

$$\begin{array}{ccc}
 \mathbb{R}^3 \setminus \{0\} & \begin{array}{c} \xrightarrow{f} \\ \xleftarrow{\iota} \end{array} & S^2 \\
 \downarrow \pi_1 & & \downarrow \pi_2 \\
 \mathbb{RP}^2 & \begin{array}{c} \xrightarrow{\tilde{f}} \\ \xleftarrow{\tilde{\iota}} \end{array} & \tilde{S}^2
 \end{array}$$