Indirect Reciprocity Simulation

A reproduction of the simulation by Nowak & Sigmund [1]

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Reputational concerns are believed to play a crucial role in explaining cooperative behavior among non-kin humans. Individuals cooperate to avoid a negative social image, if being branded as defector reduces pay-offs from future interactions. [2] Indirect reciprocity does not require two individuals to interact more than one time but nevertheless favors the ones who have helped others in the past. [1] A donor provides help if the recipient is likely to help others (which often means, if the recipient has helped others in the past). In this case, it pays to advertise cooperation, as the cost of an altruistic act is offset by an increased chance to become the recipient of an altruistic act later. Animal and human behavior may be influenced by attempting to increase image (or status) in the group. [1]

This report attempts to confirm (**i**) Cooperation can emerge from a reputation based system, resulting in a single strategy used by the whole population (**ii**) Cooperation depends on the type of the world (parameters: such as population size, number of interactions) (**iii**) Mutations can generate random drifts in the dominance of strategies, creating periods of extreme cooperative individuals or the opposite (**iv**) Different strategy selection due to changes in group size for populations without perfect information (**v**) Taking in consideration each individuals reputation and not only the interaction partner's reputation changes their behavior and strategy selection.

The challenge of cooperation

In a cooperative (or social) dilemma, there is tension between what is good for the individual and what is good for the population. The population does best if individuals cooperate, but for each individual there is a temptation to defect since it is does not impose any cost. A simple definition of cooperation is that one individual pays a cost for another to receive a benefit. Cost and benefit are measured in terms of reproductive success. One most challenging dilemma's for cooperation is the prisoner's dilemma (in which two players choose between cooperating and defecting; cooperation maximizes social welfare, but defection maximizes one's own payoff regardless of the other's choice). In a well-mixed population in which each individual is equally likely to interact and compete with every other individual, natural selection favors defection in the PD: why should you reduce your own fitness to increase that of a competitor in the struggle for survival? Defectors always out-earn cooperators, resulting in higher fitness. Selection therefore reduces the abundance of cooperators until the population consists entirely of defectors. For cooperation to arise, a mechanism for the evolution of cooperation is needed. The mechanism used was **indirect reciprocity**. [3]

Indirect Reciprocity

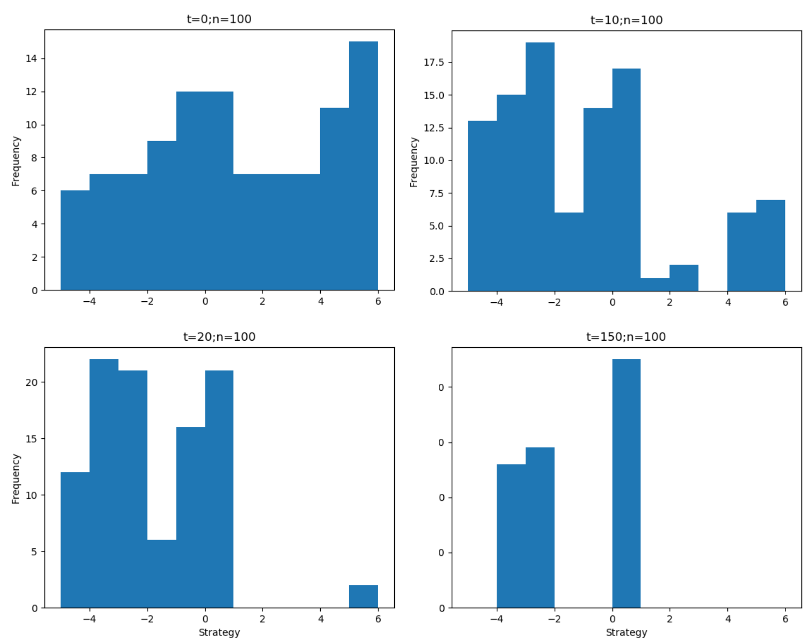
Indirect reciprocity consists of interaction structures that specify how the individuals of a population interact to receive payoffs, and how they compete for reproduction. In this model, any two players are supposed to interact at most once with each other. Each player can experience many rounds, but never with the same partner twice. The altruistic act consists in conferring a benefit b on the recipient at a cost c to the donor. If the donor decides not to help both individuals receive zero pay-off. We shall always assume that the cost is smaller than the benefit, so that if the act is returned, both individuals experience a gain. **[4]** Each player has a reputation, an image score, s which is known to every other player If a player is chosen as a donor and decides to cooperate then his (or her) image score increases by one unit; if the donor does not cooperate then it decreases by one unit. The image score of a recipient does not change. In the simulations were considered strategies where donors decide to help according to the image score of the recipient. A strategy is given by a number k. A player with this strategy provides help if, and only if, the image score of the potential recipient is at least k. **[1]**

Simulation

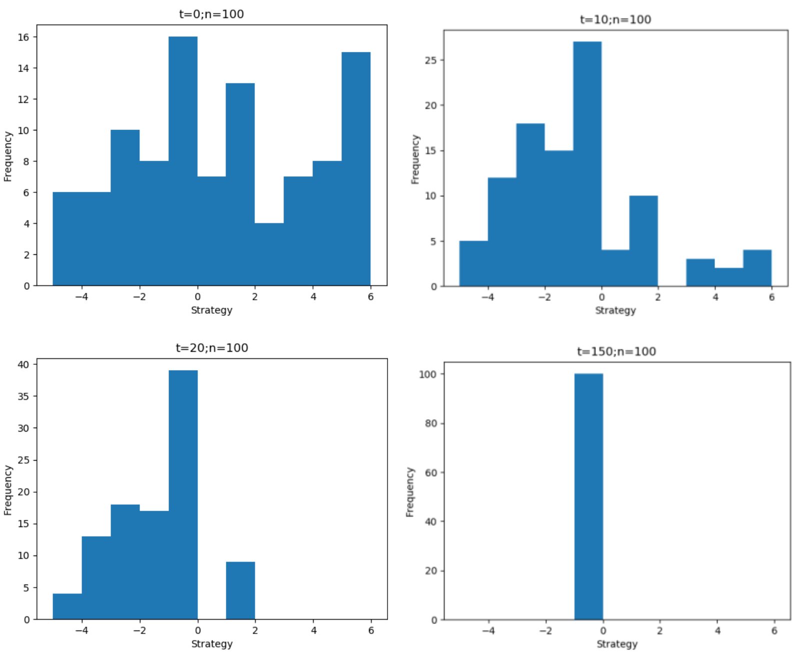
The base model of this simulation consists of a population of n players. The strategies are given by k, $-5 \leq k \leq 6$ and the image levels by s, $-5 \leq s \leq 5$. There simulation runs for t generations and m rounds, being m rounds played at each generation. At the beginning of each generation, the image levels of all players are zero. In succession, m donor recipient pairs are chosen. A donor x cooperates with a recipient, y if $k[x] \leq s[y]$. The fitness of a player is given by the total number of points ($payoff$) received during the m interactions. Some players may never be chosen, in which case their payoff from the game will be zero. At the end of each generation, players leave offspring in proportion to their fitness (ratio over global fitness). Offspring generation is divided into three phases (1) *Selection* tries to organize individuals with higher fitness from which the next generation is produced (2) *Recombination* creates new individuals based on the previously selected phase (3) *Mutation* changes the produced offspring randomly. **[5]**
If the k value of this strategy is 0 or less then cooperation is established; if the value is 1 or more then defection has won.

(i) Cooperation can emerge from a reputation based systems, resulting in a single strategy used by the whole population

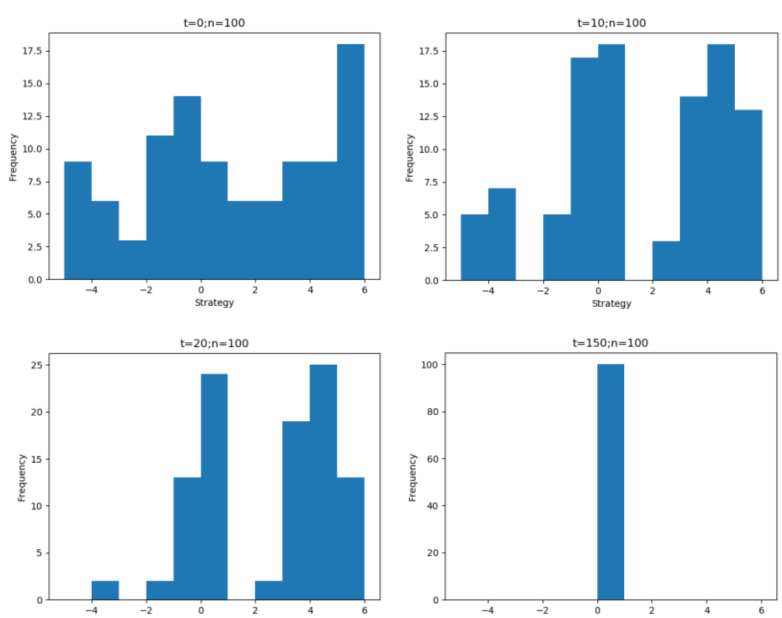
To test this observation from the original paper from *Martin A. Nowak* and *Karl Sigmund* [1] the same values were used to run the simulation, such as: $n = 100$ (population); $m = 125$ (rounds); $b = 1$ (benefit); $c = 0.1$ (cost) and $t = 166$ (generations).



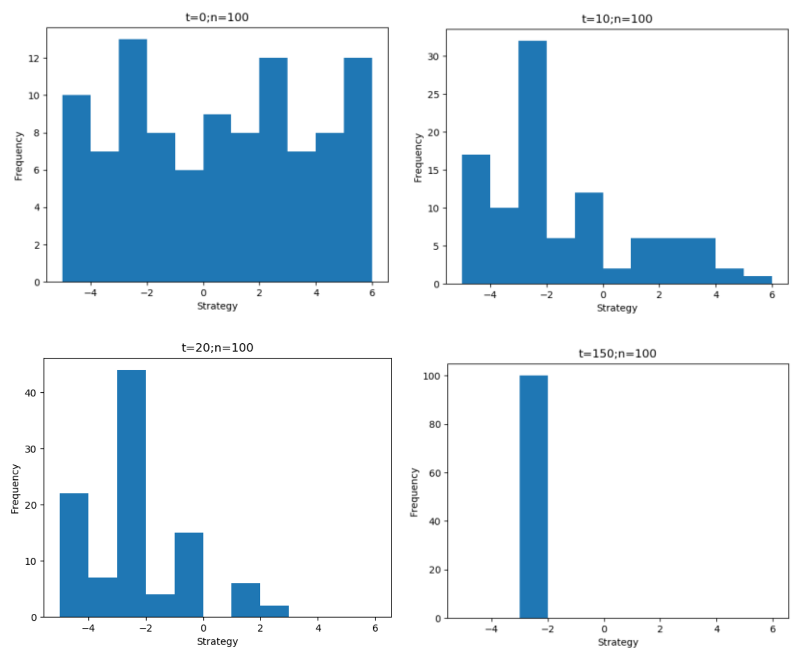
In this first run, a noticeable change in strategy is seen in the first few generations, from values of k bigger than 0 to smaller or equal to 0. This is an indicator that individuals started to cooperate from an early stage. At $t > 50$, defectors had almost gone extinct. A dominant strategy was not achieved due to the insufficient number of generations.



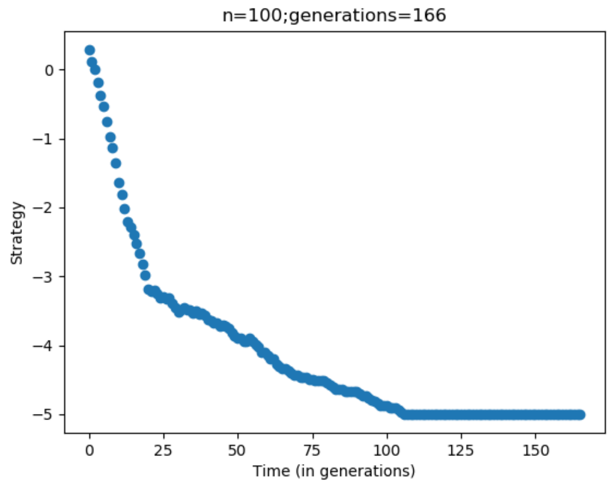
Running a second time, the result is a dominant, cooperative strategy at $t = 150$. Considering all experiments, there were very few cases of defecting being the dominant strategy.



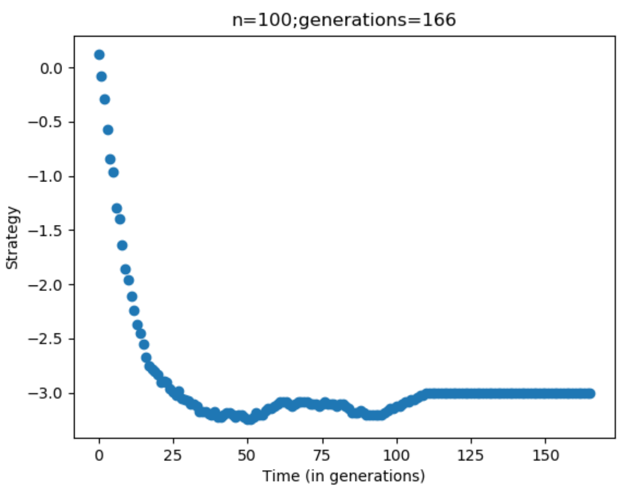
(ii) Cooperation depends on the type of the world (parameters: such as population size, number of interactions)



Every configuration value of the simulation was kept the same, except for the number of rounds, per generation, $m = 250$. From the early stages of this run, it is possible to guess, with highly probably, the resulting dominant strategy. The individual's choice of strategy seems more 'thoughtful'. Comparing the second run from **(i)** with latter, there is a subtle change in strategies frequency per generation, whereas in this run from **(ii)** it moves faster in direction to the resulting strategy. Besides this, with both round numbers, it only become stable at $t > 100$ (generations).



Average strategy per generation (i)

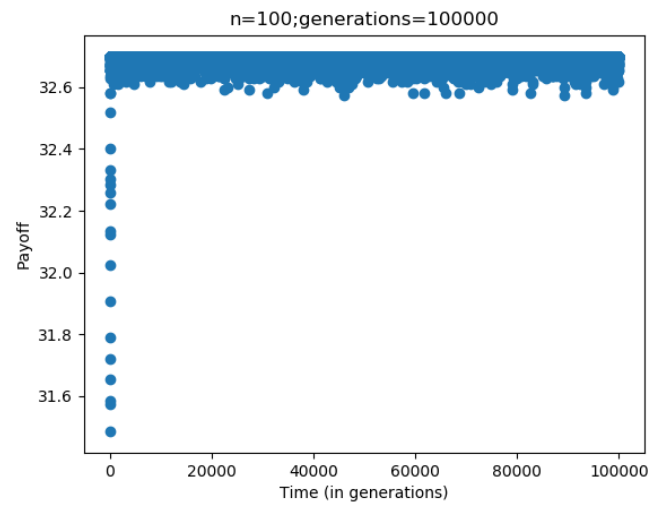


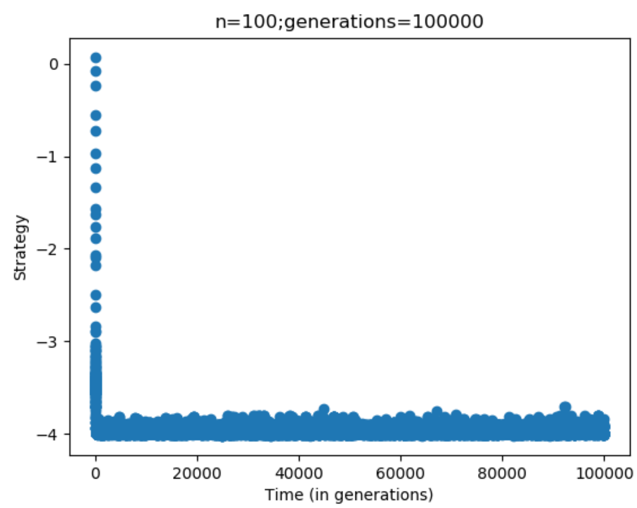
Average strategy per generation (ii)

(iii) Mutations can generate random drifts in the dominance of strategies, creating periods of extreme cooperative individuals or the opposite

A mutation was introduced, being it a probability of 0.001 that an offspring does not act like its parent and uses other randomly chosen strategy instead.

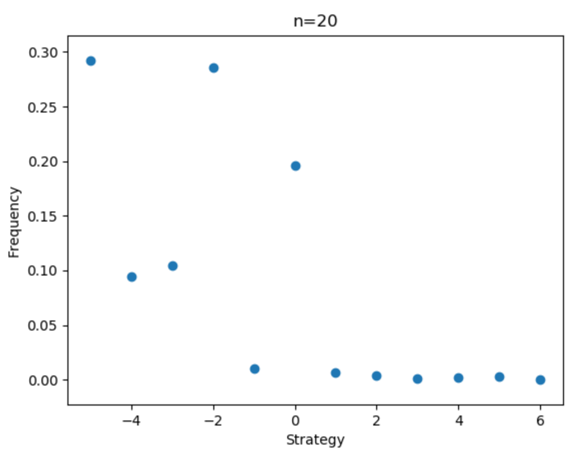
Unfortunately, random drifts creating periods of cooperation followed by defection or vice-versa were not found. Running times for $t = 10^7$ took $5h+$ with no ending prediction, unavailing this verification. For a smaller number of generations $t = 10^5$ with running times of $1h+$ no similar results were found, only small variations of payoff and strategies.

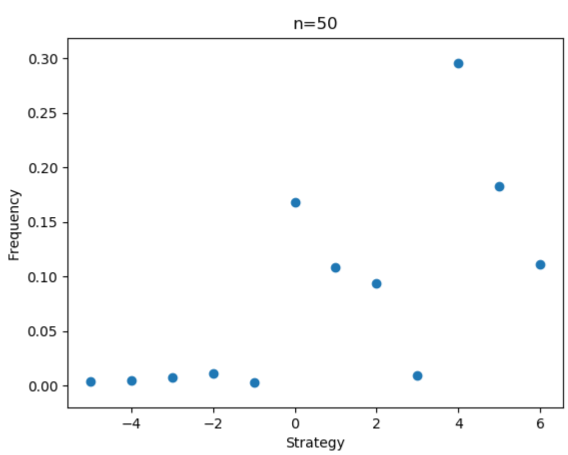


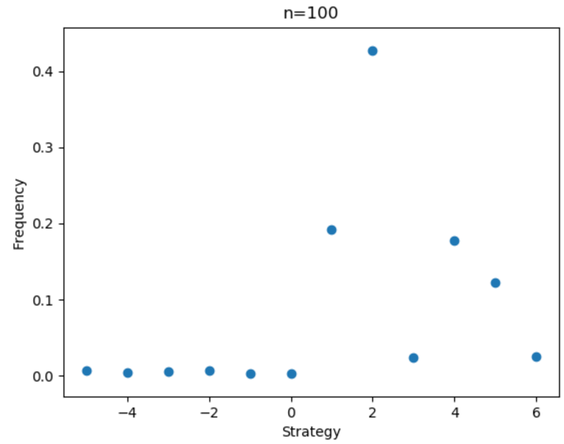


(iv) Different strategy selection due to changes in group size for populations without perfect information

Following the other runs configurations, observers were introduced. Observers are randomly chosen individuals (10 , on average, in this case) for each interaction, that participate passively just to update the reputation of the donor (from their point of view). The number of rounds was made dependent upon the population size $m = 10 \times n$ and the sampling size was 10^2 generations instead of 10^7, since it gave representative results and the running times were much smaller.



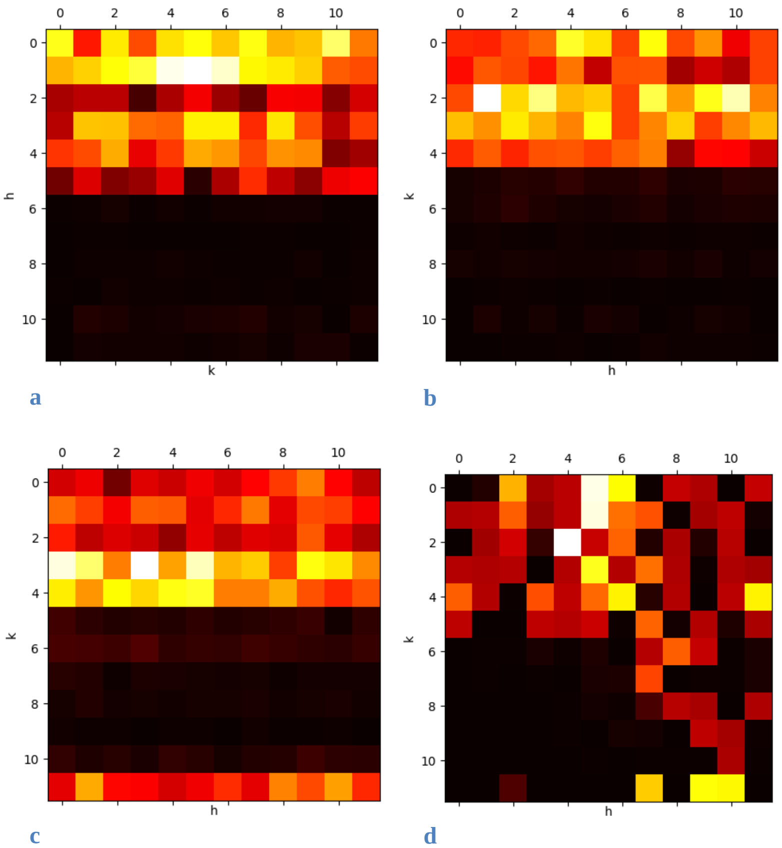




As observable, the group size influences the strategy individuals pick when in an environment with imperfect information, meaning that reputation holds cooperation levels above defection in a population and when the knowledge about one's reputation is not available, anonymity tends to increase and defectors emerge.

(v) Taking in consideration each individual's reputation and not only the interaction partner's reputation results in changes in their behavior and strategy selection.

Now a further dimension is added to the problem. Individuals cooperate not just based on the image score of the recipient but also on their own image score. **a,b,** Cooperation happens if the image score of the recipient is atleast k *and* the image score of the donor is less than h. **c,d,** Cooperation happens if the image score of the recipient is atleast k *or* the image score of the donor is less than h. h is a randomly attributed value at the beginning of each simulation and it is in the same value range as k. For running time reasons, values were sampled for 10^2 instead of 10^7. **a, c,** Runs with perfect information, population size $n = 100$. **b,d,** Runs with imperfect information, population size $n = 20$. In both, the number of rounds at each generation was $m = 500$.



a,b,c,d, Represent the frequency of h values in many runs of a simulation. Frequency is represented in colors from black to white, being less frequent and the most respectively. The real values should be obtained by subtracting 5 to the scale values.

Difference in sampling size and offspring generation with h value implementation caused the differences of h value preference in the accumulated samples. Not having a clear solution, it is still possible to observe the clearly avoided solutions, such as:

a, $k > 0$; **b,** $k > 0$;
c, $0 < k < 5$; **d,** $0 < k < 5$ with $0 < h < 4$.

Implementation

To simulate this task, Python (3.7) was the selected language. It has all the agility necessary for this project, including fantastic libraries such as *numpy* and *matplotlib*. In the original paper **[1]** some ambiguities can be found, causing some confusion to whom is implementing it, mainly in the topic of evolutionary dynamics, more specifically offspring generation and mutation. In summary, possible differences between this implementation and the original are related to the latter topics. Regarding the implementation of offspring generation, a model based on payoff accumulation between all rounds in a generation was made. A measure that reflects the quantity of new born child's using a strategy k, proportional to payoff, is obtained with the sum of the ratio of the payoff for an individual i using k in a population:

$$strength(k) = \sum_i \frac{(payoff(i) \times n)}{totalPayoff}$$

Since results are not rounded and it is not possible to factor individuals, to compensate for the missing population (it has got to be equal every generation) the individuals with higher fitness are selected. While in this process, if for each individual mutation happen $(random\ value \leq probability\ of\ mutation)$ then it will choose other random participant's strategy and add his payoff ratio to the *strength* of this new strategy.

REFERENCES

[1] Martin A. Nowak and Karl Sigmund (1998). Evolution of indirect reciprocity by image scoring. Nature, 393(6685) 573-577.

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[3] David G. Rand and Martin A. Nowak (2013). Human cooperation. Trends in Cognitive Sciences, Vol. 17, No. 8, 1364(6613), 413-425

[4] Martin A. Nowak and Karl Sigmund (2013). Evolution of indirect reciprocity, Nature, Vol. 437, 1291-1298

[5] Zelinka, I., Celikovský, S., Richter, H., Chen, G. (2010). Evolutionary Algorithms and Chaotic Systems, 73-75