## Transport equation

The convection-diffusion equation for the transport of temperature $T$ is

$$\frac{\partial T}{\partial t} = -U\frac{\partial T}{\partial x} + k\frac{\partial}{\partial x}(\frac{\partial T}{\partial x}) + S \tag{1}$$

where $U$ is velocity and $S$ a source term. For a non existing convection case, Equation (1) becomes the diffusion Equation

$$\frac{\partial T}{\partial t} = k\frac{\partial}{\partial x}(\frac{\partial T}{\partial x}) + S \tag{2}$$

that, for a steady-state, is

$$0 = k\frac{\partial}{\partial x}(\frac{\partial T}{\partial x}) + S \tag{3}$$

These equations are solved by a Finite Volume Method (FVM) and by a Finite Difference Method (FDM).

## Finite Volume Method

In the Finite Volume Method, values for the above diferential equations are calculated at discrete places on a grid of volumes, shown in Figure 1,

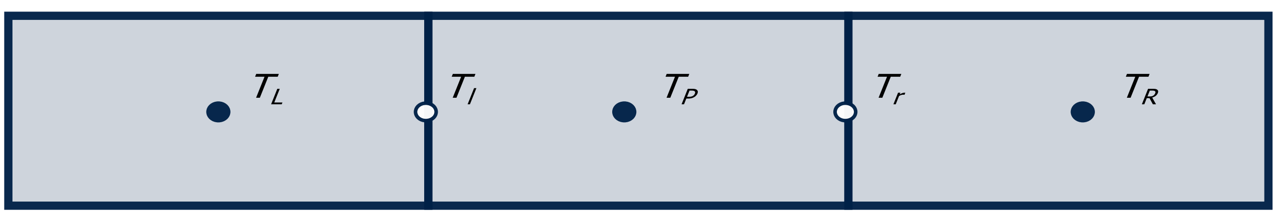


**Figure 1**: Temperature at center of cell, and on left and right cell borders.

where temperature at the center of a cell is $T_P$, $T_r$ e $T_l$ is temperature at the border between cells, right and left, $T_R$ e $T_L$ is temperature at the right and left cell centers. Integrating Equation 3 over a cell volume, and considering the rate of accumulation over the volume $V$ equal to the flow across the surfaces of the control volume, Equation (6), where $n$ is the unit normal vector pointing out of the control volume and $A$ is the cross sectional area of the volume.

$$\int \left[k\frac{\partial}{\partial x}(\frac{\partial T}{\partial x}) + S\right] dV = 0 \tag{4}$$

$$\int \left[ \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x})dV \right] + SV = 0 \tag{5}$$

$$\int (k\frac{\partial T}{\partial x}n)dA + SV = 0 \tag{6}$$

Equation (6) is written in terms of the flux leaving the right face $r$ minus the flux entering the left face $l$ in Figure (1)

$$(kA\frac{\partial T}{\partial x})_r - (kA\frac{\partial T}{\partial x})_l + SV = 0 \tag{7}$$

Equation (7) for the temperature gradient at the boundaries is dicretized in terms of the temperature at the cell centers, where $\Delta x$ is the distance between cell centers

$$kA\frac{T_R - T_P}{\Delta x} - kA\frac{T_P - T_L}{\Delta x} + SV = 0 \tag{8}$$

An individual set of equations is made for each cell.

## Finite Difference Method

The diffusion equation (2) is approximated by finite differences with

$$\frac{T_i^{n+1} - T_i^n}{\mathit{\Delta} x} = k\frac{T_{i-1}^n - 2T_i^n + T_{i+1}^n}{\mathit{\Delta} x^2} + S_i^n \tag{9}$$

## References


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