Mathematical Preliminaries

B.1 SUMMATION

Consider four numbers that we will call x_1, x_2, x_3, and x_4. If s is equal to the sum of these numbers, then we can express this fact either by writing

$$s = x_1 + x_2 + x_3 + x_4$$

or by using the summation notation $\sum$. In this latter situation we write

$$s = \sum_{i=1}^{4} x_i$$

which means that s is equal to the sum of the x_i values as i ranges from 1 to 4.

The summation notation is quite useful when we want to sum a large number of quantities. For instance, suppose that we were given 100 numbers, designated as x_1, x_2, and so on, up to x_{100}. We could then compactly express s, the sum of these numbers, as

$$s = \sum_{i=1}^{100} x_i$$

If we want the sum to include only the 60 numbers starting at x_{20} and ending at x_{79}, then we could express this sum by the notation

$$\sum_{i=20}^{79} x_i$$

That is, $\sum_{i=20}^{79} x_i$ is the sum of the x_i values as i ranges from 20 to 79.

B.2 ABSOLUTE VALUE

The absolute value of a number is its magnitude regardless of its sign. For instance, the absolute value of 4 is 4, whereas the absolute value of -5 is 5. In general, the absolute value of a positive number is that number, whereas the absolute value of a negative number is its negative. We use the symbol $|x|$ to denote the absolute

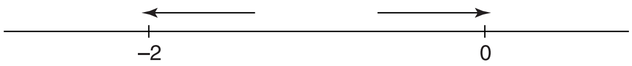

FIGURE B.1
Distance from -2 *to* 0 *is* $|-2| = 2$.

value of the number x. Thus,

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

If we represent each real number by a point on a straight line, then $|x|$ is the distance from point x to the origin 0. This is illustrated by Fig. B.1.

If x and y are any two numbers, then $|x - y|$ is equal to the distance between x and y. For instance, if $x = 5$ and $y = 2$, then $|x - y| = |5 - 2| = |3| = 3$. On the other hand, if $x = 5$ and $y = -2$, then $|x - y| = |5 - (-2)| = |5 + 2| = 7$. That is, the distance between 5 and 2 is 3, whereas the distance between 5 and -2 is 7.

B.3 SET NOTATION

Consider a collection of numbers, for instance, all the real numbers. Sometimes we are interested in the subcollection of these numbers that satisfies a particular property. Let A designate a certain property; for instance, A could be the property that the number is positive or that it is an even integer or that it is a prime integer. We express the numbers in the collection that have the property A by the notation

$$\{x\colon x \text{ has property } A\}$$

which is read as "the set of all the values x in the collection that have the property A." For instance,

$$\{x\colon x \text{ is an even integer between 1 and 7}\}$$

is just the set consisting of the three values 2, 4, and 6. That is

$$\{x\colon x \text{ is an even integer between 1 and 7}\} = \{2, 4, 6\}$$

We are sometimes interested in the set of all numbers that are within some fixed distance of a specified number. For instance, consider the set of all numbers that are within 2 of the number 5. This set can be expressed as

$$\{x\colon |x - 5| \leq 2\}$$

Because a number will be within 2 of the number 5 if and only if that number lies between 3 and 7, we have

$$\{x\colon |x - 5| \leq 2\} = \{x\colon 3 \leq x \leq 7\}$$