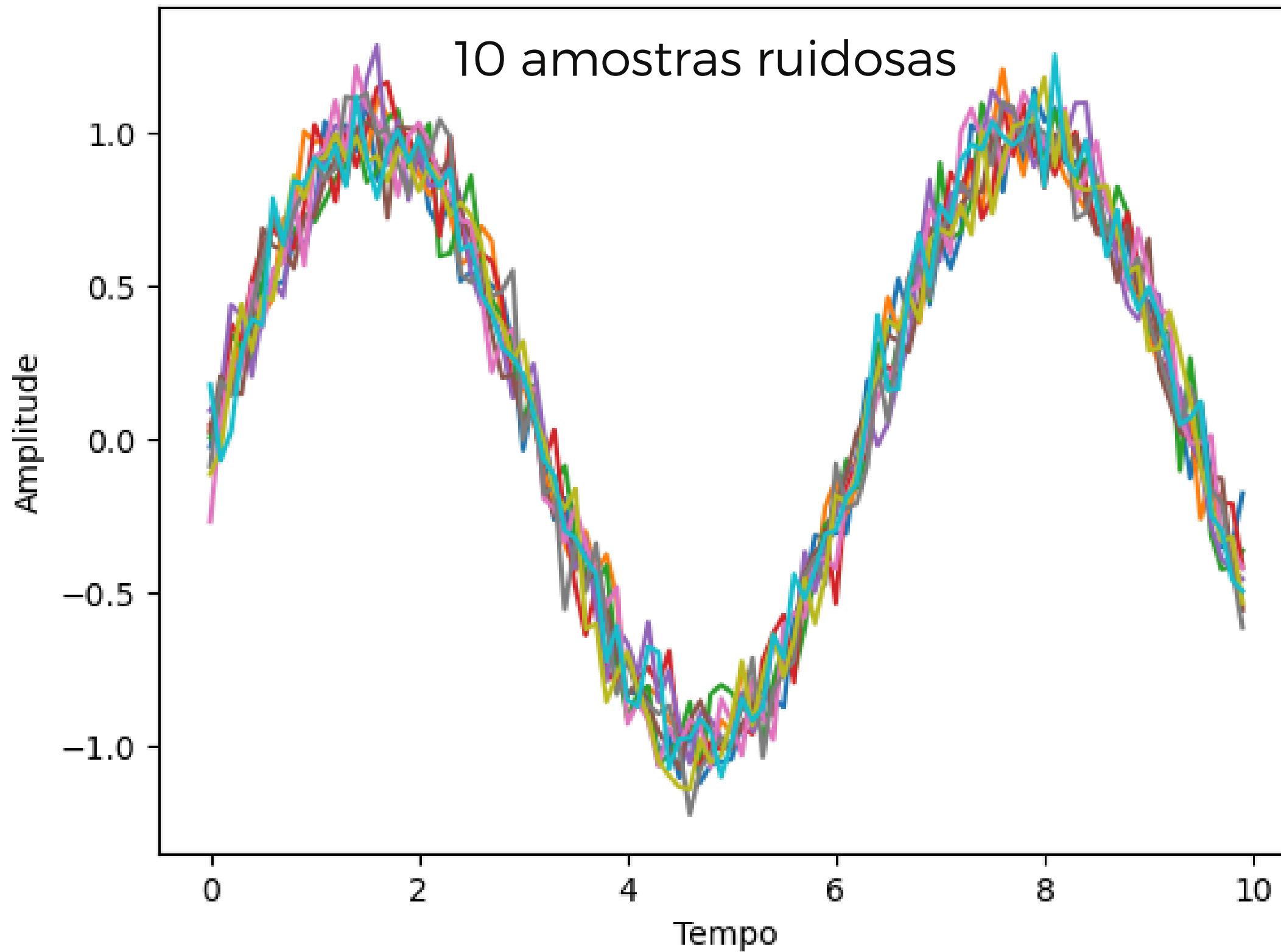
Algumas camadas escondidas

Funções de ativação iguais



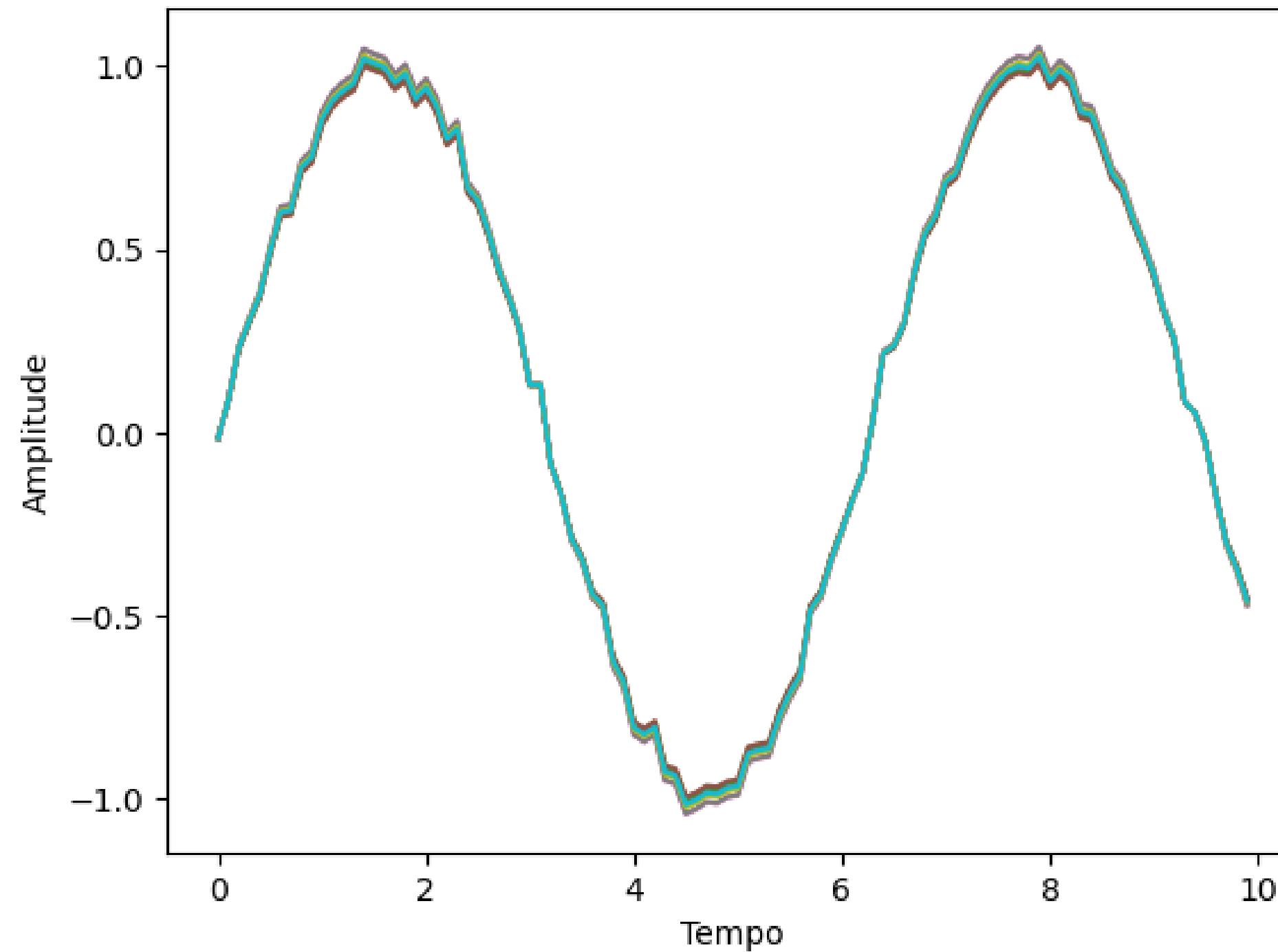
# REMOÇÃO DE RUÍDOS

Seno com ruído

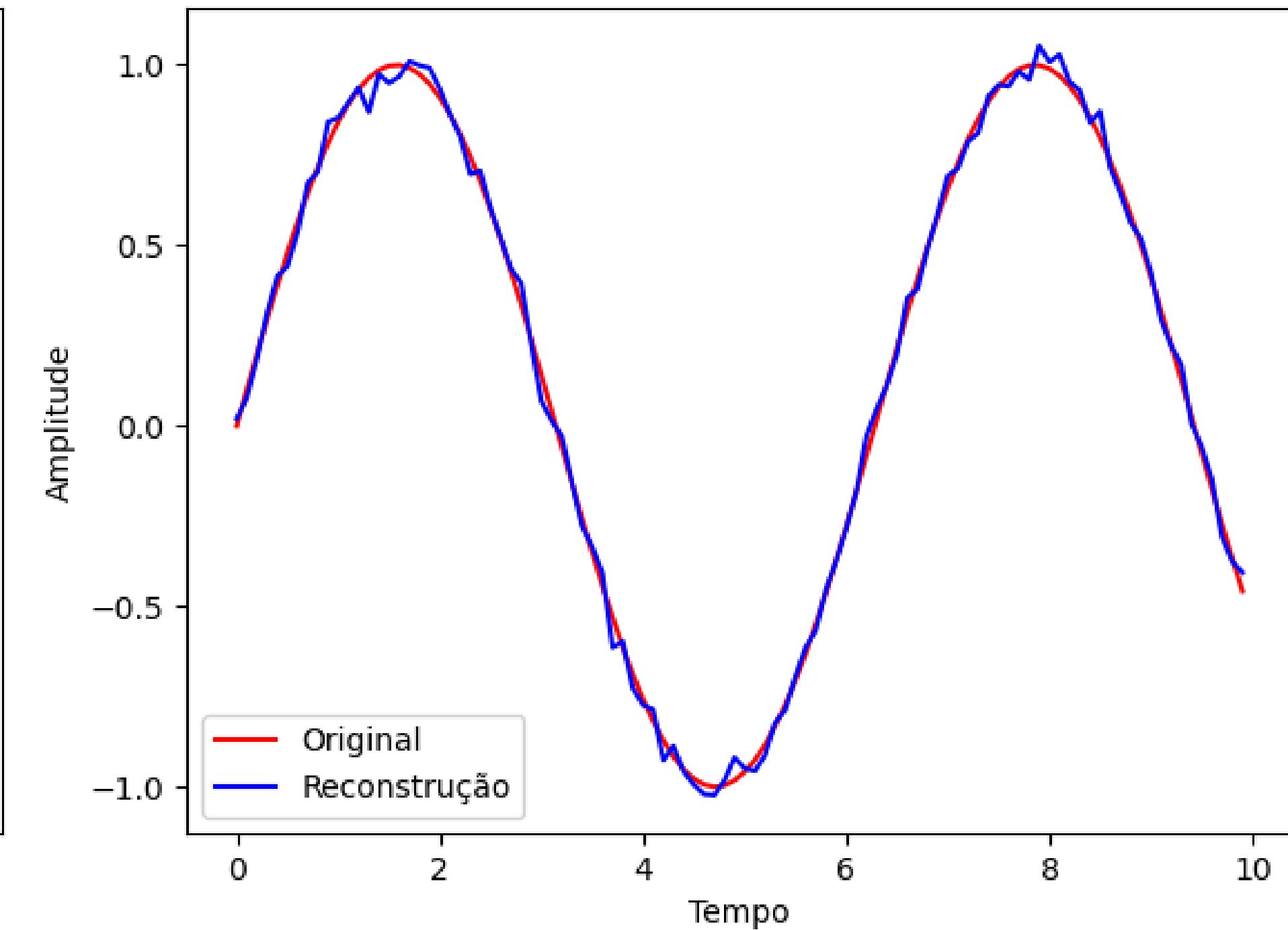


# REMOÇÃO DE RUÍDOS

Seno reconstruído - SVD posto 1

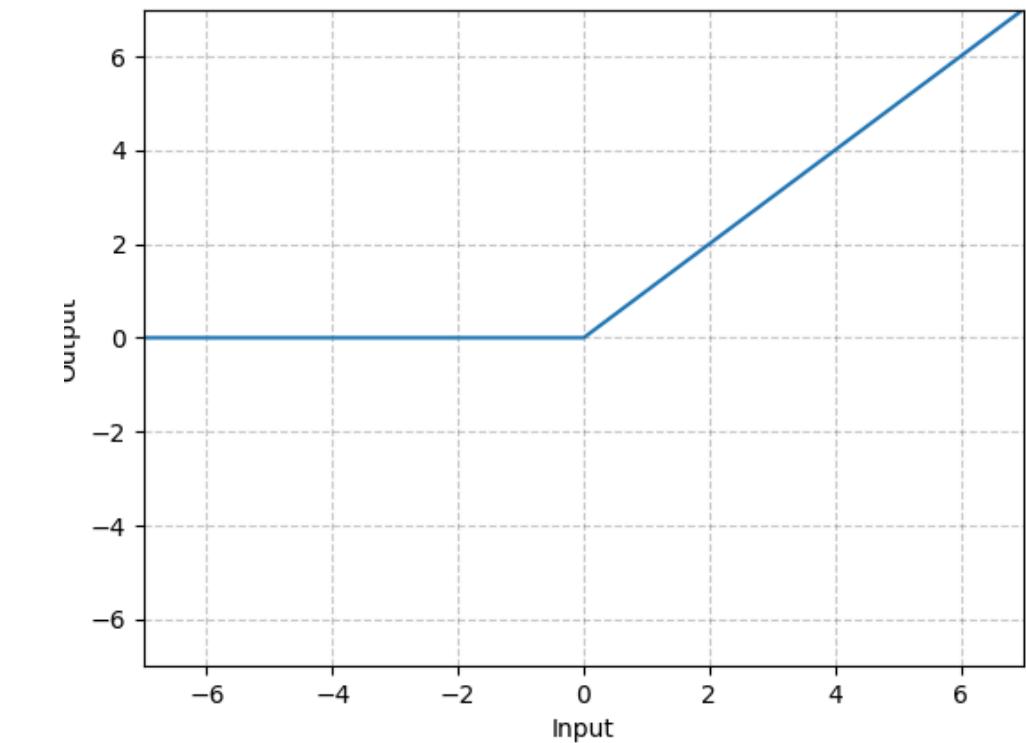


Seno reconstruído - SVD posto 1

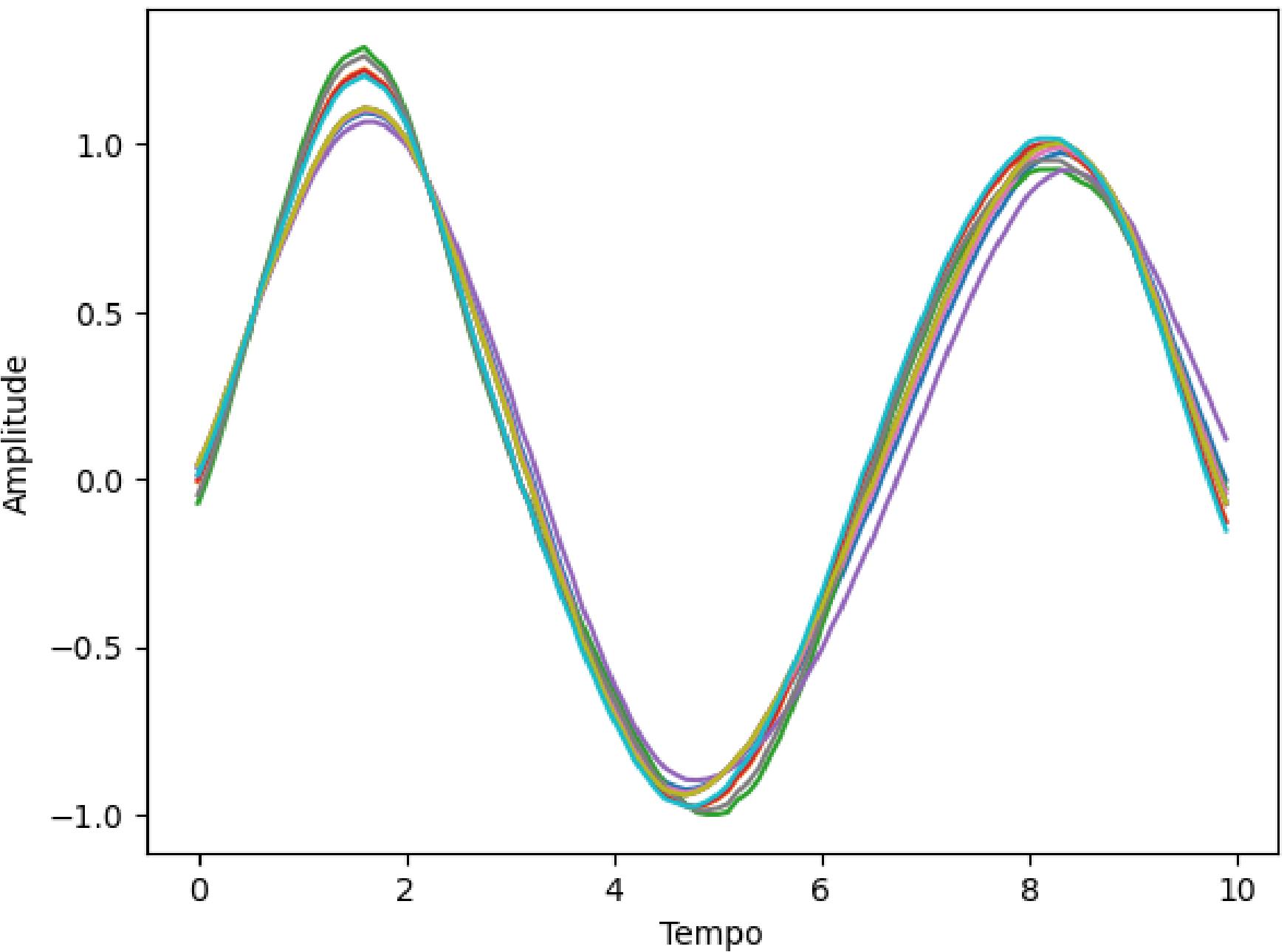


# REMOÇÃO DE RUÍDOS

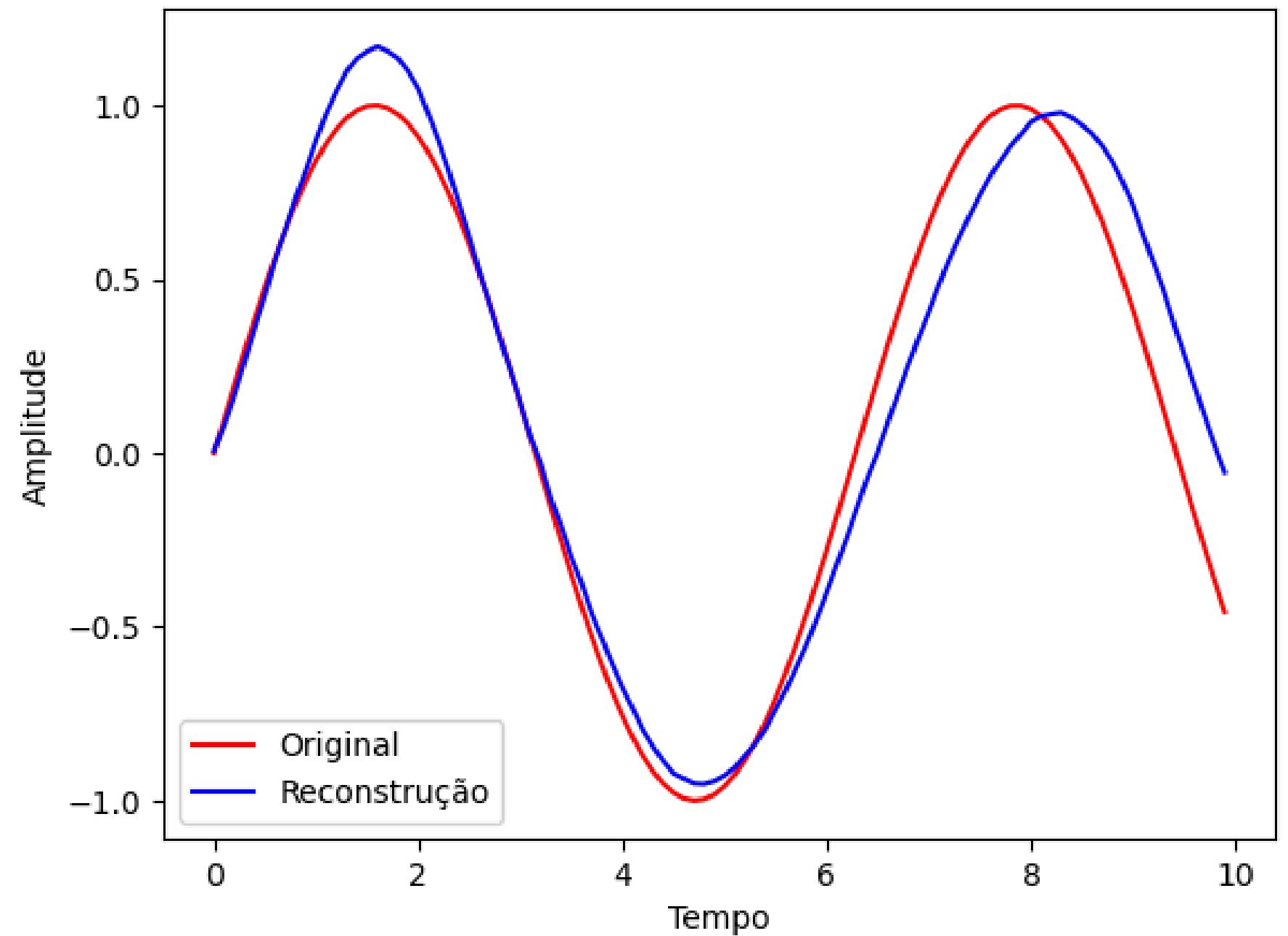
$$ReLU(x) = \max(0, x)$$

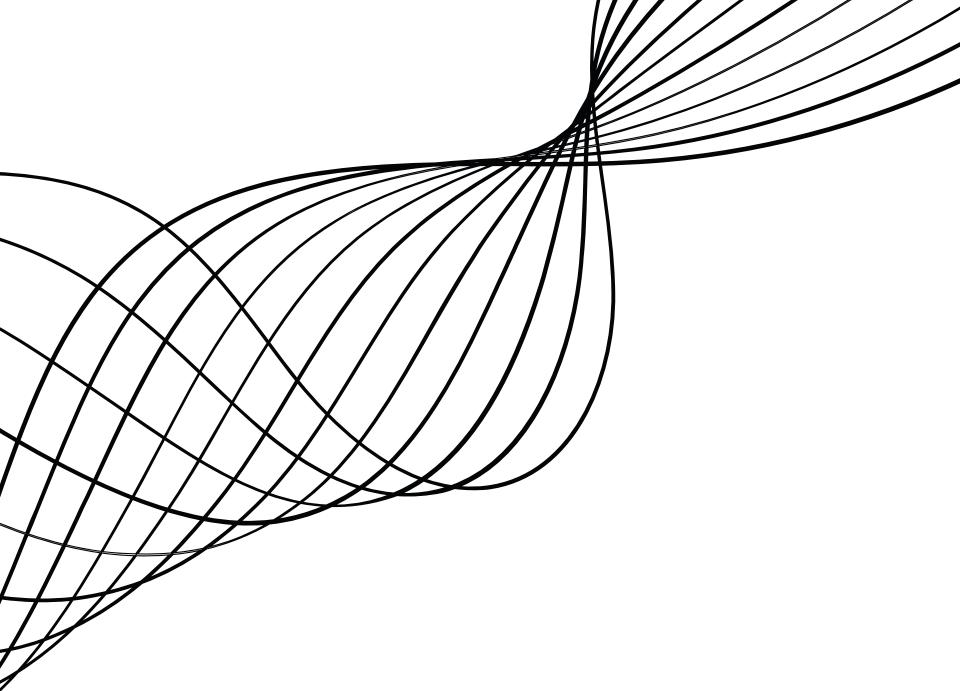


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: ReLU



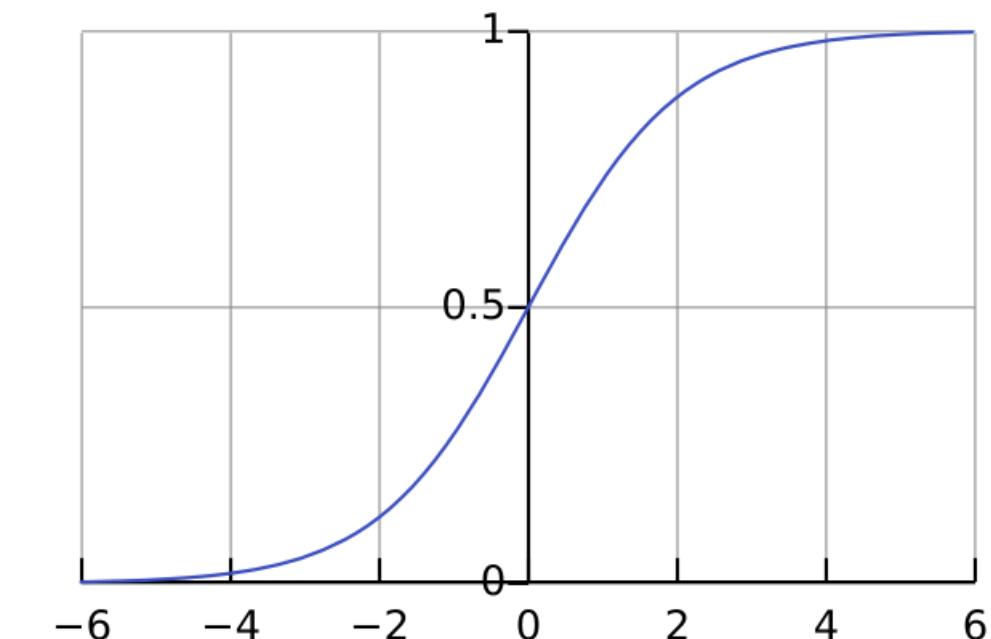
Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: ReLU



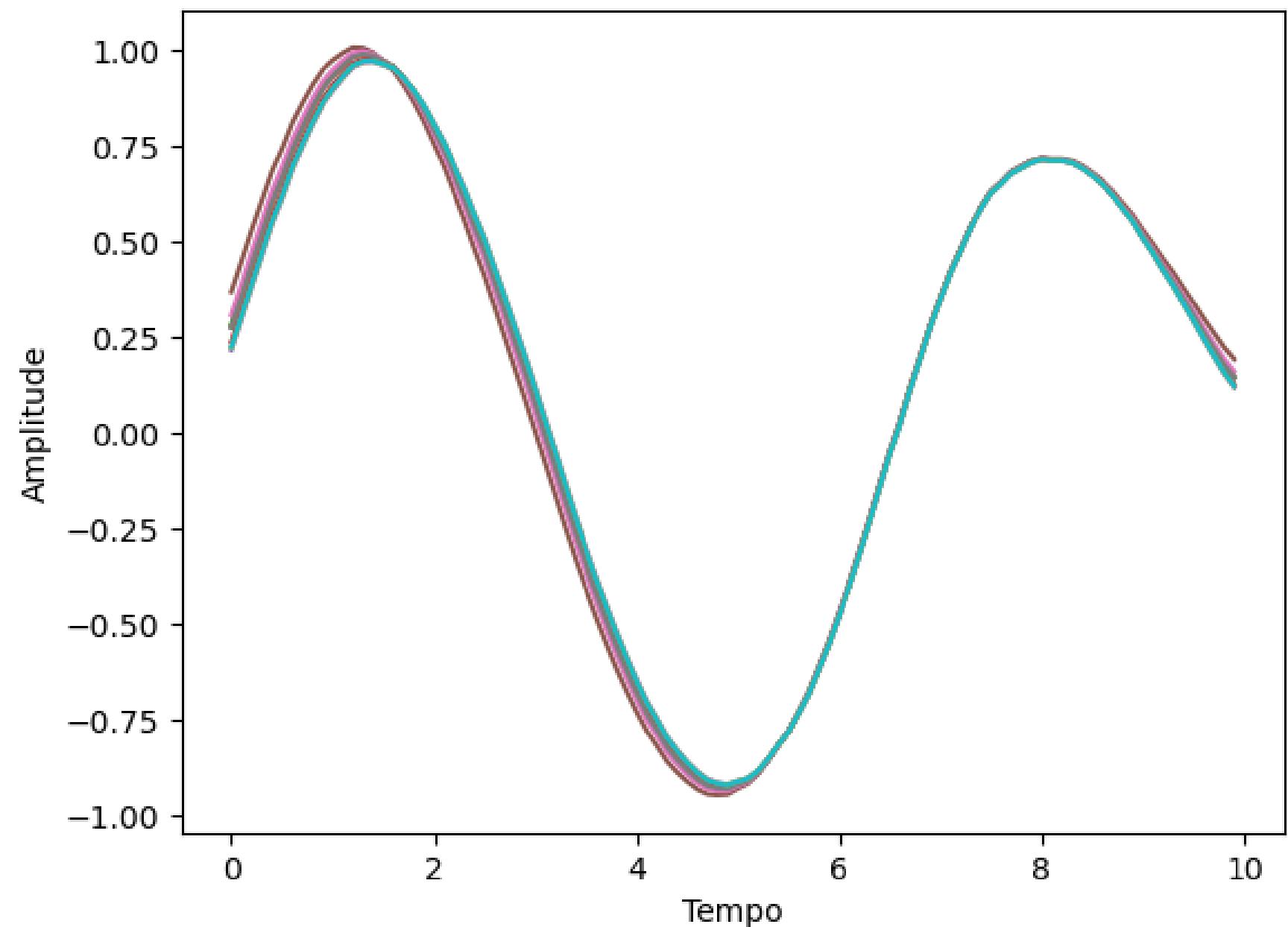


# REMOÇÃO DE RUÍDOS

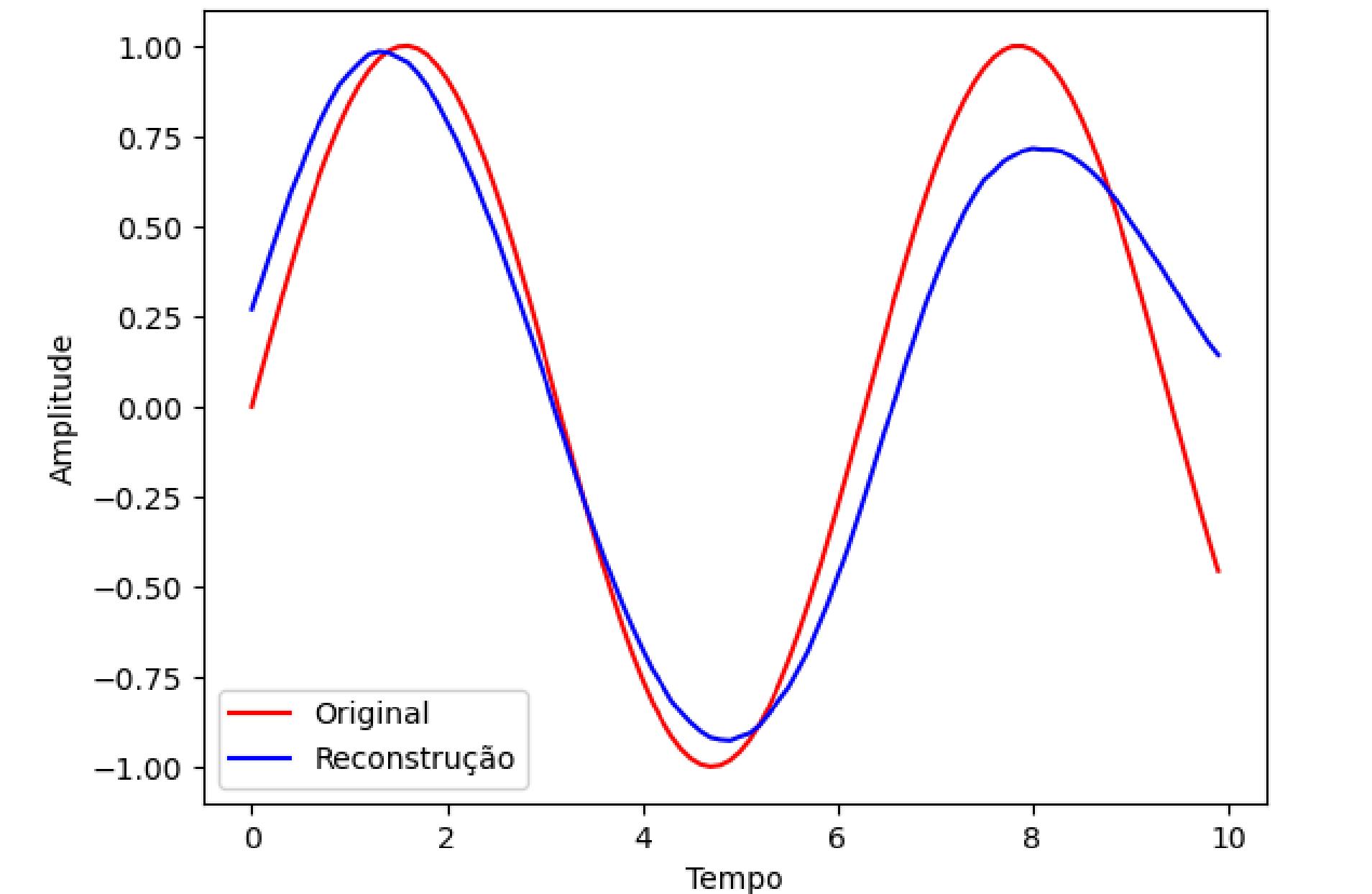
$$\text{Sigmoid}(x) = \frac{1}{1+e^{-x}}$$



Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: Sigmoid

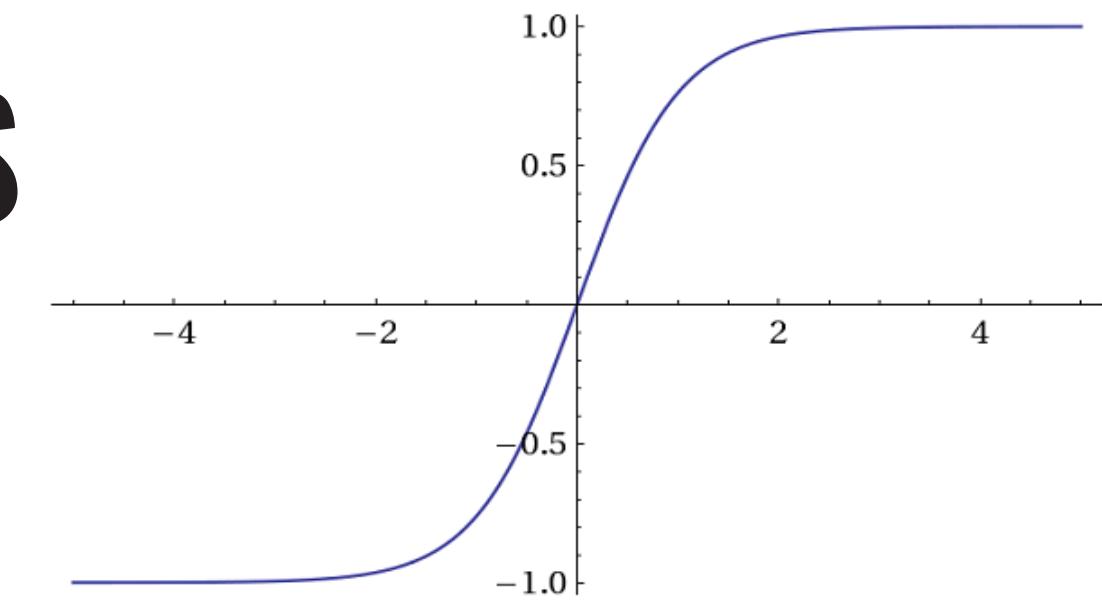


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: Sigmoid

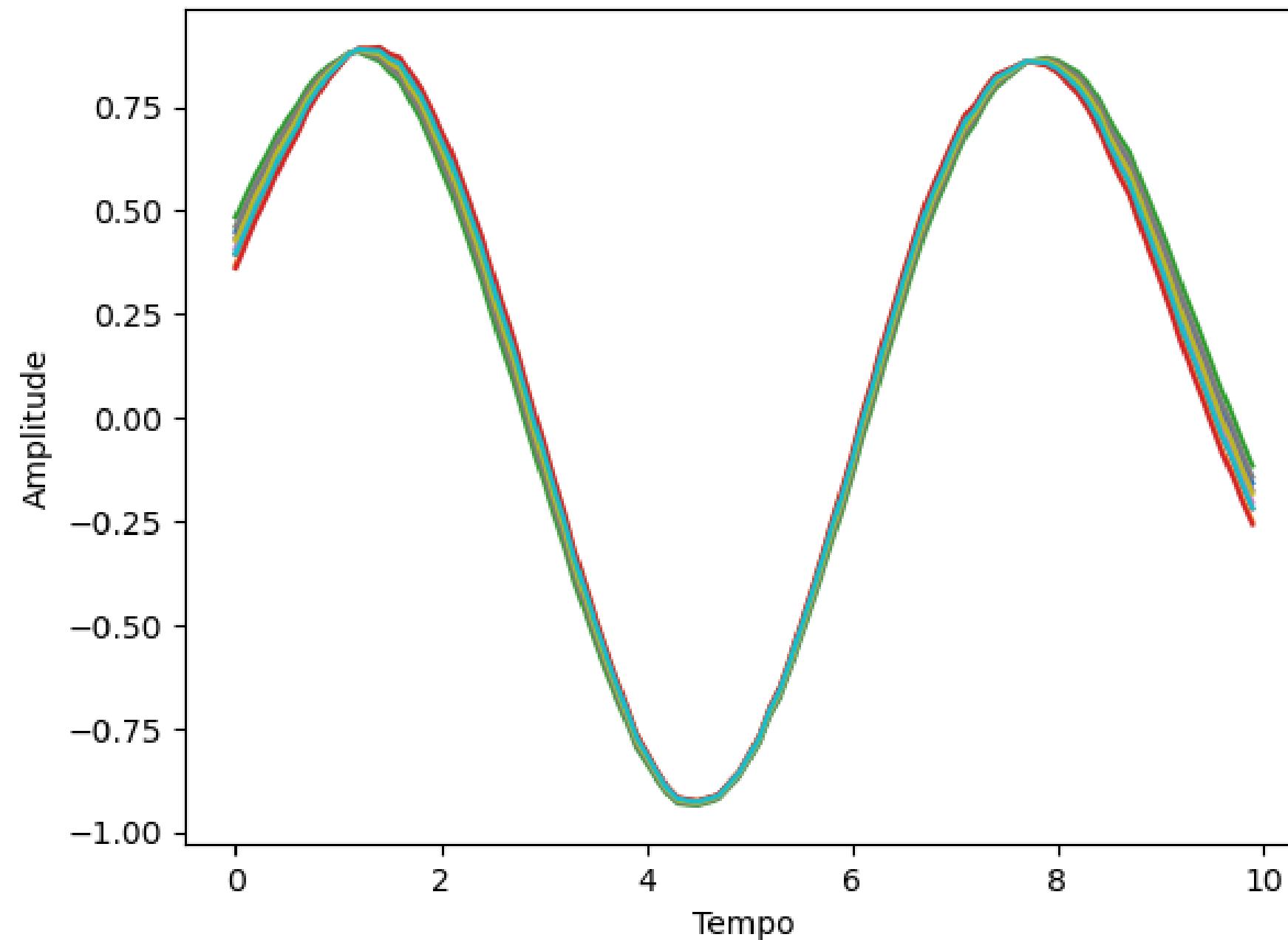


# REMOÇÃO DE RUÍDOS

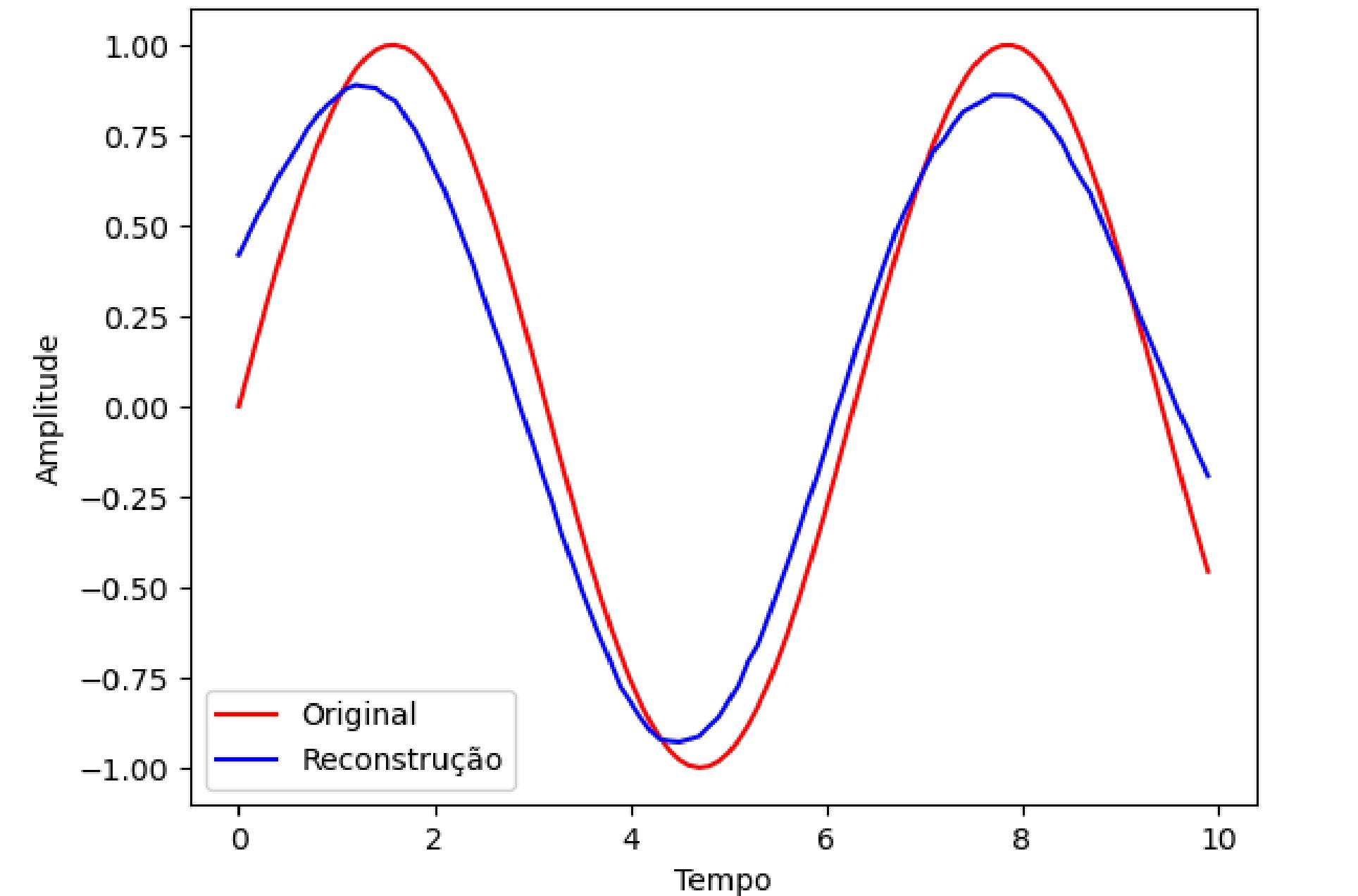
$$\Tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



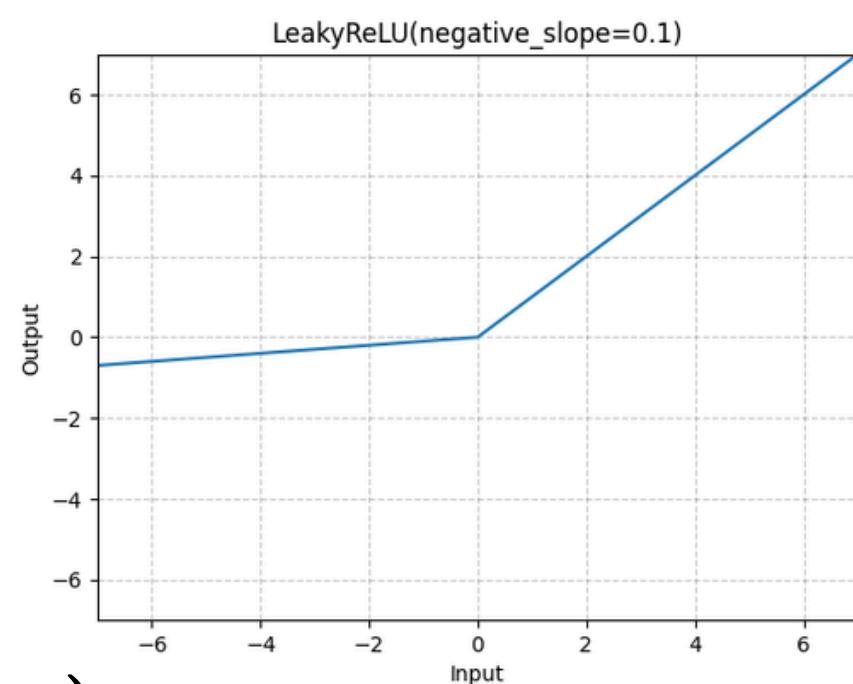
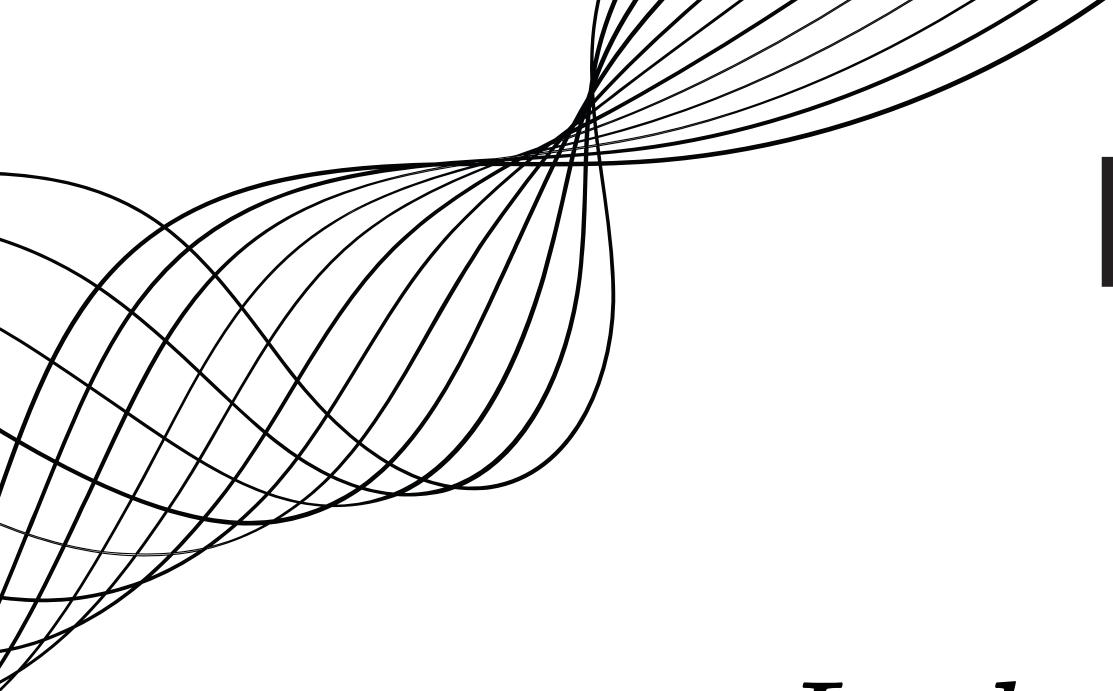
Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: Tanh



Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: Tanh

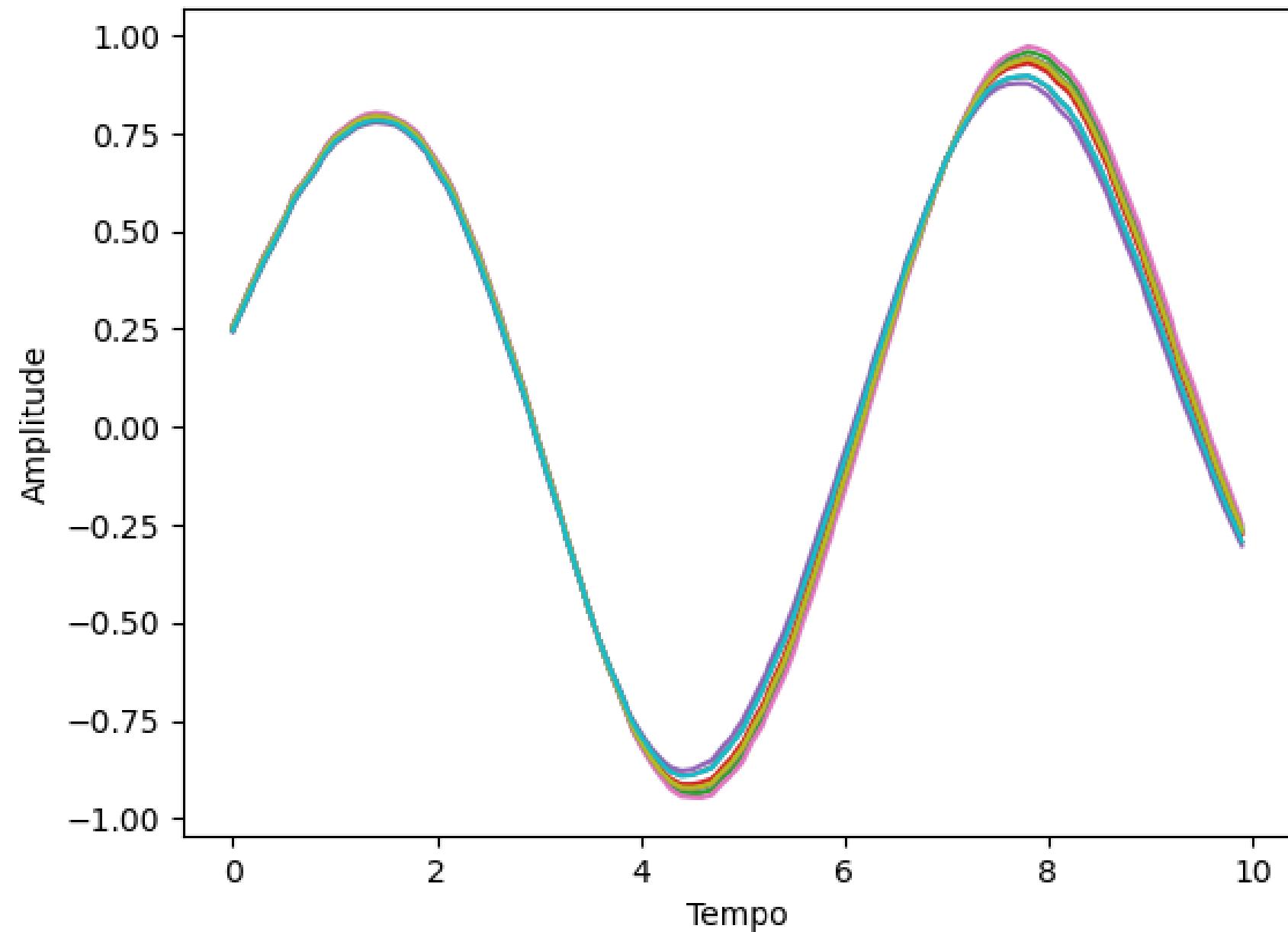


# REMOÇÃO DE RUÍDOS

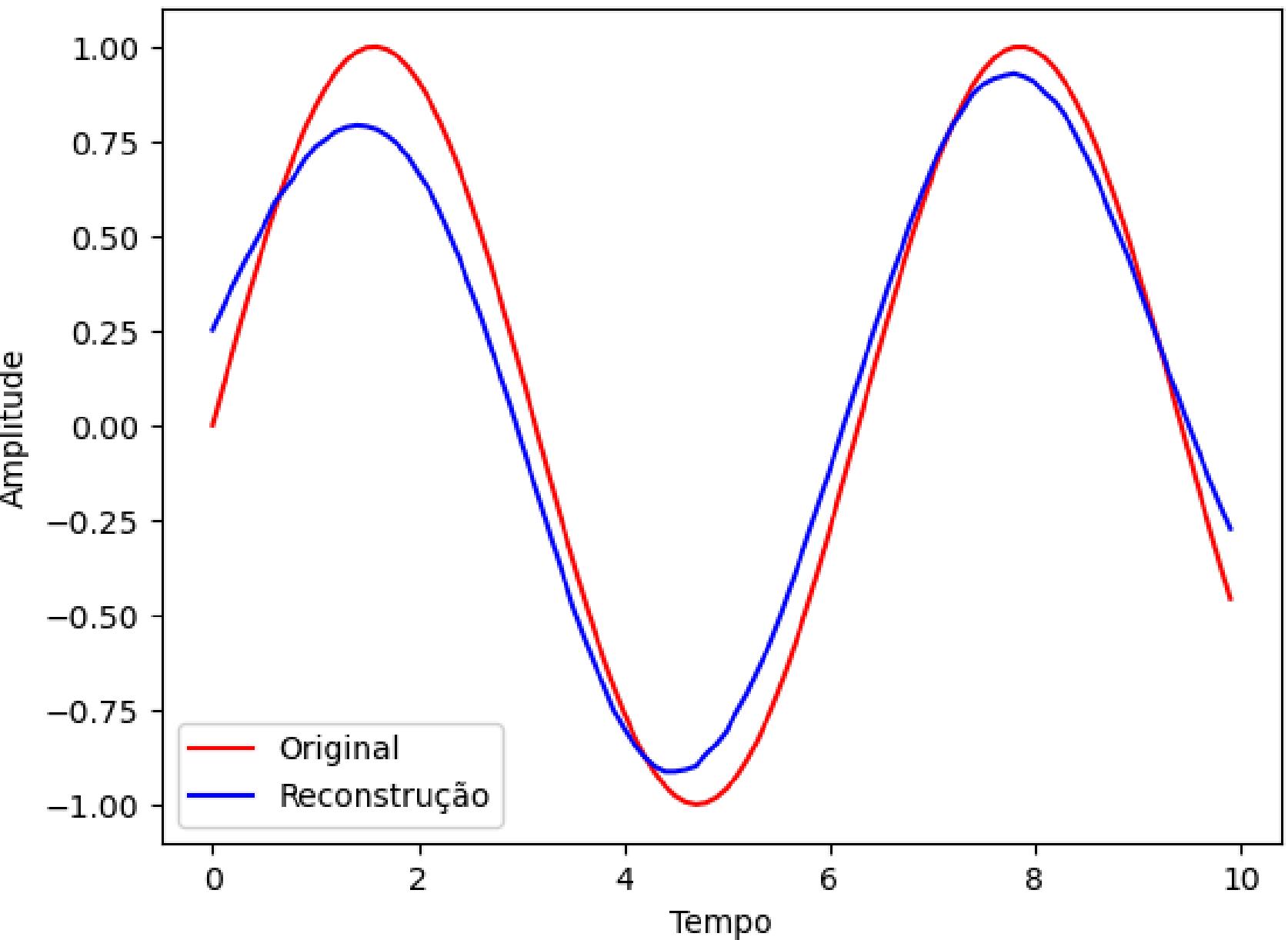


$$\text{LeakyReLU}(x) = \max(0, x) + \alpha \cdot \min(0, x)$$

Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: LeakyReLU

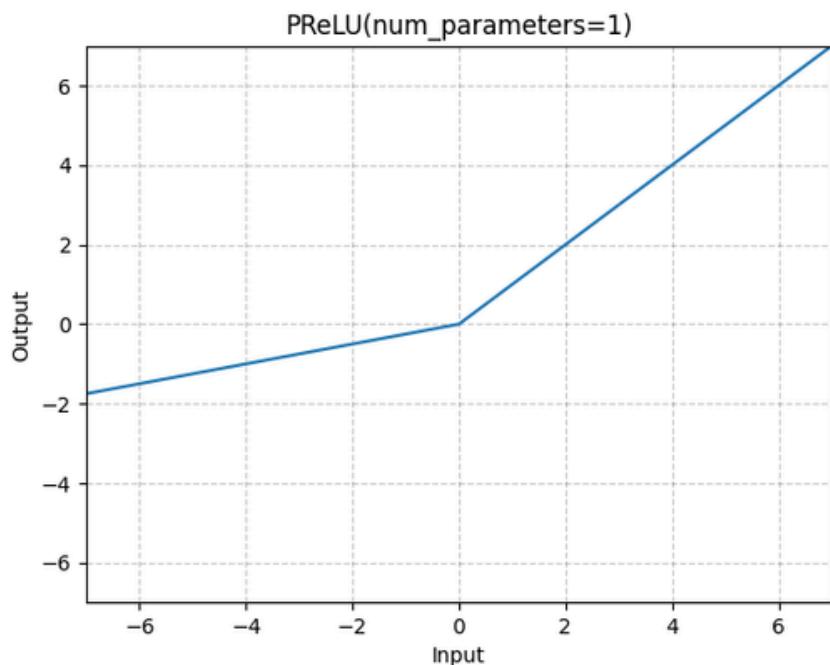


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: LeakyReLU

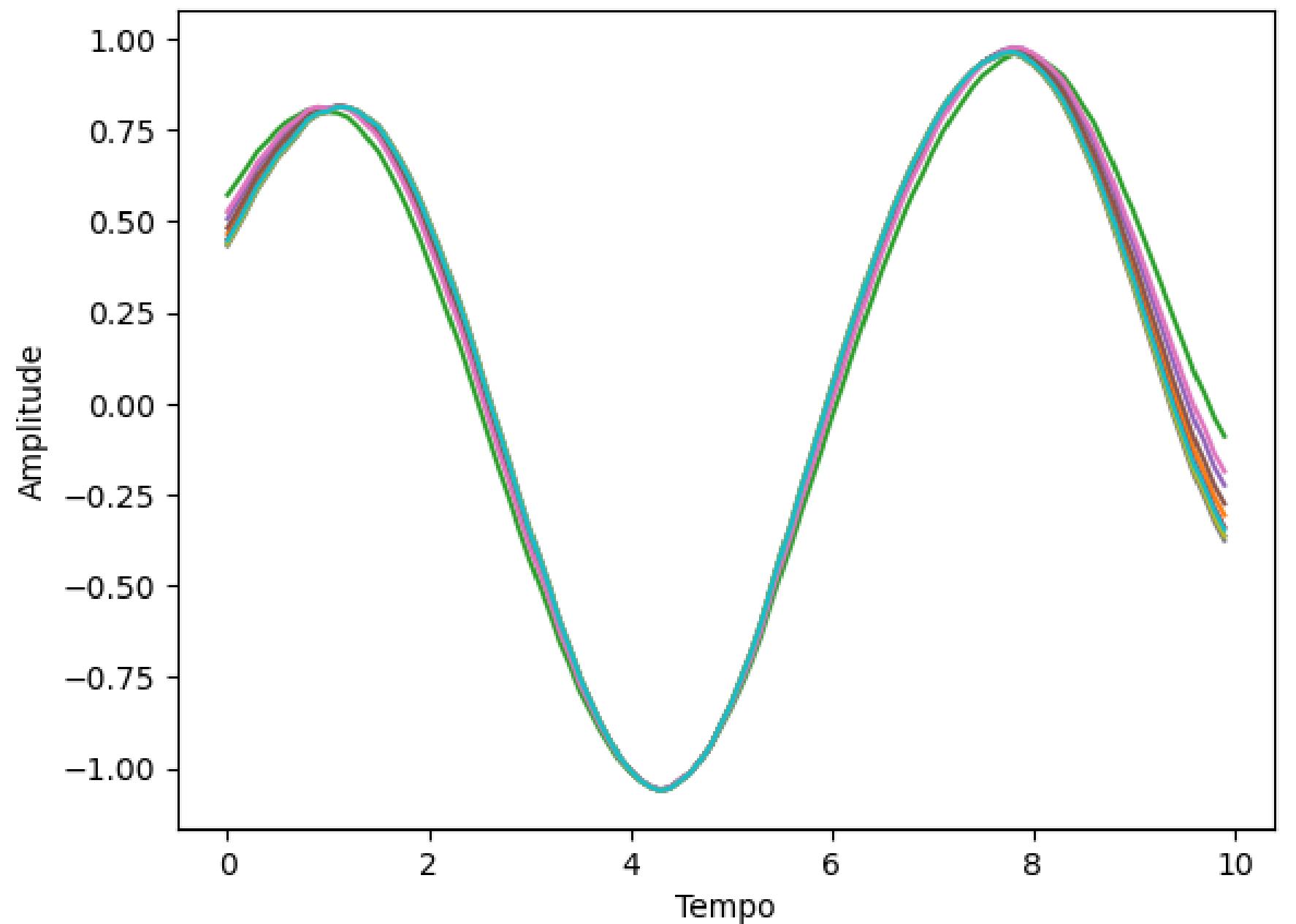


# REMOÇÃO DE RUÍDOS

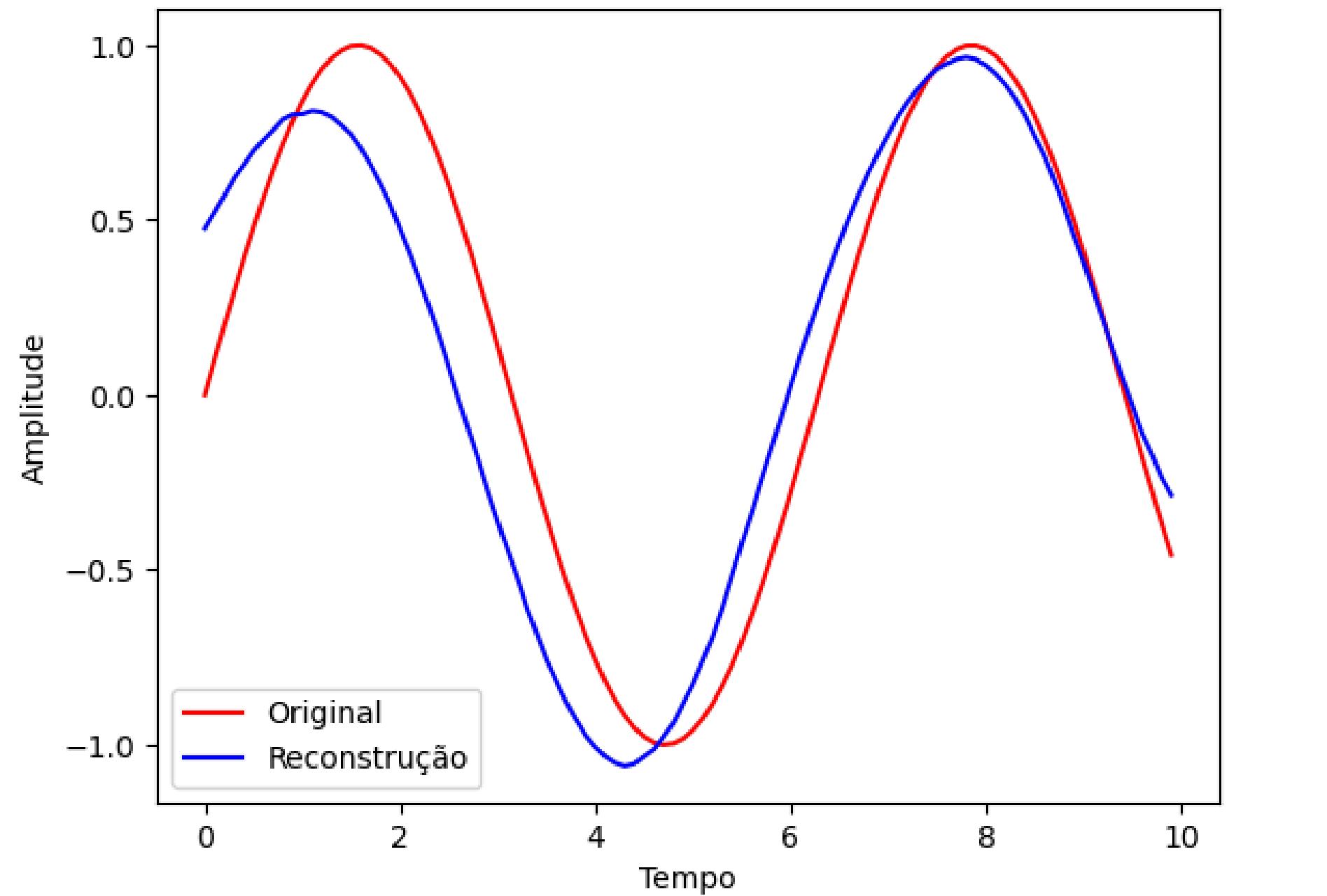
$$PReLU(x) = \max(0, x) + \alpha \cdot \min(0, x)$$



Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: PReLU

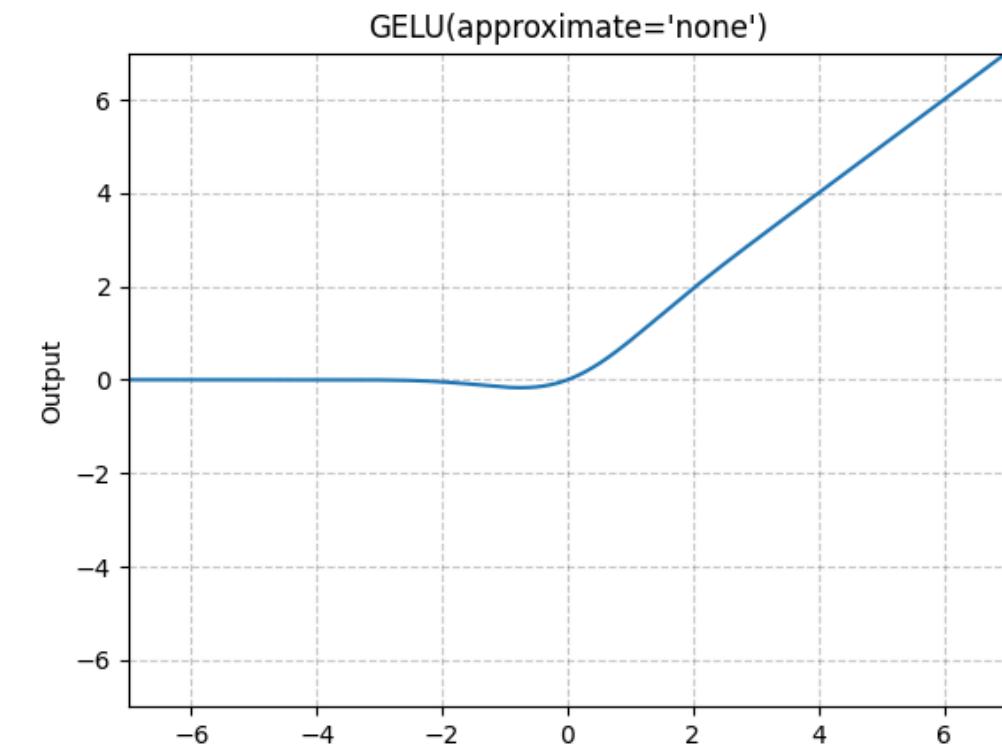


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: PReLU

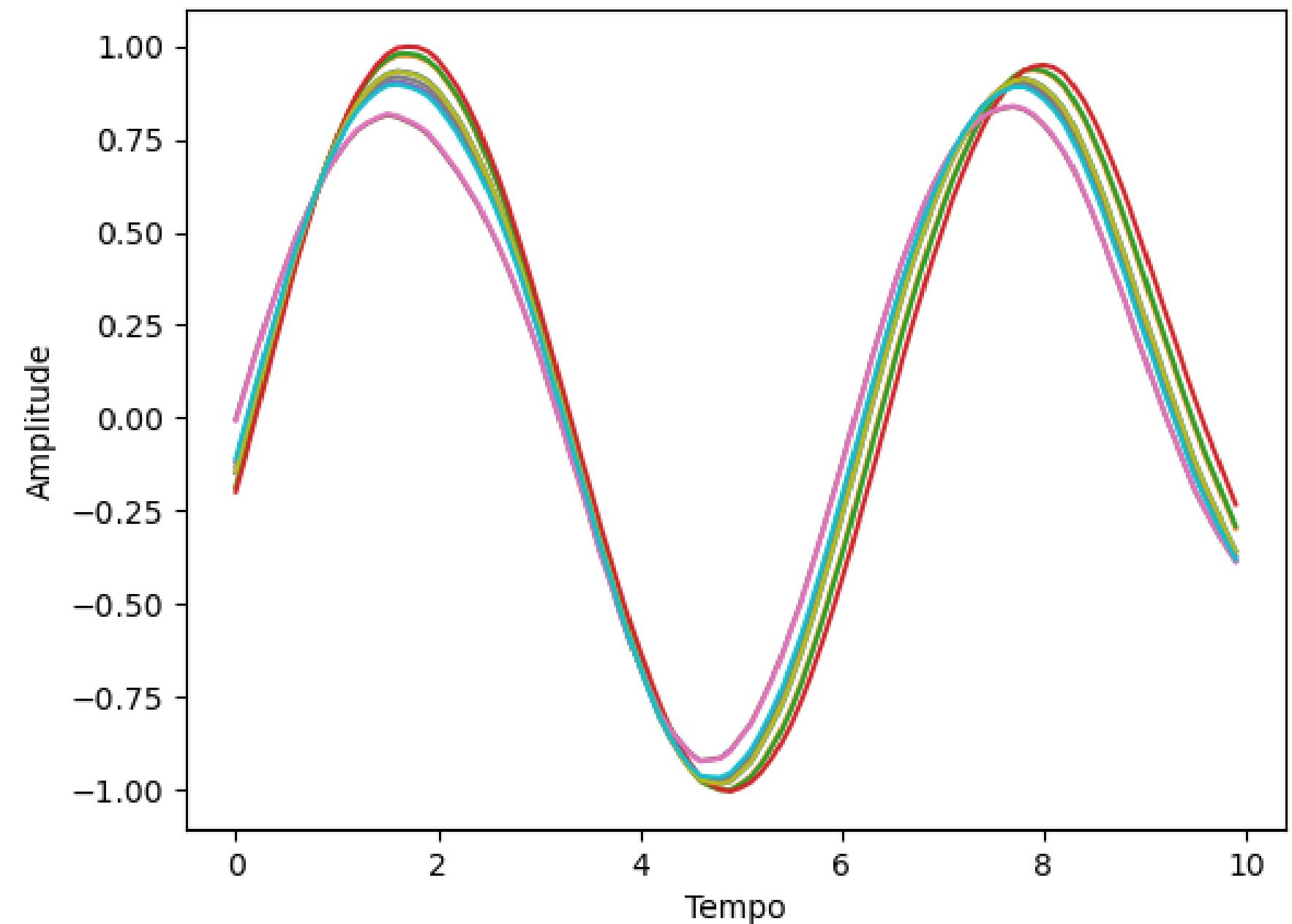


# REMOÇÃO DE RUÍDOS

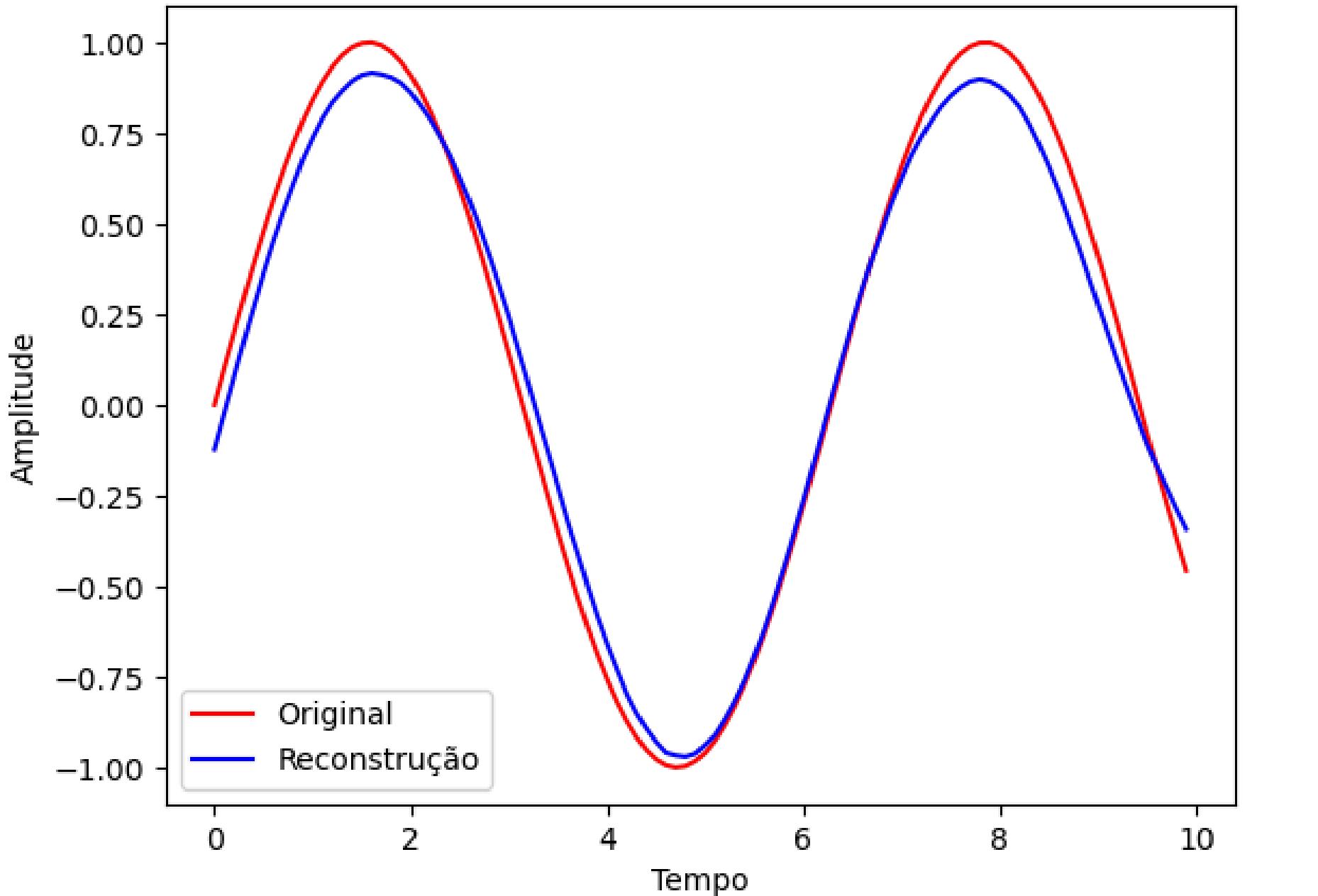
$$GELU(x) = x \cdot \Phi(x)$$

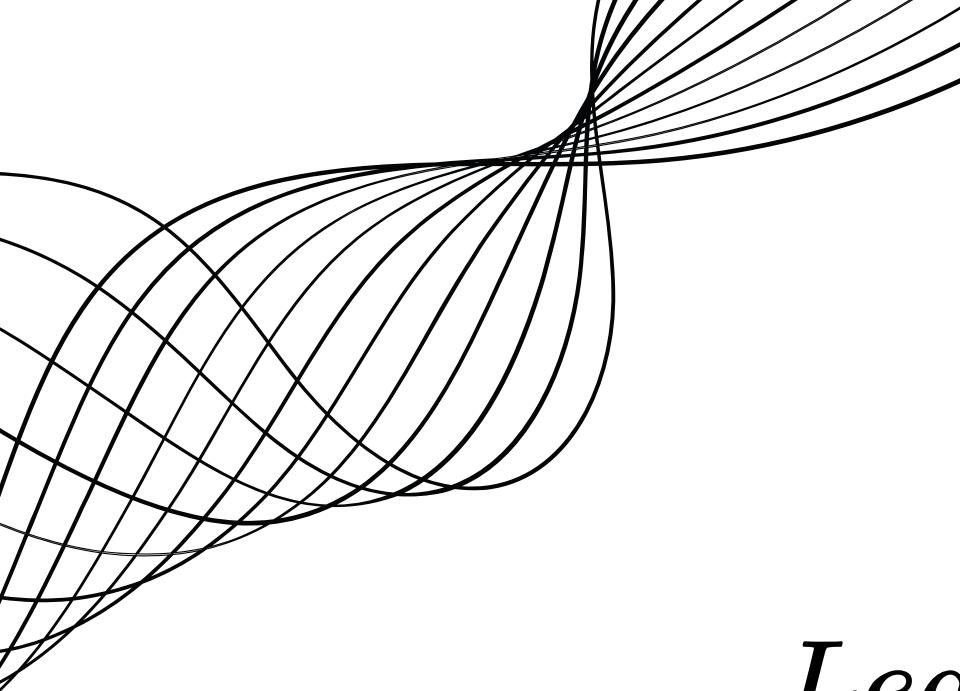


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: GELU



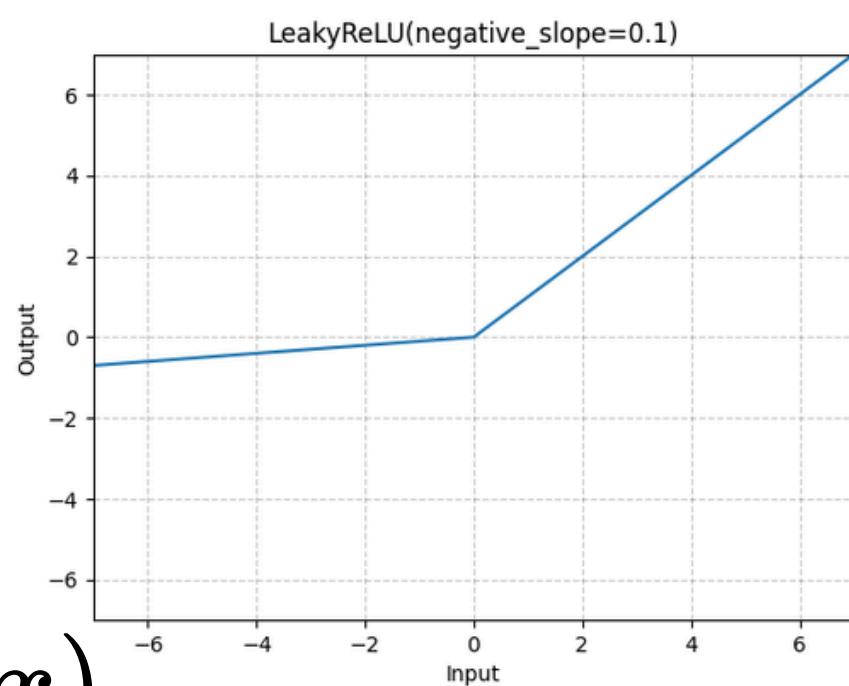
Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: GELU



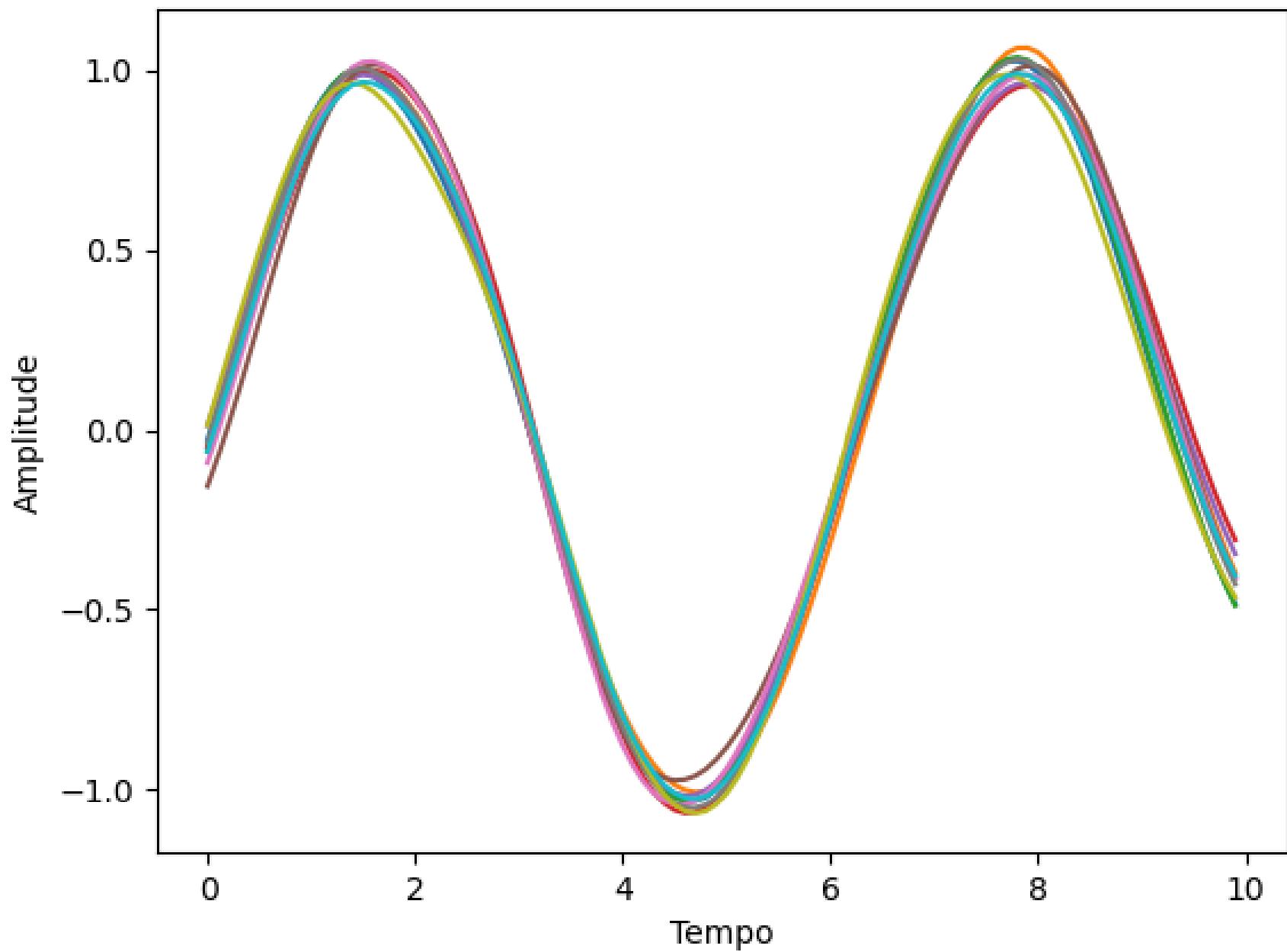


# AUTOENCODER COM ESPAÇO LATENTE MAIOR

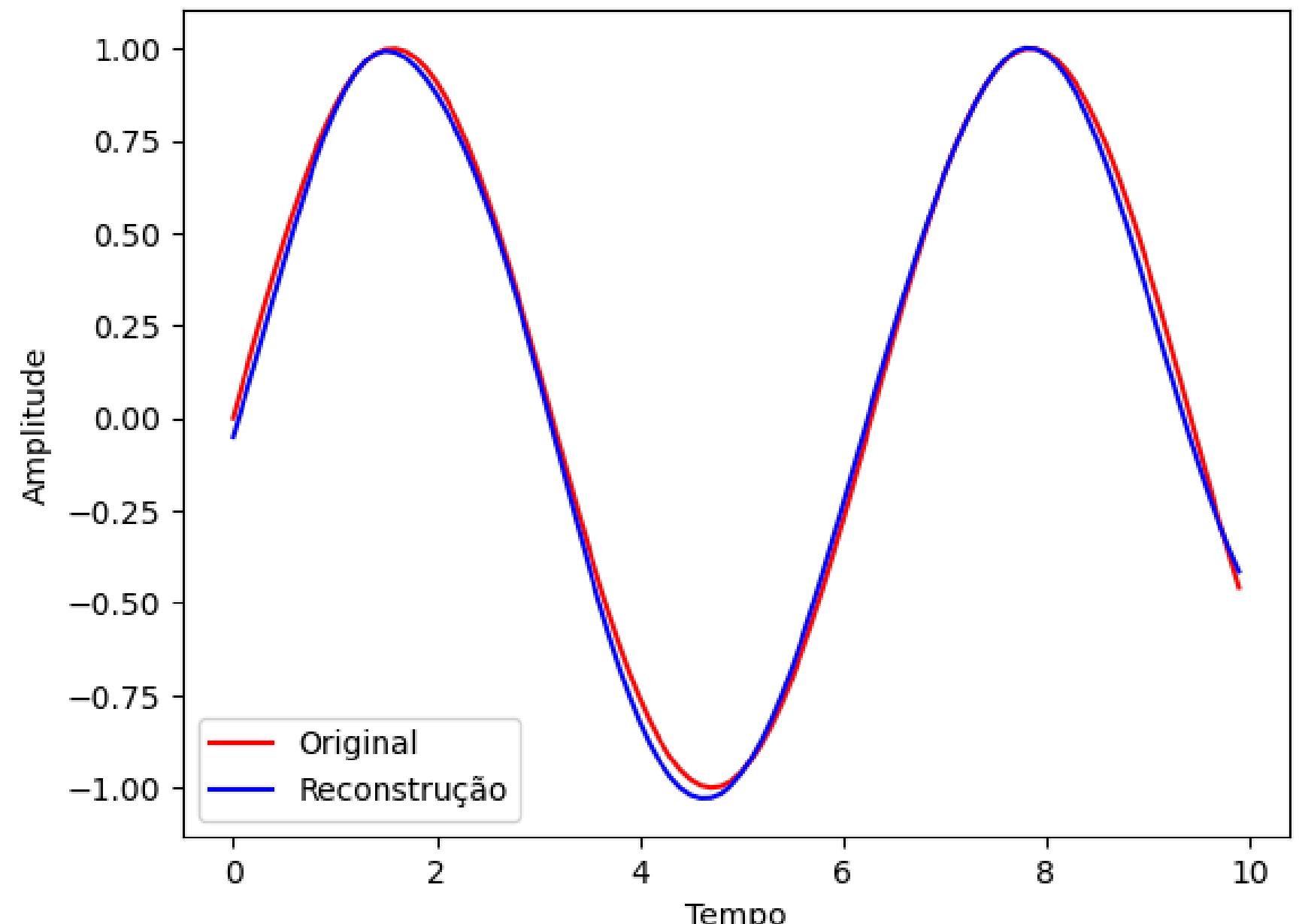
$$\text{LeakyReLU}(x) = \max(0, x) + \alpha \cdot \min(0, x)$$



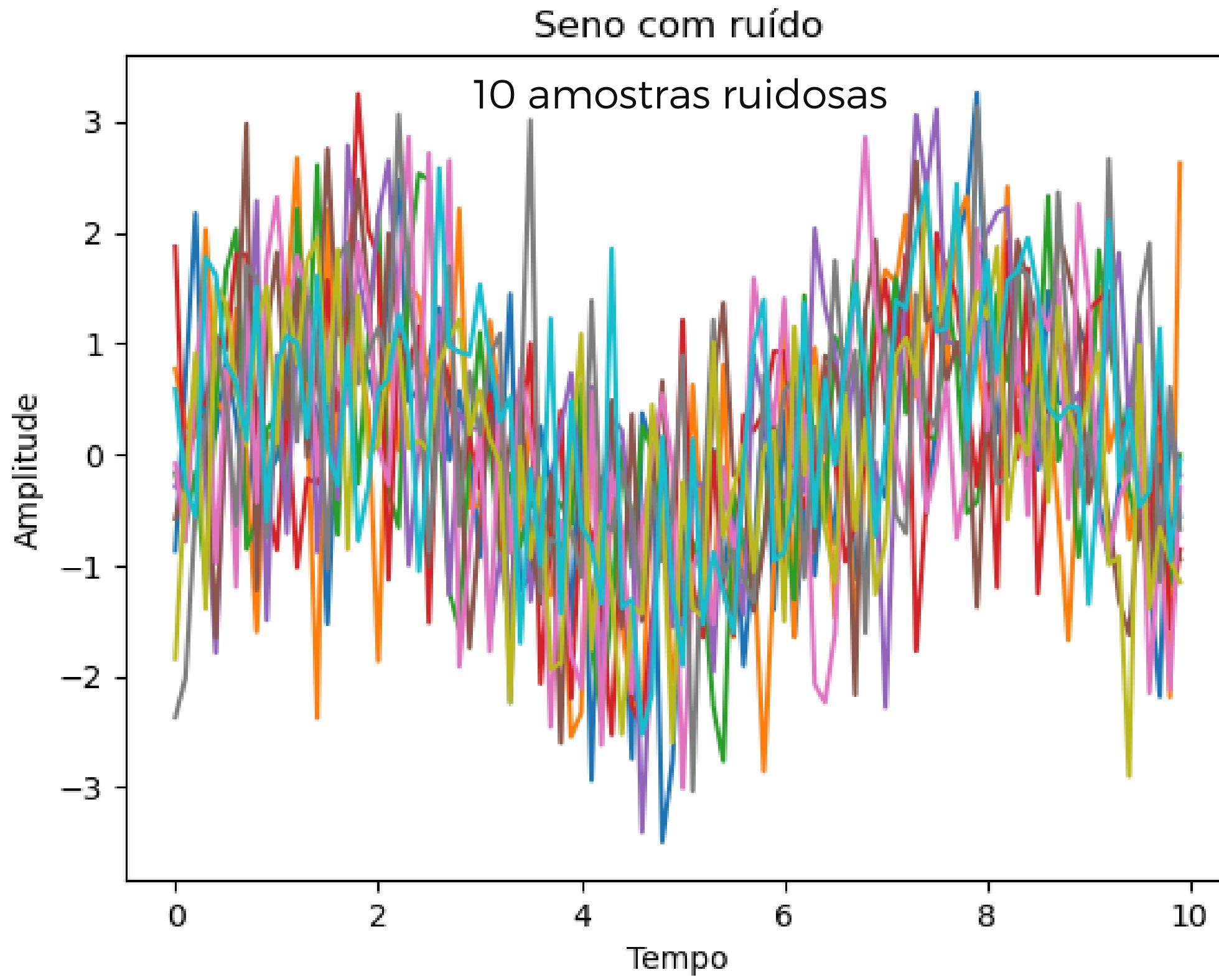
Seno reconstruído - Autoencoder - Espaço latente 5d - Ativação: LeakyReLU



Seno reconstruído - Autoencoder - Espaço latente 5d - Ativação: LeakyReLU

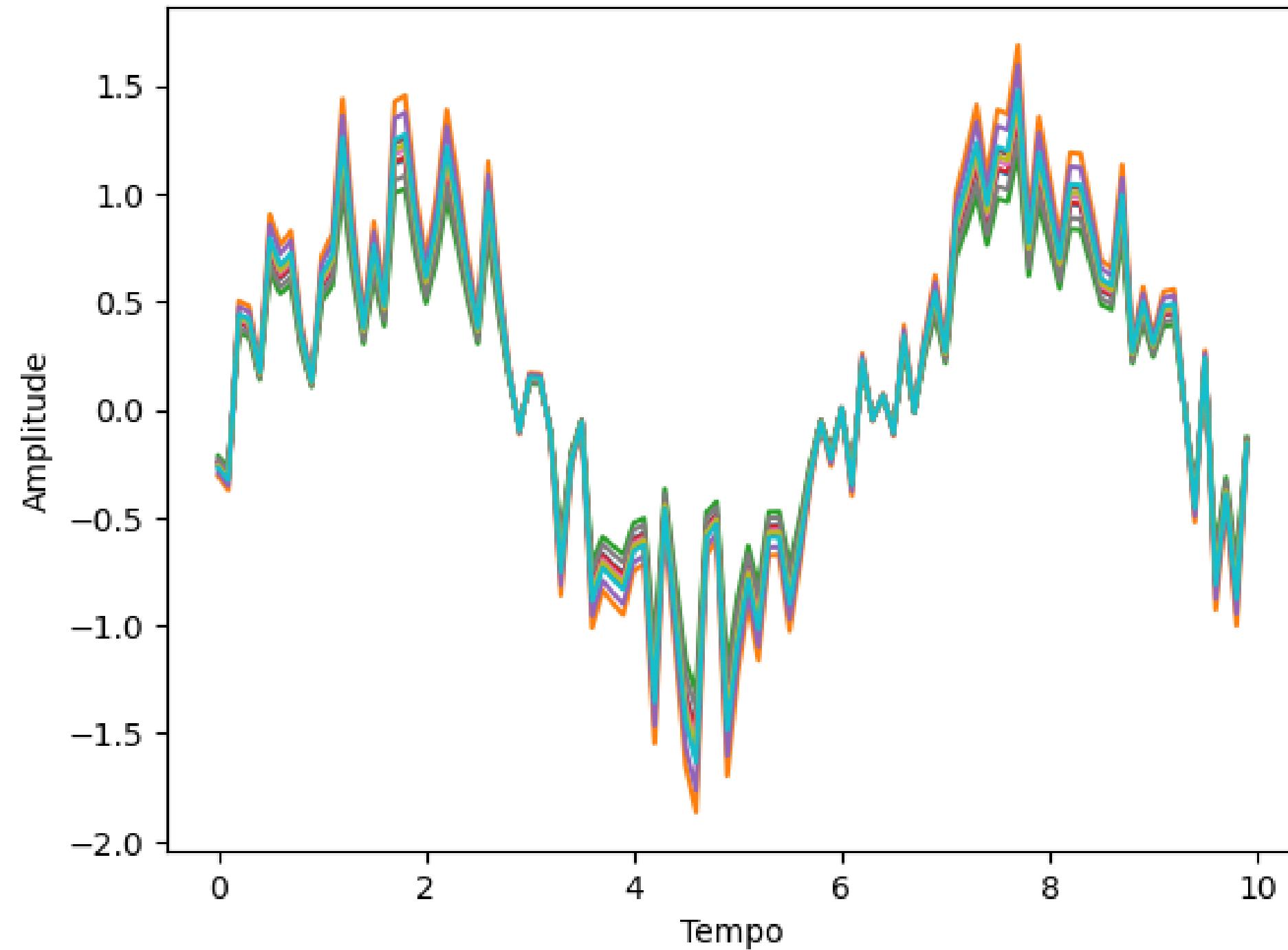


# REMOÇÃO DE RUÍDOS

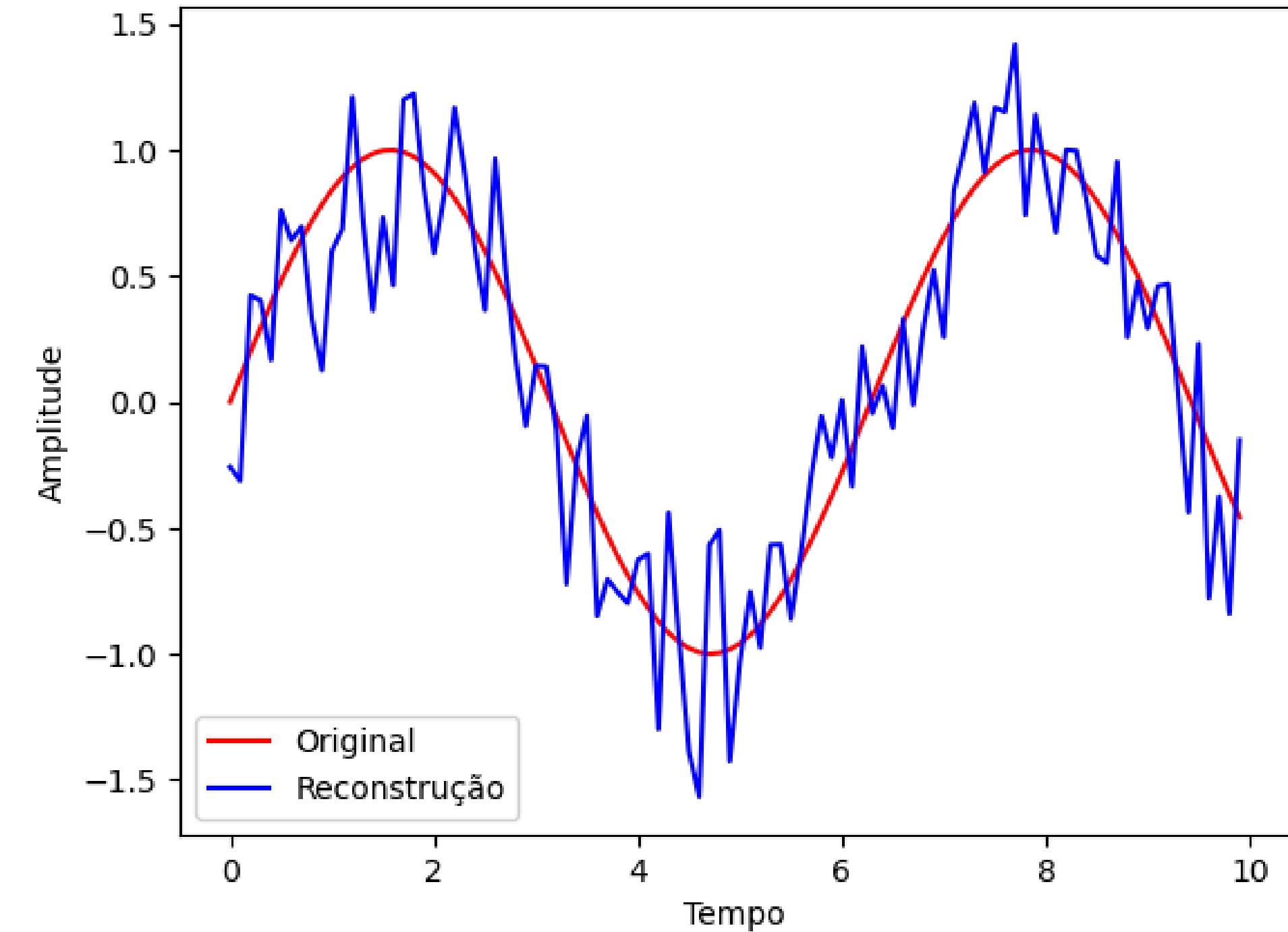


# REMOÇÃO DE RUÍDOS

Seno reconstruído - SVD posto 1

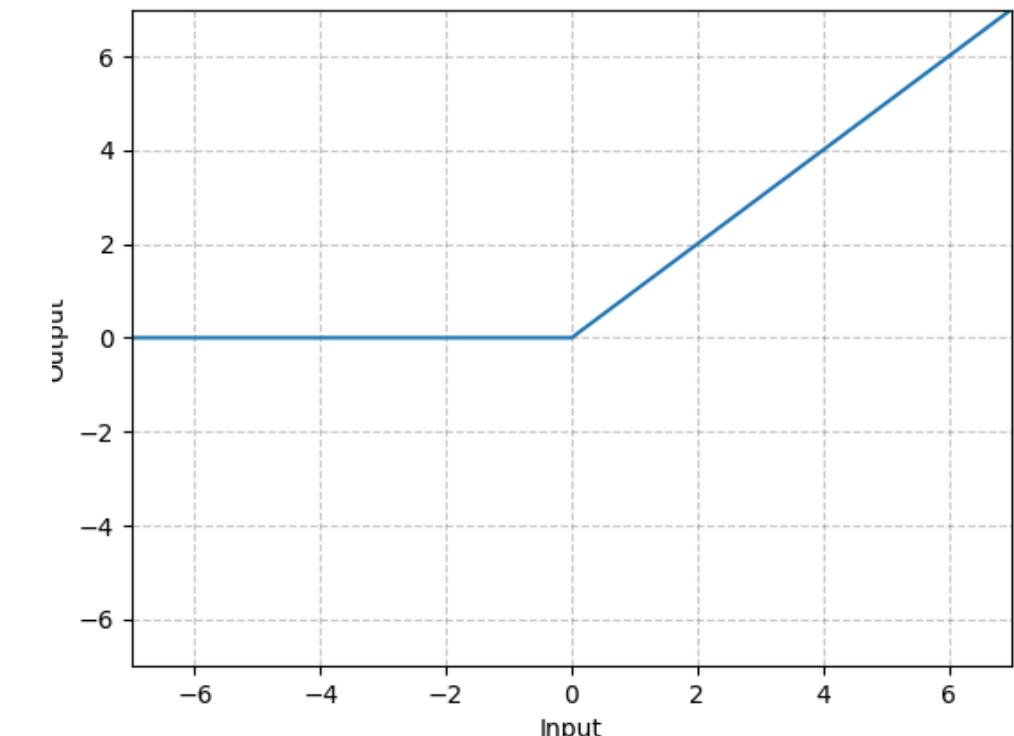
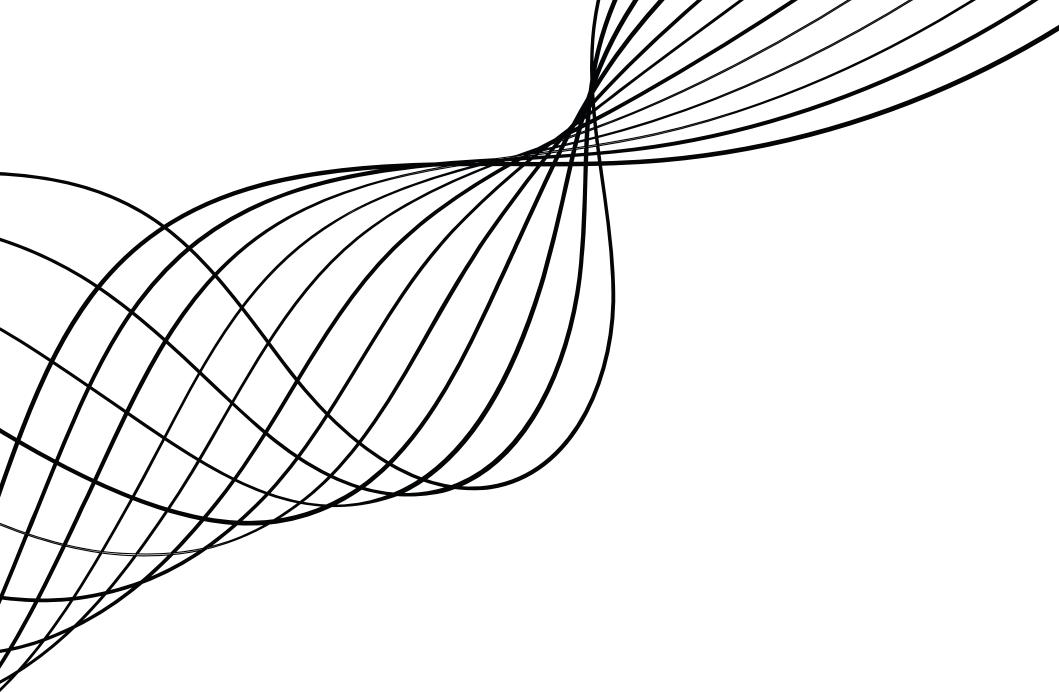


Seno reconstruído - SVD posto 1

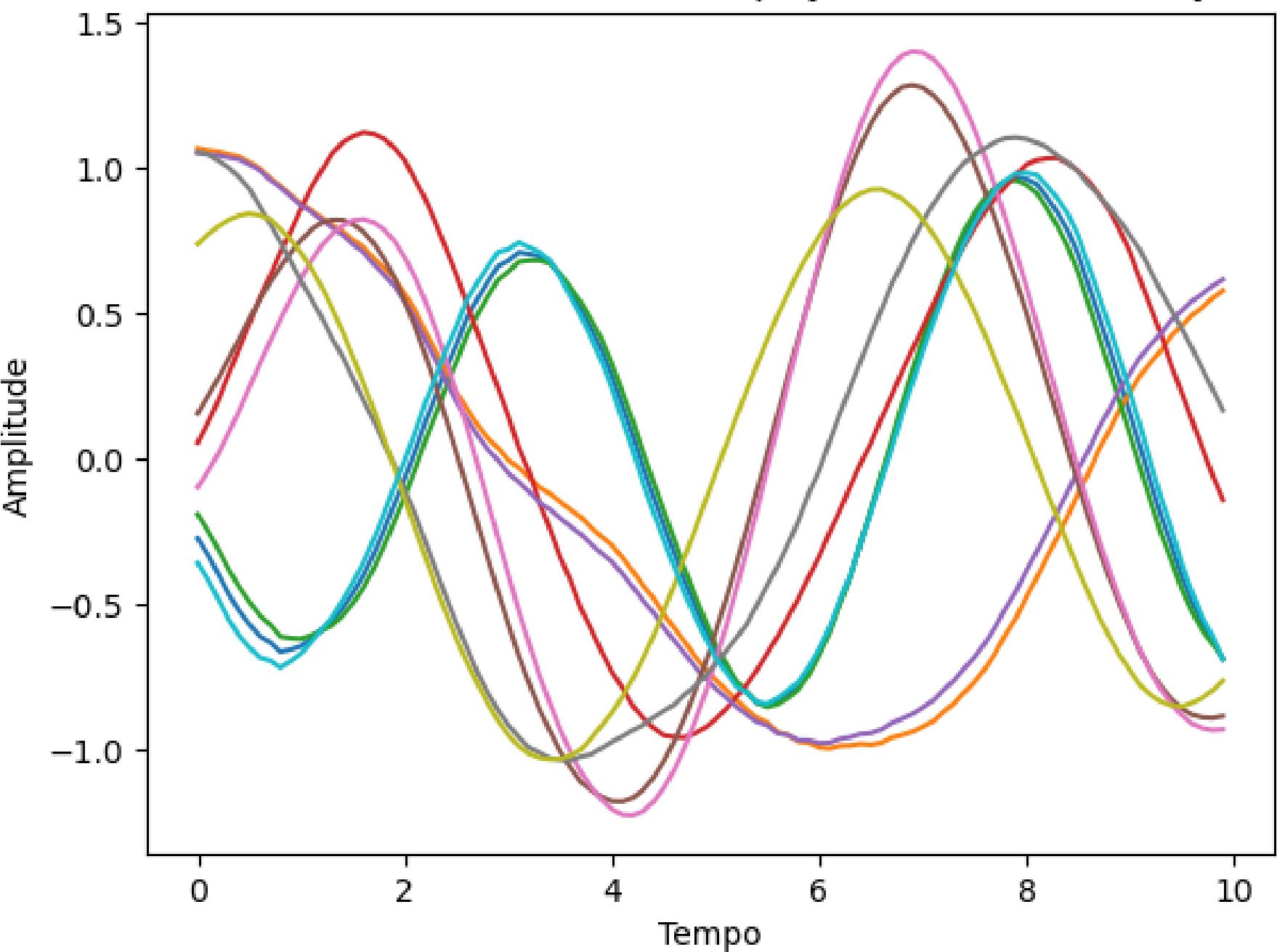


# REMOÇÃO DE RUÍDOS

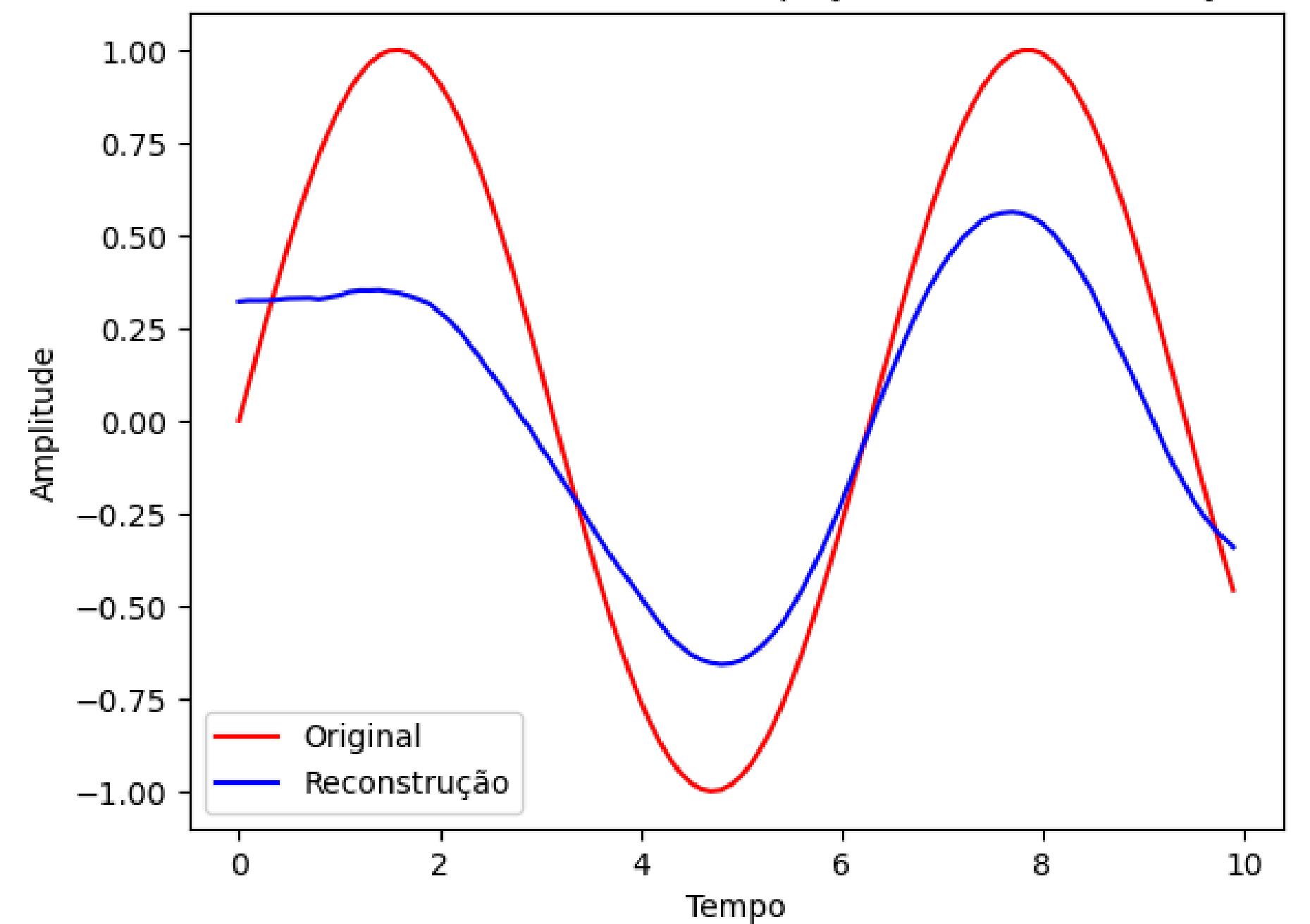
$$ReLU(x) = \max(0, x)$$

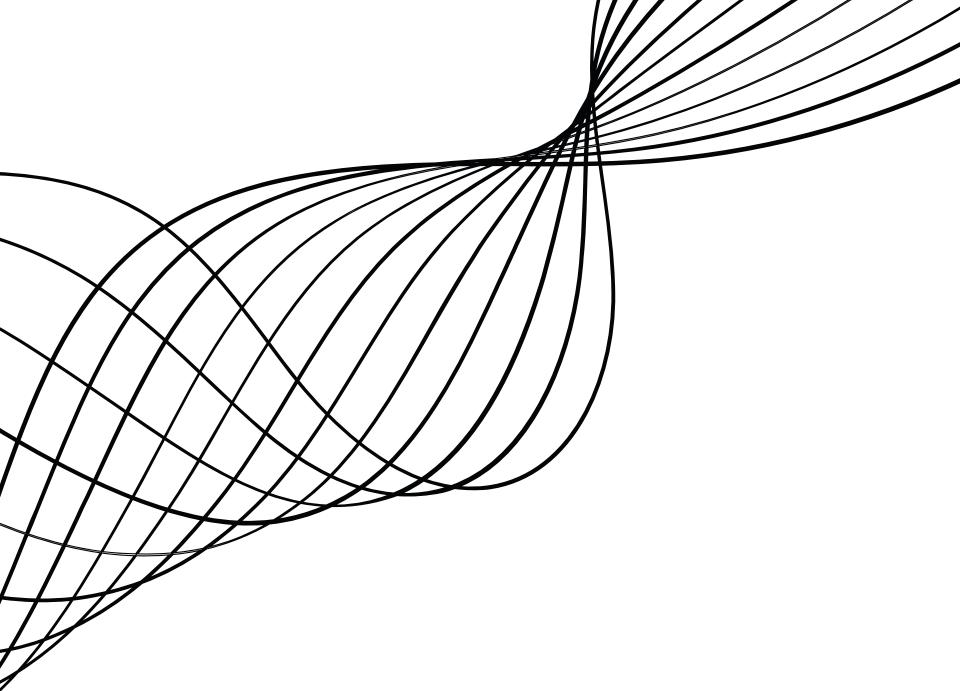


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: ReLU



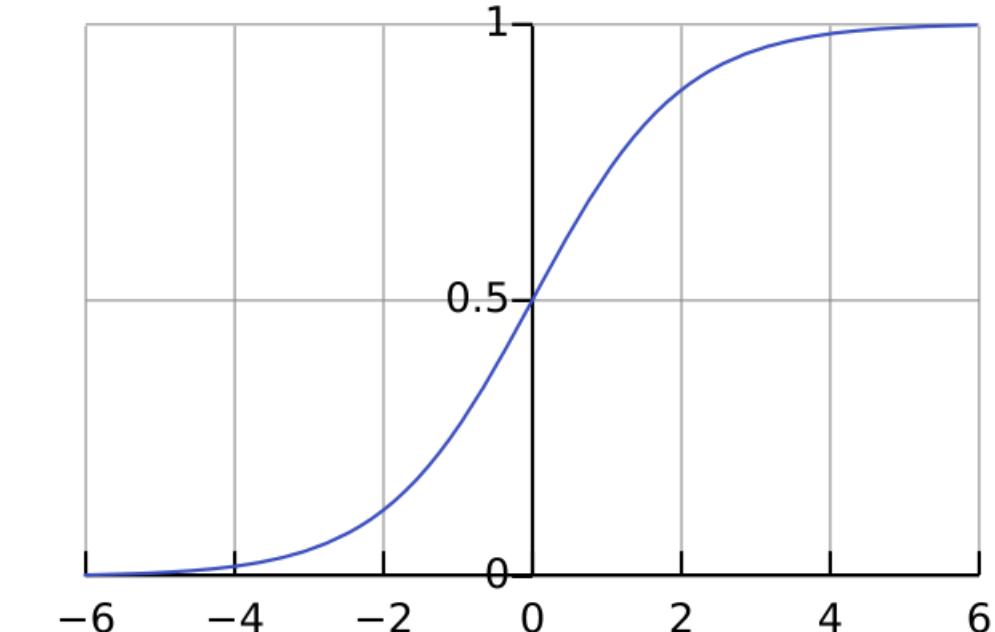
Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: ReLU



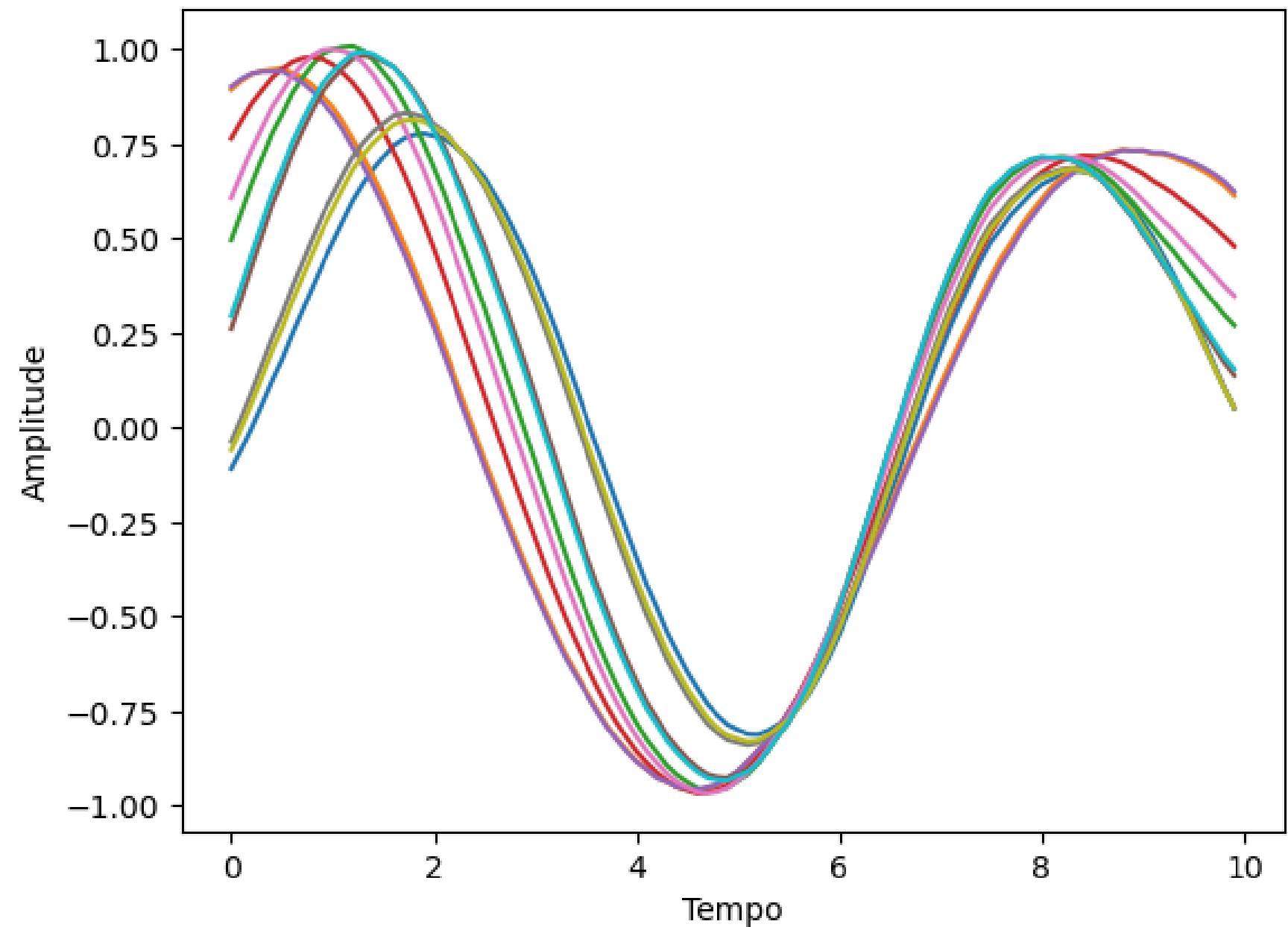


# REMOÇÃO DE RUÍDOS

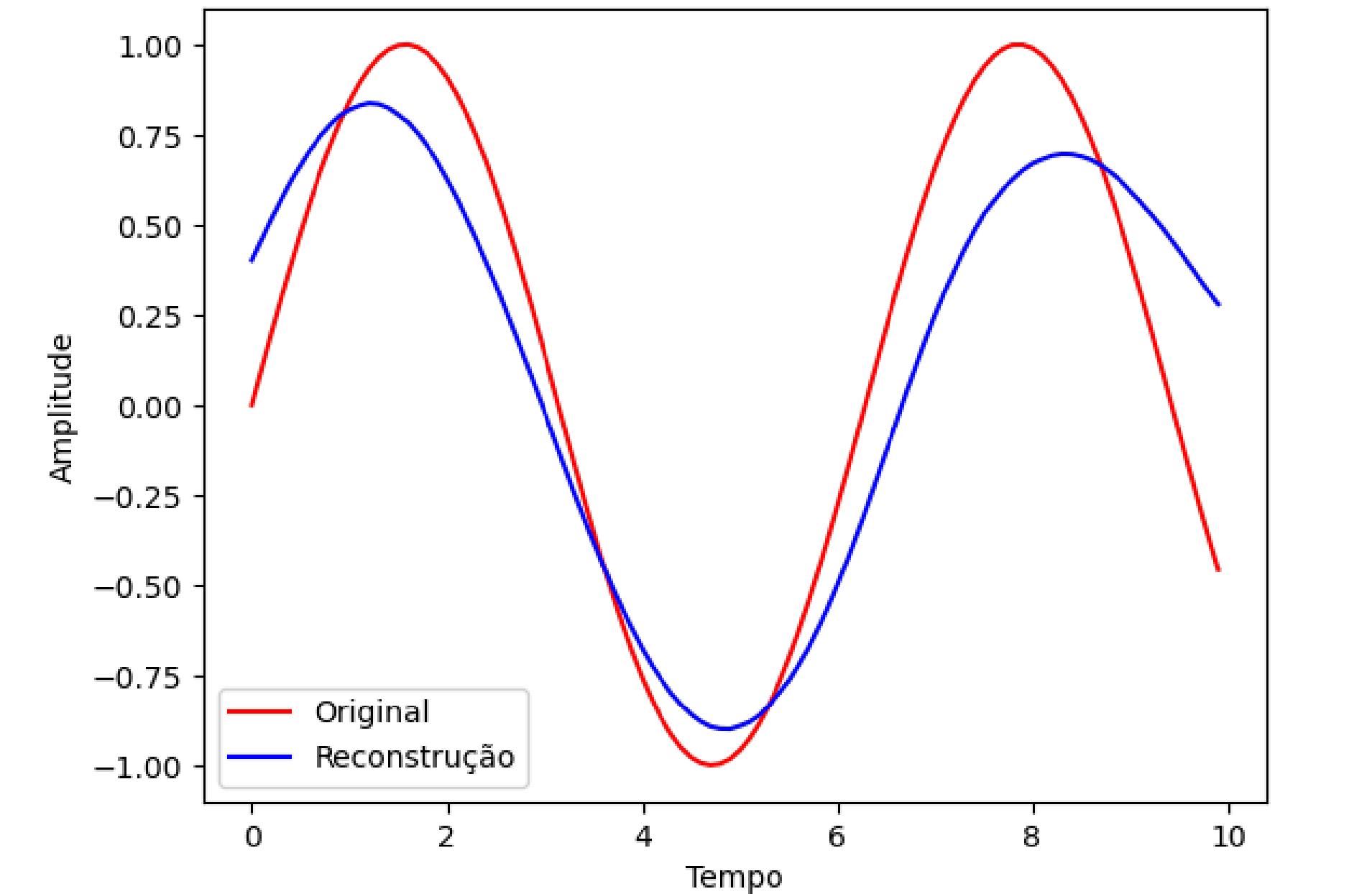
$$\text{Sigmoid}(x) = \frac{1}{1+e^{-x}}$$



Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: Sigmoid

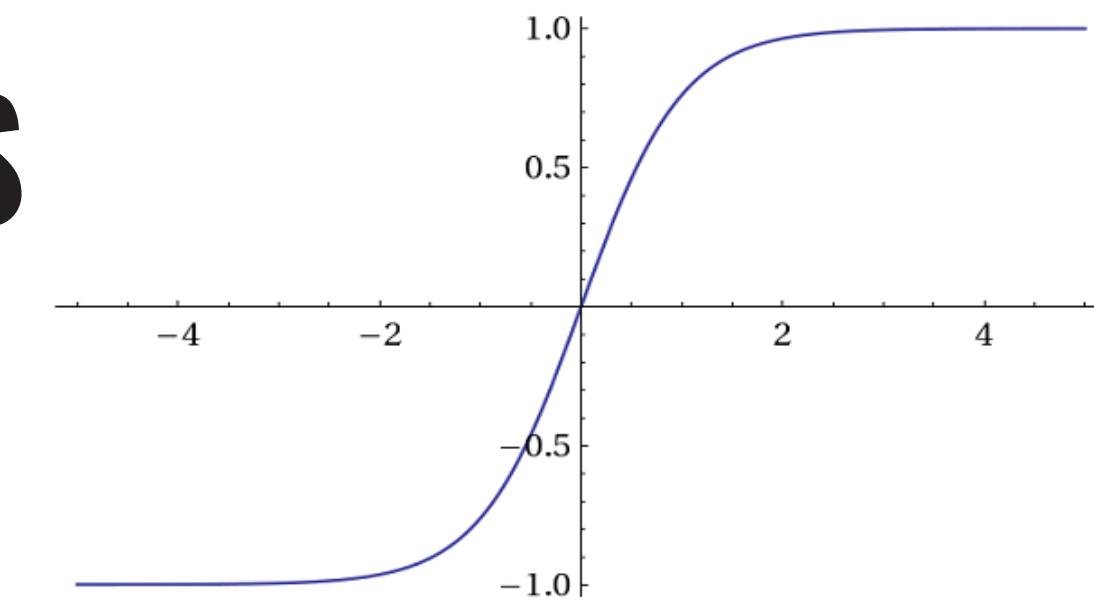


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: Sigmoid

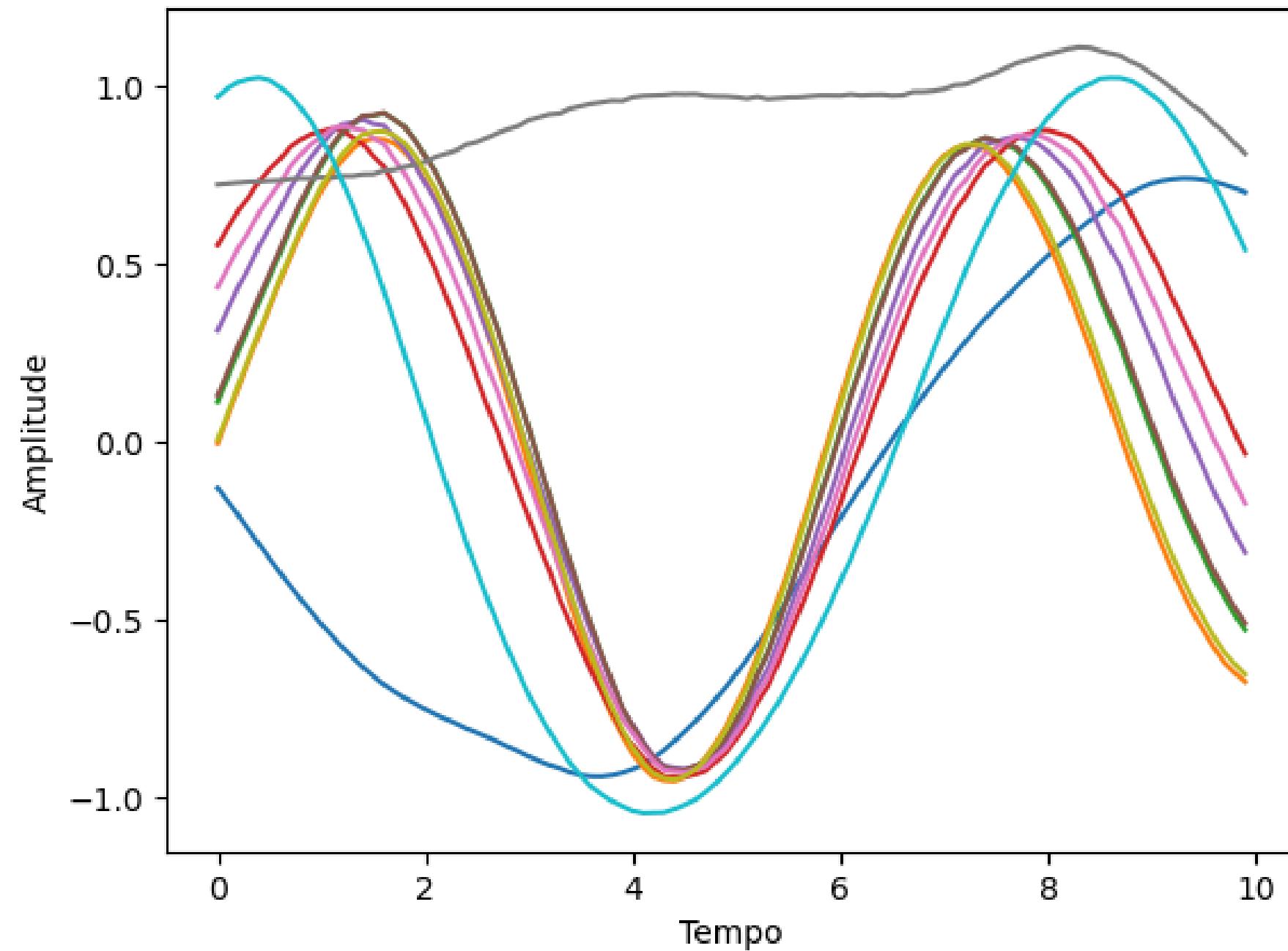


# REMOÇÃO DE RUÍDOS

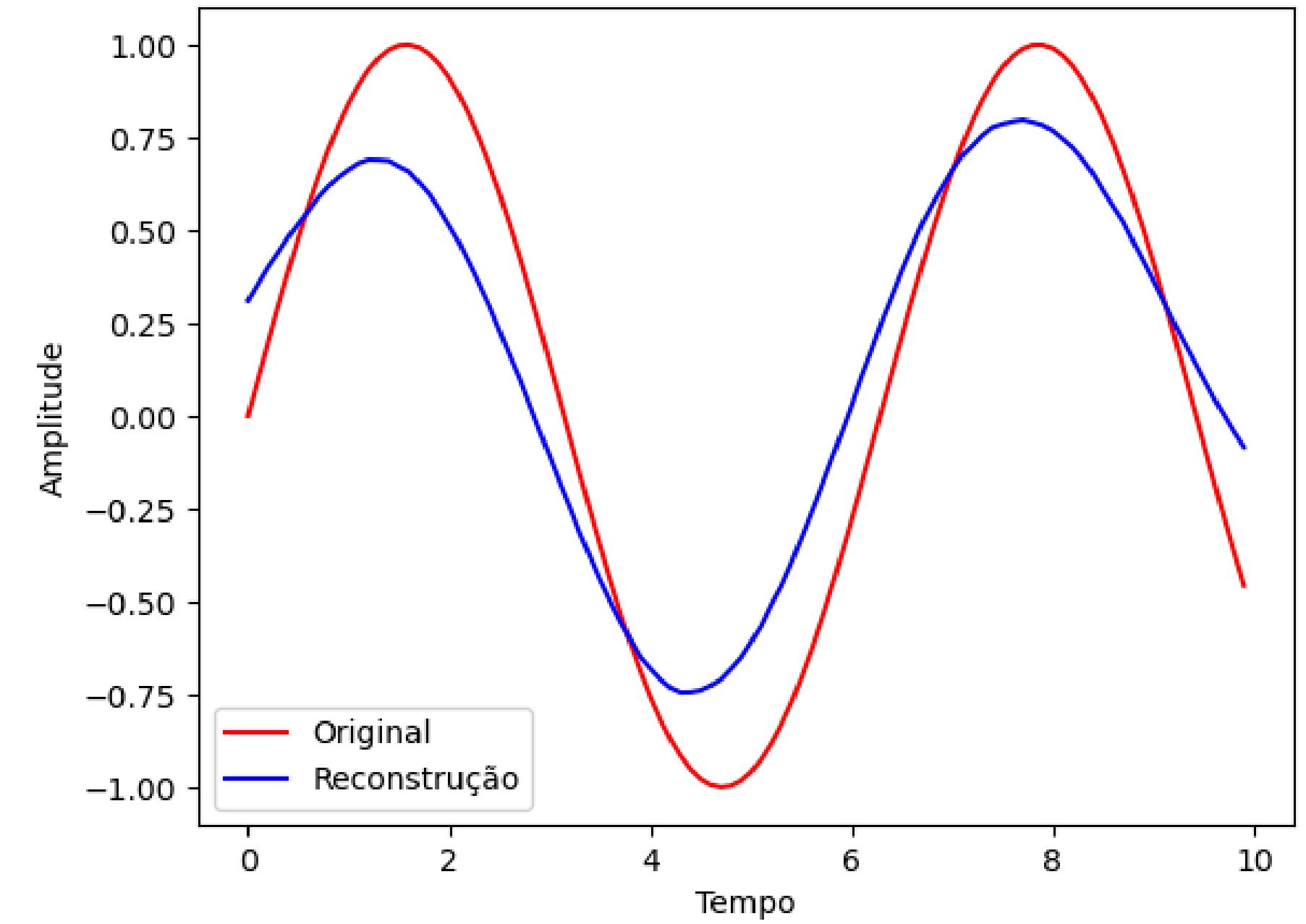
$$\Tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$



Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: Tanh

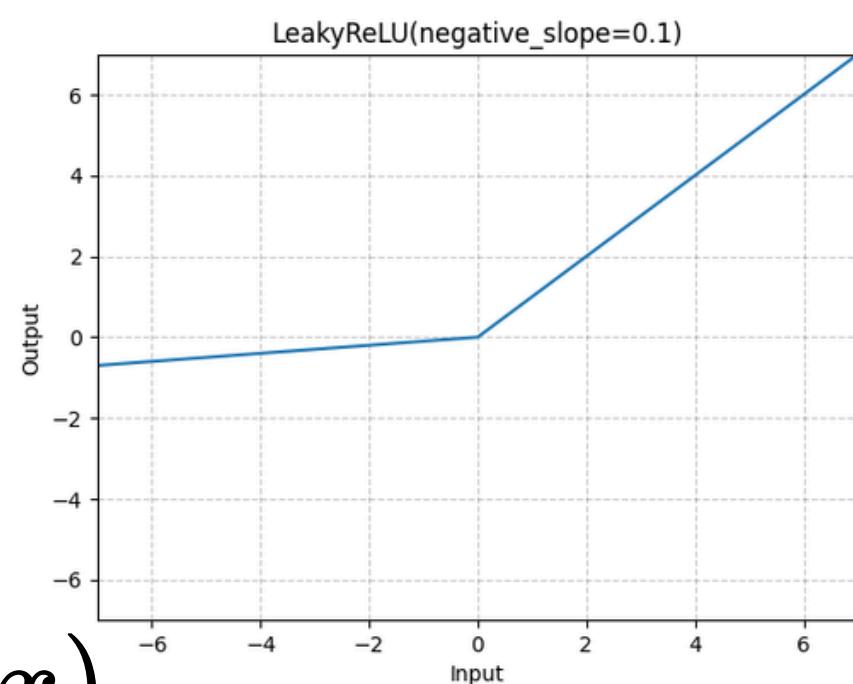


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: Tanh

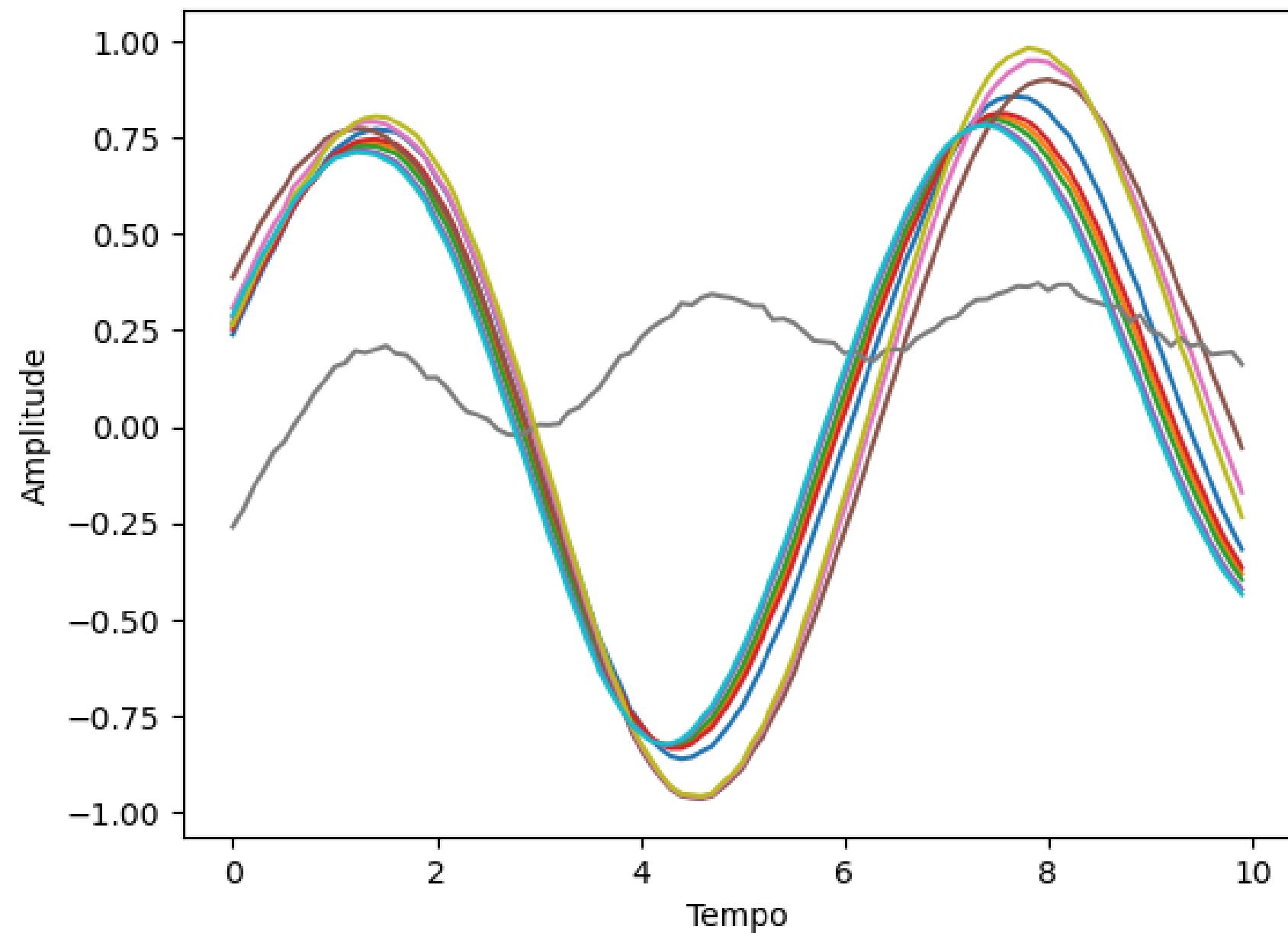


# REMOÇÃO DE RUÍDOS

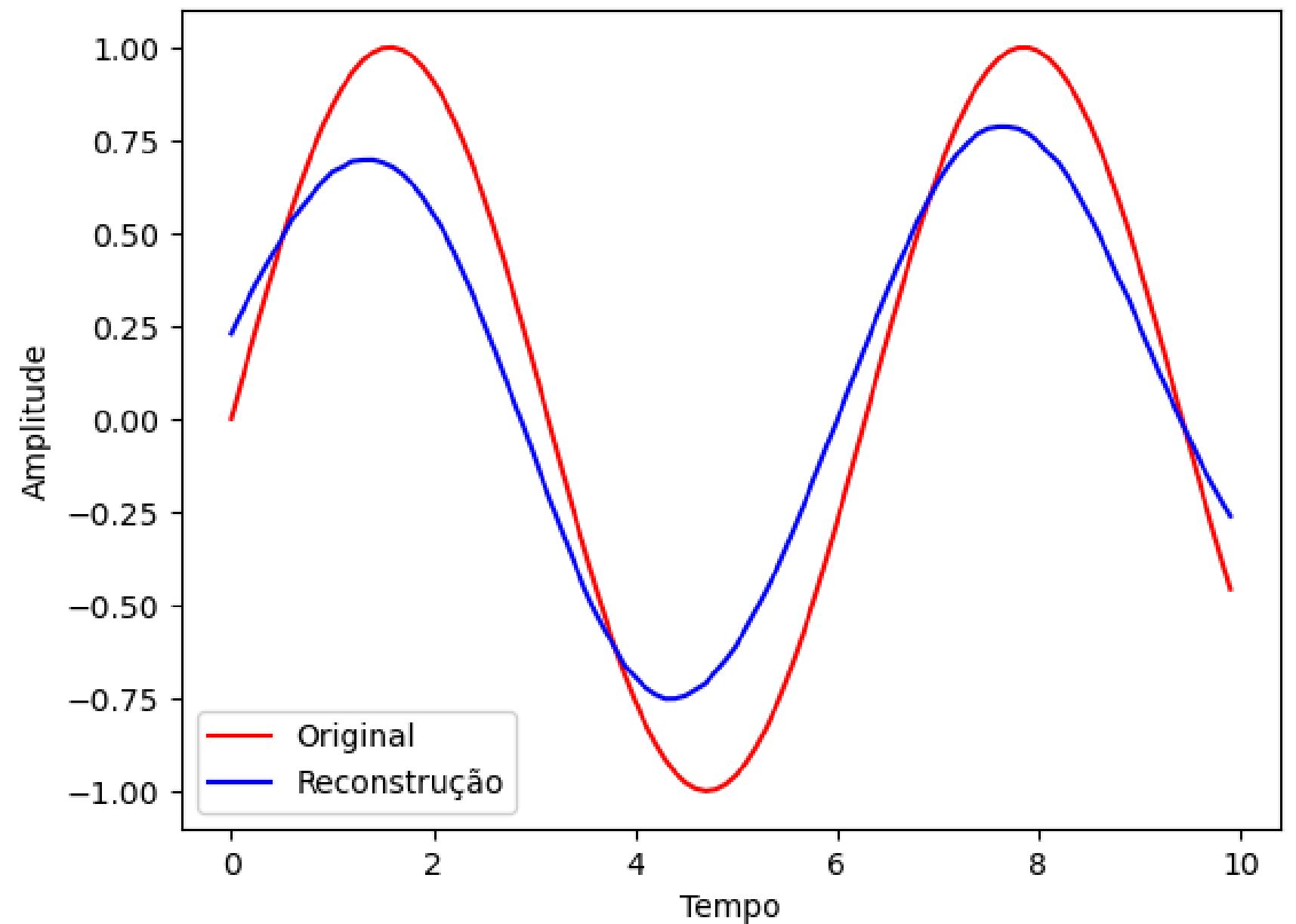
$$\text{LeakyReLU}(x) = \max(0, x) + \alpha \cdot \min(0, x)$$



Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: LeakyReLU

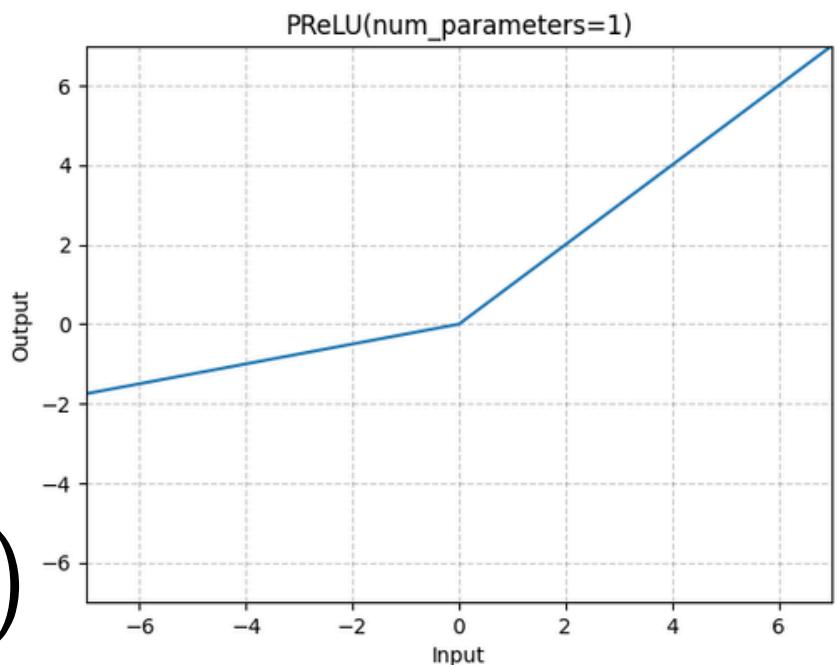


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: LeakyReLU

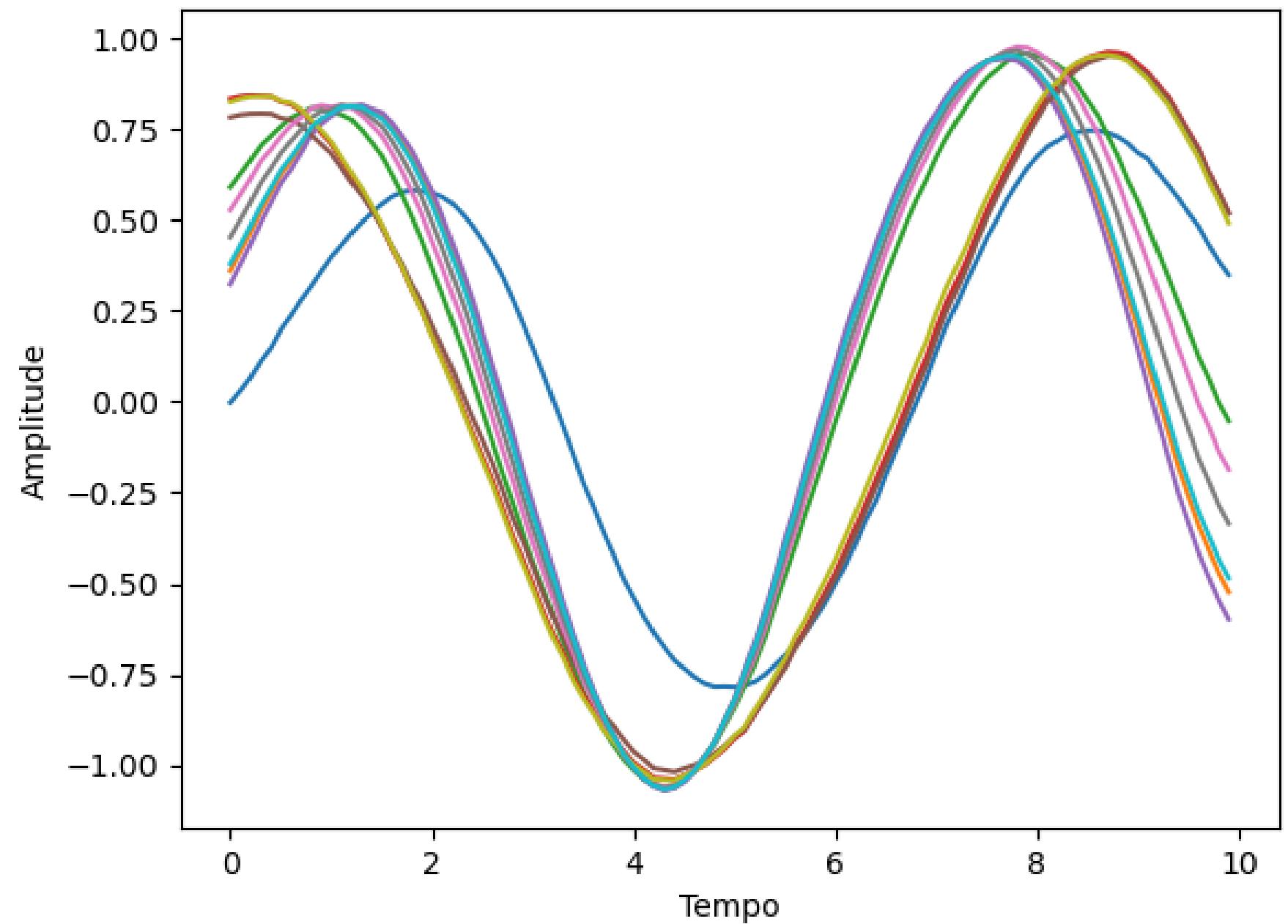


# REMOÇÃO DE RUÍDOS

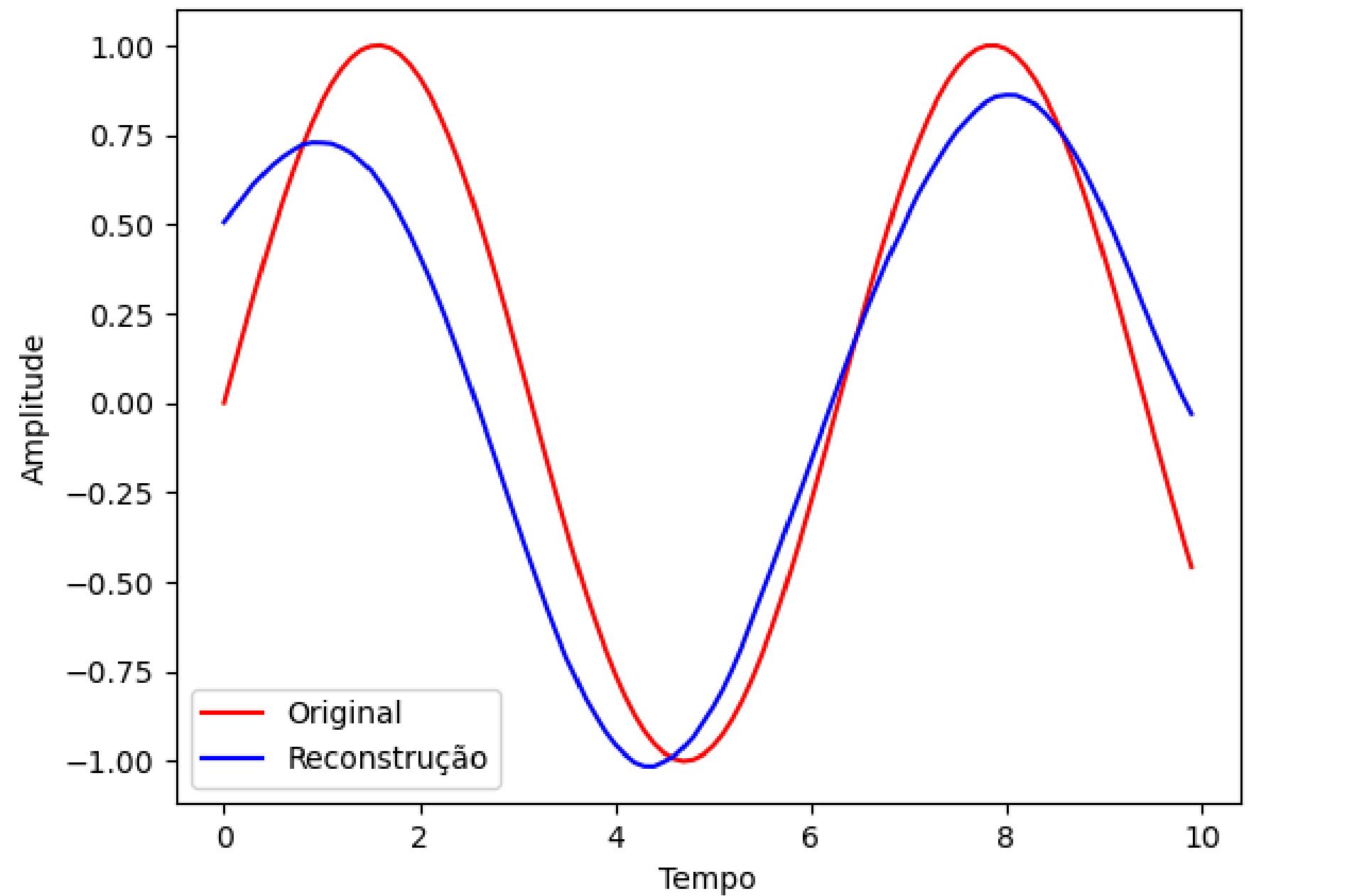
$$PReLU(x) = \max(0, x) + \alpha \cdot \min(0, x)$$



Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: PReLU

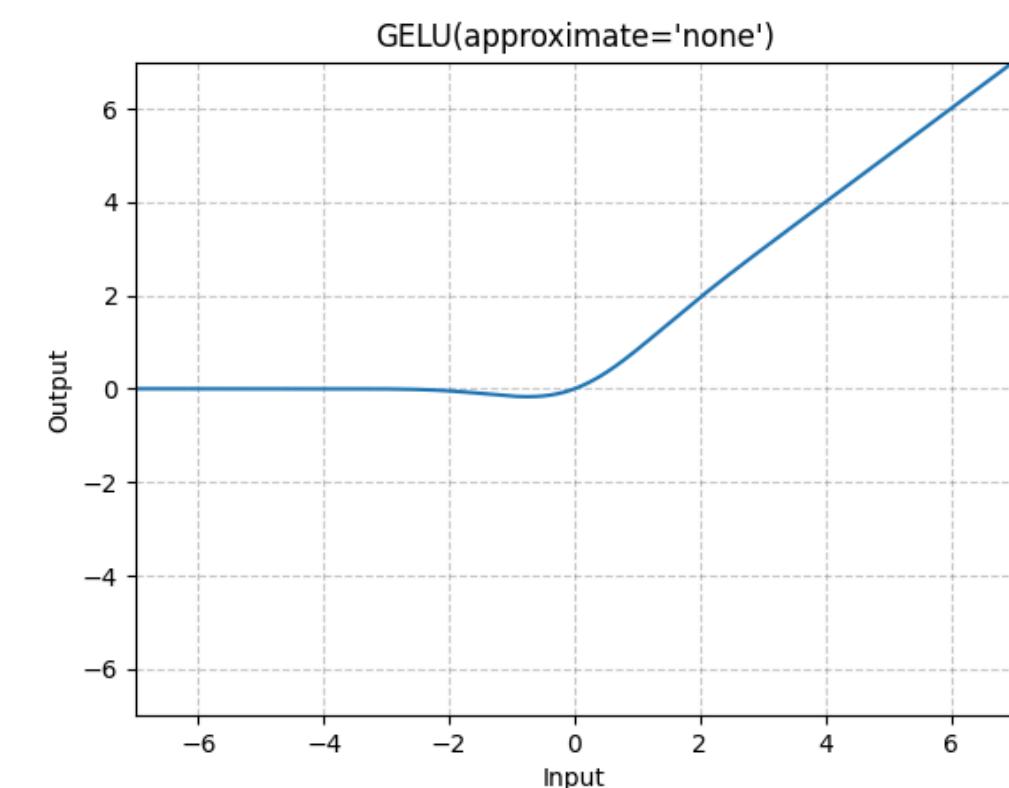


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: PReLU

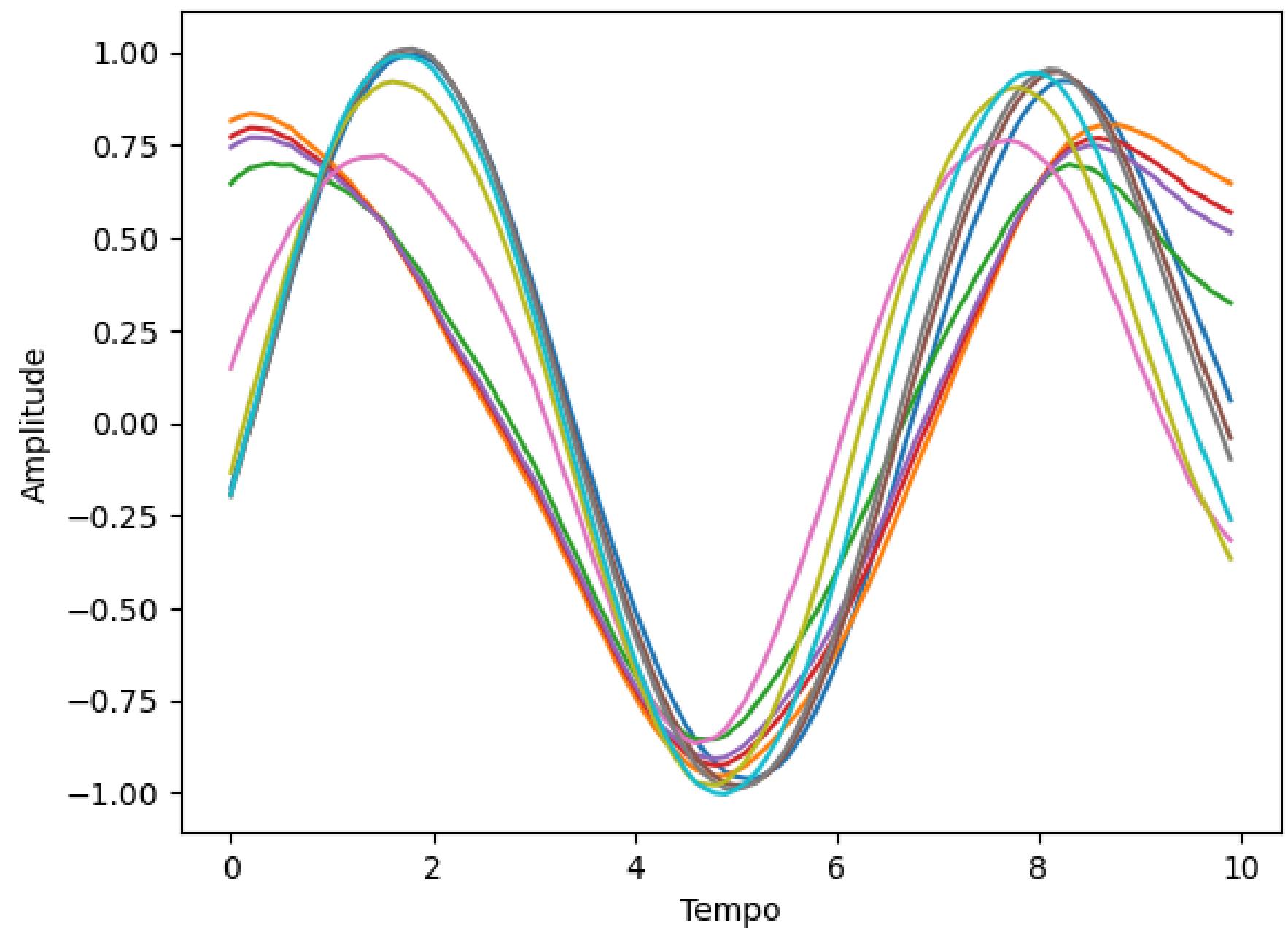


# REMOÇÃO DE RUÍDOS

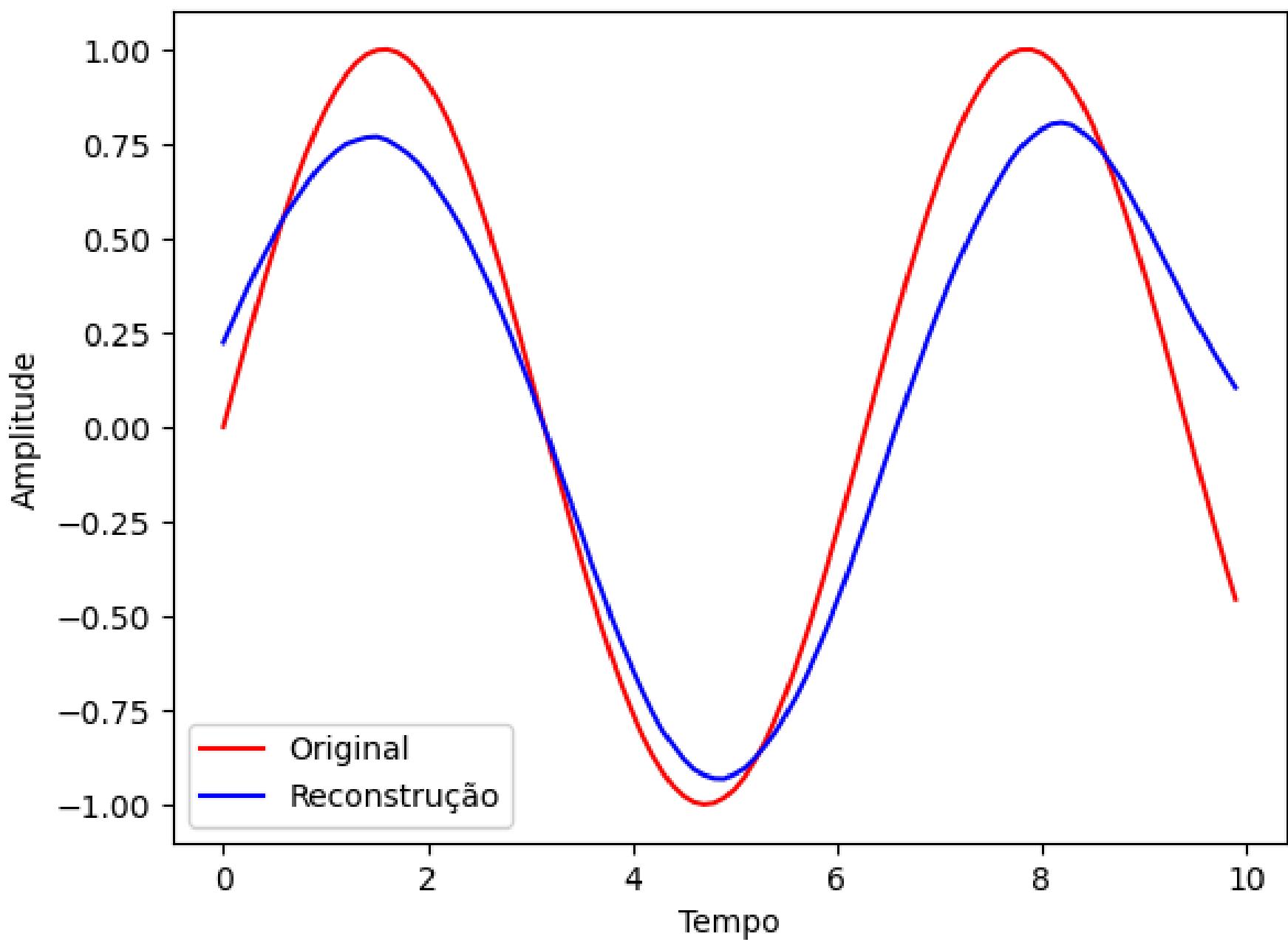
$$GELU(x) = x \cdot \Phi(x)$$



Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: GELU

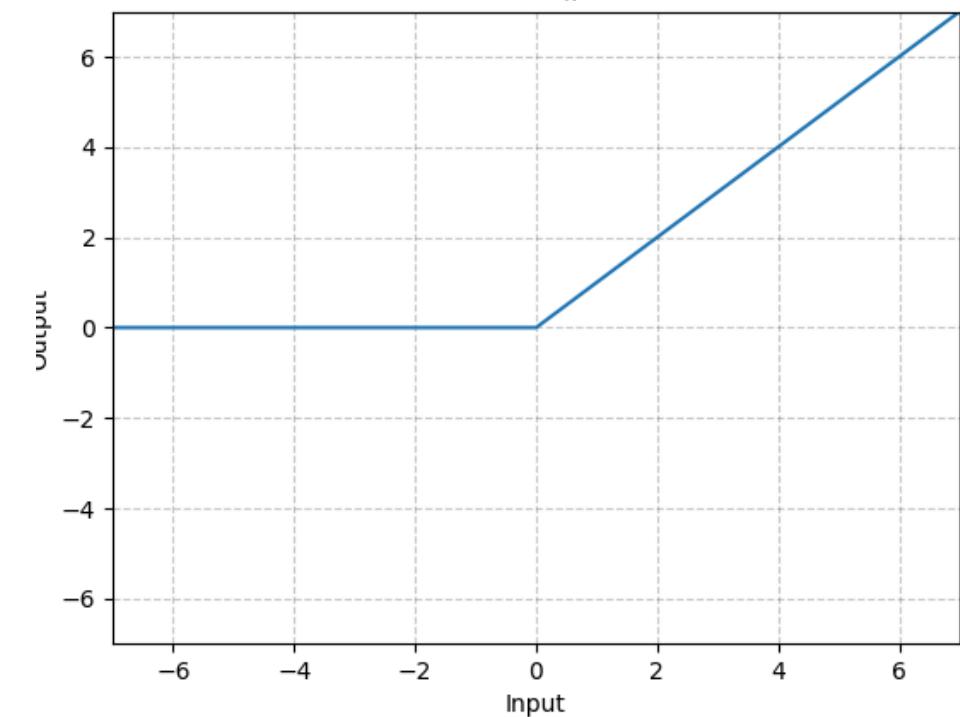


Seno reconstruído - Autoencoder - Espaço latente 1d - Ativação: GELU

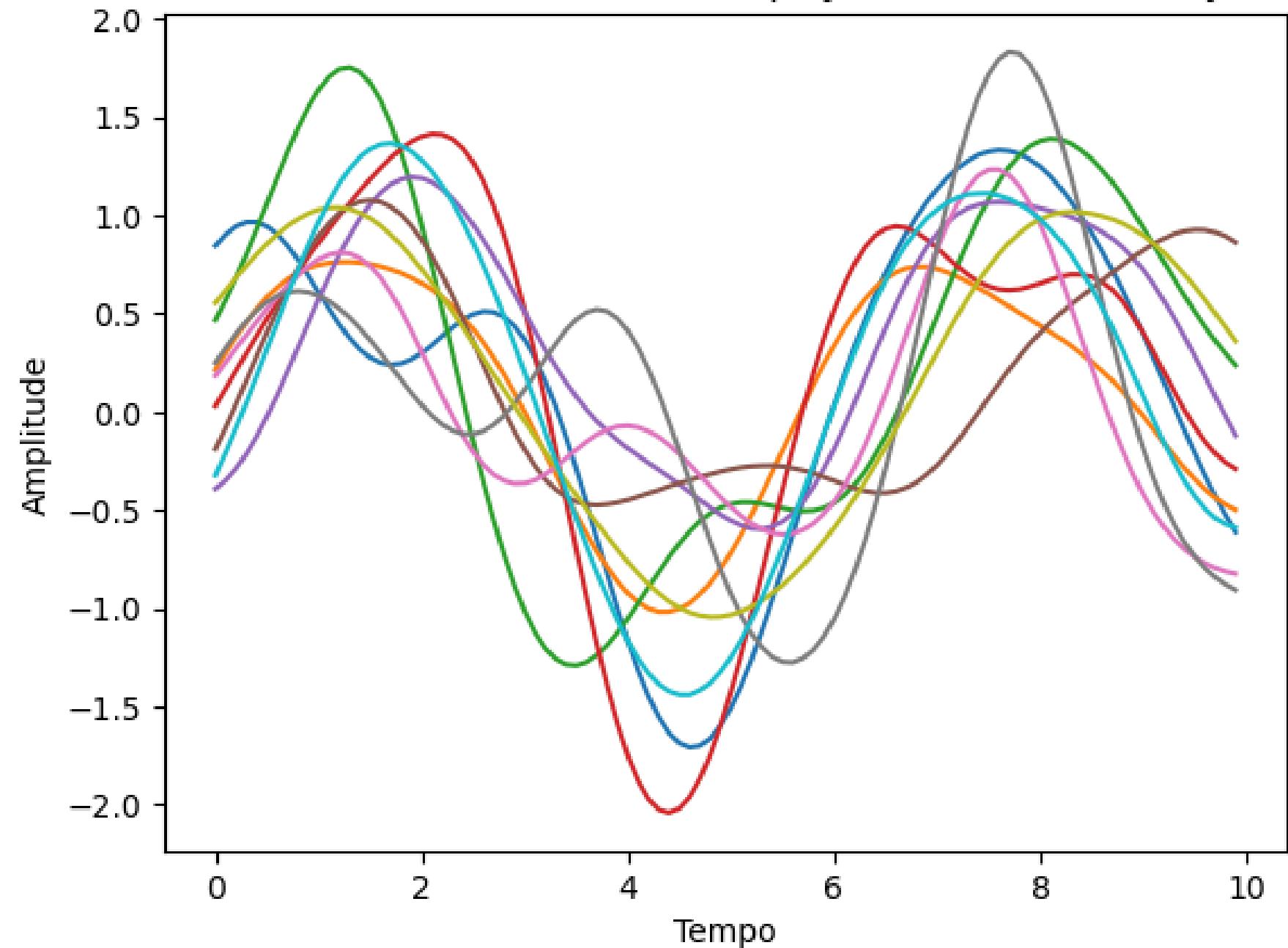


# AUTOENCODER COM ESPAÇO LATENTE MAIOR

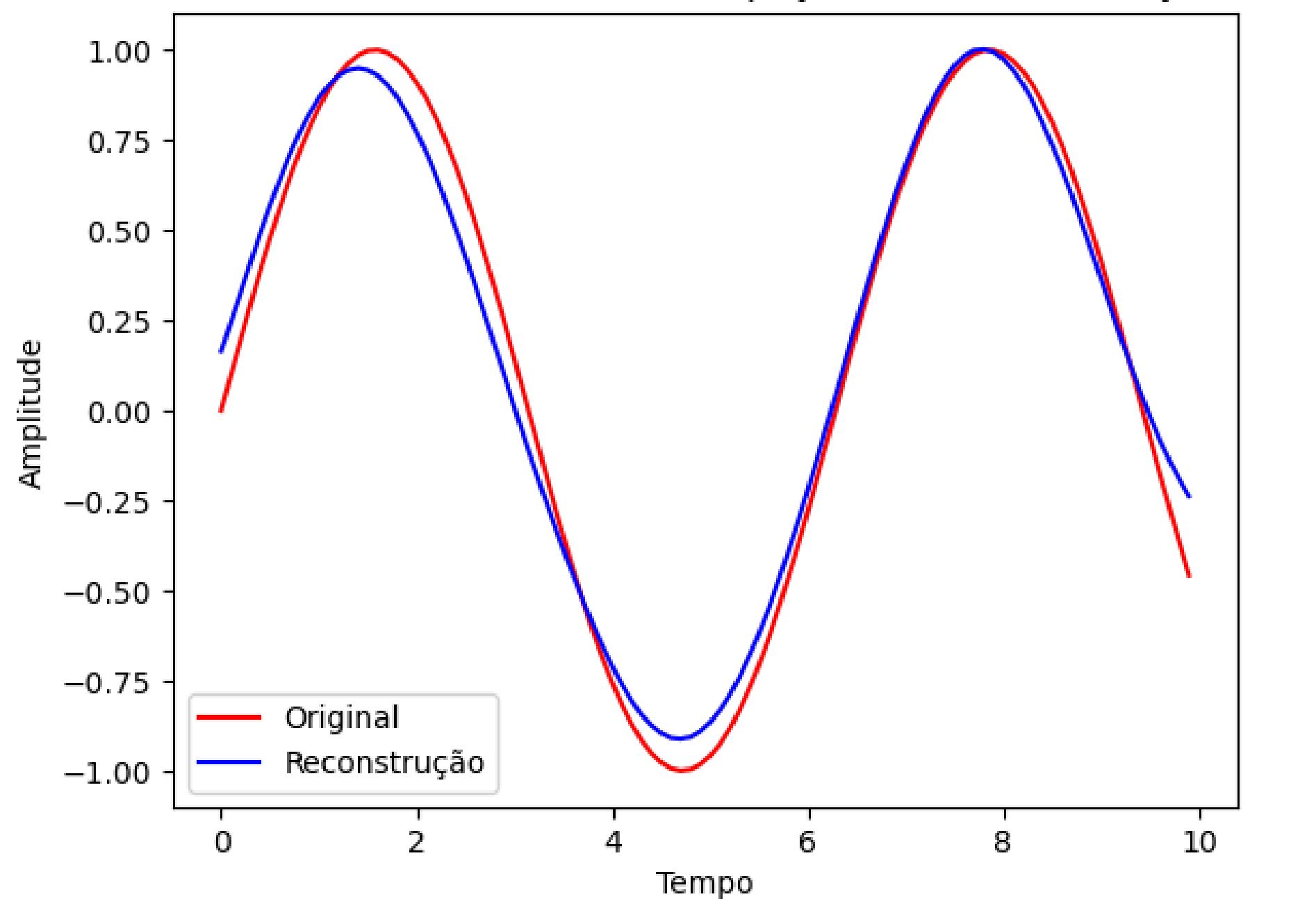
$$ReLU(x) = \max(0, x)$$

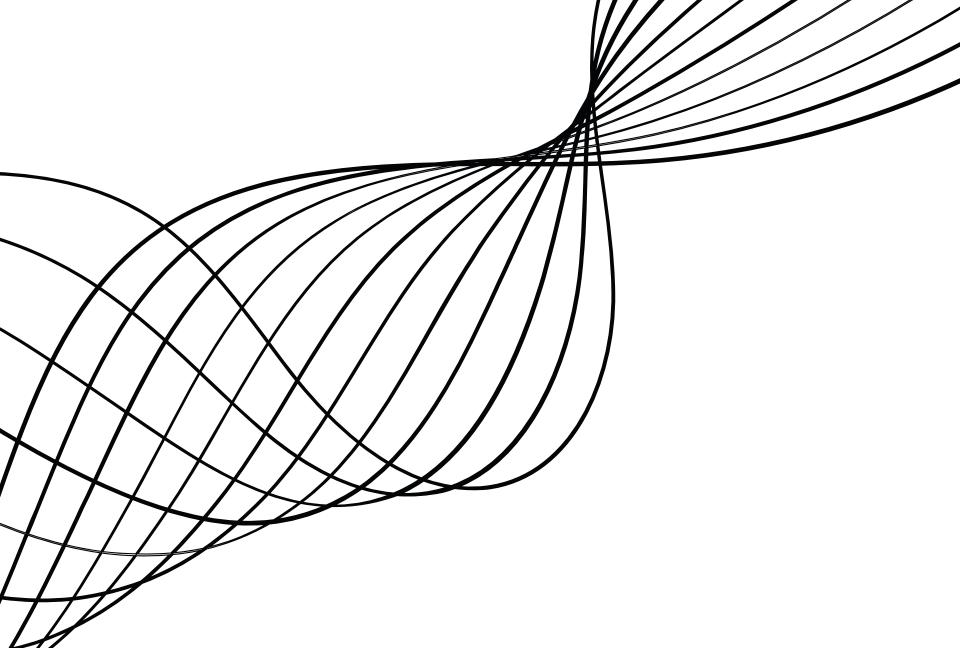


Seno reconstruído - Autoencoder - Espaço latente 5d - Ativação: ReLU



Seno reconstruído - Autoencoder - Espaço latente 5d - Ativação: ReLU

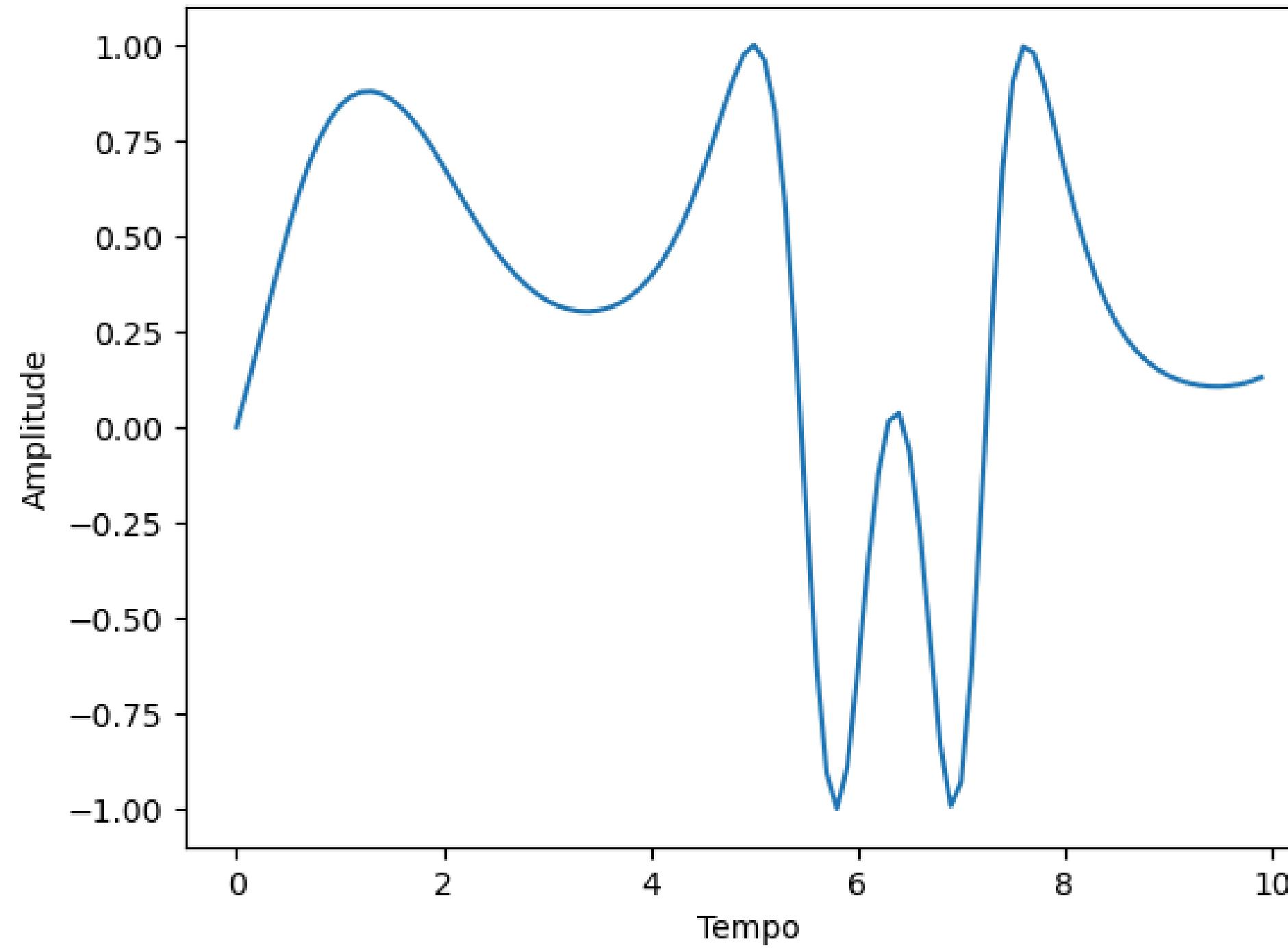




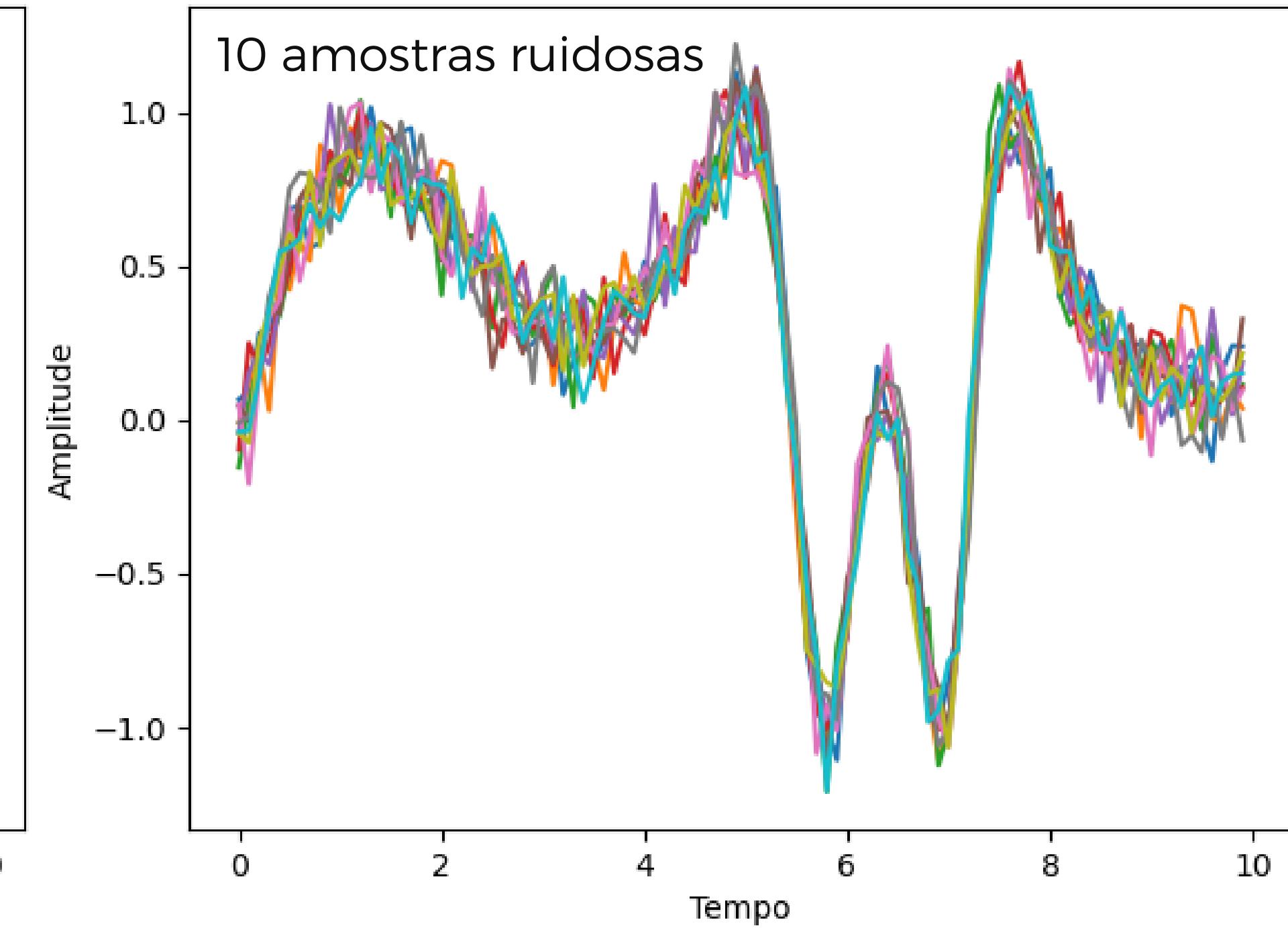
# QUANDO NÃO FUNCIONA?

$$F(x) = \sin(x^{\cos(x)})$$

F sem ruído



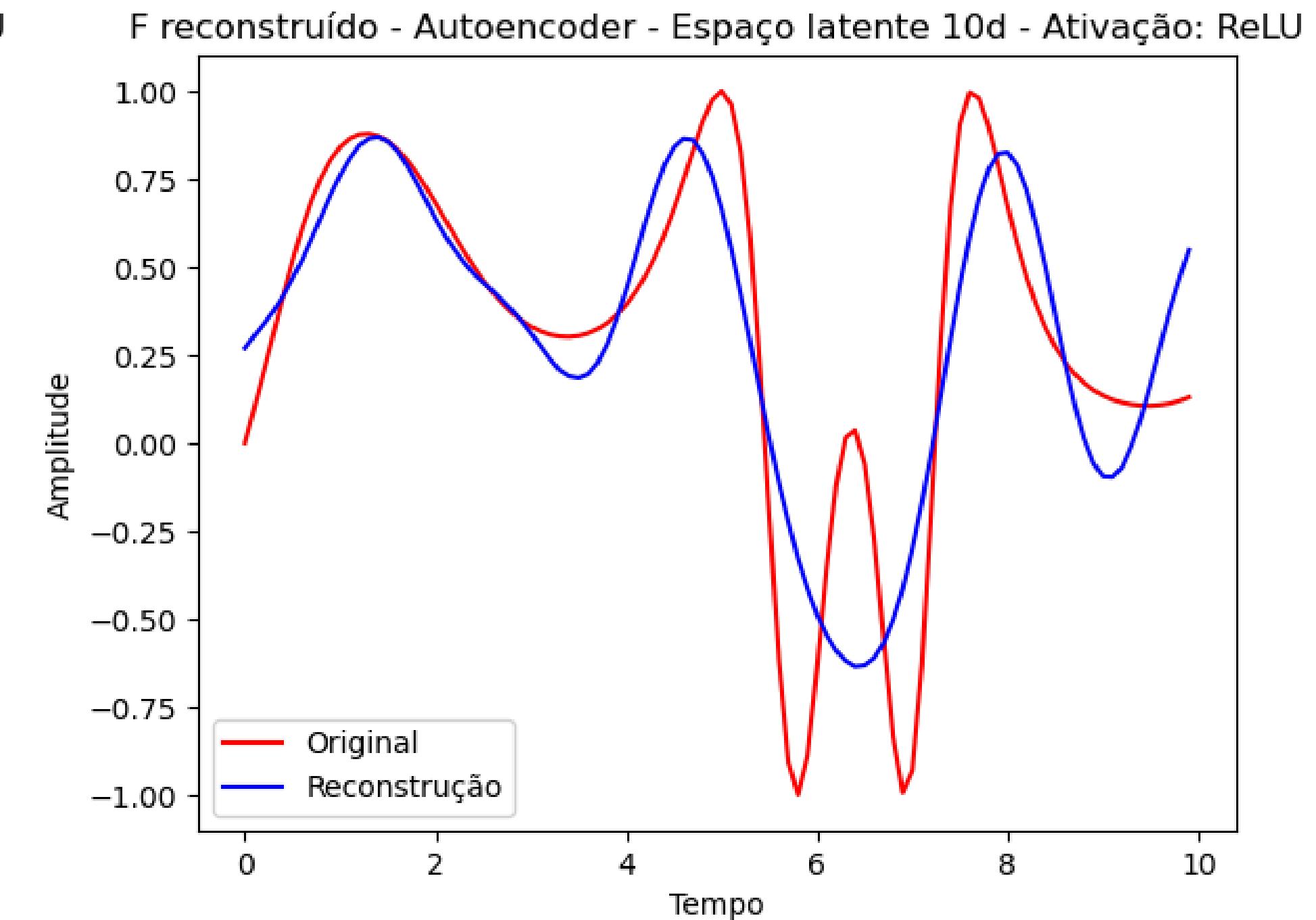
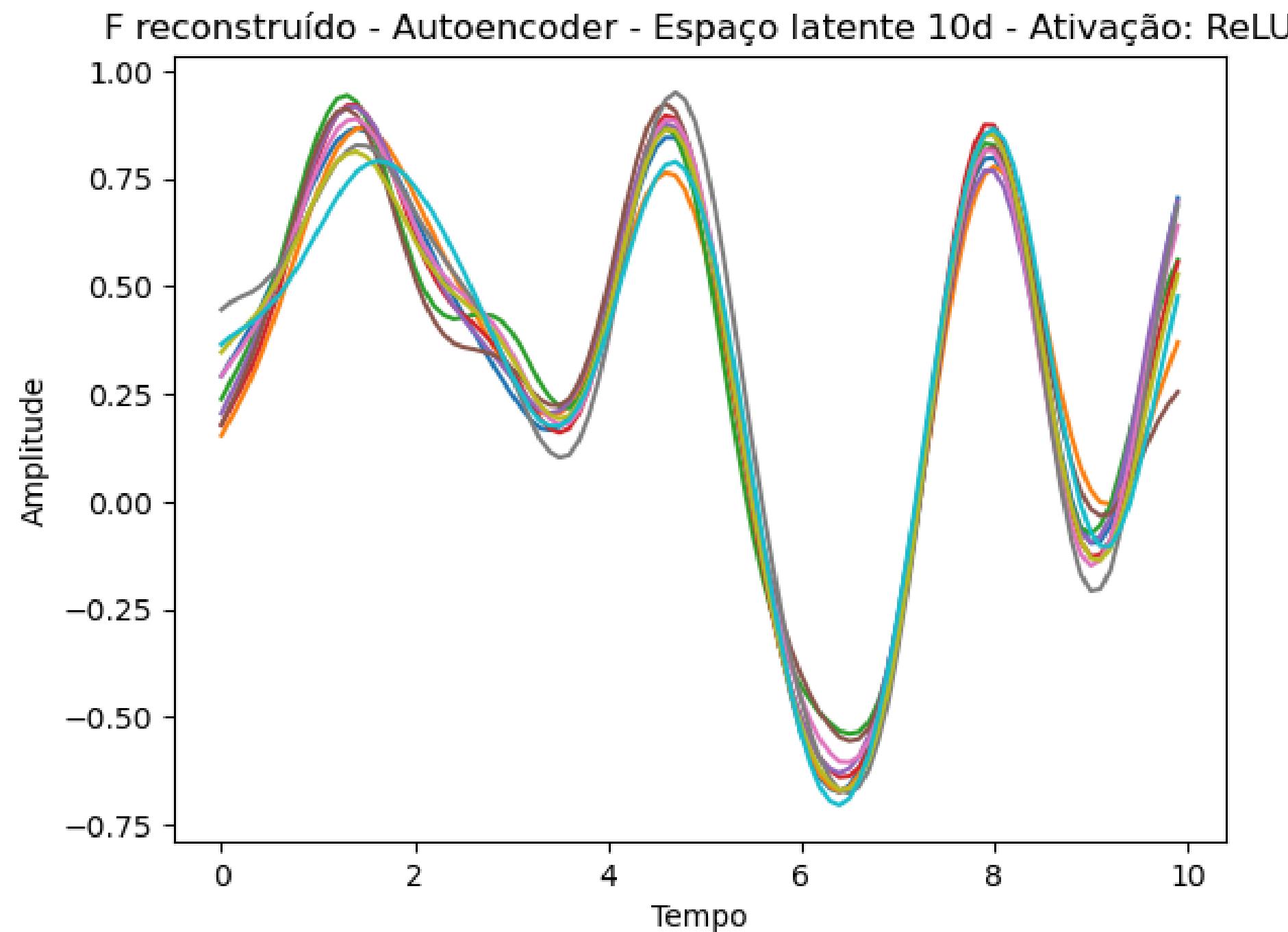
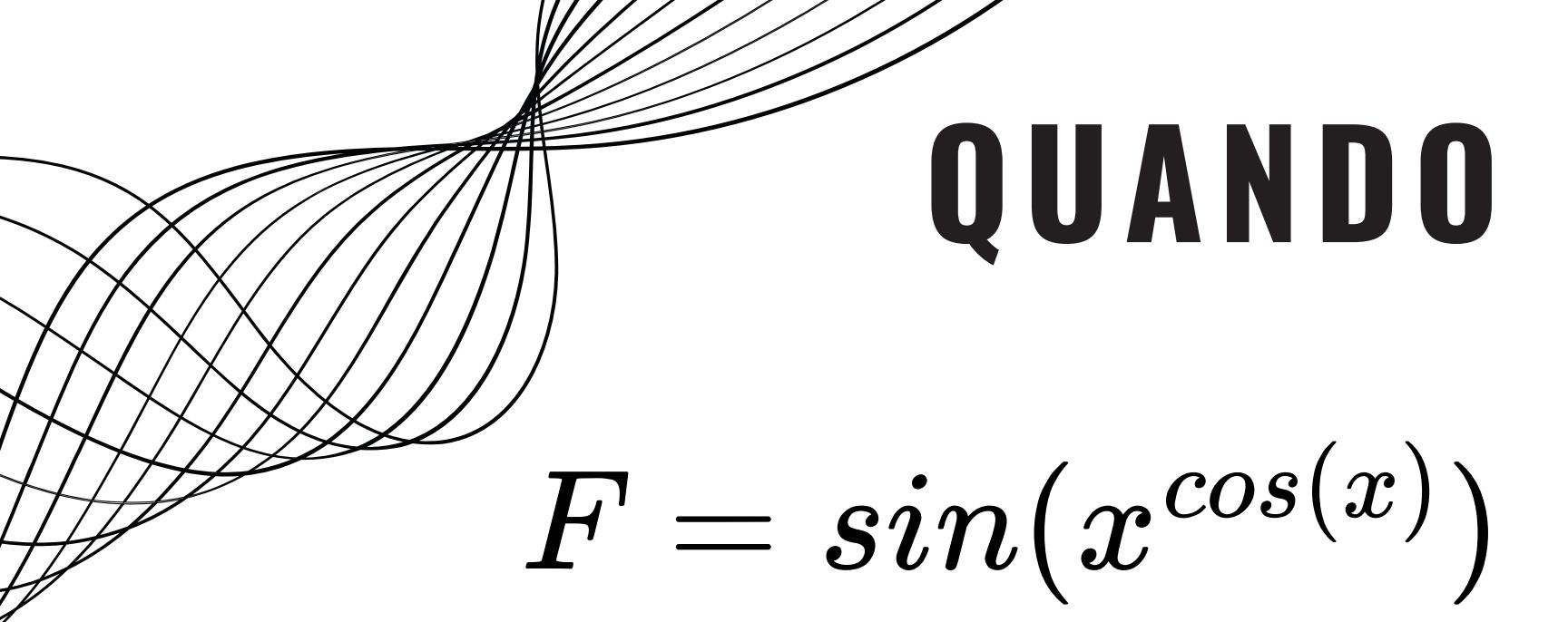
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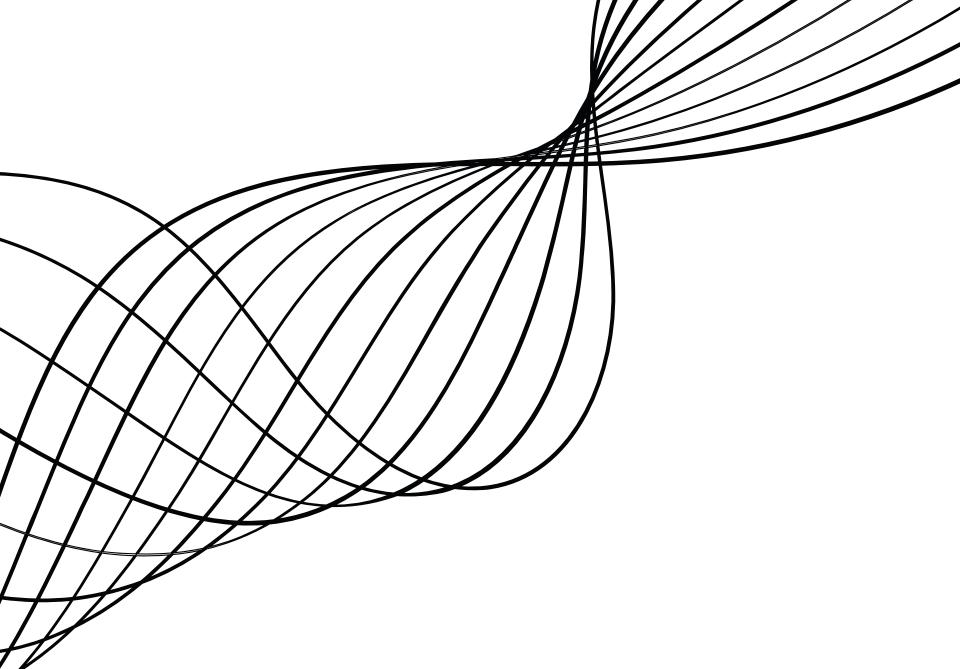


# QUANDO NÃO FUNCIONA?

$$F = \sin(x^{\cos(x)})$$

$$\text{ReLU}(x) = \max(0, x)$$

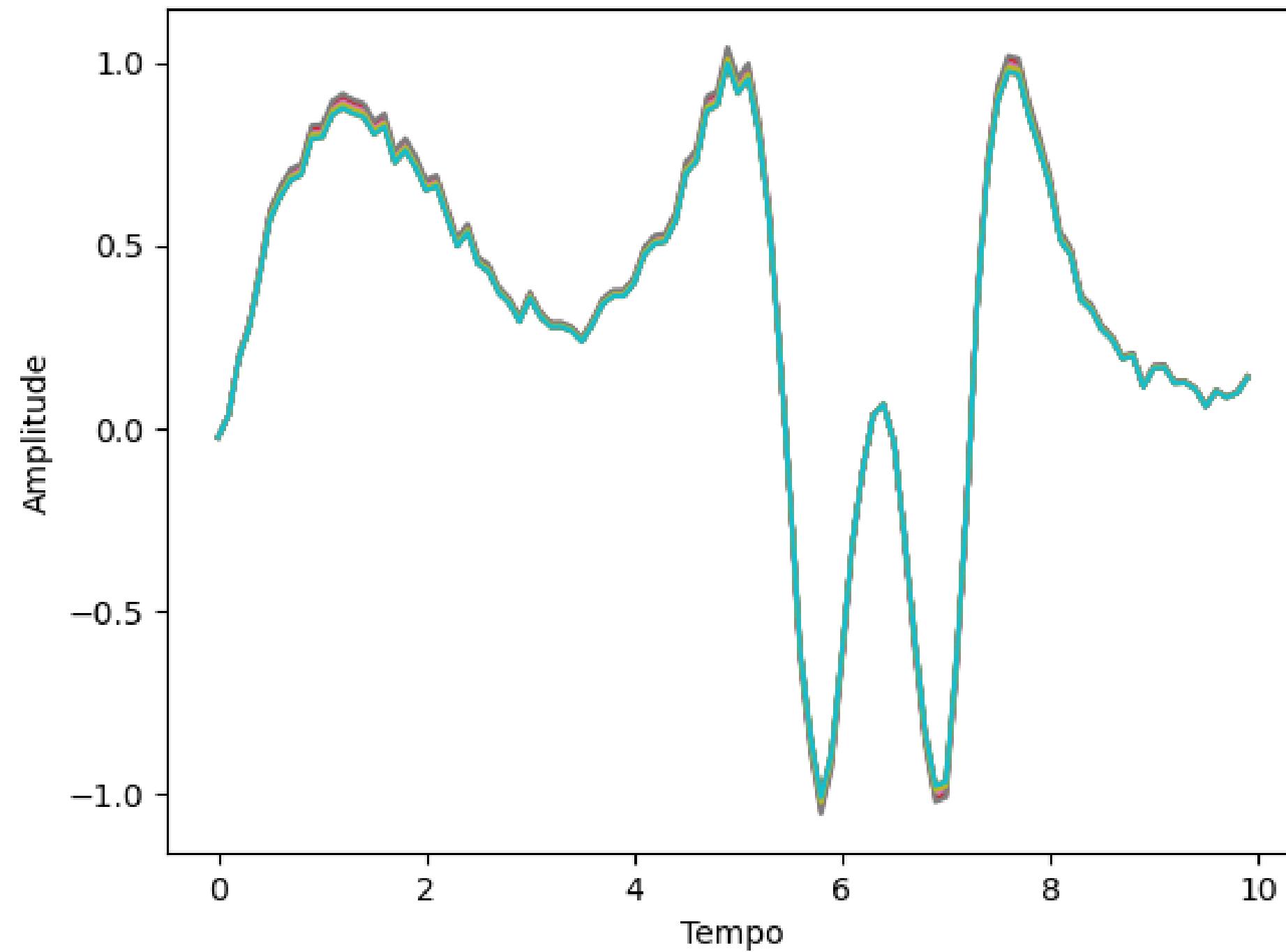




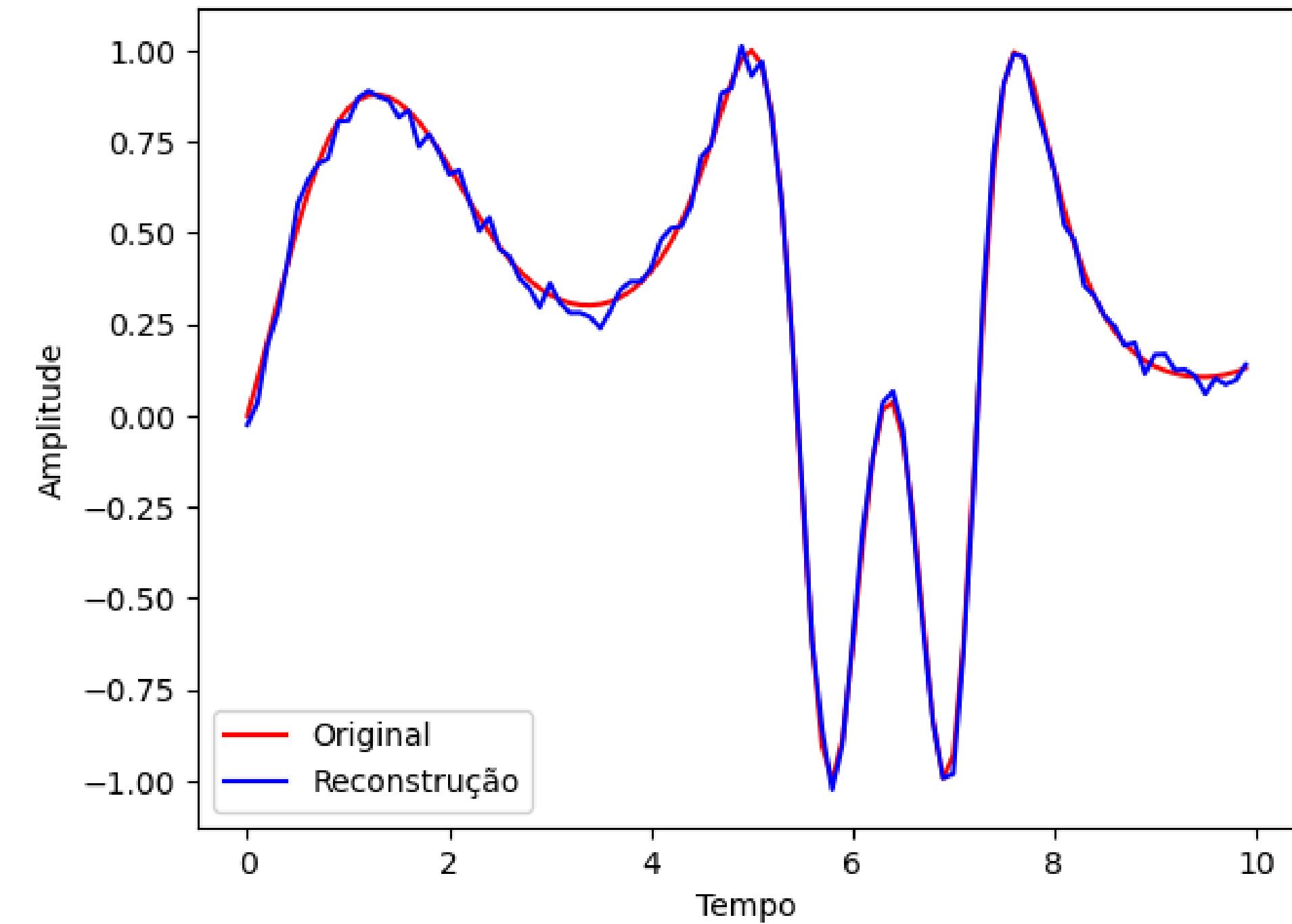
# QUANDO NÃO FUNCIONA?

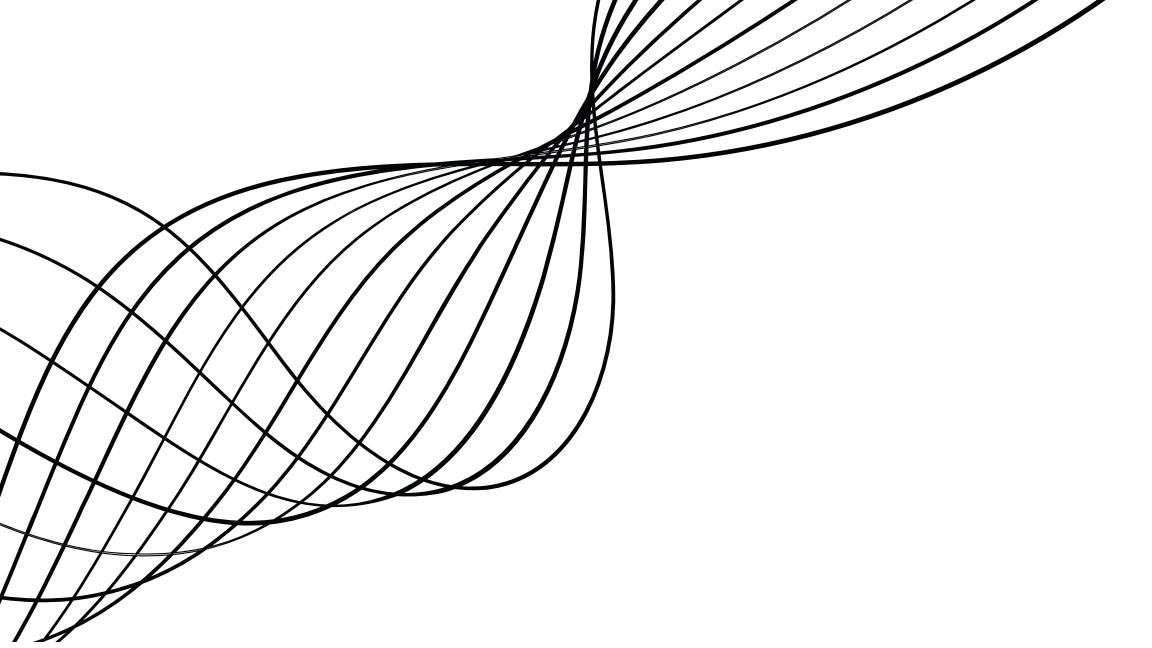
$$F = \sin(x^{\cos(x)})$$

F reconstruído - SVD posto 1



F reconstruído - SVD posto 1

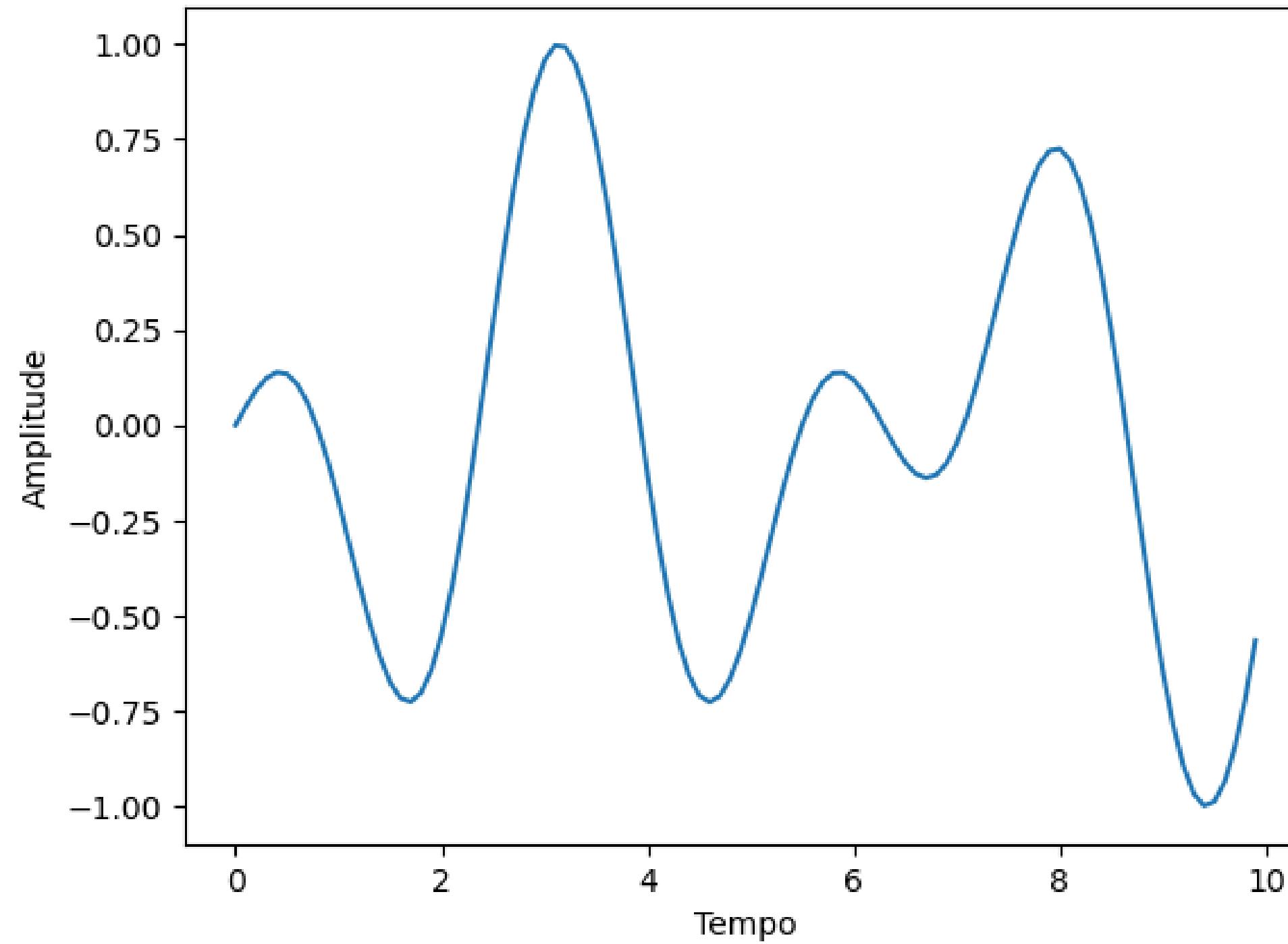




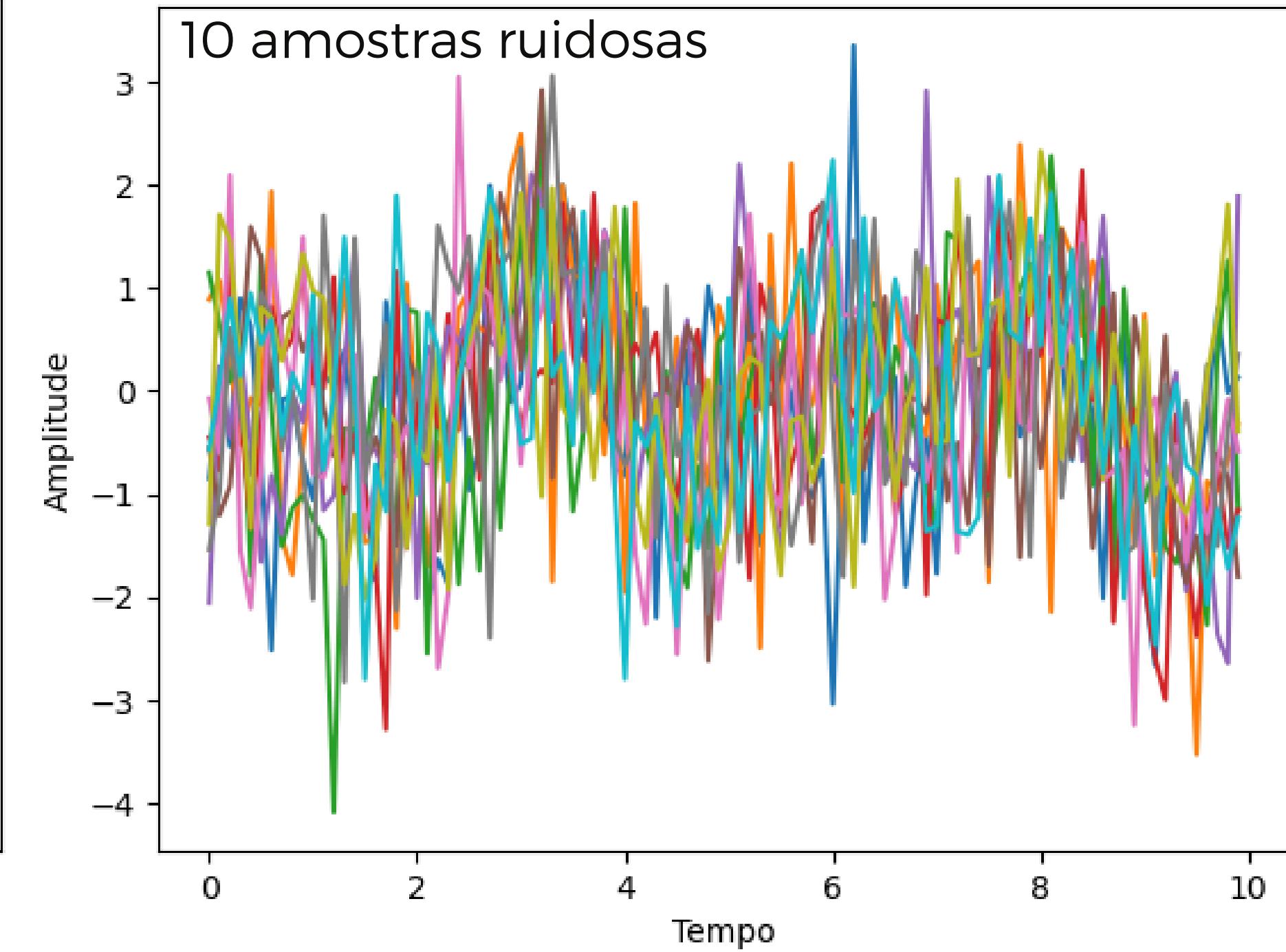
# QUANDO FUNCIONA?

$$F(x) = \sin\left(\frac{x}{2}\right) \cdot \cos(2x)$$

F sem ruído



F com ruído

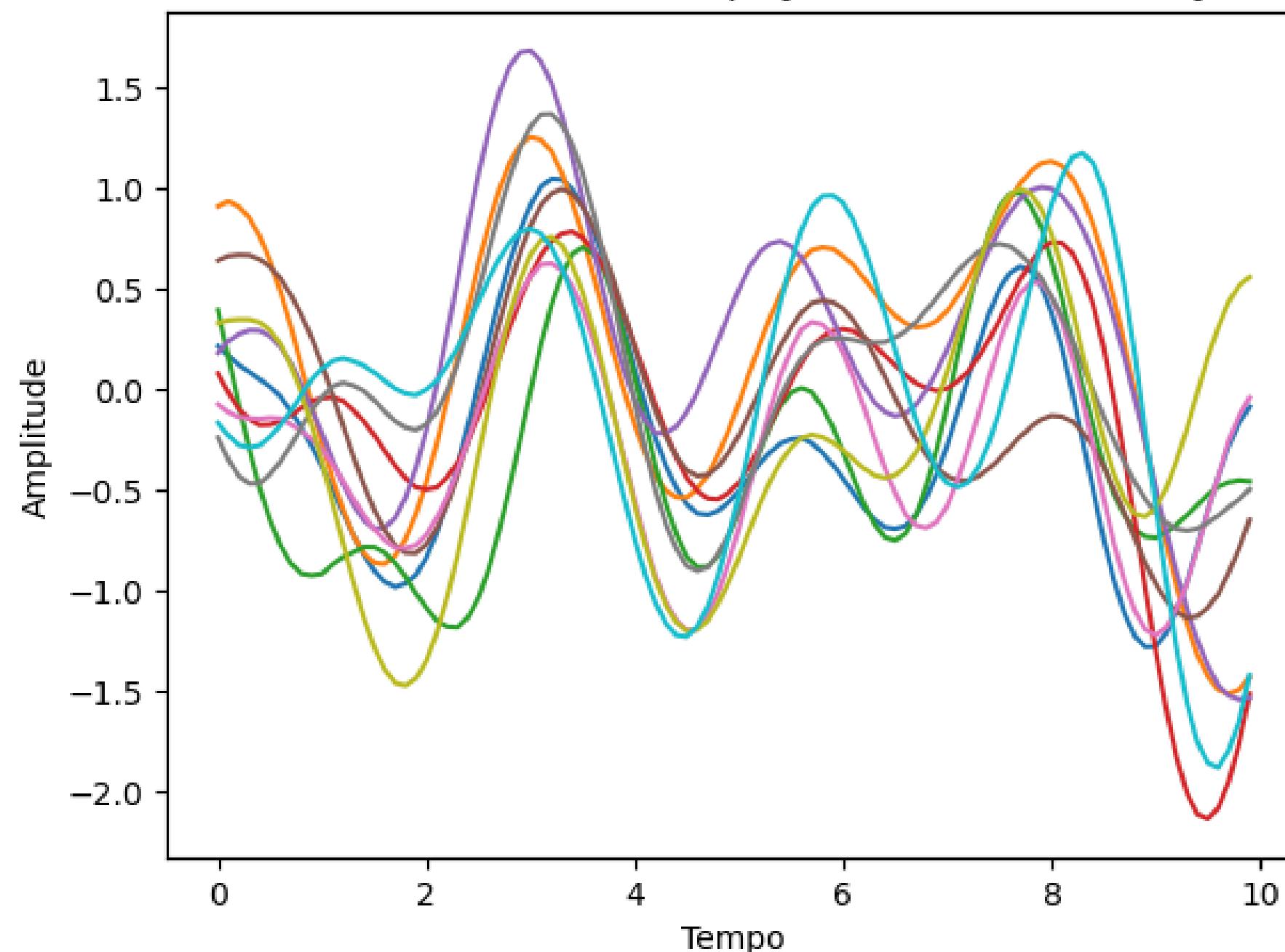


# QUANDO FUNCIONA?

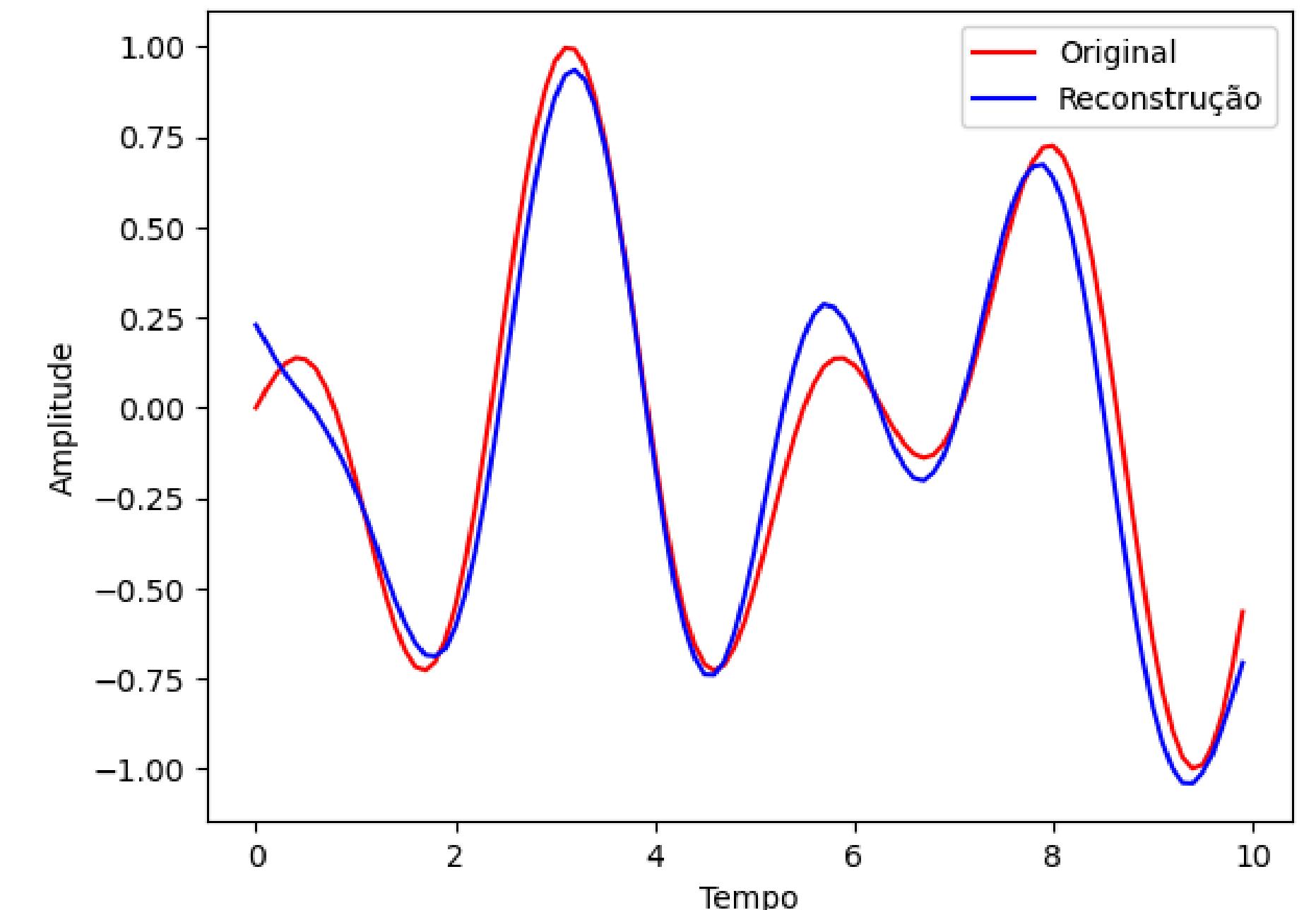
$$F = \sin\left(\frac{x}{2}\right) \cdot \cos(2x)$$

$$\text{ReLU}(x) = \max(0, x)$$

F reconstruído - Autoencoder - Espaço latente 10d - Ativação: ReLU



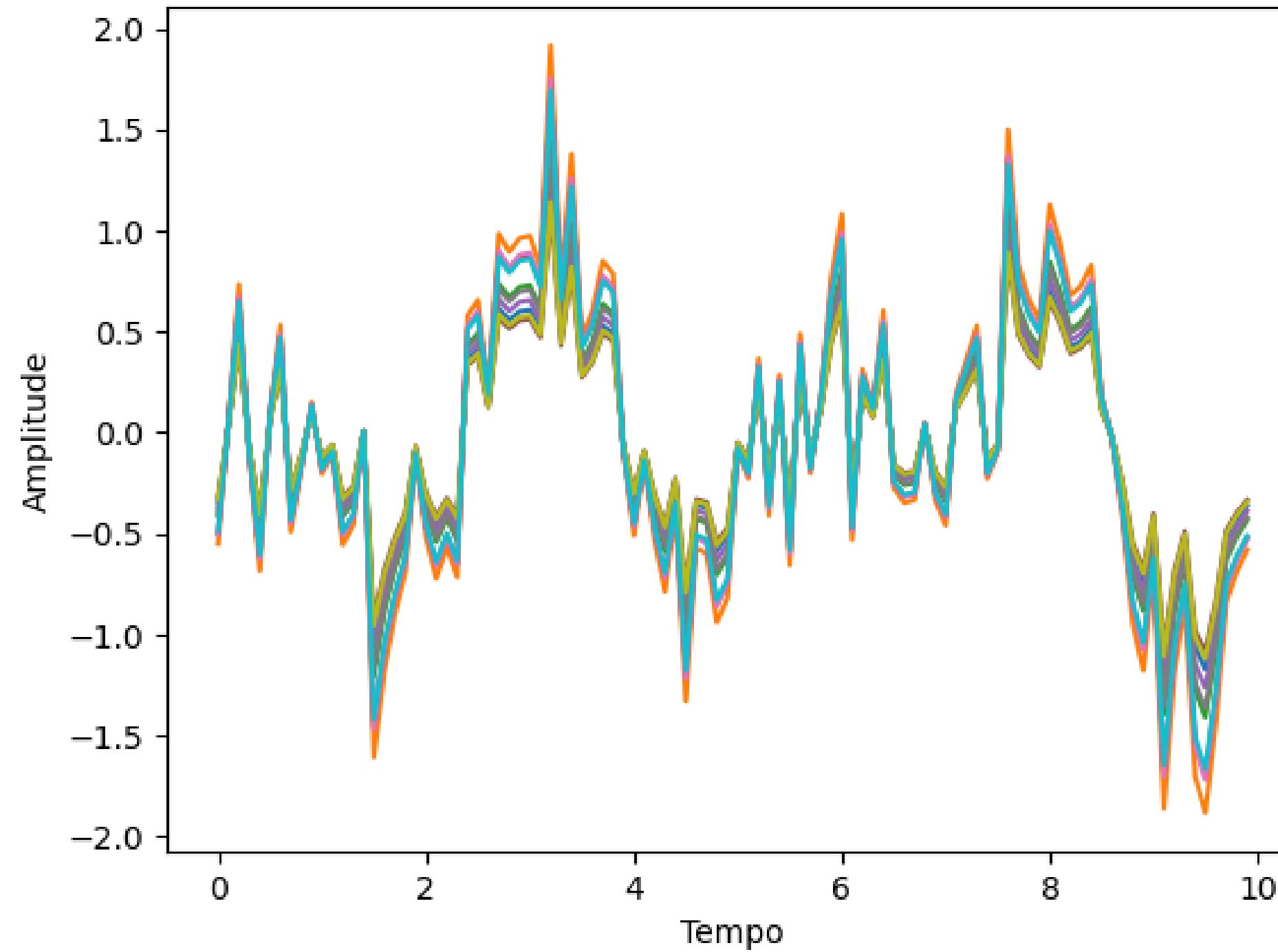
F reconstruído - Autoencoder - Espaço latente 10d - Ativação: ReLU



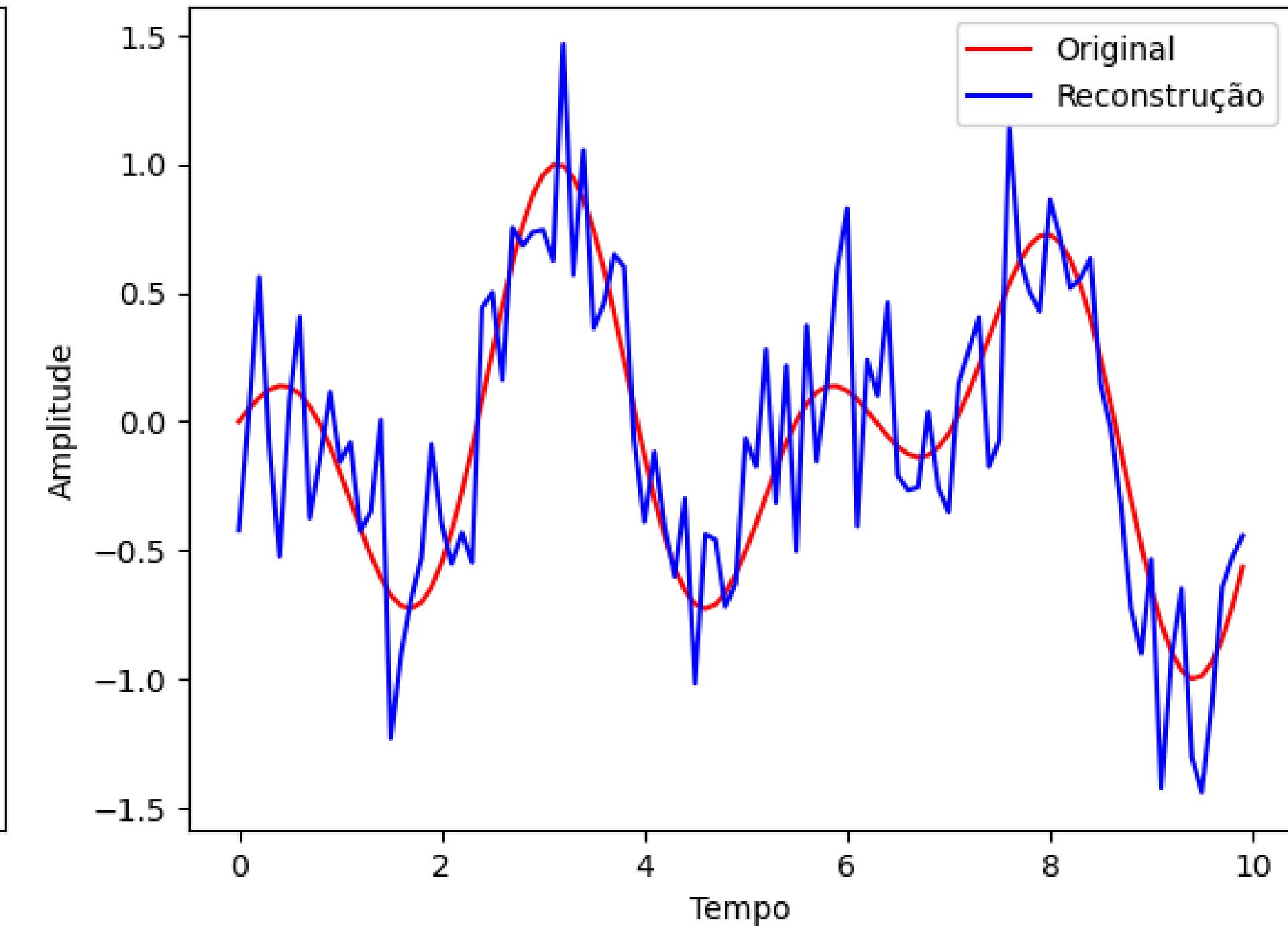
# QUANDO FUNCIONA?

$$F = \sin\left(\frac{x}{2}\right) \cdot \cos(2x)$$

F reconstruído - SVD posto 1

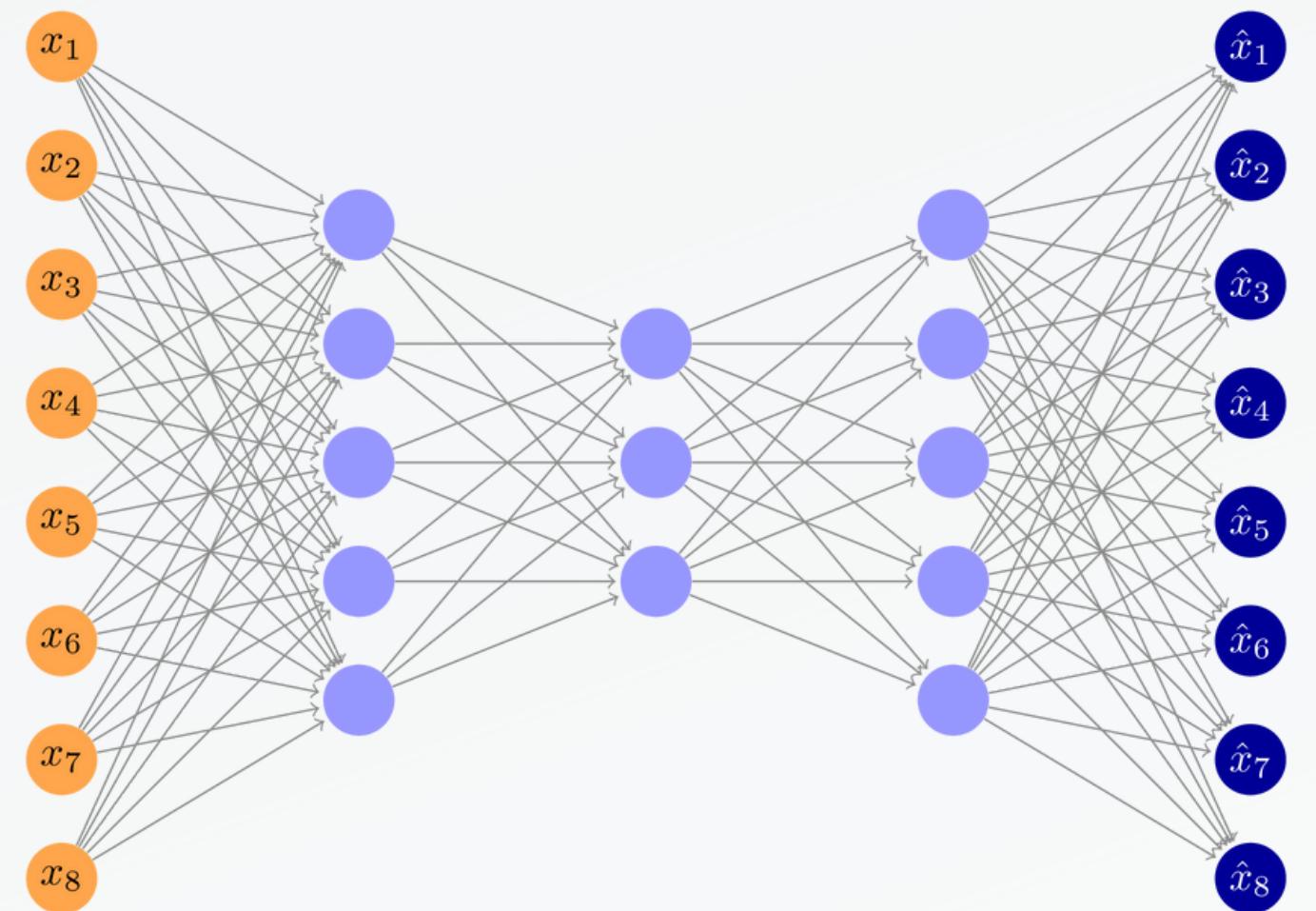


F reconstruído - SVD posto 1



# DESVANTAGENS

- Muitos hiperparâmetros:
  - Qual dimensão definir para o espaço latente?
  - Quantas camadas usar?
  - Quantos neurônios por camada?
  - Quais funções de ativação usar?
  - Qual função de custo usar?
  - Qual otimizador usar?
  - Qual learning rate usar?
- São necessárias muitas amostras para o Autoencoder performar bem
- Chance de overfitting



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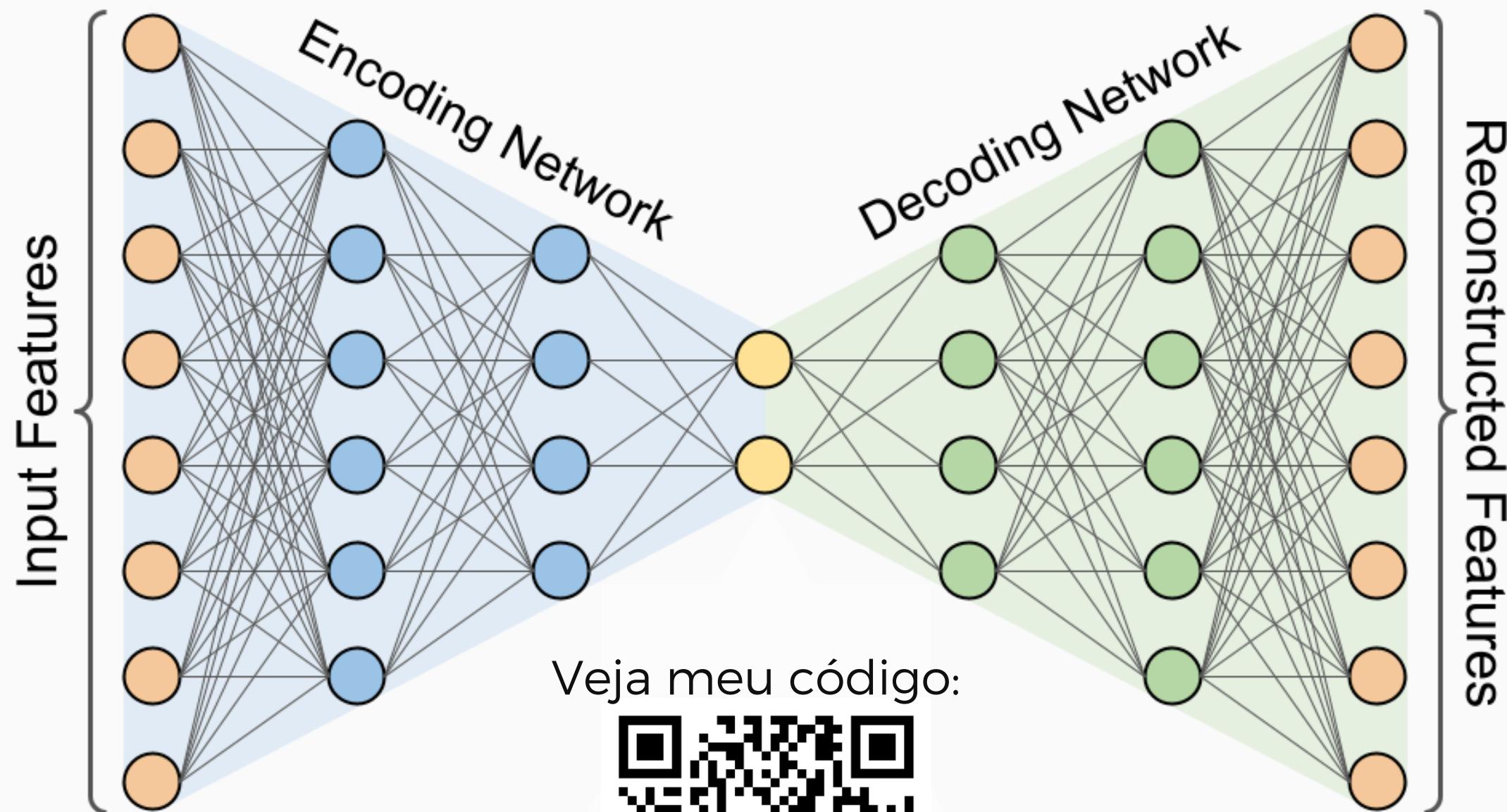
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ÁLGEBRA LINEAR APLICADA

# OBRIGADO!



PEDRO JORGE