

# ENUMERATIVE GEOMETRY

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### 1. TALK 1 — THE CHOW RING

**1.1. Algebraic varieties.** Roughly speaking, they are mathematical objects glued from zero loci of polynomials which globally satisfy a certain Hausdorffness property. They are called *projective* if they can be embedded as a closed subset in some projective space, and *irreducible* if as a topological space they cannot be expressed as a union of two proper closed subsets.

From now on we follow [?] and assume all varieties to be irreducible unless otherwise specified.

#### 1.2. The Chow group.

**Definition 1.1.** Let  $X$  be a variety. The *group of cycles* on  $X$ , denoted  $Z(X)$ , is the free abelian group generated by the set of subvarieties of  $X$ .

**Example 1.2.** If  $X$  is a curve, then any cycle  $\xi \in Z(X)$  has the form

$$n_1 P_1 + \dots + n_r P_r + mX,$$

where  $r \in \mathbb{N}$ ,  $n_1, \dots, n_r, m \in \mathbb{Z}$  and  $P_1, \dots, P_r \in X$  are points.

**Definition 1.3.** Denote by  $\text{Rat}(X) \subseteq Z(X)$  the subgroup generated by differences of the form

$$\Phi \cap (\{t_0\} \times X) - \Phi \cap (\{t_1\} \times X),$$

where  $t_0, t_1 \in \mathbb{P}^1$  are two points and  $\Phi \subseteq \mathbb{P}^1 \times X$  is a subvariety not contained in any fibre  $\{t\} \times X$ .

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We say that two cycles  $\xi_1, \xi_2 \in Z(X)$  are *rationally equivalent* if their difference is in  $\text{Rat}(X)$ .

#### REFERENCES

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