

Housing Supply Constraints and the Distribution of Economic Activity: The Case of the Twin Cities*

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Abstract

This paper analyzes the effects of city-level zoning reforms on the spatial distribution of economic activity in a metropolitan area. Using parcel-level property tax and zoning data, I use Minneapolis recent reform, which eliminated single-family zoning lots, to estimate productivity gains in the local housing development sector. Using the methodology, I find that median productivity in the housing development sector is expected to grow by 38 percent at the tract level in the city. I feed the estimated productivity gain into a quantitative spatial model of the Twin Cities, the metropolitan area which Minneapolis is a part of, to compute the effect of the reform on local wages, rents and commuting patterns in the metropolitan area. I find that housing becomes around 15 percent more affordable in Minneapolis, and rents in most of the metropolitan area fall significantly as well. As a result of people moving to Minneapolis after the zoning reform, wages in other regions of the metropolitan area increase between 10 and 15 percent, on average.

JEL Classification: R12, R13, R31, R52

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1 Introduction

Zoning is in the center of public debate. Many cities in the United States zone most of their residential areas as single-family detached houses. They account for seventy five percent of the residential land in Los Angeles, CA; seventy percent in Minneapolis, MN; and seventy nine percent in Chicago, IL, for example. This restriction on development can impact by how much a city can attract new workers, while at the same time driving up housing costs and increasing commuting times from home to work. Increase in housing costs, many local and state governments have been pushing for zoning reforms that allow more densely packed buildings to increase housing affordability and attract new workers. In recent years, cities such as Minneapolis, Seattle, and Portland have introduced or passed bills in an effort to reduce or eliminate neighborhoods exclusively zoned as single-family housing.

One aspect that is often left out of the debate is the effect of such policies on neighboring cities. For example, the population of Minneapolis and Los Angeles correspond to roughly ten and thirty percent of their respective metropolitan areas. Although the potential gains from upzoning may seem obvious for the city that implements the policy, less clear are the second-order effects that come from population reallocation across an urban area.

This paper studies the effects of upzoning when it takes place exclusively in a city inside a larger urban area, focusing on the recent zoning reform that eliminated single-family housing in the city of Minneapolis. Starting in 2020, every parcel in the city of Minneapolis can now admit at least three dwelling units. No other city inside the metro area thus far has introduced a similar plan for housing reform.

To tackle this subject, I build a quantitative spatial model of a metropolitan area where workers are allowed to commute to work across neighborhoods and cities. There are three forces at play in the model. First, each location has a housing development sector that differ from each other based on their productivity level. Second, I allow for workers to spatially sort on income by introducing nonhomothetic preferences on housing. Contrary to the case when preferences are homothetic, housing costs affects workers differently when they don't earn the same income. Finally, I introduce agglomeration effects and decreasing returns to scale in the production of the consumption good. This allows for ex-post productivity to be higher, but wages to be lower in locations where there is higher density of workers, given their innate productivity levels.

To quantitatively assess the impact of the zoning reform, I use the model to back out housing productivity at the level of the Census tract before the zoning reform. I combine

these productivity measures with zoning data available at the parcel level for Minneapolis to quantify by how much the productivity of the housing sector changes after the restriction on single-family units is lifted across the city. I project the estimated housing productivity prior to the zoning reform on each tract's share of lots zoned as single-family units, as well as the tract's distance to the city's downtown area. Because these regressors don't vary due to endogenous choices made by the agents in the model and in the data, I argue they are exogenous and thus can be used to inform how housing productivity will change in each location. Since most of the residential area of Minneapolis was zoned exclusively for single-family units, productivity increases substantially across the city. The median productivity growth is about 38 percent at the tract level.

I find that the upzoning is expected to decrease the cost of housing in Minneapolis by about fifteen percent, even with the additional influx of residents from other cities in the metropolitan area. Rents in most other tracts in the metro area are also expected to fall, given that part of their local population moves out to Minneapolis. Additionally, I predict that the policy should attract new residents and workforce to the city by six and thirty five percent, respectively. Most of the new jobs are created in Downtown Minneapolis, which also draws workers from other tracts in the city. Because these tracts lose workers, wages in those locations rise. Aggregated at the city level, wages in Minneapolis increase by almost five percent.

In addition, the model predicts the second-order effects of the policy coming from the reallocation of the workforce inside the metropolitan area. In particular, I find that the bulk of the workers that move to Minneapolis come from Scott County, a suburban part of the metropolitan area. In other counties, population from other counties decreases only marginally. More importantly, the model predicts that the population in Ramsey County, which is adjacent to Minneapolis and home to Saint Paul, Minnesota's capital, increases as a result of the policy. Population also increases in Hennepin County, where Minneapolis is located. Driving this result is both the increase in wages in locations outside Minneapolis, as well as the lower costs of commuting costs from these counties to the city.

The results described above highlight the importance of looking at zoning reforms in a broader context outside the city in which it takes place. Because individuals don't need to live and work in the same location, making housing more affordable in one location can have impacts on cities in a commuting distance to it. It also shows how heterogeneous the impacts can be across the metropolitan area.

This paper does not deal directly with the potential distributional conflicts that may arise between renters and homeowners. Zoning rules exist in the real world sometimes for reasons that are not internalized in the model. For instance, homeowners may use

zoning to intentionally reduce density around where they live, or to force higher sorting through income in their neighborhoods. They may also want to use their properties as a source of financial investment. In such case, an increase in house prices and rents due to land-use regulations is beneficial to homeowners. My model is silent about these features of the real world, choosing instead to focus on the spatial and labor market implications of the policy.

Related Literature This paper dialogues with two different fields. The field of quantitative spatial economics has been growing in the past decades, beginning with papers such as Lucas and Rossi-Hansberg (2002) and more recently synthesized in Allen and Arkolakis (2014) and Redding and Rossi-Hansberg (2017). Ahlfeldt et al. (2015) develop a quantitative model of a city building upon international trade models such as Eaton and Kortum (2002). They use the exogenous variation at the city block level of the division and reunification of Berlin to estimate and quantify the agglomeration and dispersion forces present inside a city. Tsivanidis (2020) evaluates the impact of the introduction of a faster public transportation network in a city. By introducing nonhomothetic preferences, his paper also allows for workers to sort on income spatially. Heblich et al. (2020) use data on bilateral commuting flows to inform a quantitative spatial model where commuting costs change due to the introduction of passenger steam railways in 19th century London. Their work highlights the importance of separation between workplace and residence locations for workers inside modern metropolitan areas. Owens III et al. (2020) studies the urban structure of Detroit using a model with residential externalities can generate multiple equilibria at the neighborhood level. They include neighborhood-specific fixed costs in housing development to allow for empty neighborhoods in equilibrium when few residents want to live there. Differently from this paper, model features a housing cap per neighborhood. Couture et al. (2019) find that the rise in income among the rich increased demand for luxury amenities in cities, driving housing prices up in downtown areas, pricing out many low-wage workers. They find that sorting on income is an important factor driving welfare changes in a city, concluding that previous estimates highlighting heterogeneous welfare changes among the rich and the poor were potentially understated.

The literature of land-use regulations and economics activity was recently surveyed by Glaeser and Gyourko (2018). At the city level, Kulka (2020) studies the effect of minimum lot sizes on household sorting by income. It calculates the welfare effects of reducing minimum lot sizes using data from Wake County in North Carolina. The paper finds that decreasing minimum lot sizes in rich neighborhoods bring in lower-income workers. Households with at least the area's median income benefit from the policy. Parkhomenko

(2019) and Khan (2020) study the consequences of decentralized control over land use regulations. Both papers find welfare gains in centralizing land-use regulations in higher levels of government instead of allowing them to be chosen locally.

At the national level, several papers study the role of housing supply constraints in the allocation of economic activity across space. Ganong and Shoag (2017) looks at income convergence across regions in the United States. They introduce nonhomotheticity in housing demand to capture higher housing expenditures among lower income households. They show that increases in housing supply regulations were an important factor to explain why lower wage workers are not moving to high-income places as much as they did three decades ago. Herkenhoff et al. (2018) and Hsieh and Moretti (2019) study land-use regulations and spatial misallocation in the United States. Both find negative impacts of land-use regulations on the United States' level of GDP per capita. In particular, Herkenhoff et al. (2018) model land-use regulations in a similar way as this paper, by interpreting housing productivity heterogeneity as exogenous differences in land use-restriction. Fajgelbaum and Gaubert (2018) study optimal spatial policies in the presence of local agglomeration and congestion forces. They find that spatial sorting by skill and wage inequality in larger cities in the U.S. is too high relative to efficient allocations. Martellini (2020) studies city-size wage premium in the United States and how relaxing housing regulation in cities affect the sorting of workers with different skill levels.

This paper is organized as follows. Section 2 discusses the Twin Cities metro area and the zoning reform implemented in Minneapolis. Section 3 presents the spatial model of the urban area. Section 4 discusses the calibration and estimation strategies used in this paper. The quantitative counterfactual analysis is presented in Section 5. Finally, Section 6 concludes.

2 The Twin Cities Metropolitan Area

The Minneapolis, Saint Paul and Bloomington Metropolitan Statistical Area, also known as the Twin Cities metropolitan area, is the only MSA in the state of Minnesota. It contains a total of seven counties in the State: Anoka, Carver, Dakota, Hennepin, Ramsey, Scott and Washington. The population of the metro area contains about 3.64 million people, being the third largest population-wise in the Midwest and the 16th largest metropolitan area in the United States.

The Twin Cities metro area gets its name from two neighboring cities that are considered to be the most important in the metropolitan area: Minneapolis and Saint Paul. The former is the largest and most populous city in the state, and the seat of Hennepin County,

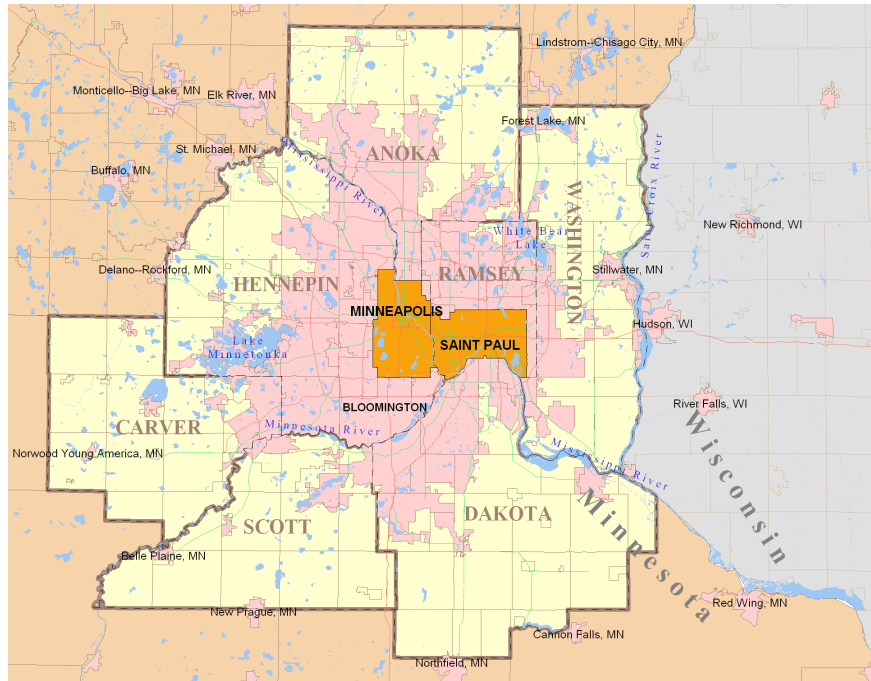


Figure 1: Map of Minneapolis, Saint Paul and Bloomington Metropropolitan Statistical Area

the state's most populous county. Outside Chicago, Minneapolis is the most densely populated city in the Midwest. The latter is the state's capital and located in Ramsey County, the state's most densely populated county. Figure 1 show the map of the Twin Cities metro area, with Minneapolis and St. Paul highlighted.

Even though Minneapolis is economically the most important city in the metropolitan area, it is far from concentrating the majority of its population and labor market. Table 1 shows the population and workplace shares in the Twin Cities. Minneapolis population is roughly ten percent of the metro area's population, and twenty one percent of the area's workforce works in the city. In fact, in other counties, at least twenty three percent of their own population work in the same county, highlighting that the economic activity in the metropolitan area is reasonably dispersed. Still, at least ten percent of the workforce living in each county works in Minneapolis, which suggests how important the city is for the overall metropolitan area.

Until January 1st 2020, about seventy percent of Minneapolis' residential zoning was composed of neighborhoods zoned exclusively for single family detached homes. This meant that each parcel could only have a house where only one family could live in, and the house had to be surrounded by lawn, and not attached to a neighboring house. Figure 2 displays in green all the city regions zoned as single-family detached units and in blue all the other strictly residential parcels.

| | Population | Workforce | Work in same location | Work in Minneapolis |
|-------------|------------|-----------|--------------------------|------------------------|
| Anoka | 12 | 7 | 31 | 18 |
| Carver | 3 | 2 | 27 | 10 |
| Dakota | 14 | 10 | 37 | 13 |
| Hennepin* | 28 | 35 | 57 | 21 |
| Minneapolis | 13 | 21 | 45 | 45 |
| Ramsey | 16 | 19 | 44 | 19 |
| Scott | 5 | 2 | 37 | 13 |
| Washington | 8 | 4 | 23 | 12 |

* Hennepin considers Hennepin County without Minneapolis

Table 1: Commuting Patterns in the Twin Cities (in %)

Starting in 2016, Minneapolis City Council proposed a twenty-year comprehensive plan to update the city’s long-term plan for itself with respect to its urban landscape, economy and climate impact. The plan, named *Minneapolis 2040*, focuses on a wide variety of topics, such as land use, transportation, housing, public health, arts and culture. Of the interest to this paper is its plan to change residential zoning in the city, allowing for substantial upzoning.

The plan was approved by the city council and, effective in January 1st, 2020, the city’s zoning code changed drastically. Population density in buildings in the downtown area was increased. Along important public transit routes, the city allowed for development of high density units. Nevertheless, the most substantial change regards single-family zoning. All neighborhoods until recently zoned as single family now allow for at most three dwelling units on an individual lot. This has the potential to triple the amount of housing units in most of the city.

An important outcome from this zoning reform will be how the economic activity, population distribution and local labor markets will be affected in the metropolitan area. The reform will not only affect Minneapolis, but all the surrounding cities. It is therefore important to analyze the policy change in the context of the entire metropolitan area, not the city itself. We can expect workers to move in to Minneapolis as a result of an increase in housing affordability, and as a consequence more jobs in Minneapolis and locations nearby. The next section presents an urban model that allows us to make predictions of what to expect in the aftermath of such policy change.

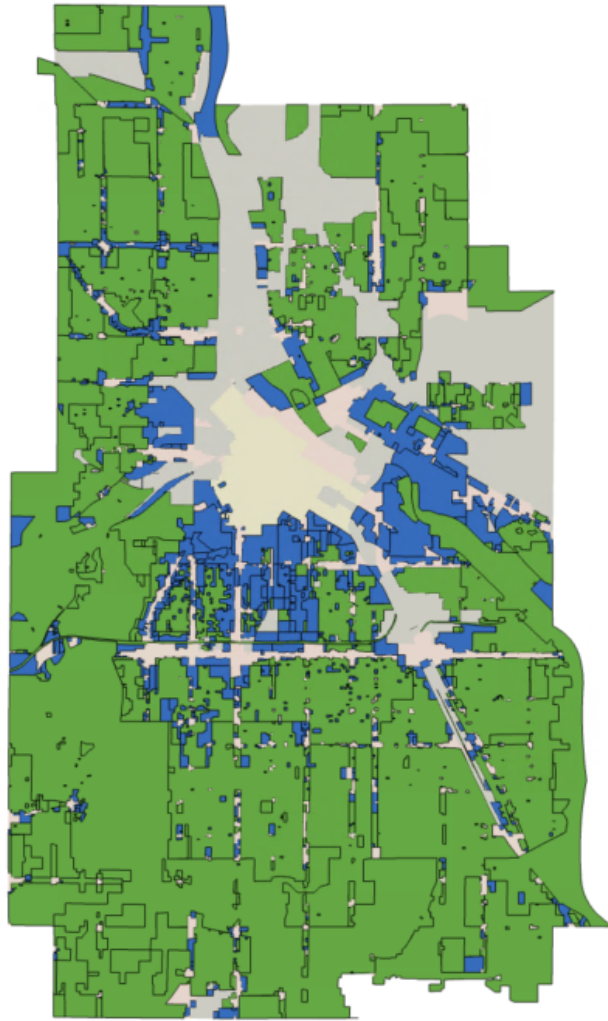


Figure 2: Residential Zoning in Minneapolis up to 2019

3 Model

To study the impacts of neighborhood upzoning, I build a quantitative spatial equilibrium model of a metropolitan area. There is a finite and discrete set Ω of neighborhoods. There are four sets of agents: workers, consumption goods producers, housing producers, and absentee land and firm owners. There are \bar{R} workers in the city who can live and work in distinct locations. They are indexed by a pair ij , where $i \in \Omega$ and $j \in \Omega$ correspond to their workplace and residence locations, respectively. Each location produces a homogeneous consumption good, produced by a representative firm. Housing is developed locally as well.

Worker's problem Worker values consumption of a single good, c , housing services, h , neighborhood amenities, s_j , and idiosyncratic preferences from living in location j and working in i , ϵ_{ij} . I represent commuting costs from j to i by adding a parameter $\kappa_{ij} \geq 1$. I use a Stone-Geary utility function to represent the worker's preferences over consumption and housing services, with a non-homothetic term \bar{h} in housing. The worker's problem is:

$$V_{ij} \equiv \max_{\{c, h, i, j\}} \frac{s_j}{\kappa_{ij}} \left(\frac{c}{\alpha} \right)^\alpha \left(\frac{h - \bar{h}}{1 - \alpha} \right)^{1-\alpha} \epsilon_{ij} \quad \text{subject to} \quad c + (1 + \tau_j)r_j h = w_i$$

where w_i is the wage in workplace i , τ_j is a tax on housing unit in j and r_j is the pre-tax rental rate of a unit of housing. I assume the worker supplies inelastically one unit of labor. Denote $\hat{V}_{ij} \equiv V_{ij} \times \epsilon_{ij}$ as the counterfactual indirect utility. We can represent it as

$$\hat{V}_{ij} = \frac{w_i s_j - (1 + \tau_j)r_j \bar{h}}{\kappa_{ij} [(1 + \tau_j)r_j]^{1-\alpha}} \times \epsilon_{ij}.$$

I assume that the worker's preference over local amenities is represented by the vector ϵ_k , which is i.i.d and drawn from a Type II extreme value (Fréchet) distribution:

$$F_{ij}(\epsilon) = \exp \left(-a_{ij} \epsilon^{-\theta} \right),$$

where a_{ij} is the location-specific amenity term, $a_{ij} > 0$ and $\theta > 1$. Worker's location choices to work and live are the ones that maximize their counterfactual indirect utility.

Production in Neighborhood i Technology given by $Y_i = A_i n_i^\beta$, $\beta \in (0, 1]$. I introduce agglomeration effects: $A_i = \bar{A}_i n_i^\eta$. There is a homogeneous good in the city and the

representative firm in each location behaves competitively. Because I allow for decreasing returns to scale in production, potential profits are claimed by absentee firm owners.

Housing Sector in j There's a representative developer that behaves competitively. Production of housing services per location is given by the Cobb-Douglas function $G_j L_j^\phi M_j^{1-\phi}$, where L_j is the quantity of land, M_j is materials and G_j is local productivity of the housing sector. The price of materials is given by ι and is homogeneous across locations. Land prices are region-specific, and given by p_j . The developer's problem is given by

$$\max_{L, M} r_j G_j L^\phi M^{1-\phi} - \iota M - p_j L$$

The price of land is derived from an ad-hoc supply function given by $p_j = (H_j/L_j)^{\bar{\psi}}$, $\bar{\psi} > 0$. Restrictions on development in each neighborhood are interpreted as changes in G_j . Land rents from the housing sector go to absentee land owners.

Amenity Supply in j I assume amenities are endogenously supplied, and financed by the tax collected from the rental market. Let R_j be the number of residents in neighborhood j and $\pi_{i|j}$ be the share of workers living in location j that commute to i to work. The supply of local amenities is given by

$$S_j = \tau_j r_j \left(R_j \sum_{i \in \Omega} \pi_{i|j} h_{ij} \right).$$

The supply of amenities is taken as given by the workers when making location choices.

3.1 Equilibrium

3.1.1 Firm's Optimization

Local wages are given by input's marginal productivity: $w_i = \beta \bar{A}_i n_i^{\beta+\eta-1}$.

3.1.2 Worker's Location Choice

Before exploring the problem, it is convenient to define the following probability:

$$G_{ij}(v) = \Pr(\hat{V}_{ij} \leq v).$$

Let $\psi_{ij} \equiv a_{ij}V_{ij}^\theta$. Using the definition of counterfactual indirect utility above and the functional form of the Fréchet distribution, we have:

$$G_{ij}(v) = \Pr \left(\epsilon_{ij} \leq \frac{v}{V_{ij}} \right) = F_{ij} \left(\frac{v}{V_{ij}} \right) = \exp \left(-\psi_{ij}v^{-\theta} \right).$$

Similarly, by independence of the draws,

$$\begin{aligned} \Pr \left(\max_{i,j \in \Omega} \{ \hat{V}_{ij} \} \leq v \right) &= \Pr \left(\cap_{i,j \in \Omega} (\hat{V}_{ij} \leq v) \right) \\ &= \prod_{i,j \in \Omega} \Pr (\hat{V}_{ij} \leq v) \\ &= \prod_{i,j \in \Omega} G_{ij}(v) \\ &= \prod_{i,j \in \Omega} \exp \left(-\psi_{ij}v^{-\theta} \right) \\ &= \exp \left(-v^{-\theta} \sum_{i,j \in \Omega} \psi_{ij} \right). \end{aligned}$$

From the law of large numbers, the fraction of workers living in location j and working in neighborhood i , π_{ij} , can be represented by:

$$\begin{aligned} \pi_{ij} &= \Pr \left(\hat{V}_{ij} \geq \max_{i',j' \in \Omega} \{ \hat{V}_{i'j'} \} \right) = \int_0^\infty \prod_{i',j' \in \Omega} G_{i'j'}(v) dG_{ij}(v) \\ &= \int_0^\infty \exp \left(-v^{-\theta} \sum_{i',j' \in \Omega} \psi_{i'j'} \right) \left(\psi_{ij} \theta v^{-\theta-1} \right) dv \\ &= \psi_{ij} \int_0^\infty \theta v^{-\theta-1} \exp \left(-v^{-\theta} \sum_{i',j' \in \Omega} \psi_{i'j'} \right) dv \\ &= \psi_{ij} \left[\frac{\exp \left(-v^{-\theta} \sum_{i',j' \in \Omega} \psi_{i'j'} \right)}{\sum_{i',j' \in \Omega} \psi_{i'j'}} \right]_0^\infty = \frac{\psi_{ij}}{\sum_{i',j' \in \Omega} \psi_{i'j'}} \\ &= \lambda a_{ij} \left(\kappa_{ij} [(1 + \tau_j)r_j]^{(1-\alpha)} \right)^{-\theta} \left(w_i s_j - (1 + \tau_j)r_j \bar{h} \right)^\theta. \end{aligned}$$

where $\lambda \equiv \left[\sum_{i',j' \in \Omega} \psi_{i'j'} \right]^{-1}$.

The equation above is a gravity equation for commuting, describing overall patterns of workers' workplace and location choices. It shows that the fraction of the population living in j and working in i is increasing in the location taste shock a_{ij} , wages paid in i ,

and amenities in j . Similarly, the share of workers is decreasing in costly it is to commute between the pair ij , how high are residential taxes in j , and rent (r_j). The latter is Reinforced by nonhomothetic parameter \bar{h} . Sensitivity to these variables depend on shape parameter θ of location taste

Summing across residential locations, we get the share of workers in location i :

$$\pi_i = \sum_{j' \in \Omega} \pi_{ij'} = \lambda \sum_{j' \in \Omega} a_{ij'} V_{ij'}^\theta.$$

The share of workers living in location j is given by:

$$\pi_j = \sum_{i' \in \Omega} \pi_{i'j} = \lambda \sum_{i' \in \Omega} a_{i'j} V_{i'j}^\theta.$$

Equivalently, the share of workers living in location j that commute to i to work is given by:

$$\pi_{i|j} = \frac{a_{ij} V_{ij}^\theta}{\sum_{i' \in \Omega} a_{i'j} V_{i'j}^\theta} = \frac{a_{ij} \left(\frac{w_i s_j - (1 + \tau_j) r_j \bar{h}}{\kappa_{ij}} \right)^\theta}{\sum_{i' \in \Omega} a_{i'j} \left(\frac{w_{i'} s_j - (1 + \tau_j) r_j \bar{h}}{\kappa_{i'j}} \right)^\theta}.$$

3.1.3 Rental Markets

Housing Demand Housing demand for residents in j commuting to i is given by

$$h_{ij} = \alpha \bar{h} + (1 - \alpha) \frac{w_i}{(1 + \tau_j) r_j}.$$

Let $\bar{w}_j = \sum_{i \in \Omega} \pi_{i|j} w_i$. Aggregating across working neighborhoods, we get the total housing demand, H_j^d :

$$H_j^d = R_j \left[\alpha \bar{h} + (1 - \alpha) \frac{\bar{w}_j}{(1 + \tau_j) r_j} \right].$$

From the equilibrium condition $s_j = S_j$,

$$s_j = \tau_j r_j R_j \left[\alpha \bar{h} + (1 - \alpha) \frac{\bar{w}_j}{(1 + \tau_j) r_j} \right].$$

Housing Supply First-order condition for materials in the housing developer problem yields $r_j = \frac{\iota}{(1-\phi)G_j} \left(\frac{M_j}{L_j}\right)^\phi$. Using the zero profit condition and substituting for r_j gives us

$$\begin{aligned} \frac{\iota}{(1-\phi)G_j} \left(\frac{M_j}{L_j}\right)^\phi G_j L_j^\phi M_j^{1-\phi} &= \iota M_j + p_j L_j \\ \frac{\iota}{(1-\phi)} M_j &= \iota M_j + p_j L_j \Rightarrow M_j = \frac{1-\phi}{\phi} \frac{p_j L_j}{\iota} \end{aligned}$$

Again, from the zero profit condition,

$$\begin{aligned} r_j &= \frac{\iota M_j + p_j L_j}{G_j L_j^\phi M_j^{1-\phi}} \\ &= \frac{\iota \frac{1-\phi}{\phi} \frac{p_j L_j}{\iota} + p_j L_j}{G_j L_j^\phi \left(\frac{1-\phi}{\phi} \frac{p_j L_j}{\iota}\right)^{1-\phi}} = \frac{\frac{1-\phi}{\phi} p_j + p_j}{G_j \left(\frac{1-\phi}{\phi} \frac{p_j}{\iota}\right)^{1-\phi}} \\ &= \underbrace{\frac{\iota^{1-\phi}}{G_j (1-\phi)^{1-\phi} \phi^\phi}}_{\equiv \rho_j} p_j^\phi = \rho_j p_j^\phi. \end{aligned}$$

Using the land supply equation, we get the relationship between housing rent, housing demand and land

$$r_j = \rho_j \left(\frac{H_j}{L_j}\right)^\psi, \quad \psi \equiv \phi \times \bar{\psi}.$$

Housing Equilibrium From the housing demand equation, the relationship between the number of residents on that neighborhood and total housing demanded is given by

$$\frac{(1-\alpha)\bar{w}_j}{1+\tau_j} \left[\frac{H_j}{R_j} - \alpha \bar{h}\right]^{-1} = \rho_j \left(\frac{H_j}{L_j}\right)^\psi.$$

3.1.4 Labor Market Clearing

In each region, the amount of labor demanded for each skill has to be equal to the amount of labor supplied. The latter is determined by the amount of workers living in each region that commutes to a specific neighborhood to work:

$$n_i = \sum_{j \in \Omega} \pi_{i|j} R_j \quad \forall i \in \Omega.$$

Definition 1. Given a geography $\{\bar{H}_i\}_{i \in \Omega}$ and local housing taxes $\{\tau_j\}_{j \in \Omega}$, the equilibrium of the model is defined by a set of location observables such that:

1. Given the number of workers in each location, the quantity produced in each region is given by the location's production function.

$$Y_i = A_i n_i^\beta.$$

2. Given wages, rents and commuting costs, the share of workers commuting from neighborhood j to i follows, $\forall i, j \in \Omega$:

$$\pi_{ij} = \lambda a_{ij} \kappa_{ij}^{-\theta} [(1 + \tau_j) r_j]^{-\theta(1-\alpha)} \left(w_i s_j - (1 + \tau_j) r_j \bar{h} \right)^\theta.$$

3. Given wages, number of residents, zoning restrictions and rents, housing supply is implicitly defined by

$$\frac{(1 - \alpha) \bar{w}_j}{1 + \tau_j} \left[\frac{H_j}{R_j} - \alpha \bar{h} \right]^{-1} = \rho_j \left(\frac{H_j}{L_j} \right)^\psi.$$

4. Given wages, commuting costs, outside-option utility, location preferences, and housing supply, the number of residents in each location follows:

$$R_j = \sum_{i \in \Omega} \pi_{ij} \bar{R}, \quad \forall j \in \Omega.$$

5. Given wages, zoning restrictions and fixed costs, rents are given by

$$r_j = \rho_j \left(\frac{H_j}{L_j} \right)^\psi.$$

6. Given the number of residents in each neighborhood and commuting probabilities, the labor supply in each neighborhood is given by

$$n_i = \sum_{j \in \Omega} \pi_{ij} \bar{R}.$$

7. Given the number of workers in each location, local output and prices, firms' first-order

conditions determine the wages.

$$w_i = \beta \bar{A}_i n_i^{\beta+\eta-1}.$$

8. Given wages, commuting probabilities and number of residents by neighborhood, local amenities are given by

$$s_j = \tau_j r_j R_j \left[\alpha \bar{h} + (1 - \alpha) \frac{\bar{w}_j}{(1 + \tau_j) r_j} \right].$$

3.2 Welfare

Using similar arguments from the section on worker's location choice, the probability of a worker to work in neighborhood i and live in neighborhood j is $\exp \left(-v^{-\theta} \sum_{i,j \in \Omega} \psi_{ij} \right)$. Consequently, the expected utility of living in the MSA for such worker is:

$$E(V) = \int_0^\infty v \left(\sum_{i,j \in \Omega} \psi_{ij} \right) v^{-\theta-1} \exp \left(-v^{-\theta} \sum_{i,j \in \Omega} \psi_{ij} \right) dv.$$

Let $x = \left(\sum_{i,j \in \Omega} \psi_{ij} \right) v^{-\theta}$ so $x \in (\infty, 0)$ for $v \in (0, \infty)$, $dx = - \left(\sum_{i,j \in \Omega} \psi_{ij} \right) \theta v^{-\theta-1} dv$ and $v = \left(\frac{x}{\sum_{i,j \in \Omega} \psi_{ij}} \right)^{-\frac{1}{\theta}}$. Then

$$\begin{aligned} E(V) &= - \int_0^\infty v \exp \left(-v^{-\theta} \sum_{i,j \in \Omega} \psi_{ij} \right) \left[- \left(\sum_{i,j \in \Omega} \psi_{ij} \right) v^{-\theta-1} dv \right] \\ &= - \int_\infty^0 \left(\frac{x}{\sum_{i,j \in \Omega} \psi_{ij}} \right)^{-\frac{1}{\theta}} \exp(-x) dx = \int_0^\infty \left(\frac{x}{\sum_{i,j \in \Omega} \psi_{ij}} \right)^{-\frac{1}{\theta}} \exp(-x) dx \\ &= \int_0^\infty x^{(1-\frac{1}{\theta})-1} \exp(-x) dx \left(\sum_{i,j \in \Omega} \psi_{ij} \right)^{\frac{1}{\theta}} \\ &= \Gamma \left(1 - \frac{1}{\theta} \right) \left(\sum_{i,j \in \Omega} a_{ij} V_{ij}^\theta \right)^{\frac{1}{\theta}} = \Gamma \left(1 - \frac{1}{\theta} \right) \left(\sum_{i \in \Omega} a_{ij} \left(\frac{w_i s_j - (1 + \tau_j) r_j \bar{h}}{\kappa_{ij} [(1 + \tau_j) r_j]^{1-\alpha}} \right)^\theta \right)^{\frac{1}{\theta}}. \end{aligned}$$

3.3 The Effects of Changing Zoning Regulations

In this model, changes in local zoning regulations are interpreted as changes in local productivity of the housing sector, G_j . Therefore, if a neighborhood is allowed to build more

housing units per parcel or decreases the minimum lot size of each parcel, the model captures these changes as increases in G_j .

In the model, the mechanism works as follows. When housing productivity goes up in a location, housing can be produced at lower marginal cost. This has the effect of lowering rents for those already residing in location, which is equivalent to a movement along the housing demand curve. As a consequence, residents already living in the location demand more housing. Residential amenities may move upwards or downwards, depending on how much rent and housing demand respond to the change.

The second-order effects of the policy change come from the general equilibrium structure of the model. Due to lower rents in the location, residents from other locations move, which is equivalent to a shift in the housing demand curve. As an effect, rents goes up. The population and rent increases unequivocally increases taxes collected in the location, making room for a higher supply of neighborhood amenities, which again reinforces the incentives to move in. Because of commuting costs, some of the new residents change their workplace location to work nearby. The possibility of a downward-sloping labor demand curve if $\beta + \eta < 1$, wages tend to fall locally and rise in locations farther away that lost residents and workers.

Other general equilibrium effects are also present. For instance, locations that lose workers producing the consumption good due to the spatial reallocation of residents will observe an increase in local wages, an unintended effect of the policy. In addition, because of commuting costs, locations close to the one which implemented the policy may observe an increase in population as well. These results highlight the importance of analyzing changes in housing policies in a broader context other than the city or county that implemented them if there are nearby regions that will be directly impacted by it.

4 Quantitative Analysis

In this section, I apply the model presented above to analyze the impact of allowing for upzoning in the Minneapolis 2040 plan. I set up the model so that it replicates patterns of the data on the Twin Cities before January 1st 2020, when the new zoning rule took place. That is, the model is supposed to replicate the commuting patterns, local population and labor force, rents and wages across the Twin Cities metro area when most of the residential part of Minneapolis was zoned as single-family, detached, units. I then use the data on higher-density areas to inform the change in housing productivity we should expect to happen when the neighborhoods are allowed to upzone.

The seven counties comprising the Twin Cities metro area contain 702 census tracts in

total. Of these tracts, 113 are in the city of Minneapolis. The objective is to use data at the tract level on housing, population, wages, commuting patterns, property taxes, rents and commuting costs to inform the model.

4.1 Mapping to Data

The main data sources used for the empirical exercise are the following. I use data on wages, residents and workers from the Longitudinal Origin-Destination Employment Statistics (LODES). They provide origin and destination data on the population of workers that at the Census block level, as well as data on wages by brackets. I use Minnesota Geospatial Commons' Metro Regional Parcel Dataset, which compiles parcel-level data for all the seven counties, as the source for zoning and rents in each Census tract. Monthly rent is calculated using the following formula:

$$\frac{r}{1+r} \frac{\text{Average Building Value/Units}}{1 - (1+r)^{-T}}, \quad r = 0.06/12, \quad T = 20 \times 12$$

Commuting costs are calculated using IRS estimate of \$0.58 cents per mile. I compute distance across locations in miles using Google Maps. I use local, county-level property tax rates to calibrate τ_j . Productivity can be obtained by inverting the model to match wages.

4.2 Gravity Equation

Following Monte, Redding and Rossi-Hansberg (2016), I regress commuting patterns on commuting costs, origin and destination fixed effects to identify the shape parameter of the Frechet distribution, θ :

$$\log \left(\frac{\pi_{ij}}{\pi_{jj}} \right) = -\theta \log \left(\frac{\kappa_{ij}}{\kappa_{jj}} \right) + \mu_i + \mu_j + u_{ij}$$

The estimated θ was 4.4, within the bounds of the literature. For location taste shock, I use $a_{ij} = \pi_{ij} \left(\frac{\kappa_{ij}}{w_i} \right)^\theta$. In the near future, I intend to use the estimates for θ and a_{ij} described above as initial guesses and update the estimates using model, since the procedure used is not fully consistent with the model.

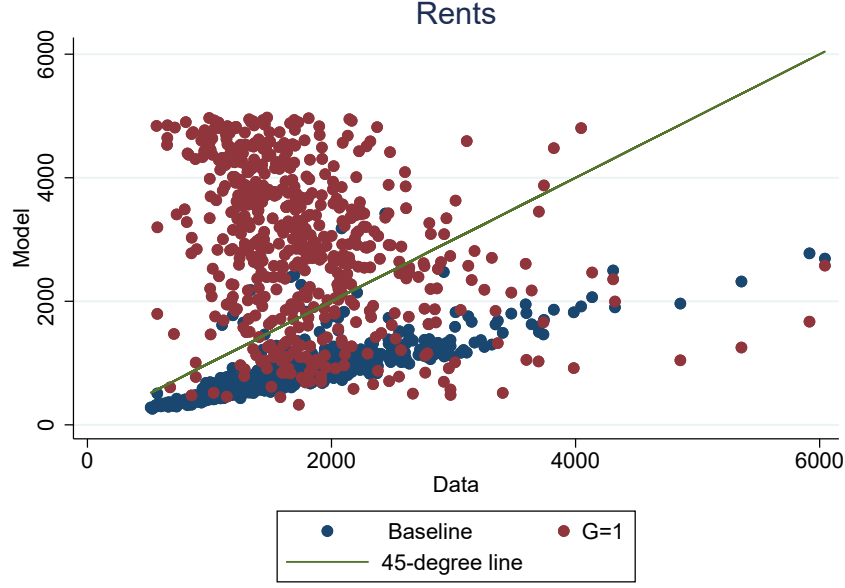


Figure 3: Model Fit–Baseline vs. Homogeneous Productivity in Housing Sector

4.2.1 Identification of Housing Productivity Parameters

From the equilibrium conditions of the model, I can write the housing productivity parameter for every tract as proportional to MSA-wide parameters from the model, rent, population density (residents per square mile) and the data equivalent of w_j :

$$G_j = \text{constant} \times \text{Rent}_j^{-1} \left\{ \left(\frac{\text{Residents}_j}{\text{Land}_j} \right) \left[\alpha \bar{h} + (1 - \alpha) \frac{\text{average wage}_j}{(1 + \tau_j) \text{rent}_j} \right] \right\}^{\psi \times \phi}.$$

Given parameters and data, I can recover G_j . Heuristically, given rents and square mileage, a higher number of residents imply that housing can be built at a lower marginal cost, suggesting higher productivity of the housing sector. Similarly, given wages and population density, a higher value for rent implies that housing is developed at a higher marginal cost, which suggests lower productivity of the housing sector in the particular tract.

In fact, pinning down G_j is crucial for model to reproduce rents as seen in data. As Figure 3 shows, if I assume that G_j is equal to one in every tract, the model does a much worse job at matching rent rates at the tract level compared to the specification in which I allow tract-level productivity differences for the housing sector.

4.3 Structural Estimation

I use Simulated Method of Moments to estimate $\alpha, \bar{h}, \beta, \eta$ and G_j . The objective is to compare the model outcomes for several endogenous variables and the data counterpart of those variables and find the set of parameters that best minimize the distance between the model outcomes and the data.

The moments I use from the data are the following: the distribution of wages $\{w_i\}_{i \in \Omega}$, rents $\{r_j\}_{j \in \Omega}$, share of non-commuting workers by location $\{\pi_{jj}\}_{j \in \Omega}$, population $\{R_j\}_{j \in \Omega}$, workers $\{n_i\}_{i \in \Omega}$. Housing productivity is computed from model inversion given parameters and data.

The estimation uses 5×702 observations to estimate $4 + 702$ parameters. Table 2 in Appendix B presents the results.

5 Quantitative Exercise—The Impact of Zoning Reform

Using the estimated model, I evaluate the impact of Minneapolis zoning reform on the equilibrium prices and allocations across the metropolitan area. In my model, a zoning reform allowing for more density is interpreted as an increase in G_j . This modeling choice assumes that allowing for the development of more housing units in a single plot of land reduces the cost per unit.

5.1 Zoning Reform in the Model

In this section, I discuss how I incorporate the zoning reform on G_j . The *Minneapolis 2040* plan rezoned the parcels previously marked as single family to allow for up to three dwelling units. As shown in Figure 2, single-family units constituted the vast majority of the residential parcels in the city. In my quantitative exercise, this reform is interpreted as an increase in G_j for the locations j that are affected by the reform.

To perform the exercise, I match the Census tracts with Minneapolis zoning map prior to the housing reform. The mapping between Census tracts and municipal zoning is not one-to-one, which means that zoning is not homogeneous for each tract. Restricting attention to residential zoning, about ten percent were fully zoned as single family. Figure 4 plots the histogram of the distribution of tracts by share of lots zoned as single family.

I then implement the following empirical strategy: with the estimated housing pro-

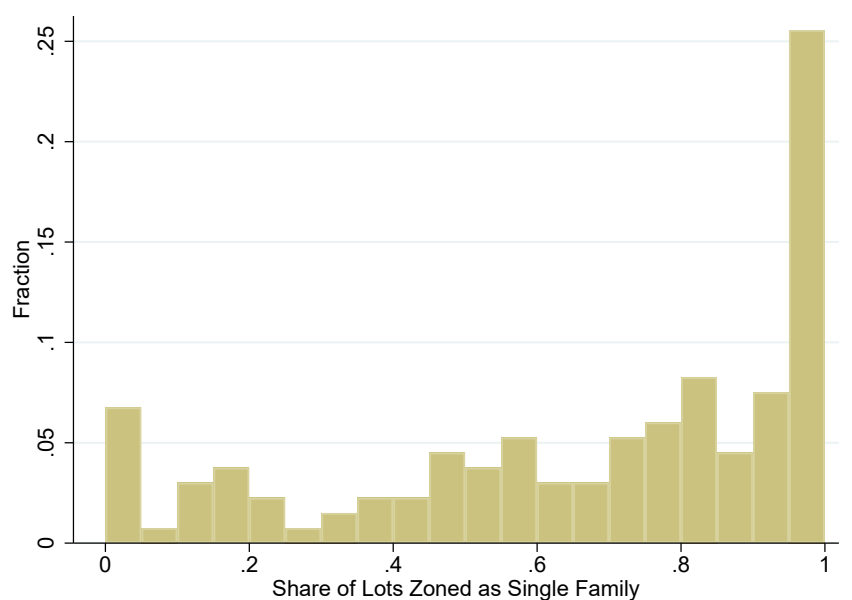


Figure 4: Distribution of Tracts by Share of Lots Zoned as Single Family

ductivities G_j , I run the following regression:

$$G_j = \beta_0 + \beta_1 \times \text{share of single-family units}_j + \beta_2 \times \text{dist}_j + \beta_3 \times \text{dist}_j^2 + u_j,$$

where $\text{share of single-family units}_j$ represents the tract's share of residential parcels previously zoned as single family, and dist_j is the tract j 's distance, in miles, to the most central tract in Downtown Minneapolis. When I run this regression, the coefficient for share of single-family units and distance squared are negative. The fact that the former is negative serves as validation of the model.

I discipline the G_j 's after policy in the following way: I use estimated equation above and set the share of single-family units to the minimum that I observe in the data, which is 0.03%. Because $\beta_1 < 0$ and different tracts have different shares of parcels zoned as single family, housing productivity will increase more in the tracts with higher initial shares of single-family units. Moreover, since the share comes from the zoning code, not de facto development, this method ensures that the productivity increase I introduce in the model primitives are exogenous. In addition, the inclusion of the quadratic polynomial form for distance allows my counterfactual housing productivities to increase more in tracts that are closer to Downtown Minneapolis. Adding this quadratic form in the construction of the counterfactual assumes that building at a lower marginal cost is likely increase in places that are already close to dense locations. Using the methodology, I find that median productivity in the housing development sector is expected to grow by 38 percent

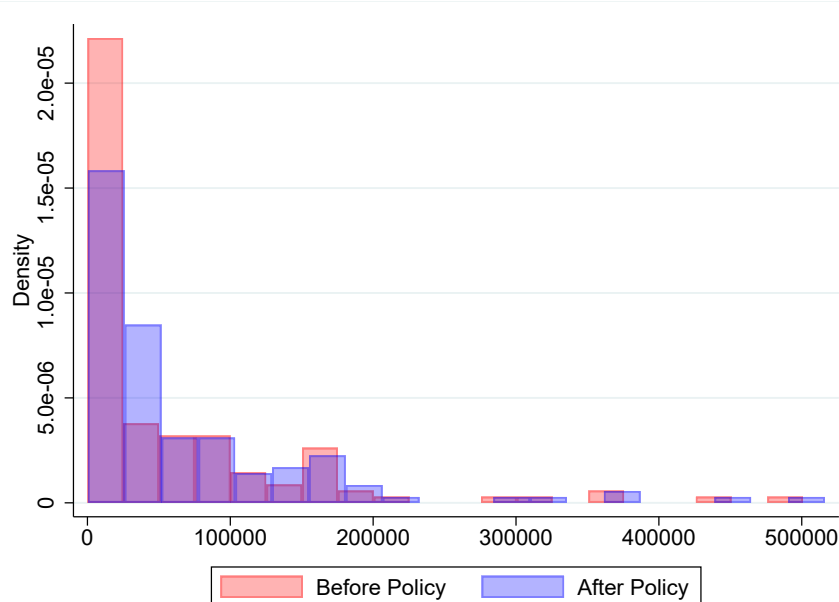


Figure 5: Distribution of Housing Productivity in Minneapolis Before and After The Zoning Reform

in the city. Figure 5 shows the change in the distribution of housing productivities inside Minneapolis.

5.2 Results

I now present the results of the quantitative exercise where I interpret the zoning reform in Minneapolis as a change in the productivity of the housing development sector. While the model is computed at the tract level, most of the results presented below are aggregated to the county level, with the exception being Minneapolis, where I aggregate to the city level. I also present the results holding amenities fixed. Because I highlight Minneapolis in the figures and in the results discussion, Hennepin County is aggregated without Minneapolis.

Figure 6 shows the impact of upzoning on rents. Overall, rent falls about 15 percent in Minneapolis. At the same time, population increases by about six percent in the city, as shown in Figure 7. Due to population reallocation, rents fall in other locations. In addition, population increases in nearby counties, such as Hennepin and Ramsey. The effects of the housing reform on rents is heterogeneous across tracts. As Figure 8 shows, this is true even for tracts outside Minneapolis. This highlights the importance of taking these general equilibrium effects across the metropolitan area: a decrease in marginal cost of producing housing in Minneapolis attracts workers to the city. This affects these

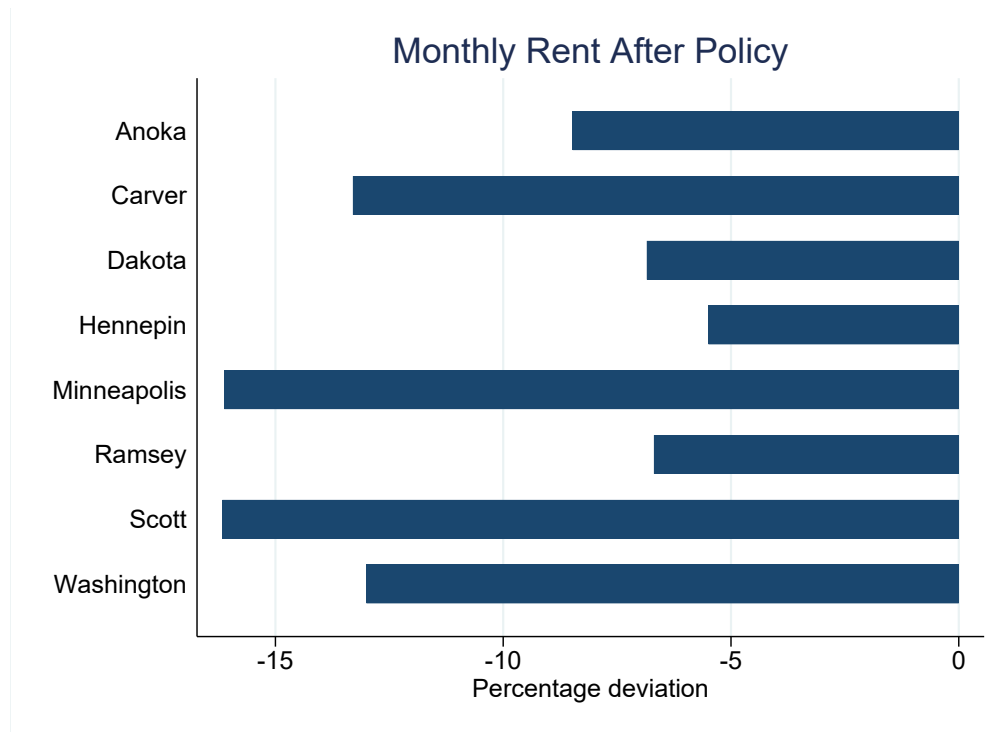


Figure 6: Impact on Rents

workers' workplace decision, which consequently affects wages. In turn, wages and rents in other locations also respond to this migration decision, which further generates other migration decisions in other locations in the metropolitan area as well.

Inside Minneapolis, the change in rents is only mildly positively correlated with the population changes, as reflected in Figure 9. Many tracts experience a large inflow of residents, but still experience a drop in rents. As Figure 10 suggests, the mechanism explaining this large influx couples with rent drops in many tracts is largely explained by the negative association between the variation in rents and the tract's pre-policy share of lots zoned as single-family units. Intuitively, since these tracts are now allowed to develop more housing units per lot due to the zoning reform, they can supply more housing to residents at a substantially lower cost. Finally, the drop in rents is also negatively associated with the tract's distance to Downtown Minneapolis.

Upzoning in Minneapolis also has effects on wages across the metropolitan area. Figure 12 presents the predicted wage changes from the baseline resulting from the upzoning. At the aggregate level, wages rise in all counties, as well as in Minneapolis. Wages rise the most in Hennepin County, where they rise by about 18 percent. As Figure 13 shows, the wage increases outside Minneapolis are explained by a significant drop in the number of agents working in those locations. Just as Hennepin County experienced the

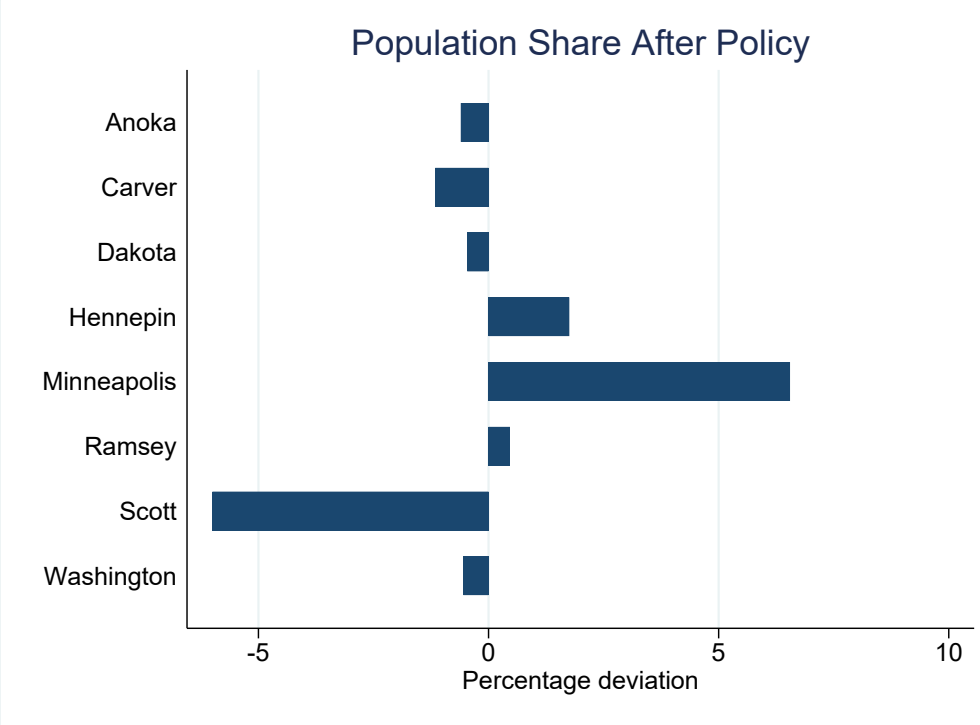


Figure 7: Distribution of Population

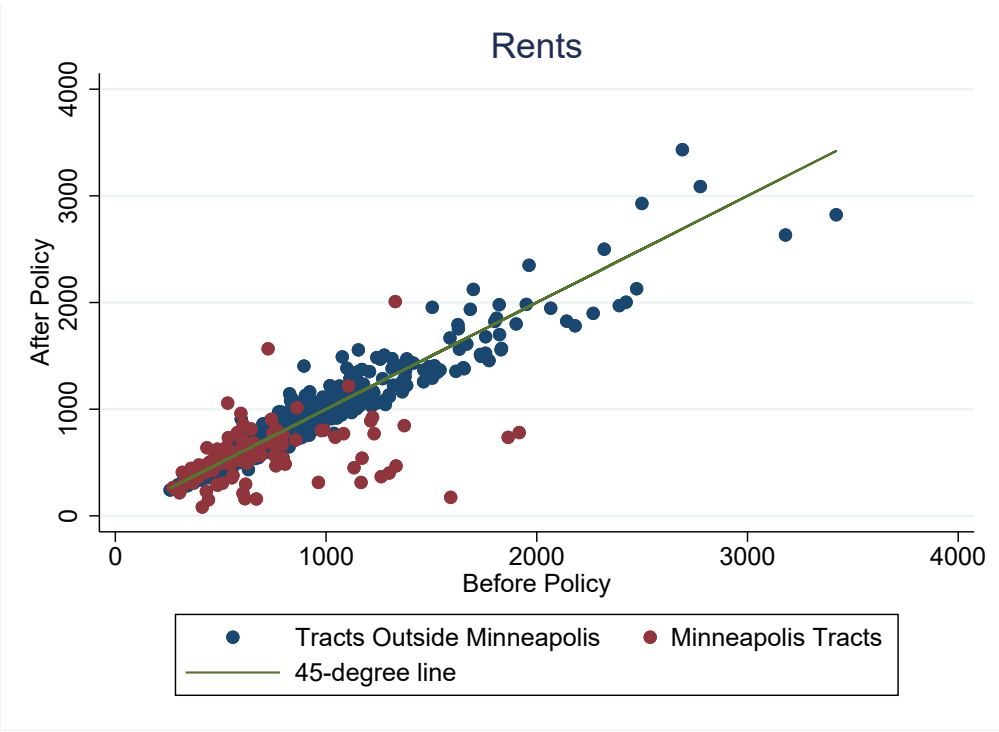


Figure 8: Impact on Rents–Tract Level

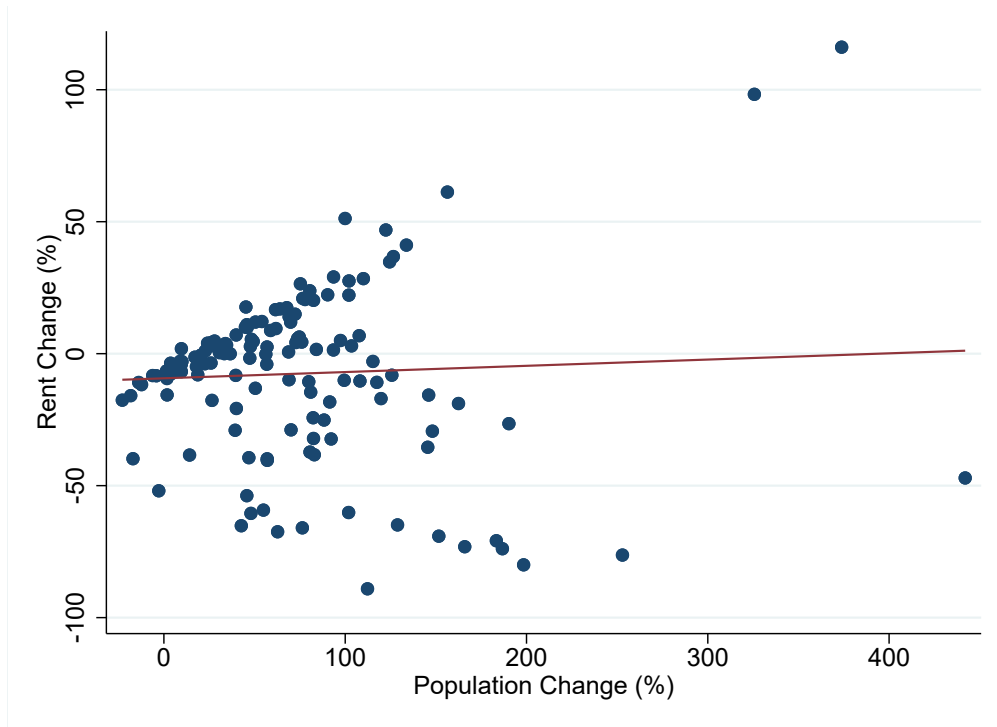


Figure 9: Relationship Between Population and Rent Changes
Scatter Plot and Trend Line

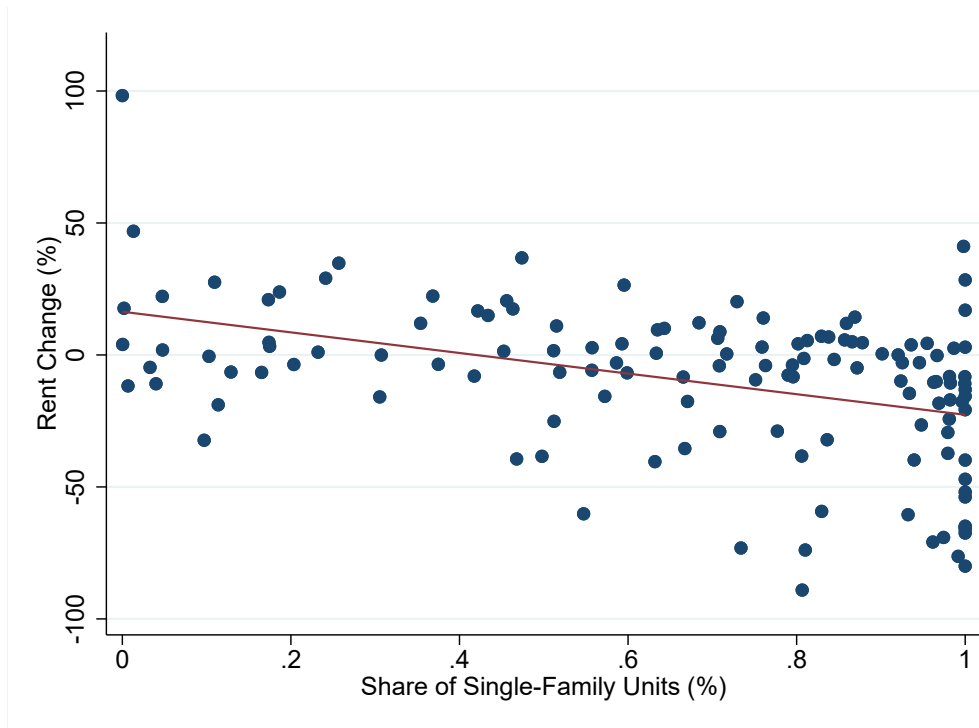


Figure 10: Relationship Between Rent Change and Share of Single-Family Units
Scatter Plot and Trend Line

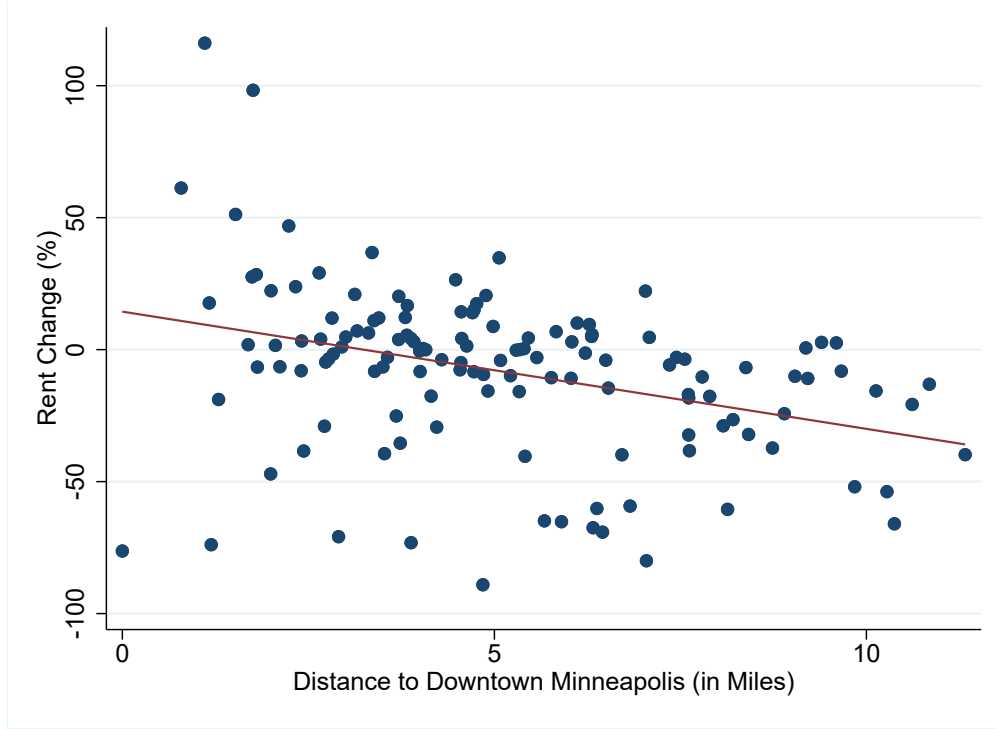


Figure 11: Relationship Between Rent Change and Distance to Downtown Minneapolis
Scatter Plot and Trend Line

highest wage increases, it also sees the highest decline in its share of workers.

Looking at the results at the tract level allows us to inspect the mechanism more carefully. As Figure 14 shows, while wages increase in most tracts, many of them experience a wage drop. In addition, Figure 15 shows that the model implies one particular tract, the one in the middle of Downtown Minneapolis, experiences a sharp increase in its number of workers: the tract's share of total employment jumps from about 1.5 percent of total employment in the metropolitan area to about 33 percent after the zoning reform. Two forces are at play here. First, the estimated parameters for the production function show that labor demand is downward sloping. Therefore, less workers in most locations implies higher wages for the workers that keep working in such places, even in the presence of agglomeration effects. Finally, because many agents decide to move to Minneapolis to reside in the city due to the zoning reform, many decide to work in the city's center, even if their wages are not as high as before. This is explained by the higher commuting costs many workers would incur if they chose to work somewhere else. In addition, the estimated set of location pair preferences from the Frechet distribution, a_{ij} , capture a strong preference for workers who live in Minneapolis to work in the city's Downtown area. Because many workers move to Minneapolis after housing became cheaper across most of the city, they decide to work in downtown, explaining the tract's significant rise

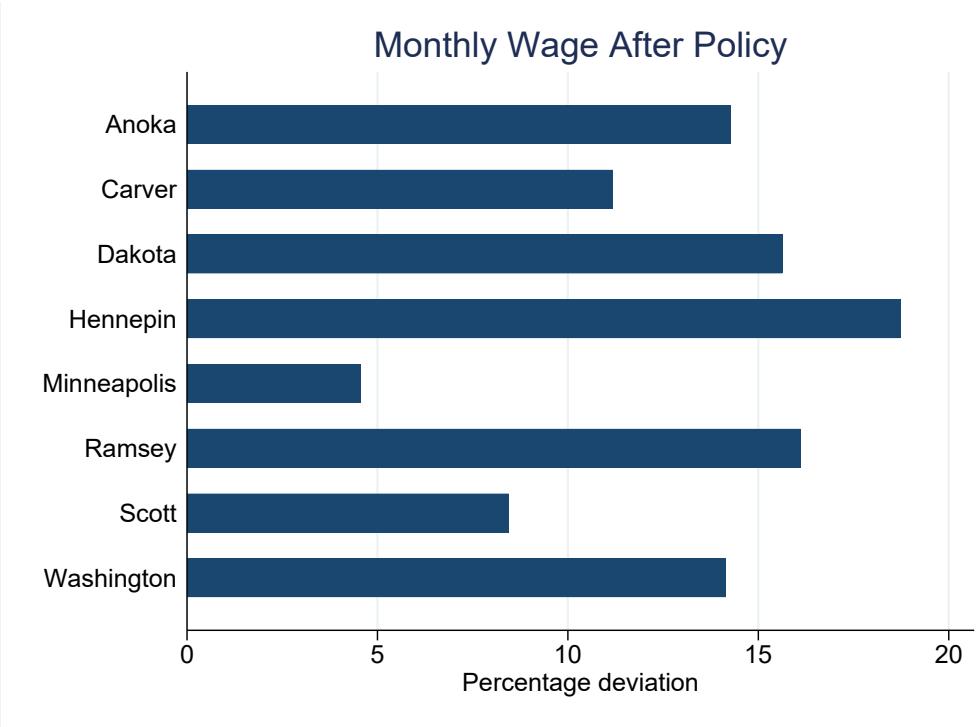


Figure 12: Impact on Wages

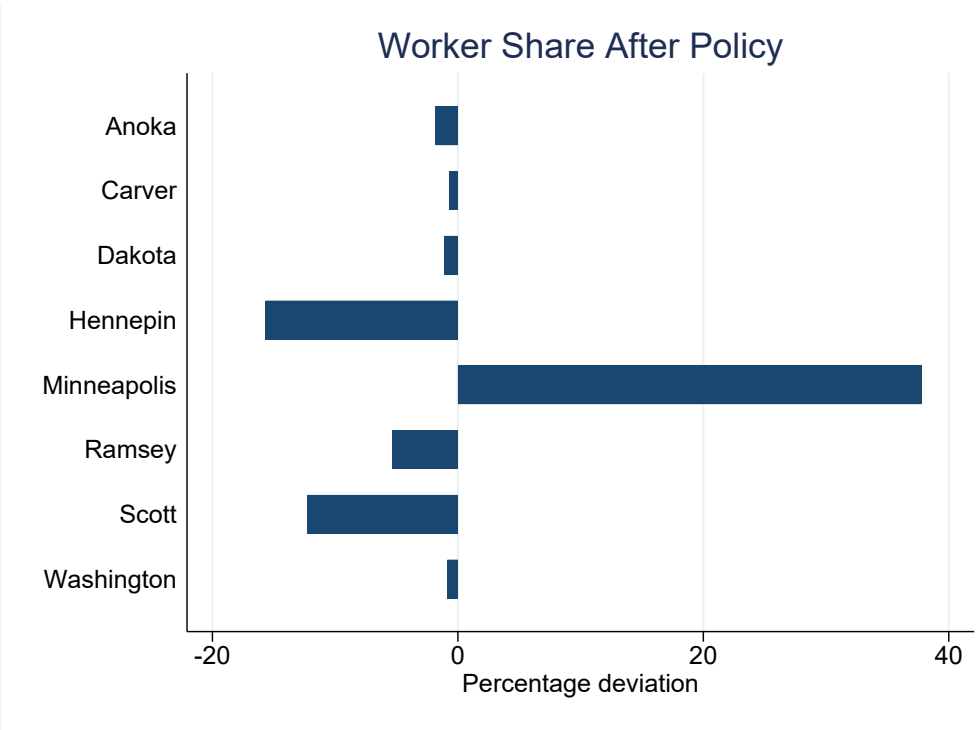


Figure 13: Distribution of Workers

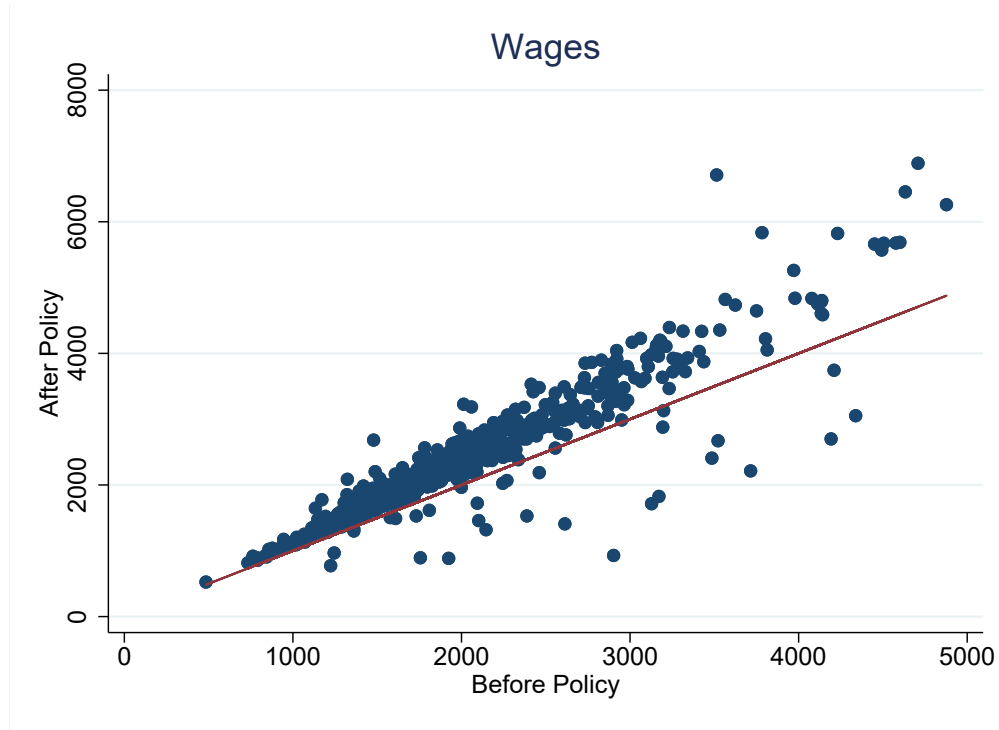


Figure 14: Impact on Wages–Tract Level
Scatter Plot and 45-degree Line

in its share of overall workforce in the metropolitan area.

6 Conclusion

In this paper, I analyzed the general equilibrium effects of upzoning in a city from the perspective of the metropolitan area. I built a spatial model with heterogeneous locations, amenities, productivities, agglomeration and congestion forces, and commuting to quantify the impact of the policy. Local housing policies affect the equilibrium outcomes not only of the city that implements the aforementioned policy, but also of the ones directly connected to it in the greater urban area.

I find quantitatively important effects throughout the metropolitan area. Housing becomes more affordable in the desired location, but this effect also spills over most of other counties as well. At the same time, I find that upzoning is likely to attract more workers, but at the cost of lowering wages due the increase in labor supply locally. In general, the whole metropolitan area benefits from the policy, not just the city which implemented the policy.

The results from this paper highlight the importance of analyzing housing reforms

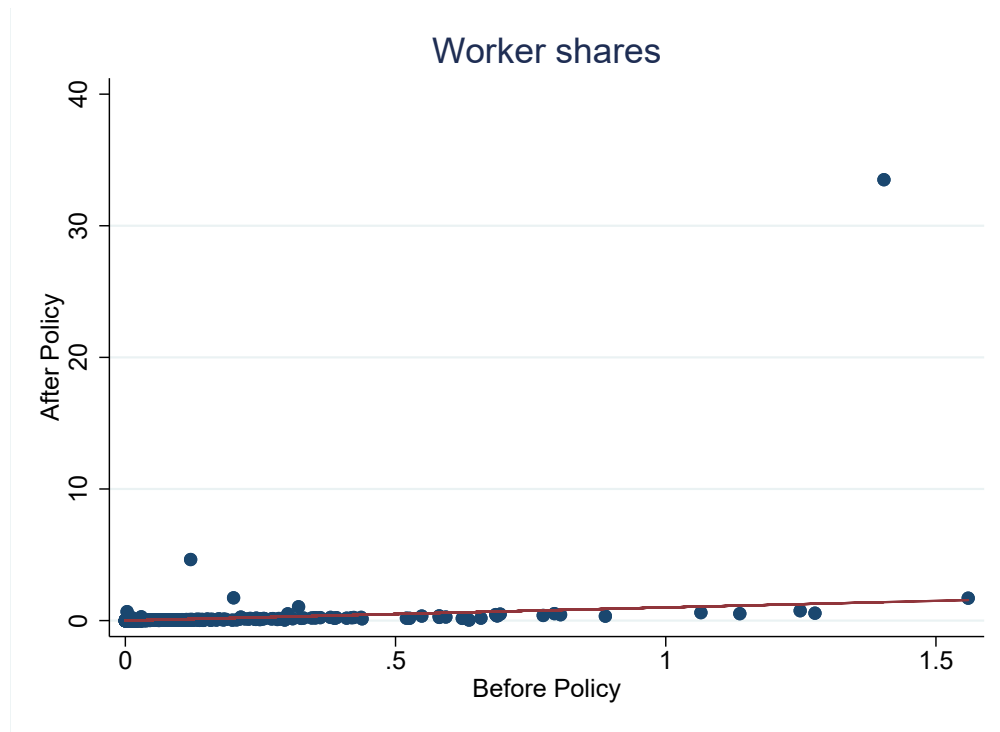


Figure 15: Distribution of Workers–Tract Level
Scatter Plot and 45-degree Line

in the perspective of a larger metropolitan area. They may have unexpected benefits and losses to nearby cities that could potentially be taken into account when discussing housing policies.

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Appendix

A Tract-Level Housing Productivity and Zoning Reform

Suppose there's a developer that wants to build in a parcel with land size L_i in location j . They combine L_i with materials M_i to create total square footage in the parcel, with encompasses both floorspace by unit and number of units in the parcel. Developer chooses both u_i , the number of units in the parcel, and a_i , floorspace by unit. The parcel-level productivity is given by g_i . Their maximization problem is:

$$\max_{u, M} r_j u g_i L_i^{(1-\phi)} M^\phi - \iota M - p_j \left(\frac{u}{L_i} \right).$$

Given u , the supply of floorspace per unit is given by $g_i^{\frac{1}{1-\phi}} u^{\frac{\phi}{1-\phi}} \left(\frac{\phi r_j}{\iota} \right)^{\frac{\phi}{1-\phi}} L_i$. After maximizing with the respect to M , the developer then picks $u \in 0, 1, \dots, \bar{u}$ that maximizes profits. Denote this u as $u^*(\bar{u})$. The upper bound, \bar{u} , is dictated by local zoning laws.

Similarly, suppose there is a representative developer at the tract level, with productivity G_j and land L_j . The supply function of this developer is given by $G_j^{\frac{1}{1-\phi}} \left(\frac{\phi r_j}{\iota} \right)^{\frac{\phi}{1-\phi}} L_j$.

The sum of total housing supplied by land, in both cases, is given by

$$\begin{aligned} \sum_{i \in j} h_i &= \left(\frac{\phi r_j}{\iota} \right)^{\frac{\phi}{1-\phi}} \sum_{i \in j} g_i^{\frac{1}{1-\phi}} u_i^*(\bar{u})^{\frac{\phi}{1-\phi}} L_i \\ H_j &= \left(\frac{\phi r_j}{\iota} \right)^{\frac{\phi}{1-\phi}} G_j^{\frac{1}{1-\phi}} L_j. \end{aligned}$$

If we want to use the aggregate housing production function to represent the sum of housing supply by lot, internal consistency requires that

$$G_j = \left[\sum_{i \in j} \frac{L_i}{L_j} g_i^{\frac{1}{1-\phi}} u_i^*(\bar{u})^{\frac{\phi}{1-\phi}} \right]^{1-\phi}.$$

That is, tract-level productivity is the land-weighted sum of lot-level productivity and optimal unit choice. For the counterfactual exercise presented in the main body of the text to make sense conceptually, it is sufficient that the choice of $u^*(\bar{u}_{pre}) < u^*(\bar{u}_{post})$, where \bar{u}_{pre} is the upper bound on development before the zoning reform and \bar{u}_{post} is the upper bound after the zoning reform. In such case, an increase in number of units per lot is equivalent to a productivity increase in the tract level.

B Parameter Estimates

| Parameter | Estimate | Standard Error |
|-----------|----------|----------------|
| α | 7.9e-5 | 0.001 |
| \bar{h} | 2.631 | 5.592 |
| β | 0.787 | 0.005 |
| η | 5.6e-9 | 0.005 |

Table 2: Parameter Estimates

C Computational strategy

1. Guess array \mathbb{V}^0 ;
2. Compute $\lambda = \left[\sum_{i,j \in \Omega} a_{ij} V_{ij}^0 \right]^{-1}$;
3. Calculate residents and workers by neighborhood using

$$R_j = \lambda \sum_{i \in \Omega} a_{ij} (V_{ij}^0)^\theta \bar{R}$$

$$n_i = \lambda \sum_{j \in \Omega} a_{ij} (V_{ij}^0)^\theta \bar{R}.$$

4. Derive neighborhood wages using

$$w_i = \beta \bar{A}_i n_i^{\beta+\eta-1}.$$

5. Compute \bar{w} and find housing by neighborhood using

$$\frac{(1-\alpha)\bar{w}_j}{1+\tau_j} \left[\frac{H_j}{R_j} - \alpha \bar{h} \right]^{-1} = \rho_j \left(\frac{H_j}{L_j} \right)^\psi.$$

6. Calculate rent using

$$r_j = \rho_j \left(\frac{H_j}{L_j} \right)^\psi.$$

7. Find local amenities:

$$s_j = \tau_j r_j R_j \left[\alpha \bar{h} + (1 - \alpha) \frac{\bar{w}_j}{(1 + \tau_j) r_j} \right].$$

8. Update indirect utility \mathbb{V}^1 , where:

$$V_{ij}^1 = \frac{w_i s_j - (1 + \tau_j) r_j \bar{h}}{\kappa_{ij} [(1 + \tau_j) r_j]^{1-\alpha}}$$

9. Check if $||\mathbb{V}^1 - \mathbb{V}^0|| < 10^{-6}$. Stop if true. If not, set $\mathbb{V}_{new}^0 = .25\mathbb{V}^1 + .75\mathbb{V}_{old}^0$.