Transport equation

The convection-diffusion equation for the transport of temperature T is

$$\frac{\partial T}{\partial t} = -U\frac{\partial T}{\partial x} + k\frac{\partial}{\partial x}(\frac{\partial T}{\partial x}) + S \tag{1}$$

where U is velocity and S a source term. For a non existing convection case, Equation (1) becomes the diffusion Equation

$$\frac{\partial T}{\partial t} = k\frac{\partial}{\partial x}(\frac{\partial T}{\partial x}) + S \tag{2}$$

that, for a steady-state, is

$$0 = k\frac{\partial}{\partial x}(\frac{\partial T}{\partial x}) + S \tag{3}$$

These equations are solved by a Finite Volume Method (FVM) and by a Finite Difference Method (FDM).

Finite Volume Method

In the Finite Volume Method, values for the above diferential equations are calculated at discrete places on a grid of volumes, shown in Figure 1, where temperature at the center of a cell is T_P, and T_R e T_L is temperature at the border between cells, right and left. Integrating Equation 3 over a cell volume, and considering the rate of accumulation over the volume V equal to the flow across the surfaces of the control volume, Equation (6), where n is the unit normal vector pointing out of the control volume and A is the cross sectional area of the volume.

$$\int \left[k\frac{\partial}{\partial x}(\frac{\partial T}{\partial x}) + S\right] dV = 0 \tag{4}$$

$$\int \left[\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x})dV\right] + SV = 0 \tag{5}$$

$$\int (k\frac{\partial T}{\partial x}n)dA + SV = 0 \tag{6}$$

Equation (6) is written in terms of the flux leaving the right face r minux the flux entering the left face l in Figure (1)

$$(kA\frac{\partial T}{\partial x})_r - (kA\frac{\partial T}{\partial x})_l + SV = 0 \tag{7}$$

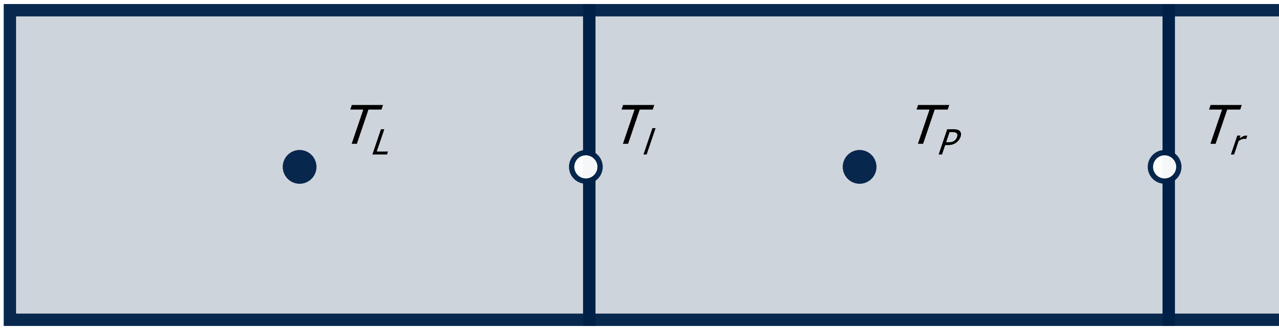

Figure 1: Temperature at center of cell, and on left and right cell borders.

Equation (7) for the temperature gradient at the boundaries is dicretized in terms of the temperature at the cell centers, where Δx is the distance between cell centers

$$kA\frac{T_R - T_P}{\Delta x} - kA\frac{T_P - T_L}{\Delta x} + SV = 0 \tag{8}$$

An individual set of equations is made for each cell.

Finite Difference Method

The diffusion equation (2) is approximated by finite differences with

$$\frac{T_i^{n+1} - T_i^n}{\Delta x} = k\frac{T_{i-1}^n - 2T_i^n + T_{i+1}^n}{\Delta x^2} + S_i^n \tag{9}$$

References

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3. Finite Difference Computing with PDEs. A Modern Software Approach. H. Langtangen, S. Linge. 2016.