THE THOM ISOMORPHISM

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ABSTRACT. Script for a talk of the Wednesday Seminar of the GK1821 at Freiburg during the Summer Semester 2021. The main reference is [Ati67, §2].

CONTENTS

—parts in gray will be omitted during the talk—

1. SETTING AND CONVENTIONS

- We work with complex vector spaces and complex vector bundles only.
- We use the usual word *rank* instead of *dimension*, which is the one used in [Ati67].
- We use $\mathrm{Vect}(X)$ to denote the set of isomorphism classes of vector bundles on X, and $\mathrm{Vect}_n(X)$ to denote the subset of $\mathrm{Vect}(X)$ given by bundles of dimension n.
- We denote by $\mathcal{C}$ the category of compact spaces, by $\mathcal{C}^+$ the category of compact spaces with distinguished basepoints and by $\mathcal{C}^2$ the category of compact pairs.

2. RECOLLECTION FROM PREVIOUS TALKS

2.1. **Definition of $K(X)$.** If X is any space, the set $\mathrm{Vect}(X)$ has the structure of an abelian semigroup, where the additive structure is defined by direct sum.

We have a functor $\mathcal{C}^2 \to \mathcal{C}^+$ that sends a pair (X, Y) to X/Y, with basepoint Y/Y. If $Y = \varnothing$, then we interpret the resulting object as X with a disjoint basepoint. We also have a functor $\mathcal{C} \to \mathcal{C}^2$ sending $X \mapsto (X, \varnothing)$. Hence, the composition of the two functors gives $X \mapsto X^+$, where X^+ is the disjoint union of X with a basepoint.

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For X in $\mathcal{C}^+$ we define $\tilde{K}(X)$ to be the kernel of the map $i^*\colon K(X) \to K(x_0)$, where $i\colon x_0 \to X$ is the inclusion of the basepoint. If $c\colon X \to x_0$ is the collapsing map, then c^* induces a splitting

$$K(X) \cong \tilde{K}(X) \oplus K(x_0).$$

Indeed, we need...

References

[Ati67] M. F. Atiyah. *K-theory*. Lecture notes by D. W. Anderson. W. A. Benjamin, Inc., New York-Amsterdam, 1967.

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