ENUMERATIVE GEOMETRY WEDNESDAY SEMINAR

GK1821 "COHOMOLOGICAL METHODS IN GEOMETRY"

List of Talks (assuming 1h 30min / Talk)

Some suggestions that apply to all the talks:

- Follow *3264 and all that*, by Eisenbud and Harris [EH16].
- Work only over $\mathbb{C}$, even if it is not necessary at many points.
- Try to avoid scheme-theoretic technical details.
- Try to draw many pictures and focus on examples.

But feel free to do otherwise if you want/need at some point!

1. Talk 1 — The Chow ring. Affine spaces.

1.1. **Algebraic varieties.** These have already appeared often in past Wednesday seminars, so hopefully we can keep this to a very brief introduction/recollection. Possible things that may be useful to say:

- Huge open subsets (no strict Hausdorffness, no local $\mathbb{A}^n$-ness).
- The union of two intersecting lines is connected but not irreducible.
- Intersection of irreducible stuff is not necessarily irreducible.
- Dimension of varieties is defined by chains of irreducibles.

1.2. **Chow groups** [EH16, §1.2.1 and 1.2.2]**.** Cycles and rational equivalence. [EH16, Prop. 1.4] and [EH16, Prop. 1.10] are good to know. A nice picture to see what can happen is [EH16, Fig. 1.2].

1.3. **Ring structure** [EH16, Thm. 1.5]**.** Generic transversality and moving lemma [EH16, Thm. 1.6]. What goes wrong without the smoothness assumption? Example in [EH16, p. 20].

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1.4. **Chow groups of affine spaces.** For any variety X, the equivalence class $[X]$ is a free generator of $A^0(X)$. This can be argued using irreducibility and dimension. In the case of affine spaces, this free cyclic group is all there is [EH16, Prop. 1.13]. A nice picture to visualise the proof is [EH16, Fig. 1.7].

1.5. **Functoriality [EH16, §1.3.6].** Proper pushforward and flat pullback without technical details. I wouldn't define properness and flatness too seriously, but it is good to know that inclusions of open subsets are flat morphisms and that any morphism between projective varieties is proper. Degree map [EH16, Prop. 1.21]. [EH16, Thm. 1.23] without details of the proof.

2. Talk 2 — Affine stratifications. Projective spaces.

2.1. **Brief recollection of previous talk.** Cycles, rational equivalence, functoriality.

2.2. **Mayer–Vietoris and excision [EH16, §1.3.4].**

2.3. **Affine stratifications [EH16, §1.3.5].** With examples of what is or isn't a stratification, examples of quasi-affine stratifications that are not affine, etc. Totaro's theorem [EH16, Thm. 1.18] is nice to know, although it won't be used later on.

2.4. **Chow ring of projective space.** [EH16, Thm. 2.1] and corollaries [EH16, Cor. 2.2 and Cor. 2.3]. Bézout's theorem as a consequence of [EH16, Thm. 2.1].

3. Talk 3 — Grassmannian of lines in space

3.1. **Kleiman's transversality:** [EH16, Thm. 1.7] and its proof in the case of $\mathrm{GL}_n(\mathbb{C})$.

3.2. **Grassmannians.** definition and projective spaces as a particular case. Plücker embedding and affine open cover already in the case of the Grassmannian of lines in $\mathbb{P}^3$. Schubert cycles and stratification of $\mathbb{G}(1,3)$.

3.3. **Computation of the Chow ring:** [EH16, Thm. 3.10]. Mentioning explicitly how to use Kleiman's transversality and the method of undetermined coefficients during the proof.

As an immediate consequence of the previous point and transversality: how many lines in $\mathbb{P}^3$ intersect 4 general lines?

4. Talk 4 — Specialisation and Knutson–Tao puzzles

Schubert calculus studies how to compute products of Schubert classes. There are many different approaches to this, e.g. using Young diagrams [EH16, §4.5]. In this talk we will instead discuss an approach due to Knutson and Tao, which is very visual and combinatoric as well.

4.1. **Brief recollection of previous talk.** Using the method of undetermined coefficients to compute the square σ_1^2 of the Schubert class of lines intersecting a given line.

4.2. **(Static) specialisation** [EH16, §3.5.1]**.** This is another useful technique to compute products of Schubert classes. As an example we can use it to compute σ_1^2 in a different way. This will require describing the tangent space of the Schubert cycle of lines intersecting a given line L at a point different from L [EH16, Exe. 3.26].

4.3. **Knutson–Tao puzzles.** These give a nice visual tool to compute products of Schubert classes. Besides of the original article [add ref], there is also a nice YouTube video discussing them [add link]. Again, one can use the computation of σ_1^2 as an example, as is done in the video already.

5. Talk 5 — Chern classes and lines on a cubic surface

Use them to sketch the fact that there are 27 lines on a cubic surface. The number 27 should come out, but one should also explain clearly what remains to be shown in order to have an actual proof.

References

[EH16] David Eisenbud and Joe Harris. *3264 and all that—a second course in algebraic geometry*. Cambridge University Press, Cambridge, 2016.

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