TALK ON SITES AND TOPOLOGIES

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ABSTRACT. Notes for a talk on Sites and Topologies as part of the seminar on Topos theory and Logic organized by Luca Terenzi at the University of Freiburg during the Winter Term 2021/2022.

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The main references for this talk are [SGA4] and [Sta21]. When in doubt, e.g. about notation and conventions, we will try to follow the more modern reference [Sta21] instead of [SGA4].

1. RECOLLECTIONS FROM PREVIOUS TALKS

- Recall the notion of presite and pretopology from previous talk, and the example that different pretopologies on the same categories may have the same associated category of sheaves.
- Using the Yoneda Lemma, explain how to every covering family of an object in a presite can be attached a sub-functor of the corresponding representable functor, and introduce the notion of *sieve* on an object in a category.

2. GROTHENDIECK TOPOLOGIES

- Define the notion of a *Grothendieck topology* on a category and of *site*.
- Give examples including the *chaotic* topology, the *discrete* topology, and the topology *associated* to a given pretopology.

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3. Topologies and sheaves

- Define the topology *generated* by a family of sieves. Dually, define the *finest* topology for which all presheaves in a given family are separated/sheaves.
- Show that this defines an order-reversing correspondence between Grothendieck topologies and categories of sheaves. Deduce that a pretopology and the associated topology define the same category of sheaves.
- Introduce the notion of *canonical* and *sub-canonical* topology. Explain how to characterize them via the Yoneda embedding.

References

[SGA4] *Théorie des topos et cohomologie étale des schémas. Tome 1: Théorie des topos*. Lecture Notes in Mathematics, Vol. 269. Séminaire de Géométrie Algébrique du Bois-Marie 1963–1964 (SGA 4), Dirigé par M. Artin, A. Grothendieck, et J. L. Verdier. Avec la collaboration de N. Bourbaki, P. Deligne et B. Saint-Donat. Springer-Verlag, Berlin-New York, 1972, pp. xix+525.

[Sta21] The Stacks project authors. *The Stacks project*. https://stacks.math.columbia.edu. 2021.

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