# Lecture slides with LATEX A user friendly template

#### Pedro Fonseca



ISEG Lisbon School of Economics & Management

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#### Before we start



#### In this lecture you will see:

- a graph
- some formulas
- some code
- some citations (Kass and Raftery, 1995)

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## A scatterplot



This picture is in the "figures" folder:

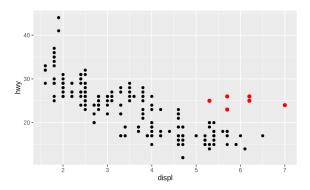


Figure 1: Fuel consumption (Hwy) vs engine size (displ)

#### Model selection



- Likelihood:  $\mathbf{x}|\mathbf{\theta} \sim \mathcal{M}_k(N,\mathbf{\theta})$
- Hypotheses:  $H_0: \theta = \theta_0$  vs  $H_1: \theta \neq \theta_0$
- lacksquare Prior:  $\pi( heta|H_0)=1_{ heta_0}( heta_0)$  and  $heta|H_1,lpha\sim \mathsf{Dir}_k(lpha)$
- Marginal density:  $m_i(\mathbf{x}) = \int_{\Theta_i} f(\mathbf{x}|\boldsymbol{\theta}) \pi(\boldsymbol{\theta}|H_i) d\boldsymbol{\theta}, \quad i = 0, 1$

$$m_0(\mathbf{x}) = \frac{N!}{\prod_{i=1}^{k+1} x_i!} \prod_{i=1}^{k+1} \theta_{0i}^{x_i}$$

Bayes factor:

$$B_{01}(\mathbf{x}) = \frac{m_0(\mathbf{x})}{m_1(\mathbf{x})} = \frac{\prod_{i=1}^{k+1} (\theta_{0i}^{x_i}) \prod_{i=1}^{k+1} [\Gamma(\alpha_i)] \Gamma[\sum_{i=1}^{k+1} (\alpha_i + x_i)]}{\Gamma(\sum_{i=1}^{k+1} \alpha_i) \prod_{i=1}^{k+1} \Gamma(\alpha_i + x_i)}$$

• For an interesting application see Pericchi and Torres (2011).

#### Parameter estimation



- $lacktriangleq \mathsf{Prior} \colon m{ heta} | m{lpha} \sim \mathsf{Dir}_k(m{lpha})$
- Posterior:  $\theta | \alpha, \mathbf{x} \sim \mathsf{Dir}_k(\alpha + \mathbf{x}) \to \mathsf{Mean}$ , mode, credible intervals.
- For an interesting application see Ley (1996).

### Model selection



- Likelihood:  $x \sim Bin(N, \theta)$
- Hypotheses:  $H_0: \theta = \theta_0$  vs  $H_1: \theta \neq \theta_0$
- Prior:  $\pi(\theta|H_0)=1_{\theta_0}(\theta)$  and  $\theta|H_1\sim \mathsf{Beta}(\mathsf{a},\mathsf{b})$
- ullet Marginal density:  $m_i(x) = \int_0^1 f(x| heta)\pi( heta|H_i)d heta, \ i=0,1$ 
  - $m_0(x) = \binom{N}{x} \theta_0^x (1 \theta_0)^{N-x}$
- Bayes factor:  $B_{01}(x) = \frac{m_0(x)}{m_1(x)} = \frac{\theta_0(1-\theta_0)^{N-x} \Gamma(a) \Gamma(b) \Gamma(n+a+b)}{\Gamma(a+b) \Gamma(n+a-x) \Gamma(x+a)}$

Binomial - Beta Model

#### Parameter estimation



- Prior:  $\theta \sim \text{Beta}(a, b)$
- Posterior:  $\theta | x \sim \text{Beta}(a + x, b + N x) \rightarrow \text{Mean, mode, credible intervals}$

Python code

# Some python code



Lets print the Fibonacci sequence.

```
nterms = int(input("How many terms? "))
n1, n2 = 0, 1
count = 0
print("Fibonacci sequence:")
while count < nterms:
    print(n1)
    nth = n1 + n2
    n1 = n2
    n2 = nth
    count += 1
```

R code

#### Now some R code



I also included a a verbatim environment with background color:

```
getmode <- function(v) {
  uniqv <- unique(v)
  uniqv[which.max(tabulate(match(v, uniqv)))]
}</pre>
```

With this R code you can build a function that calculates the mode.

## References



- Kass, RE and AE Raftery (1995). Bayes factors. *Journal of the American Statistical Association* **90**(430), 773–795.
- Ley, E (1996). On the peculiar distribution of the US stock indexes' digits. The American Statistician  ${\bf 50}(4),\,311{-}313.$
- Pericchi, L and D Torres (2011). Quick anomaly detection by the Newcomb—Benford law, with applications to electoral processes data from the USA, Puerto Rico and Venezuela. Statistical Science 26(4), 502–516.