SOLUTIONS TO EXERCISES

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The exercises below are taken from [Har77, Chapter IV].

Exercise 1.1. Let X be a curve, and let $P \in X$ be a point. Then there exists a nonconstant rational function $f \in K(X)$, which is regular everywhere except at P.

Solution. We need to show that $l(nP) > 1$ for some non-zero $n \in \mathbb{N}$. Indeed, Riemann–Roch implies that this holds for all large enough $n \in \mathbb{N}$, as explained in [Har77, Remark 1.3.2]. ■

References

[Har77] Robin Hartshorne. *Algebraic geometry*. Graduate Texts in Mathematics, No. 52. Springer-Verlag, New York-Heidelberg, 1977, pp. xvi+496. ISBN: 0-387-90244-9.

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