Ondas

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1 Hasegawa-Mima equation

$$\frac{\partial}{\partial t}\Big(\nabla^2\phi - \phi\Big) - [(\nabla\phi\times\hat{\mathbf{z}})\cdot\nabla]\Big[\nabla^2\phi - \ln\Big(\frac{n_0}{\omega_{ci}}\Big)\Big] = 0 \tag{1}$$

The Hasegawa-Mima equation 1 describes the drift-wave propagation of waves in plasma and the emergence of cross-field transport, the equation is a starting point in the study of instability in plasma drift waves and how it can decay in two or more coupled waves, that emerge when the initial waves reach a limit value

The equation is based on the propagation of electrostatic waves with frequency $\omega \ll \omega_{ci}$, where $\omega_{ci} = eB/m_i$ is the ion cyclotron frequency. The wave is propagated in a magnetized and inhomogeneous plasma medium. The wave is then treated as a drift wave.

One of the solutions to this equation is using to consider the potential as an infinite sum of waves in the form of

$$\phi(\mathbf{r},t) = \frac{1}{2}\sum_{\mathbf{k}=1}^{\infty}\Big[\phi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{r}} + \phi_{\mathbf{k}}^*(t)e^{-i\mathbf{k}\cdot\mathbf{r}}\Big] \tag{2}$$

the equation that describes $\phi_{\mathbf{k}}$ is

$$\frac{\mathrm{d}\phi_{\mathbf{k}}}{\mathrm{d}t} = -i\omega_{\mathbf{k}}\phi_{\mathbf{k}} + \sum_{\mathbf{k}=\mathbf{k}_2+\mathbf{k}_3}\Lambda_{\mathbf{k}_2,\mathbf{k}_3}\phi_{\mathbf{k}_2}^*\phi_{\mathbf{k}_3}^* + \gamma_{\mathbf{k}}\phi_{\mathbf{k}} \tag{3}$$

where Λ is a constant factor that depend on $\mathbf{k},\mathbf{k}_2,\mathbf{k}_3$, and γ is a dissipative term.

2 $\mathbf{E}\times\mathbf{B}$ drift

Considering a test particle embedded in a nonuniform plasma and using the slab approximation. The movement of the particle is given by

$$\mathbf{v}_E = c\frac{\mathbf{E}\times\mathbf{B}}{B^2} \tag{4}$$

The nonuniformity is given by electrostatic potential in the plasma, $\mathbf{E} = \boldsymbol{\nabla}\phi(\mathbf{r}, t)$. Considering a uniform magnetic field in the $\hat{z}$ direction and a perpendicular electric field the velocity became

$$\mathbf{v}_E = -\frac{1}{B_0}\boldsymbol{\nabla}\phi(x, y, t) \times \hat{z} \tag{5}$$

then the x and y velocities are

$$v_x = \frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{B_0}\frac{\partial}{\partial y}\phi(x, y, t) \qquad \text{and} \qquad v_y = \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{1}{B_0}\frac{\partial}{\partial x}\phi(x, y, t) \tag{6}$$

Comparing these equations with the Hamilton equation, it's notable that

$$H(x, y, t) = \frac{\phi(x, y, t)}{B_0} \tag{7}$$

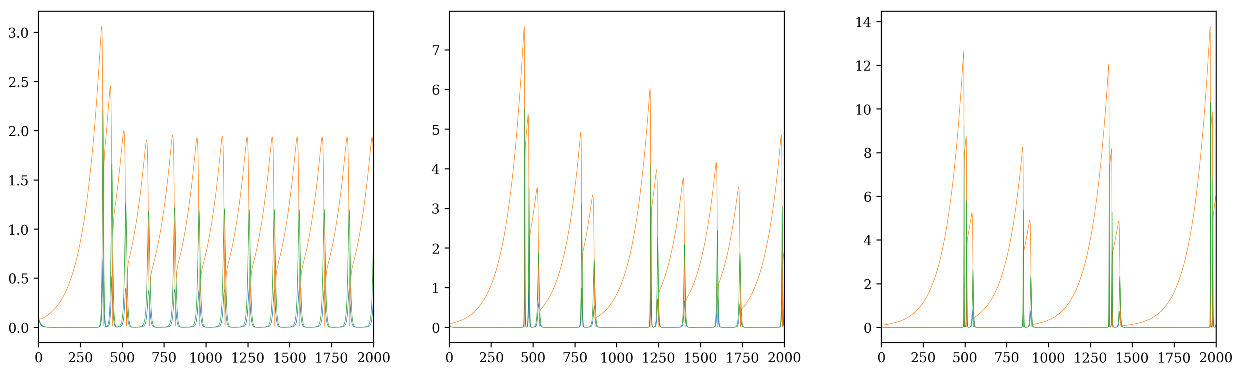
3 Particle interaction

With this set of equations, we can plug the solution of the Hasegawa-Mima equation into the Horton equation to particles to study how the particles inside a tokamak are affected by drift waves described by the equation 1. The model is given by

$$\phi(x, y, t) = \phi_0(x) + \sum_i A_i \sin(k_{xi}x)\cos(k_{yi}y - \omega_i t) \tag{8}$$

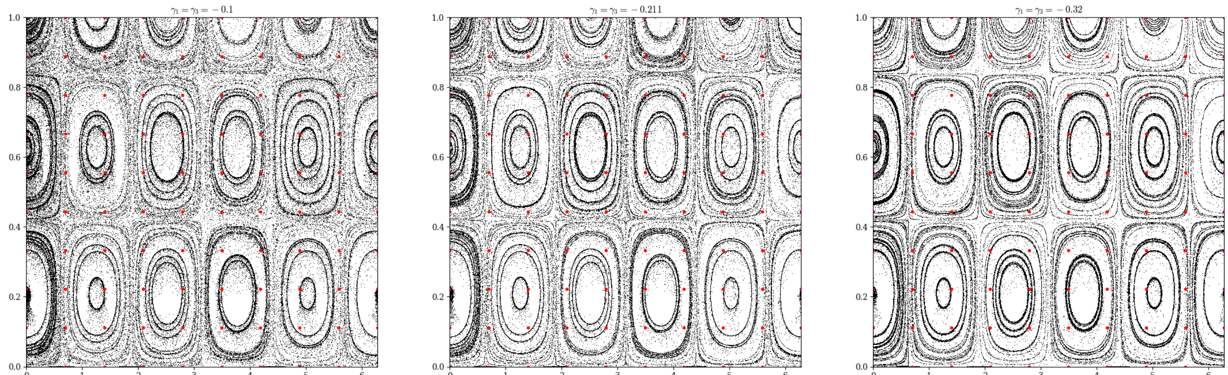
applying $|\phi_i|$ as the wave amplitude in the model for three waves we can have some results fixing the values of $\mathbf{k}, \omega_i, \Lambda$ and varing the $\gamma_1 = \gamma_3$.

The three values of γ selected are the ones that induse more caos into the waves and the wave amplitudes can be seen in figure 1.



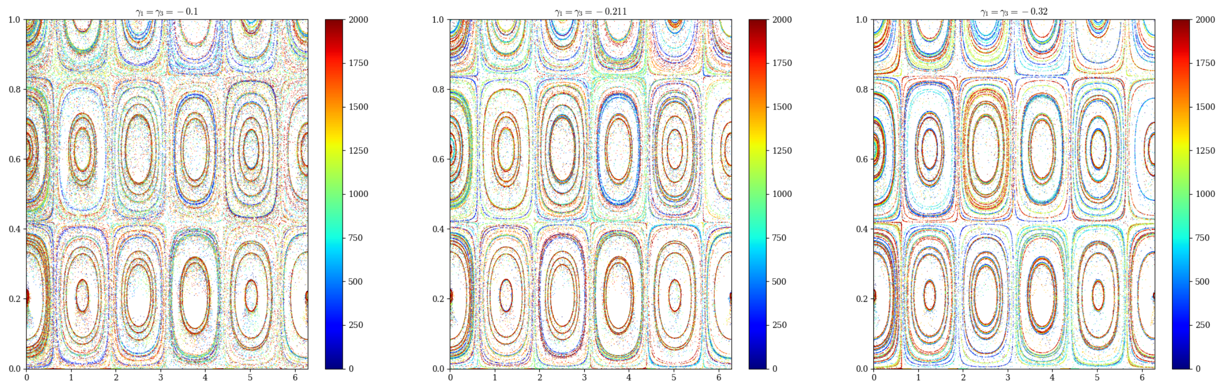
(a) $\gamma = -0.1$ (b) $\gamma = -0.211$ (c) $\gamma = -0.32$

Figura 1: Waves



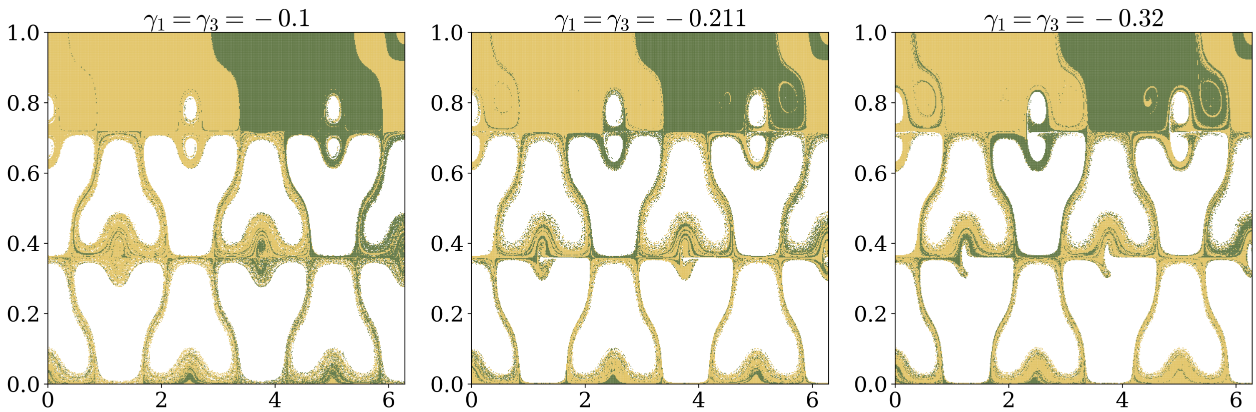
(a) $\gamma = -0.1$ (b) $\gamma = -0.211$ (c) $\gamma = -0.32$

Figura 2: Phase Spaces



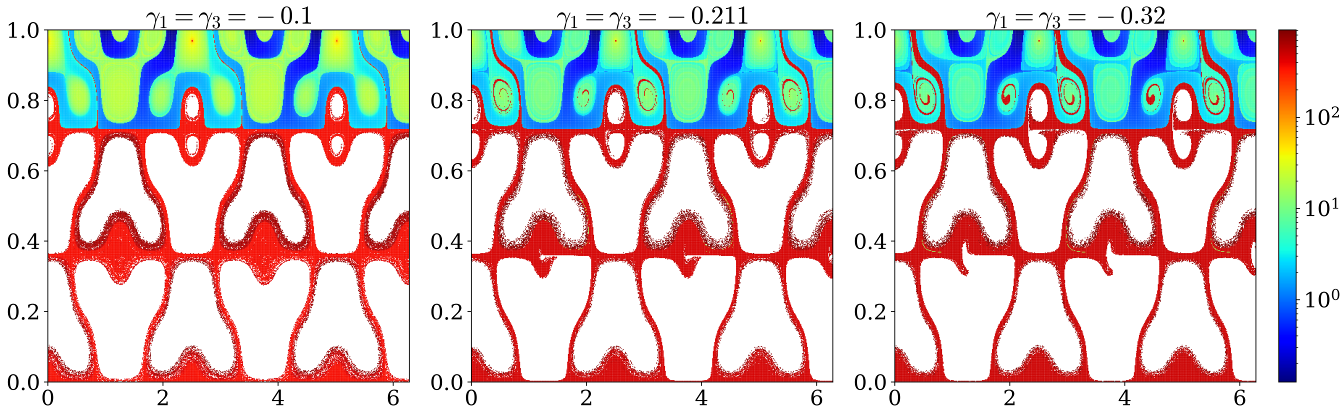
(a) $\gamma = -0.1$ (b) $\gamma = -0.211$ (c) $\gamma = -0.32$

Figura 3: Phase Spaces with time scale



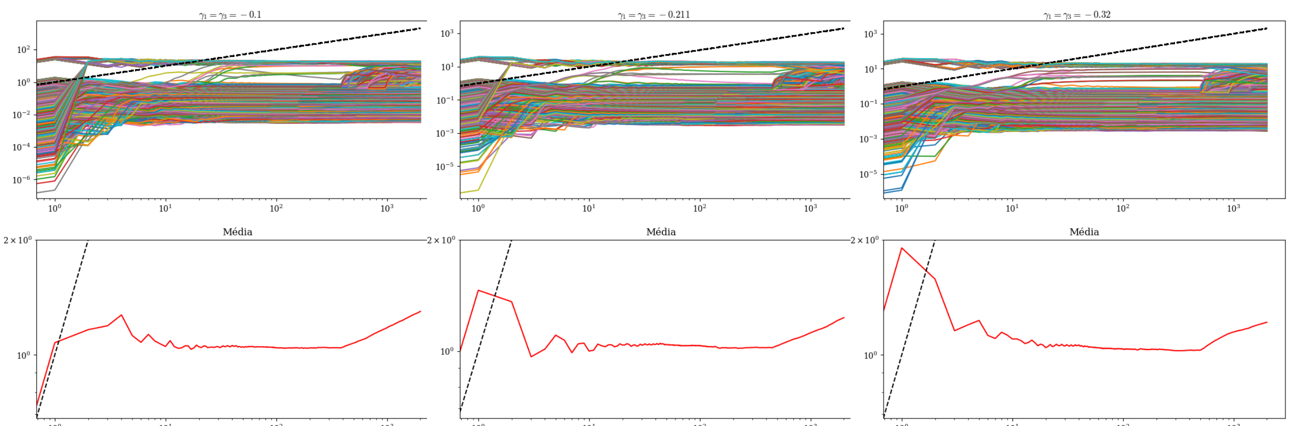
(a) $\gamma = -0.1$ (b) $\gamma = -0.211$ (c) $\gamma = -0.32$

Figura 4: Escape basins



(a) $\gamma = -0.1$ (b) $\gamma = -0.211$ (c) $\gamma = -0.32$

Figura 5: Escape time



(a) $\gamma = -0.1$ (b) $\gamma = -0.211$ (c) $\gamma = -0.32$

Figura 6: MSD to $x = 0.2$