Teacher Answer Key — All multiple regressions now guaranteed inv

regressions solved without matrices.

Simple Regression — Q1: Study Hours and Exam Grade

DATA:

Study Hours	Exam Grade
2	55
4	60
6	72
8	80

STEP-BY-STEP:

n = 4

Sums and means: x_bar=5.000000, y_bar=66.750000

Sxx = 20.000000, Sxy = 87.000000

beta1 = Sxy/Sxx = 4.350000

beta0 = y_bar - beta1*x_bar = 45.000000

Residuals: [1.3, -2.4, 0.9, 0.2]

SSE = 8.300000

sigma^2 = 4.150000

Var-Cov matrix (beta0,beta1):

[[6.225 -1.0375]

 [-1.0375 0.2075]]

Simple Regression — Q2: Advertising and Sales

DATA:

Advertising (000)	*Sales* (000)
2	25
4	35
6	45
8	55

STEP-BY-STEP:

n = 4

Sums and means: x_bar=5.000000, y_bar=40.000000

Sxx = 20.000000, Sxy = 100.000000

beta1 = Sxy/Sxx = 5.000000

beta0 = y_bar - beta1*x_bar = 15.000000

Residuals: [0.0, 0.0, 0.0, 0.0]

SSE = 0.000000

sigma^2 = 0.000000

Var-Cov matrix (beta0,beta1):

```
[[ 0. -0.]
 [-0.  0.]]
```

Simple Regression — Q3: Experience and Wage

DATA:

Experience (years)	Wage ($/hr)
1	8
3	10
5	13
7	15

STEP-BY-STEP:

n = 4

Sums and means: x_bar=4.000000, y_bar=11.500000

Sxx = 20.000000, Sxy = 24.000000

beta1 = Sxy/Sxx = 1.200000

beta0 = y_bar - beta1*x_bar = 6.700000

Residuals: [0.1, -0.3, 0.3, -0.1]

SSE = 0.200000

sigma^2 = 0.100000

Var-Cov matrix (beta0,beta1):

[[0.105 -0.02]

 [-0.02   0.005]]

Simple Regression — Q4: Price and Quantity Demanded

DATA:

Price	Quantity
10	100
12	90
14	80
16	70

STEP-BY-STEP:

n = 4

Sums and means: x_bar=13.000000, y_bar=85.000000

Sxx = 20.000000, Sxy = -100.000000

beta1 = Sxy/Sxx = -5.000000

beta0 = y_bar - beta1*x_bar = 150.000000

Residuals: [0.0, 0.0, 0.0, 0.0]

SSE = 0.000000

sigma^2 = 0.000000

Var-Cov matrix (beta0,beta1):

[[0. -0.]

 [-0. 0.]]

Simple Regression — Q5: Age and Medical Expenses

DATA:

Age	Expenses ($)
30	200
40	300
50	400
60	500

STEP-BY-STEP:

n = 4

Sums and means: x_bar=45.000000, y_bar=350.000000

Sxx = 500.000000, Sxy = 5000.000000

beta1 = Sxy/Sxx = 10.000000

beta0 = y_bar - beta1*x_bar = -100.000000

Residuals: [0.0, 0.0, 0.0, 0.0]

SSE = 0.000000

sigma^2 = 0.000000

Var-Cov matrix (beta0,beta1):

[[0. -0.]

 [-0. 0.]]

Simple Regression — Q6: Hours Worked and Weekly Income

DATA:

Hours	Income ($)
20	250
30	300
40	350
50	400

STEP-BY-STEP:

n = 4

Sums and means: x_bar=35.000000, y_bar=325.000000

Sxx = 500.000000, Sxy = 2500.000000

beta1 = Sxy/Sxx = 5.000000

beta0 = y_bar - beta1*x_bar = 150.000000

Residuals: [0.0, 0.0, 0.0, 0.0]

SSE = 0.000000

sigma^2 = 0.000000

Var-Cov matrix (beta0,beta1):

```
[[ 0. -0.]
 [-0.  0.]]
```

Simple Regression — Q7: Years of Schooling and Income

DATA:

Years of Schooling	Monthly Income ($)
8	1200
10	1500
12	1800
14	2100

STEP-BY-STEP:

n = 4

Sums and means: x_bar=11.000000, y_bar=1650.000000

Sxx = 20.000000, Sxy = 3000.000000

beta1 = Sxy/Sxx = 150.000000

beta0 = y_bar - beta1*x_bar = 0.000000

Residuals: [0.0, 0.0, 0.0, 0.0]

SSE = 0.000000

sigma^2 = 0.000000

Var-Cov matrix (beta0,beta1):

```
[[ 0. -0.]
 [-0.  0.]]
```

Simple Regression — Q8: Coffee Price and Cups Sold

DATA:

Price ($)	Cups Sold
1	90
2	80
3	70
4	60

STEP-BY-STEP:

n = 4

Sums and means: x_bar=2.500000, y_bar=75.000000

Sxx = 5.000000, Sxy = -50.000000

beta1 = Sxy/Sxx = -10.000000

beta0 = y_bar - beta1*x_bar = 100.000000

Residuals: [0.0, 0.0, 0.0, 0.0]

SSE = 0.000000

sigma^2 = 0.000000

Var-Cov matrix (beta0,beta1):

[[0. -0.]

 [-0. 0.]]

Simple Regression — Q9: Temperature and Ice Cream Sales

DATA:

Temperature (°C)	Sales ($)
20	150
25	200
30	250
35	300

STEP-BY-STEP:

n = 4

Sums and means: x_bar=27.500000, y_bar=225.000000

Sxx = 125.000000, Sxy = 1250.000000

beta1 = Sxy/Sxx = 10.000000

beta0 = y_bar - beta1*x_bar = -50.000000

Residuals: [0.0, 0.0, 0.0, 0.0]

SSE = 0.000000

sigma^2 = 0.000000

Var-Cov matrix (beta0,beta1):

[[0. -0.]

 [-0. 0.]]

Simple Regression — Q10: Years in Business and Profit

DATA:

Years	Profit (000$)
1	10
3	20
5	30
7	40

STEP-BY-STEP:

n = 4

Sums and means: x_bar=4.000000, y_bar=25.000000

Sxx = 20.000000, Sxy = 100.000000

beta1 = Sxy/Sxx = 5.000000

beta0 = y_bar - beta1*x_bar = 5.000000

Residuals: [0.0, 0.0, 0.0, 0.0]

SSE = 0.000000

sigma^2 = 0.000000

Var-Cov matrix (beta0,beta1):

[[0. -0.]

 [-0. 0.]]

Multiple Regression — Q1: Wage, Education, and Experience

DATA:

Education	Experience	Wage
8.0	1.001680	9.0
9.0	1.999067	10.0
11.0	3.995834	13.0
13.0	5.999333	16.0

STEP-BY-STEP:

Design X'X matrix:

```
[[  4.          41.          12.99591331]
 [ 41.         435.         147.95054033]
 [ 12.99591331 147.95054033  56.95831577]]
```

(X'X)^-1:

```
[[3643122.39734924 -520493.79757913  520760.4073713 ]
 [-520493.79757913   74363.09726484  -74401.21938417]
 [ 520760.4073713   -74401.21938417   74439.42891293]]
```

beta estimates (beta0, beta1, beta2):

[400.0556, -56.102847, 57.556119]

Residuals: [0.114386, -0.188503, 0.091043, -0.016925]

SSE = 0.057193

sigma^2 = 0.057193

Var-Cov matrix = sigma^2 * (X'X)^-1:

```
[[208360.62318404 -29768.5337462   29783.7819252 ]
 [-29768.5337462    4253.03890401  -4255.2192174 ]
 [ 29783.7819252   -4255.2192174    4257.40452998]]
```

Multiple Regression — Q2: Sales, Advertising, and Price

DATA:

Advertising (000)	*Price*	*Sales* (000)
2.0	10.0	42.0
3.0	9.0	51.0
4.0	8.0	63.0
6.0	7.0	80.0

STEP-BY-STEP:

Design X'X matrix:

```
[[  4.  15.  34.]
 [ 15.  65. 121.]
 [ 34. 121. 294.]]
```

(X'X)^-1:

```
[[744.83333333 -49.33333333 -65.83333333]
 [-49.33333333   3.33333333   4.33333333]
 [-65.83333333   4.33333333   5.83333333]]
```

beta estimates (beta0, beta1, beta2):

[62.5, 7.0, -3.5]

Residuals: [0.5, -1.0, 0.5, -0.0]

SSE = 1.500000

sigma^2 = 1.500000

Var-Cov matrix = sigma^2 * (X'X)^-1:

```
[[1117.25  -74.    -98.75]
 [ -74.      5.      6.5 ]
 [ -98.75    6.5     8.75]]
```

Multiple Regression — Q3: House Price, Size, and Distance

DATA:

Size (m2)	Distance (km)	Price (000$)
80.0	15.0	200.0
95.0	10.0	250.0
120.0	5.0	300.0
150.0	2.0	360.0

STEP-BY-STEP:

Design X'X matrix:

[[4.0000e+00 4.4500e+02 3.2000e+01]
 [4.4500e+02 5.2325e+04 3.0500e+03]
 [3.2000e+01 3.0500e+03 3.5400e+02]]

(X'X)^-1:

[[1.42843532e+02 -9.28427576e-01 -4.91324555e+00]
 [-9.28427576e-01 6.07281177e-03 3.16034082e-02]
 [-4.91324555e+00 3.16034082e-02 1.74670798e-01]]

beta estimates (beta0, beta1, beta2):

[162.587142, 1.37103, -4.701782]

Residuals: [-1.742835, 4.182804, -3.601859, 1.16189]

SSE = 34.856700

sigma^2 = 34.856700

Var-Cov matrix = sigma^2 * (X'X)^-1:

[[4.97905418e+03 -3.23619217e+01 -1.71259527e+02]
 [-3.23619217e+01 2.11678180e-01 1.10159053e+00]
 [-1.71259527e+02 1.10159053e+00 6.08844764e+00]]

Multiple Regression — Q4: Profit, Employees, and Capital

DATA:

Employees	Capital (000)	*Profit*(000)
5.0	10.0	40.0
12.0	15.0	70.0
18.0	25.0	110.0
25.0	40.0	160.0

STEP-BY-STEP:

Design X'X matrix:

```
[[   4.   60.   90.]
 [  60. 1118. 1680.]
 [  90. 1680. 2550.]]
```

(X'X)^-1:

```
[[ 1.28378378 -0.08108108  0.00810811]
 [-0.08108108  0.09459459 -0.05945946]
 [ 0.00810811 -0.05945946  0.03927928]]
```

beta estimates (beta0, beta1, beta2):

[4.594595, 2.972973, 2.036036]

Residuals: [0.18018, -0.810811, 0.990991, -0.36036]

SSE = 1.801802

sigma^2 = 1.801802

Var-Cov matrix = sigma^2 * (X'X)^-1:

```
[[ 2.31312393 -0.14609204  0.0146092 ]
 [-0.14609204  0.17044071 -0.10713416]
 [ 0.0146092  -0.10713416  0.07077348]]
```

Multiple Regression — Q5: Crop Yield, Fertilizer, and Rainfall

DATA:

Fertilizer (kg)	Rainfall (mm)	Yield (tons)
20.0	50.0	12.0
45.0	65.0	18.0
70.0	75.0	24.0
95.0	90.0	30.0

STEP-BY-STEP:

Design X'X matrix:

[[4.0000e+00 2.3000e+02 2.8000e+02]
 [2.3000e+02 1.6350e+04 1.7725e+04]
 [2.8000e+02 1.7725e+04 2.0450e+04]]

(X'X)^-1:

[[3.2291e+02 4.1520e+00 -8.0200e+00]
 [4.1520e+00 5.4400e-02 -1.0400e-01]
 [-8.0200e+00 -1.0400e-01 2.0000e-01]]

beta estimates (beta0, beta1, beta2):

[7.2, 0.24, -0.0]

Residuals: [0.0, 0.0, 0.0, 0.0]

SSE = 0.000000

sigma^2 = 0.000000

Var-Cov matrix = sigma^2 * (X'X)^-1:

[[1.56508957e-19 2.01240342e-21 -3.88715690e-21]
 [2.01240342e-21 2.63667501e-23 -5.04070222e-23]
 [-3.88715690e-21 -5.04070222e-23 9.69365811e-23]]

Multiple Regression — Q6: Car Price, Age, and Mileage

DATA:

Age (yrs)	Mileage (000 km)	Price (000$)
1.0	10.0	50.0
2.0	22.0	46.0
3.0	35.0	42.0
5.0	55.0	30.0

STEP-BY-STEP:

Design X'X matrix:

[[4.000e+00 1.100e+01 1.220e+02]
 [1.100e+01 3.900e+01 4.340e+02]
 [1.220e+02 4.340e+02 4.834e+03]]

(X'X)^-1:

[[1.16438356 -1.54794521 0.10958904]
 [-1.54794521 30.49315068 -2.69863014]
 [0.10958904 -2.69863014 0.23972603]]

beta estimates (beta0, beta1, beta2):

[56.191781, -13.972603, 0.794521]

Residuals: [-0.164384, 0.273973, -0.082192, -0.027397]

SSE = 0.109589

sigma^2 = 0.109589

Var-Cov matrix = sigma^2 * (X'X)^-1:

[[0.12760368 -0.16963783 0.01200976]
 [-0.16963783 3.34171514 -0.29574029]
 [0.01200976 -0.29574029 0.02627135]]

Multiple Regression — Q7: Productivity, Training, and Experience

DATA:

Training Hours	Experience	Output (units/hr)
5.0	1.0	11.0
12.0	4.0	18.0
18.0	6.0	24.0
25.0	10.0	36.0

STEP-BY-STEP:

Design X'X matrix:

[[4. 60. 21.]
 [60. 1118. 411.]
 [21. 411. 153.]]

(X'X)^-1:

[[5.15217391 -1.32608696 2.85507246]
 [-1.32608696 0.41304348 -0.92753623]
 [2.85507246 -0.92753623 2.10628019]]

beta estimates (beta0, beta1, beta2):

[8.753623, -0.376812, 3.647343]

Residuals: [0.483092, -0.821256, 0.144928, 0.193237]

SSE = 0.966184

sigma^2 = 0.966184

Var-Cov matrix = sigma^2 * (X'X)^-1:

[[4.97794581 -1.28124344 2.75852412]
 [-1.28124344 0.39907582 -0.89617027]
 [2.75852412 -0.89617027 2.03505333]]

Multiple Regression — Q8: Demand, Income, and Price

DATA:

Income (000$)	Price	Quantity
20.0	5.0	100.0
30.0	4.0	140.0
40.0	3.0	180.0
60.0	2.0	260.0

STEP-BY-STEP:

Design X'X matrix:

```
[[4.0e+00 1.5e+02 1.4e+01]
 [1.5e+02 6.5e+03 4.6e+02]
 [1.4e+01 4.6e+02 5.4e+01]]
```

(X'X)^-1:

```
[[ 2.32333333e+02 -2.76666667e+00 -3.66666667e+01]
 [-2.76666667e+00  3.33333333e-02  4.33333333e-01]
 [-3.66666667e+01  4.33333333e-01  5.83333333e+00]]
```

beta estimates (beta0, beta1, beta2):

[20.0, 4.0, 0.0]

Residuals: [-0.0, -0.0, -0.0, -0.0]

SSE = 0.000000

sigma^2 = 0.000000

Var-Cov matrix = sigma^2 * (X'X)^-1:

```
[[ 3.58396592e-19 -4.26785038e-21 -5.65618725e-20]
 [-4.26785038e-21  5.14198841e-23  6.68458493e-22]
 [-5.65618725e-20  6.68458493e-22  8.99847972e-21]]
```

Multiple Regression — Q9: Energy Use, Temperature, and House Si

DATA:

Temperature (°C)	Size (m2)	Energy Use (kWh)
15.0	80.0	210.0
22.0	105.0	270.0
28.0	125.0	330.0
35.0	160.0	420.0

STEP-BY-STEP:

Design X'X matrix:

[[4.000e+00 1.000e+02 4.700e+02]
 [1.000e+02 2.718e+03 1.261e+04]
 [4.700e+02 1.261e+04 5.865e+04]]

(X'X)^-1:

[[14.13475177 2.18794326 -0.58368794]
 [2.18794326 0.4858156 -0.12198582]
 [-0.58368794 -0.12198582 0.03092199]]

beta estimates (beta0, beta1, beta2):

[-0.957447, 0.638298, 2.489362]

Residuals: [2.234043, -4.468085, 1.914894, 0.319149]

SSE = 28.723404

sigma^2 = 28.723404

Var-Cov matrix = sigma^2 * (X'X)^-1:

[[405.99818923 62.84517881 -16.76550475]
 [62.84517881 13.95427795 -3.50384789]
 [-16.76550475 -3.50384789   0.8881847]]

Multiple Regression — Q10: Life Satisfaction, Income, and Leisure

DATA:

Income (000$)	Leisure Hours	Life Satisfaction
20.0	10.0	5.0
35.0	15.0	6.0
50.0	25.0	8.0
70.0	30.0	10.0

STEP-BY-STEP:

Design X'X matrix:

[[4.000e+00 1.750e+02 8.000e+01]
 [1.750e+02 9.025e+03 4.075e+03]
 [8.000e+01 4.075e+03 1.850e+03]]

(X'X)^-1:

[[1.95945946 0.04864865 -0.19189189]
 [0.04864865 0.02162162 -0.04972973]
 [-0.19189189 -0.04972973 0.11837838]]

beta estimates (beta0, beta1, beta2):

[2.608108, 0.07027, 0.078378]

Residuals: [0.202703, -0.243243, -0.081081, 0.121622]

SSE = 0.121622

sigma^2 = 0.121622

Var-Cov matrix = sigma^2 * (X'X)^-1:

[[0.23831264 0.00591673 -0.0233382]
 [0.00591673 0.00262966 -0.00604821]
 [-0.0233382 -0.00604821 0.01439737]]