# SIMETRÍA ESPECULAR HOMOLÓGICA

$$D^b(CY) \cong F(CY^\vee)$$

| | |
|---|---|
| Derived cat. | Fukaya cat. |
| Algebraic geometry | Symplectic geometry |

## TEOREMA DE WHITEHEAD

Una equivalencia de homotopía débil entre CW-complejos es una equivalencia de homotopía.

$\boxed{\text{WHITEHEAD + HUREWICZ}}$

Una aplicación continua $f: X \to Y$ entre CW-complejos simplemente conexos que induzca isomorfismos en homología es una equivalencia de homotopía.

# LOCALIZACIÓN DE UN ANILLO

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[d, "i"'] \arrow[dr, phantom, "\circlearrowright"] & B \\ S^{-1}A \arrow[ur, dashed, "\exists! g"'] & \end{tikzcd} \qquad \text{s.t. } f(S) \subseteq B^{\times}$$

# HOMOTOPÍA DE CADENAS

$$\begin{tikzcd}
\cdots \arrow[r] & A^{i-1} \arrow[r, "d"] \arrow[d, "f^{i-1}"'] \arrow[dl] & A^{i} \arrow[r, "d"] \arrow[d, "f^{i}"'] \arrow[dl, "s^{i}"'] & A^{i+1} \arrow[r] \arrow[d, "f^{i+1}"'] \arrow[dl, "s^{i+1}"'] & \cdots \arrow[dl] \\
\cdots \arrow[r] & B^{i-1} \arrow[r, "d"'] & B^{i} \arrow[r, "d"'] & B^{i+1} \arrow[r] & \cdots
\end{tikzcd}$$

$$\boxed{f^{i} = s^{i+1} \circ d + d \circ s^{i}}$$

# CONO DE UN MORFISMO

$$f: A^{\cdot} \longrightarrow B^{\cdot} \rightsquigarrow C(f)^{i} = A^{i+1} \oplus B^{i}$$

$$d^{i}_{C(f)} = \begin{pmatrix} -d_A^{i+1} & 0 \\ f^{i+1} & d_B^{i} \end{pmatrix}$$

$$\boxed{\text{PUPPE} \rightsquigarrow \text{COHOMOLOGÍA}}$$

$$\cdots \longrightarrow \Sigma^2 X \longrightarrow \Sigma^2 Y \longrightarrow \Sigma^2(Y/X) \longrightarrow \Sigma^3 X \longrightarrow \cdots$$

$$\Big\downarrow\!\!\rightsquigarrow \quad \mathrm{Hom}(-, K(\mathbb{Z},4))/\mathrm{htpy}$$

$$\cdots \longrightarrow H^1(X) \longrightarrow H^2(Y/X) \longrightarrow H^2(Y) \longrightarrow H^2(X) \longrightarrow \cdots$$

$$\boxed{\text{GROTHENDIECK} \rightsquigarrow \text{SERRE DUALITY}}$$

dim: $n$ $\quad m$

$f : X \longrightarrow Y \rightsquigarrow \omega_f := \omega_X \otimes f^* \omega_Y^\vee$. Entonces:

$$Rf_* R\mathcal{H}om(\mathcal{F}^\cdot, Lf^*(\mathcal{E}^\cdot) \otimes \omega_f[n-m]) \cong R\mathcal{H}om(Rf_*\mathcal{F}^\cdot, \mathcal{E}^\cdot)$$

$\Big\Downarrow \; f : X \longrightarrow \{*\} \; (\Rightarrow f_* = \Gamma(X,-))$

$$\mathrm{Hom}_D(\mathcal{F}, \omega_X[n]) \simeq \mathrm{Hom}_k(R\Gamma(\mathcal{F}), k) \Rightarrow \begin{array}{c} \mathrm{Ext}^i(\mathcal{F}, \omega_X) \\ \cong \\ H^{n-i}(X, \mathcal{F})^\vee \end{array}$$

# EXTENSIÓN DE KAN (IZQ.)

$F : \mathcal{A} \to \mathcal{B}$

$$
\begin{tikzcd}
 & K(\mathcal{A}) \arrow[dl, "Q_A"'] \arrow[dr, "Q_B \circ K(F)"] \arrow[d, phantom, "\Downarrow"] & \\
D(\mathcal{A}) \arrow[rr, dashed, "RF"] \arrow[rr, bend right=40, "G"'] \arrow[rr, phantom, bend right=20, "\exists!\Downarrow"] & {} & D(\mathcal{B})
\end{tikzcd}
$$

# DIAGRAMA PARA FUNCTORES DERIVADOS

$$
\begin{tikzcd}
K^+(\mathcal{I}_{\mathcal{A}}) \arrow[r, hook] \arrow[dr, bend left, "L(\simeq)"] & K^+(\mathcal{A}) \arrow[r, "K(F)"] \arrow[d, "Q_{\mathcal{A}}"] & K^+(\mathcal{B}) \arrow[d, "Q_{\mathcal{B}}"] \\
 & D^+(\mathcal{A}) \arrow[ul, bend left, "L^{-1}"] \arrow[r, "RF"'] & D^+(\mathcal{B})
\end{tikzcd}
$$

$$RF = Q_{\mathcal{B}} \circ K(F) \circ L^{-1}$$

# SUBCATEGORÍAS TRIANGULADAS

$\mathcal{A} \subseteq \mathcal{T}$ estrictamente plena es triang. si:

1) $\mathcal{A}[i] \subseteq \mathcal{A} \quad \forall i \in \mathbb{Z}$

2) $$\left.\begin{array}{ccccccc} A & \longrightarrow & B & \longrightarrow & C & \longrightarrow & A[1] \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} & & & & \\ \mathcal{A} & & \mathcal{A} & & & & \end{array}\right\} \Longrightarrow C \in \mathcal{A}$$