ENUMERATIVE GEOMETRY

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Contents

1. Talk 1 — The Chow ring

1.1. **Algebraic varieties.** Roughly speaking, they are mathematical objects glued from zero loci of polynomials which globally satisfy a certain Hausdorffness property. They are called *projective* if they can be embedded as a closed subset in some projective space, and *irreducible* if as a topological space they cannot be expressed as a union of two proper closed subsets.

From now on we follow [Har77] and assume all varieties to be irreducible unless otherwise specified.

1.2. **The Chow group.**

Definition 1.1. Let X be a variety. The *group of cycles* on X, denoted $Z(X)$, is the free abelian group generated by the set of subvarieties of X.

Example 1.2. If X is a curve, then any cycle $\xi \in Z(X)$ has the form

$$n_1P_1 + \ldots + n_rP_r + mX,$$

where $r \in \mathbb{N}$, $n_1, \ldots, n_r, m \in \mathbb{Z}$ and $P_1, \ldots, P_r \in X$ are points.

Definition 1.3. Denote by $\mathrm{Rat}(X) \subseteq Z(X)$ the subgroup generated by differences of the form

$$\Phi \cap (\{t_0\} \times X) - \Phi \cap (\{t_1\} \times X),$$

where $t_0, t_1 \in \mathbb{P}^1$ are two points and $\Phi \subseteq \mathbb{P}^1 \times X$ is a subvariety not contained in any fibre $\{t\} \times X$.

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We say that two cycles $\xi_1, \xi_2 \in Z(X)$ are *rationally equivalent* if their difference is in $\mathrm{Rat}(X)$. The quotient group $Z(X)/\mathrm{Rat}(X)$ is called the *Chow group* of X, denoted by $A(X)$.

Remark 1.4.

(1) It follows from Krull's principal ideal theorem that $A(X)$ is still graded by dimension [EH16, Proposition 1.4], hence also by codimension.
(2) One can equivalently describe $\mathrm{Rat}(X)$ as the subgroup generated by all divisors of rational functions on all subvarieties of X [EH16, Proposition 1.10].
(3) In particular, linear and rational equivalence agree for divisors and
$$A^1(X) = \mathrm{Cl}(X).$$
This explains —I think— the intuition for varying an effective divisor within its linear equivalence class as "moving it around our variety".

References

[EH16] David Eisenbud and Joe Harris. *3264 and all that—a second course in algebraic geometry*. Cambridge University Press, Cambridge, 2016. ↑ 1, 2

[Har77] Robin Hartshorne. *Algebraic geometry*. Springer-Verlag, New York-Heidelberg, 1977. Graduate Texts in Mathematics, No. 52. ↑ 1.1

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