

## Transport equation

The convection-diffusion equation for the transport of temperature  $T$  is

$$\frac{\partial T}{\partial t} = -U \frac{\partial T}{\partial x} + k \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + S \quad (1)$$

where  $U$  is velocity and  $S$  a source term. For a non existing convection case, Equation (1) becomes the diffusion Equation

$$\frac{\partial T}{\partial t} = k \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + S \quad (2)$$

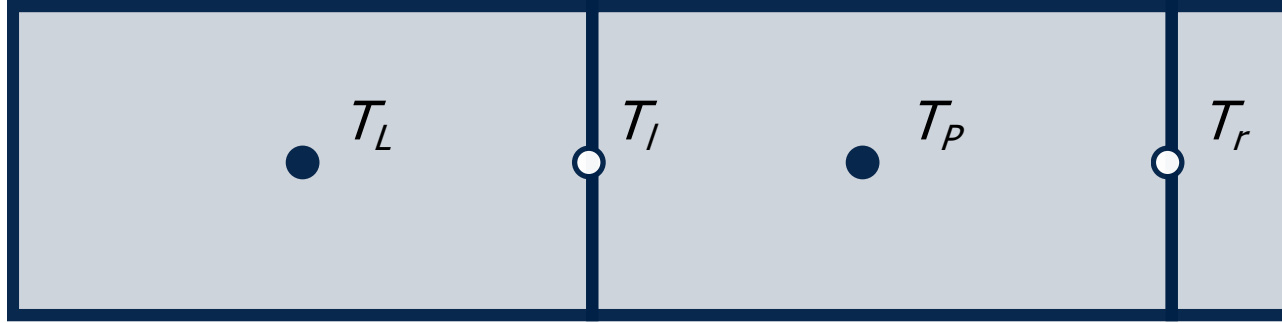
that, for a steady-state, is

$$0 = k \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + S \quad (3)$$

These equations are solved by a Finite Volume Method (FVM) and by a Finite Difference Method (FDM).

## Finite Volume Method

In the Finite Volume Method, values for the above differential equations are calculated at discrete places on a grid of volumes, shown in Figure 1,



**Figure 1:** Temperature at center of cell, and on left and right cell borders.

where temperature at the center of a cell is  $T_P$ , and  $T_R$  e  $T_L$  is temperature at the border between cells, right and left. Integrating Equation 3 over a cell volume, and considering the rate of accumulation over the volume  $V$  equal to the flow across the surfaces of the control volume, Equation (6), where  $n$  is the unit normal vector pointing out of the control volume and  $A$  is the cross sectional area of the volume.

$$\int \left[ k \frac{\partial}{\partial x} \left( \frac{\partial T}{\partial x} \right) + S \right] dV = 0 \quad (4)$$

$$\int \left[ \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) dV \right] + SV = 0 \quad (5)$$

$$\int \left( k \frac{\partial T}{\partial x} n \right) dA + SV = 0 \quad (6)$$

Equation (6) is written in terms of the flux leaving the right face  $r$  minus the flux entering the left face  $l$  in Figure (1)

$$\left( kA \frac{\partial T}{\partial x} \right)_r - \left( kA \frac{\partial T}{\partial x} \right)_l + SV = 0 \quad (7)$$

Equation (7) for the temperature gradient at the boundaries is discretized in terms of the temperature at the cell centers, where  $\Delta x$  is the distance between cell centers

$$kA \frac{T_R - T_P}{\Delta x} - kA \frac{T_P - T_L}{\Delta x} + SV = 0 \quad (8)$$

An individual set of equations is made for each cell.

## Finite Difference Method

The diffusion equation (2) is approximated by finite differences with

$$\frac{T_i^{n+1} - T_i^n}{\Delta x} = k \frac{T_{i-1}^n - 2T_i^n + T_{i+1}^n}{\Delta x^2} + S_i^n \quad (9)$$

## References

1. Computational Fluid Dynamics Fundamentals Course. A. Wimshurst. 2019.
2. An Introduction to Computational Fluid Dynamics: The Finite Volume Method. H. Versteeg, W. Malalasekera. 2007.
3. Finite Difference Computing with PDEs. A Modern Software Approach. H. Langtangen, S. Linge. 2016.