Who Wants To Be a Millionaire... Statistically

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Abstract

Who wants to be a millionaire is a famous TV show in which participants go through questions of different topics with increasing monetary values addressed to each question. The final question is worth $ 1 million m.u, according to the country the show is presented on. The scenario created for this paper is one of which the player is in the final question and have two life-lines remaining: Ask the Audience and 50/50. This paper seeks to evaluate which strategy results in a greater likelihood of the best outcome (winning the game). Using Monte Carlo simulations, strategies based on intervals achieve a greater mean return relative to probability. The strategies 50/50 followed by Ask the Audience and Ask the Audience followed by 50/50 are simpler, but require a large percentage of votes to achieve the best outcome.

Key-words: Strategy, Likelihood, Monte Carlo

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1 Variables and Assumptions

Its necessary to ... So, propositions will be used to define the assumption used to model the strategies.

Following the same structure as in the official game, a question has four alternatives, only one of which is correct. Therefore, for the final question, a set A - short for alternatives - has four elements/alternatives: a, b, c and d.

The show has an audience of any given length. The audience has two groups: the group of people who know the answer and the group of people that do not know the answer. So, the following proposition states:

Definition 1. Given an audience of size **N**, the number of people that know the answer is **n** and **N - n** is the number of people that do not know the answer.

Definition 2. Each outcome has a monetary value associated. Official rules of the original show format state that the prizes are:

$$Outcomes = \begin{cases} \text{If Player Wins} = \omega_1 = \$1000000 \\ \text{If Player Loses} = \omega_0 \ = \ |\$32000 \\ \text{If Player Stops} = \beta \ = \ \$500000 \end{cases}$$

In this context, the player has three possible outcomes. The player can either (i) answer correctly; (ii) answer incorrectly; (iii) do not answer and leaves with the value of the last question.

An assumption has to be made about how the audience as a whole answers the question. Therefore, the following proposition states:

Proposition 1. *X_i denotes the answer of an individual i, **K** denotes the set of people that know the answer, **DK** represents the set of people that do not know the answer and **p** is the probability of answering the question correctly. So, the assumptions regarding the answers are:*

$$\textit{(1)} \quad Cov[X_i, X_j] = 0 \quad \forall i, j \in N, \quad i \neq j;$$
$$\textit{(2)} \quad p_i = p_j \quad \forall i, j \in K, \quad i \neq j;$$
$$\textit{(3)} \quad p_a = p_b \quad \forall a, b \in DK, \quad a \neq b$$

Proposition 1.1 assumes independence across all the answers in the audience. In social terms, the audience, the player and the host do not share information publicly or privately about the question in a way that influences the answer of another individual. Proposition 1.2 and 1.3 assume equiprobability of answering the question correctly for each group[1]. This assumption

[1] The probabilities are not the same for both groups, but they are the same for each member within the group.

declares that no individual from set K answers incorrectly on purpose and all individuals from set DK answer equally random.

Definition 1 and Proposition 1 are the main assumptions of which the models for each strategy are based on. The following sections will elaborate the statistical properties of each strategy.

2 Strategy 50/50 - Ask the Audience

The strategy 50/50 followed by Ask the Audience, denoted as S_1, reduces in half the number of alternatives then proceeds to evaluate the audience's knowledge of the question. Intuitively, it seems a very reasonable strategy because the audience will have to think about only two alternatives instead of four and if the player is lucky, the correct answer will show itself. To find if that is the case, the next propositions state:

Proposition 2. *Let X_k denote the distribution of answers of the **n** individuals that know the answer. Therefore, the distribution of each answer in X_k follows a Bernoulli distribution with probability p_k:*

$$X_i \sim Bern\{p_k\}, \quad p_k = 1, \quad \forall i \in K$$

Proposition 3. *Let X_{dk} denote the distribution of answers of the **N - n** individuals that do not know the answer. X_{dk} follows a Bernoulli distribution with probability p_{dk}:*

$$X_j \sim Bern\{p_{dk}\}, \quad p_{dk} = 0.5, \quad \forall j \in DK$$

Proposition 4. *Let X denote the distribution of answers of the **N** individuals in the audience as the sum of the distribution given by propositions 2 and 3. Thus, X follows a shifted Binomial distribution[2] (henceforth, SB):*

$$X = X_{dk} + X_k \sim \textit{ shifted Binomial}\{N, \overline{p} = \frac{(N+n)}{2N}\}, \quad \textit{where:}$$

It is possible to rewrite proposition 4 such that the number of people in the audience that knows the answer is represented as a proportion of the number of people in the audience. Therefore:

Proposition 5. *Let $\alpha \in [0,1]$, such that, $n = \alpha N$. Then:*

$$S_n \sim SB\{N, \overline{p} = \frac{N(1+\alpha)}{2N}\}$$

Two scenarios are highlighted. The first scenario is when none of the individuals in the audience knows the answer. So:

[2]The properties of the shifted Binomial are given by Peköz et al. (2009). The authors make an approximation for the Binomial distribution.

$$n = 0 \implies \alpha = 0 \implies X \sim B\{N, 0.5\}$$

The second scenario is when all the people in the audience knows the answer. This implies that:

$$n = N \implies \alpha = 1 \implies X = n$$

Given proposition 4, the expectation function of the distribution for a given alternative is positively correlated with the number of people that know the answer. Notice that this information is unknown to the player, but it's possible to invert the expected value function such that the player can have an estimate of the amount of people in the audience that do know the answer[3]. Following proposition 5, we have:

Proposition 6. *Let $\alpha \in [0,1]$ and $E[X] = \mu$. Placing α in the left side of the equation. Then, the value of the share of people in the audience that know the answer is:*

$$\overline{\alpha} = \frac{2 \cdot \mu - N}{N} \tag{1}$$

Proposition 7. *Based on proposition 6, the variance of the share of people in the audience that know the answer is:*

$$\sigma^2 = \frac{N - \mu}{2}, \quad Var[X] = \sigma^2 \tag{2}$$

Definition 2 and proposition 6 summarise the strategy 50/50 followed by ATA. Since all people who know the answer respond correctly and people who do not know how to respond randomly, the player can have an estimate of the number of people in audience that know the answer through the number of votes for the highest answered alternative. Thus, the next step is to use the prizes and find the value of α that equals the expected value of playing and not playing. The following proposition states:

Proposition 8. *The expected value of the strategy 50/50 followed by ATA is:*

$$E[S_1] = \overline{\alpha} \cdot \omega_1 + (1 - \overline{\alpha}) \cdot \omega_0 \tag{3}$$

If the expected value of playing, given by proposition 8, equals the value of not playing (not answering), the resulting value of α:

$$\alpha^* = \frac{\beta - \omega_0}{\omega_1 - \omega_0} \tag{4}$$

After we input the prize's values for each outcome, we find that α^* = 0.4834. This result is imputed in equation (3), which returns that μ^* = 0.7417*N. These values conclude the entire strategy-formation process. The number of votes an alternative should have for a player

[3]Appendix A details the statistical properties of this step.

to be confident about his chances of winning the prize is, at least, 74.17% of the total votes of the audience. Any percentage bellow 74.17% of votes for one of the alternatives, the player is better, in terms of expected return, not playing and leaving the show with the amount of money obtained from the previous question.

3 Strategy Ask the Audience - 50/50

The strategy ATA followed by 50/50, henceforth as S_2, is inherently a more complex approach to the problem, mainly because it involves two stages: In the first stage, the audience have 4 alternatives to vote for instead of 2. In the second stage, there are correct answer and a randomly selected incorrect answer and the player has to make a decision[4].

The construction of the logic behind the strategy S_2 closely resembles strategy S_1. However, S_2 requires additional assumption and parameters for a complete analysis. The main body of propositions are:

Proposition 9. *Let Y_k denote the distribution of answers of the **n** individuals that know the answer. Given that the game is a win-lose situation, Y_k follows a Binomial distribution with probability q_k:*

$$Y_k \sim Bern\{n, q_k\}, \quad q_k = 1$$

Proposition 10. *Let Y_{dk} denote the distribution of answer of the **N - n** individuals that do not know the answer. Y_{dk} follows a Binomial distribution with probability q_{dk}:*

$$Y_{dk} \sim Bern\{N-n, q_d k\} \quad q_{dk} = 0.25$$

Proposition 11. *Let Y denote the distribution of answer of the **N** individuals in the audience as the sum of the distribution given by propositions 9 and 10.*

$$Y = Y_k + Y_{dk} \sim SB\{N, \overline{q} = \frac{(N+3n)}{4}\}$$

Rewriting proposition 11 in terms of proportion regarding the size of the audience, we obtain:

Proposition 12. *Let $\lambda \in [0,1]$, such that, $n = \lambda N$. Then:*

$$Y = Y_k + Y_{dk} \sim SB\{N, \frac{N(1+3\lambda)}{4}\}$$

Suppose that nobody in the audience knows the answer and all the audience knows the answer, the results are, respectively:

[4]Ignoring the assumptions made in section 2, a player might chose this strategy if he/she is not entirely unsure of the answer. Perhaps, it serves as a way of reassuring his/hers initial choice.

$$n = 0 \implies \lambda = 0 \implies Y \sim B\{N, \overline{q} = \frac{N}{4}\} \tag{5}$$

$$n = N \implies \lambda = 1 \implies Y = n \tag{6}$$

Proposition 13. *Let $\lambda \in [0,1]$ and $E[Y] = \mu$. Placing λ in the left side of the equation. Then, the value of the share of people in the audience that know the answer is:*

$$\overline{\lambda} = \frac{4 \cdot \mu - N}{3 \cdot N} \tag{7}$$

Proposition 14. *The variance of the share of people in the audience that know the answer is:*

$$\sigma^2 = \frac{N - \mu}{4}, \quad Var[Y] = \sigma^2 \tag{8}$$

The first round of the strategy S_2 is embodied in the propositions 9 to 14. Relative to the strategy S_1, the mean and the variance are smaller but it does not necessarily help the player. It helps the player if there are only two concurrent alternatives - the others have less than one quarter of the votes - because any combination that does not involve these alternatives ends up in the same conclusion as if it did. However, if there are three concurrent alternatives, the implications differ quite a lot. For now, we will be analysing the first case.

It is necessary to formulate another assumption about how the alternatives are selected for the second round. For this, the following definitions state:

Definition 3. Let **A** be the set of alternatives. After the first round, each element of A has a frequency value relative to **N** number of votes. Define $F(A)$ as the set that contains the frequency for each alternative, such that:

$$F(A) = \{f_a, f_b, f_c, f_d\}$$

Definition 4. Let **B** and $F(B)$ be the set of alternatives selected for the next round and their frequencies, respectively, that inherits elements of both **A** and $F(A)$, also respectively:

$$B = \{i, j\}, \quad i \neq j, \quad B \subset A;$$
$$F(B) = \{f_i, f_j\}, \quad f_i \geq f_j, \quad F(B) \subset F(A)$$

Proposition 15. *The rule of selection of alternatives for the next round is given by:*

$$\forall i \in F(A): \quad i \geq 0.25 \implies i \in F(B) \subset F(A);$$
$$if \quad |F(B)| > 2 \quad and \quad f_i \geq f_j \geq f_k \implies F(B) = \{f_i, f_j\}, \quad f_i \neq f_j$$

In summary, the rule of selection removes the alternatives from $F(A)$ whose frequencies are bellow the predicted mean value if everyone in the audience choose the answers randomly and stores it in $F(B)$. If the size of $F(B)$ is greater than 2, then the frequencies are sorted and the first two highest frequencies are selected.

In the second round, there are two alternatives which were chosen based on the rule of selection given in proposition 15. Both of these alternatives have a percentage of the total votes. Similar to the previous strategy, the highest voted answer is the first analysed by the player and given enough evidence, it is the only analysed alternative. The first step of investigation is the expected value of the game, similar to proposition 8:

Proposition 16. *The expected value of the S_2 is:*

$$E[S_2] = \lambda \cdot \omega_1 + (1-\lambda) \cdot \omega_0 \tag{9}$$

Given the value of α^* = 0.4834, we find that the expected mean value of the numbers of votes for the highest voted answer using strategy S_2 is μ^* = 0.6125*N, Therefore, the number of votes an alternative should have for a player to be confident about his chances of winning the prize, after two alternatives are removed, is 61.25% of the total votes of the audience[5]

4 Overlapping Strategies

Strategies S_1 and S_2 do not require any calculation. The player receives the question, chooses a strategy, sees the percentage of votes of the highest voted alternative and makes a decision. The strategies have different threshold values but, in essence, provide the same choices, either play or not play.

However, there is new set of strategies based on the previous ones that follow the same principles but strikingly differ in complexity. These strategies are called **overlapping strategies.**

Overlapping strategies required that the set of the two remaining alternatives be analysed and their intervals individually calculated. This procedure demands another parameter – the number of standard deviations – and excludes the calculation of the expected value of the strategy, given by proposition 8 and 16 for S_1 and S_2, respectively.

The overlapping strategy is constructed by the following definitions and propositions bellow:

Definition 5. Let **C** and $F(C)$ be the final set of pair-alternatives for any strategy:

$$C = \{i, j\}, \quad i \neq j, \quad C \subset A;$$
$$F(C) = \{f_i, f_j\}, \quad f_i \neq f_j$$

Therefore, the number of votes for alternative i: $\mu_i = N \cdot f_i$.

[5]. When we S_1 and S_2 in a purely random game, we find that:

$$P(X \geq 74|N = 100, p = 0.5) = P(Y \geq 61|N = 100, p = 0.5) = 0$$

. Thus, both strategies are equally unlikely to happen is nobody in the knows the answer.

Definition 6. Given the number of votes for the alternatives of C, the interval of each alternative is given by:

$$I_g = [\mu_g - k \cdot \sigma_g, \mu_g + k \cdot \sigma_g], \quad g \in \{i, j\}, \quad k > 0$$

Where: **k** is the number of standard deviations from the mean.

The intervals for each alternative are a measure of how spread out is the distribution of answers for a given k. The next proposition introduces the concept of overlap used in this paper.

Proposition 17. *Given the set $C = \{i, j\}$, s.t., $F(C) = \{f_i, f_j\}$ and $f_i > f_j$, then the intervals overlap if exists intersection between the two intervals. Therefore:*

$$\exists I_i \cap I_j \implies I_i \text{ and } I_j \text{ overlap} \tag{10}$$

Similarly to the rule of selection in S_2, the rule of decision of the overlap strategy defines an threshold value that separates two groups, but in this case, two decisions for the player. It has several steps, but it is summarised in the following proposition:

Proposition 18. *Given the intervals of definition 6 and properties of proposition 17. Define the lower and upper bound of each I_g, $g \in \{i, j\}$ as:*

$$l_g = \mu_g - k \cdot \sigma_g \tag{11}$$

$$u_g = \mu_g + k \cdot \sigma_g \tag{12}$$

So that:

$$I_i = [l_i, u_i]; \quad I_j = [l_j, u_j]$$

Define the amplitude of the interval as:

$$Amplitude = u_i - l_j, \quad u_i > l_j \tag{13}$$

If the intervals ***do not overlap****, s.t., $l_i > u_j$, then define the distance between the intervals as:*

$$Distance = l_i - u_j, \quad l_i > u_j \tag{14}$$

Given the distance and the amplitude of the intervals, define the Distance-to-Amplitude ratio as:

$$\text{Distance-to-Amplitude Ratio} = DAR = \frac{distance}{amplitude} \tag{15}$$

The rule of decision of the overlap strategy is given by:

$$\text{Rule of decision} = \begin{cases} \text{if DAR} \geq 0.5, \text{ player should choose highest voted answer;} \\ Otherwise, \text{ player should choose not playing} \end{cases} \tag{16}$$

The rule of decision uses the threshold value of 50% as an indicator of probability of the best outcome. It is worth mentioning that this parameter value was arbitrarily chosen.

5 Simulations

In this section, we simulate the result using Monte Carlo Simulation. Monte Carlo Simulation are very often used for studies of the properties of a distribution of a stochastic variable, optimization problems and other fields. A structured algorithm with well defined assumptions is one of the most important aspects when performing these simulations. Therefore, the next sections will present the logic behind the algorithm.

The simulation for the first strategy starts by generating games following the same properties defined in section 2. The function used for generating games has two parameters: Number of people in the audience and number of people in the audience that know the answer. A sample of the data generated is presented here:

Table 1: Random game generated for S_1

ID	Knows?	Answer
1	yes	c
2	no	a
3	no	a
...	...	...
N	no	c

After the games are generated, the frequencies of each alternative are taken and the highest voted answer is analysed by the player. The decision-making process for the player is based on equation 9, which the player has to have a estimate of the number of people in the audience. In each iteration, the number of people in the audience that know the answer increases monotonically until the N-esime iteration, which $n = N$. At each iteration, the prizes value and the number of votes for the highest answer are used to calculate the mean return value of S_1 at each level of α (another representation for n).

The simulation for the S_2 follows the same steps as in S_1. An additional part consider the rule of selection of the two alternatives that are kept for the second round. A sample of the data generated for S_2 is given by the table bellow:

Table 2: Random game generated for S_2

ID	Knows?	Answer
1	yes	c
2	no	a
3	no	b
4	no	d
...	...	...
N	no	c

After the rule of selection (given by proposition 15), the two alternatives with the largest number of votes are used in the decision-making process for the player. The same steps used in S_1 are applied here: The prizes value and the number of votes for the highest answer are used to calculate the mean return value of S_2 at each level of λ.

One way to evaluate the success of the strategies is to present a counterfactual example. In this case, the counterfactual would be simulating the same games but not following any strategy. So, the player would vote for the highest voted alternative outside an interval of standard deviation of the purely random outcome (nobody in the audience knowing the answer).

The overlap strategies are done for S_1 and S_2, which are denoted as S_1^O and S_2^O, respectively. It follows the same structure for both cases, only adding the rule of selection part of S_2 in the algorithm.

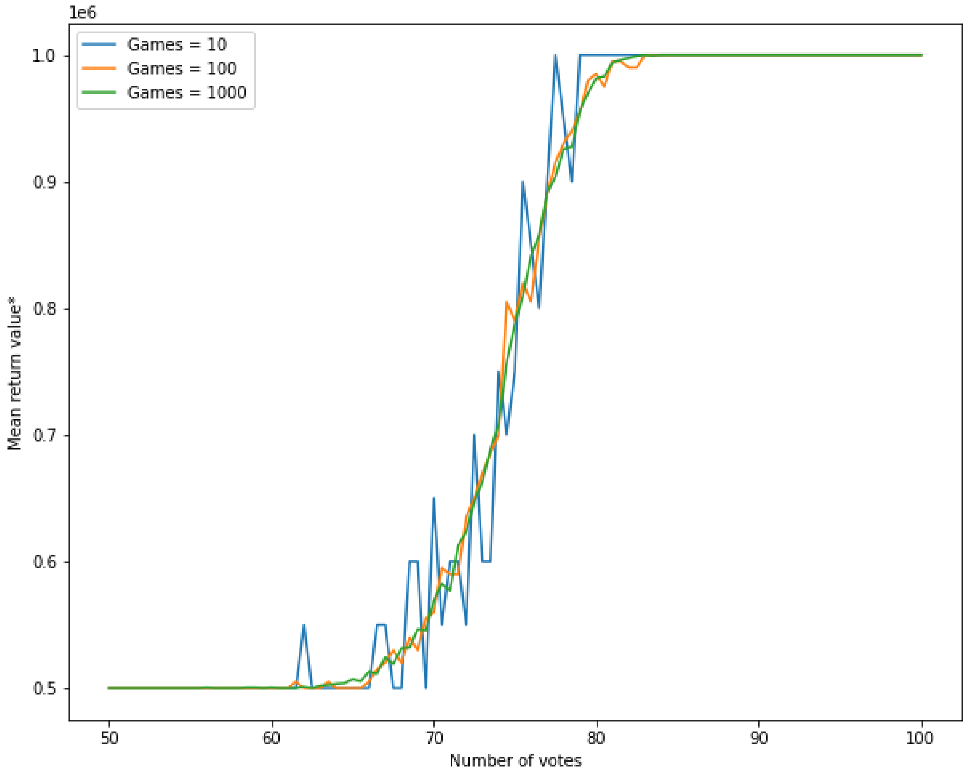
6 Results

6.1 First Strategy Results

As it was mentioned in the previous section, the algorithm used for the construction of the results bellow involves simulating games with an increasing number of people that know the answer at each iteration, for a fixed value of N. Following the strategy, the average prize after the iterations is calculated along with the percentage of votes for the highest voted answer. Three scenarios were tested: 10 games[6], 100 games and 1000 games. As we increase the number of games, the smoother the relation between the number of votes and the mean return value of the strategy becomes. With 1000 games, the output resembles the binomial cumulative distribution, mainly because it is. The results for strategy S_1 are displayed by figure 1.

[6]Each game has 100 samples with 100 testing values for **n**, so for 10 games, we have 10*100*100 iterations.

Figure 1: Mean return value of strategy S_1 for an audience of 100 people



Note: *Mean return value of the strategy is represented in a million monetary units.

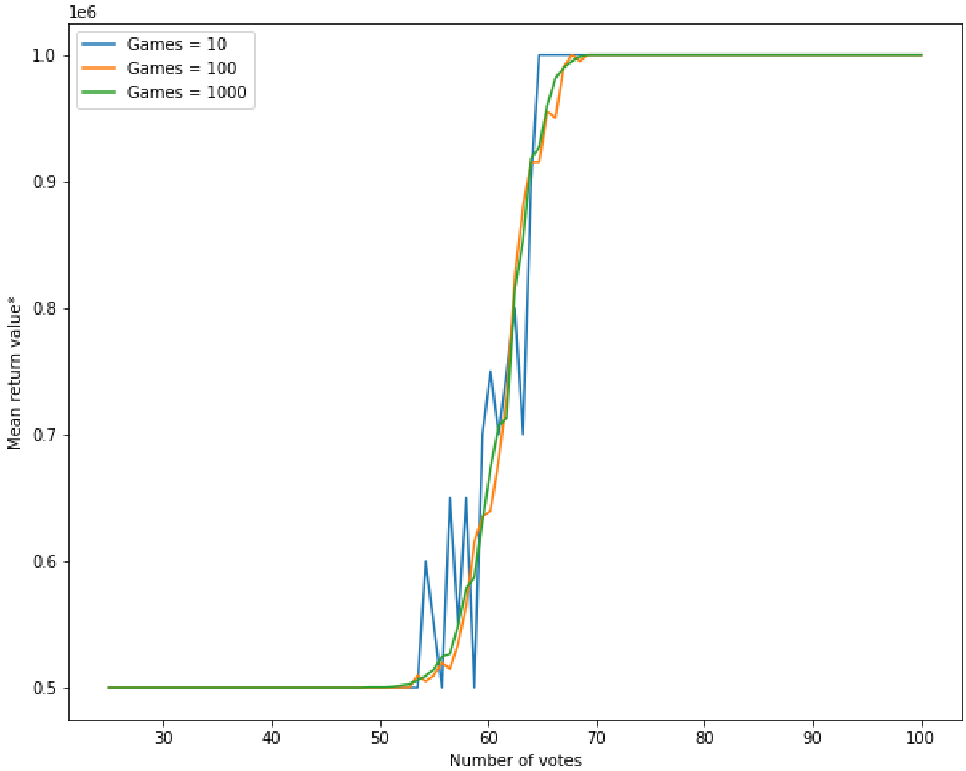
With 10 games, there is a considerable amount of variance in the simulations, with some of the iterations having a lower mean return value even after the 74%-vote threshold. With 100 and 1000 games, the variances reduces but the expected monotonically increasing relationship is strengthen.

An interesting point is that the theoretical threshold value for the percentage of votes for the highest voted answer was 74% of the total votes of the audience. However, the predicted value of $E[X]$ that guarantees, with a high degree of probability, the player winning it the 1 million dollars, for example, was **84%** of the total votes – a 10% difference between winning in "certainty" and indifference of playing and not playing" – each in turn would translate to 68% of the share of people in the audience that know the answer.

6.2 Second Strategy Results

This section follows the same steps as in 6.1. Figure 2 presents the results for strategy S_2.

Figure 2: Mean return value of strategy S_2 for an audience of 100 people

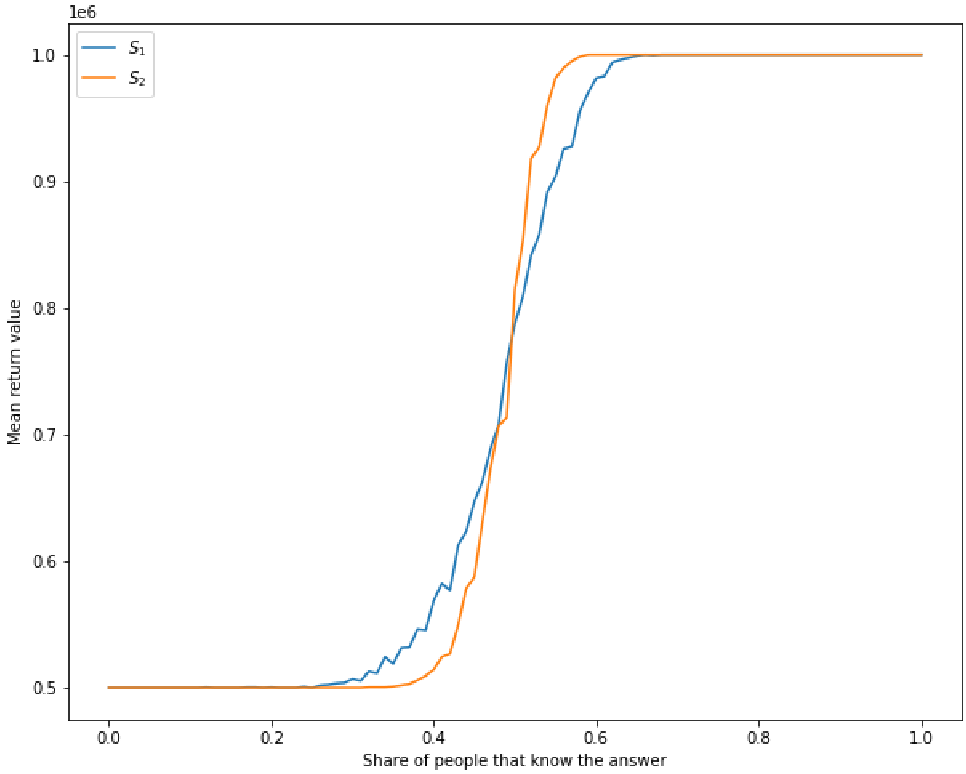


Note: *Mean return value of the strategy is represented in a million monetary units. Every alternative bellow 25% of the votes is disregarded.

Relative to strategy S_1, S_2 increases in mean return value more rapidly in relation to the number of votes, about 1.5 times faster. In the small samples, variance plays a huge play of the strategy, but as the number of games increases, the distribution tends to be smoother, as in S_1.

Figure 3 presents the relationship between S_1 and S_2 for 1000 games. The distribution intersect at 50% level, which is a good sign. If less then 50% of the audience knows the answer, then S_1 is a better strategy, since it has a higher mean return value relative to S_2. However, above 50%, S_2 becomes a more viable solution.

Figure 3: Mean return value of strategies S_1 and S_2 for an audience of 100 people



Note: Mean return value of the strategy is represented in a million monetary units.

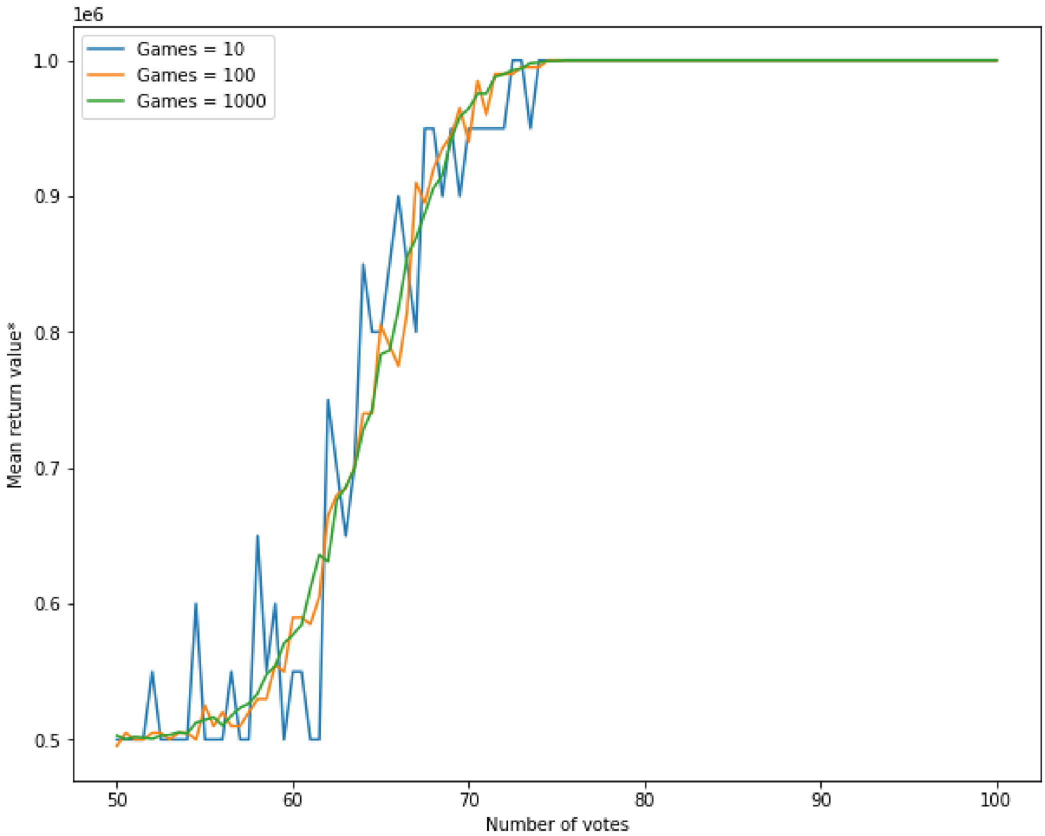
The theoretical threshold value given by proposition 16 stated that if the highest alternative had 61.25% of the votes, then the player would be indifferent between to playing or not playing. The simulations predict that if an answer has 69.25% of the votes – which would have resulted in 59% of audience knowing the answer –, then the mean return value of the strategy S_2 is the winning prize value. Hence, there is 8% difference between winning in "certainty" and indifference, which is 2% smaller then the predicted value for S_1.

In this sense, even if the player has no knowledge of the probabilities of each person in the audience answering the question correctly, if the player chooses S_2 as his strategy, the player has two advantages: On average, the player needs that only 59% of the audience to know the answer, unlike S_1, that needs 68%, a 9% differential. Secondly, the player, on average, has 2% more chance of winning the game than he would have if he had chosen S_1.

6.3 Overlap Strategy Results

Each original strategy has its overlap format. Therefore, the results for S_1 and S_2 are presented sequentially. First, we analyse the overlap strategy of S_1 results shown in figure 4.

Figure 4: Mean return value of S_1^O for an audience of 100 people and 1 standard deviation differential

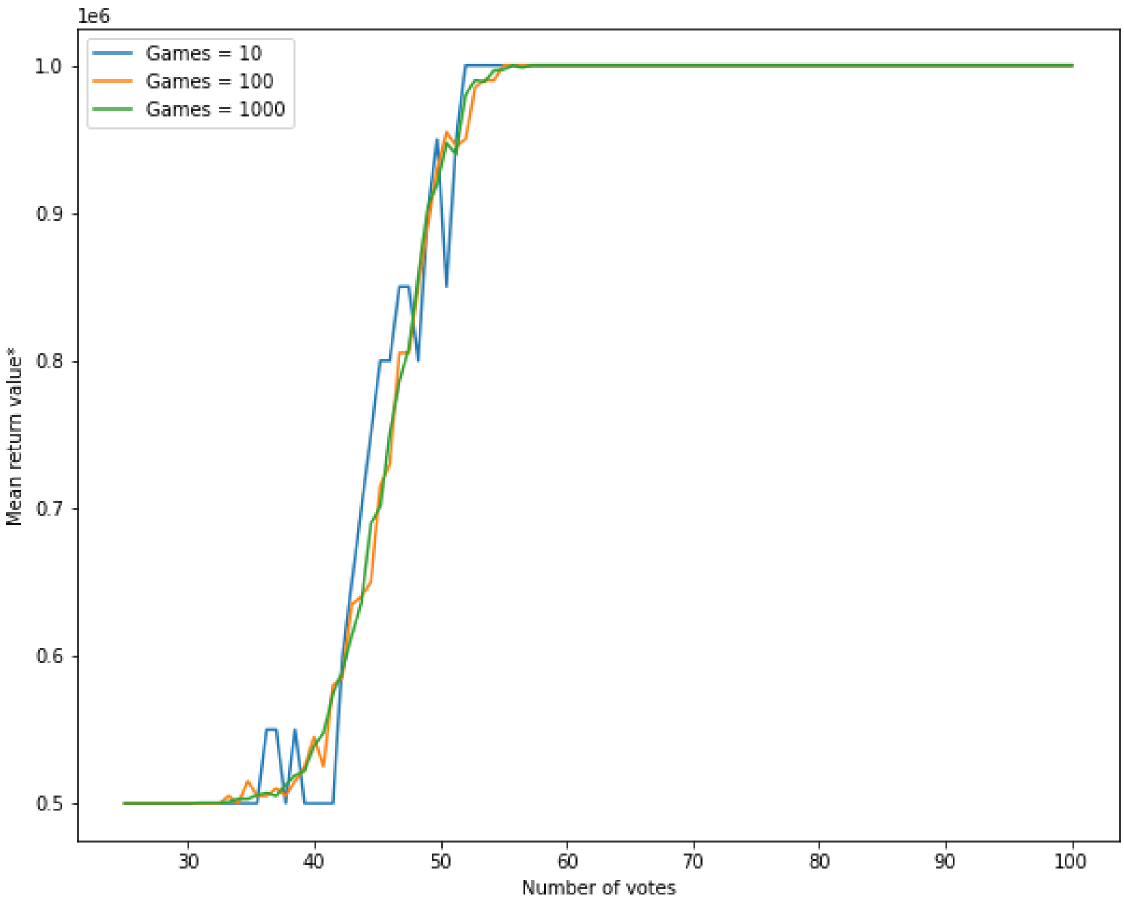


Note: *Mean return value of the strategy is represented in a million monetary units

Figure 4 demonstrates the same trend in terms of growth rate as in figure 1, even though the variance is visually the same for small samples. Differently of S_1, S_1^O reaches the mean return value of 1 million monetary units for all samples with 75.5% of the total votes of the audience. This number translates to 51% of the audience knowing the answer. Relative to S_1, there is 8.5% decrease in the number of votes for the highest voted alternative and 17% decrease in the share of audience that knows the audience. What the player looses in simplicity, the player gains in percentages.

The case is even more drastic with S_2^O. Figure 5 summarises the results for the overlap strategy of S_2.

Figure 5: Mean return value of S_2^O for an audience of 100 people and 1 standard deviation differential

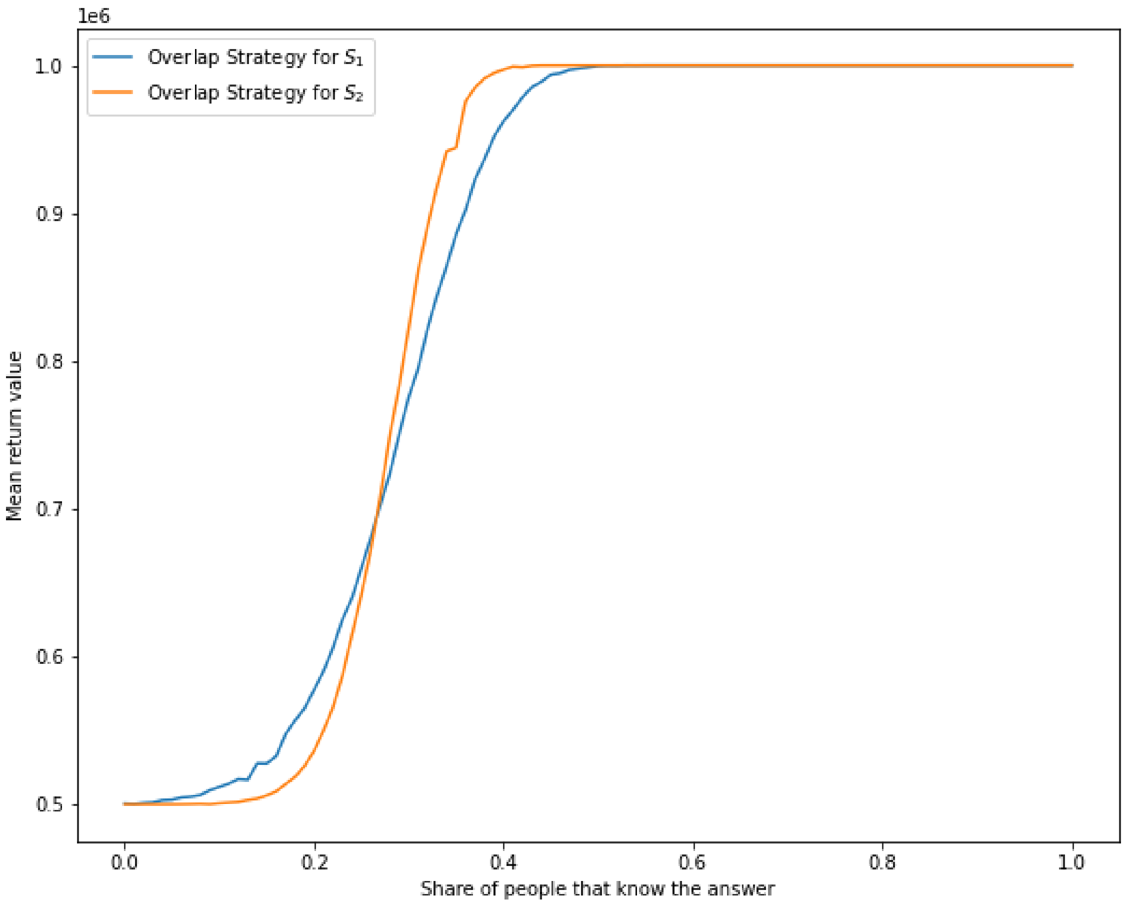


Note: *Mean return value of the strategy is represented in a million monetary units

Following this strategy, the player consistently wins the game in all samples sizes when the highest voted alternative has 57.25% of the total votes. According to proposition 13, this would indicate that only 43% of audience has to know the answer. If we compare these results to the ones we obtained in S_2, the share of votes for the highest voted answer reduces in 12% and the share of the audience that knows the answer reduces in 16%.

To finalize this section, we can cross-compare the two overlap strategies. Figure 6 compares the S_1^0 and S_2^O according to their mean return value and the share of the audience that knows the answer using 10000 iterated-games.

Figure 6: Mean return value of S_2^O for an audience of 100 people and 1 standard deviation differential



Note: *Mean return value of the strategy is represented in a million monetary units

The same pattern found in figure 3 is present in figure 6. In figure 3, the mean return value of each strategy intersected at approximately the level of 50% of audience. However, in figure 6, S_1^0 and S_2^0 intersect at approximately the 27% level, which compared to the previous cross-comparison, represents a 23% decrease of the share of people in the audience.

This results implies that the S_2^0 becomes the most profitable strategy more rapidly – meaning that need a smaller amount of people in the audience to know the answer – relative to S_1^0, but with the same growth rate as in figure 3, about 1.5 times faster.

7 Counterfactual results

A strategy is successful if it meets its end goal. If not following any strategy meets the same goal with same consistency, then the strategy is worthless. On the basis of this statement, the counterfactual results seeks to evaluate if the strategies proposed are any good. Given the previous results, the counterfactual results will be tested for S_1 and S_2. If the result are positive for these strategies, then the counterfactual analysis for the overlap strategies will not be necessary.

One might perceive that not following a strategy as a risk seeking behavior, while the opposite would be a risk aversion behavior. The traditional measure of risk is the standard variation of the return of the strategy.

Sharpe (1994) proposed a index that measures the expected return of a portfolio, relative to the risk of the portfolio and a risk-free asset[7]. We can adapt the idea from Sharpe (1994) to fit our case. The risk-free return, given our framework, is the monetary value obtained from the previous question, which the player gets if he/she chooses not to answer the final question. Table 3 presents the mean, standard variation and Sharpe ratio for each strategy[8].

Table 3: Counterfactual analysis of strategy S_1

	S_1			Highest voted			Randomly		
	Mean	Deviation	$SR^{1.}$	Mean	Deviation	SR	Mean	Deviation	SR
n = 0	5.000	0	-	0.2	0.1	0	0.5	0.2	0.000
n = 10	5.000	0	-	5.000	3.000	0.000	5.000	4.000	0.000
n = 25	5.007	3.604	0.020	6.346	4.684	0.287	5.166	4.839	0.034
n = 48	7.000	2.000	0.000	7.000	4.000	0.624	6.000	4.000	0.000
n = 50	7.000	2.000	0.000	7.000	4.000	0.000	6.000	4.000	0.000
n = 75	10.0	0.0	-	10.0	0.0	-	5.000	4.000	0.000
n = 90	10.0	0.0	-	10.0	0.0	-	5.369	4.854	0.070
n = 100	10.0	0.0	-	10.0	0.0	-	5.1 432	4.8671	0.039
n = 0-100	6.922	2.993	0.642	7.555	4.177	0.611	5.1583	4.8400	0.0327

Notes: Values presented in the table are simple averages of all iterations and are in $100.00 monetary units. Simulations have 100 people in the audience. Estimates obtained by 100000 random games. Standard variation equals zero implies that the player does not change the strategy's best decision.
[1.] Sharpe ratio (SR) with no data implies that expected return of strategy does not change in all iterations.

The results from table 3 point out that following the strategy S_1 is the optimum if there are less than 25% people or more than 50% people in the audience that know the answer. The overall average of the strategy S_1 is lower across all values of n, but relative to the risk, achieves a higher Sharpe Ratio that the counterfactual strategy. However, in the interval between 25% and 50% of the audience appears to favor the highest voted answer. In this interval, it would lead to the highest voted alternative having a percentage of votes in the range of 62.5% and 75%. This specific range provides a support for the use of the overlap strategies. To prove a point, follow the example: The player chooses S_1 and, after the audience has chosen the answers, the final distribution of alternatives is 65% and 35%.[9] By allowing approximations in the calculations of the intervals, the DAR would be 0.5, which would suggest that the player should choose to play.

[7]The Sharpe ratio formula is: Sharpe Ratio $= (\mu_S - r_f) / \sigma_S$

[8]Appendix A.3 details these results considering the Sharpe-Ratio of the risk-free return as 1.

[9]The probability of having 65% votes in an equally random binomial process is 0.08%.

Table 4: Counterfactual analysis of strategy S_2

	S_1			Highest voted			Randomly		
	Mean	Deviation	$SR^{1.}$	Mean	Deviation	SR	Mean	Deviation	SR
n = 0	5.000	0	-	2.9364	4.269	-0.4923	2.9364	4.3011	-0.4797
n = 10	5.000	0	-	3.3386	4.4912	-0.3707	3.0616	4.3633	-0.4443
n = 25	5.009	0.007	0.014	6.142	4.5671	0.268	5.3472	4.8675	0.082
n = 48	7.082	2.474	0.841	7.5800	4.224	0.624	6.4458	4.6897	0.309
n = 50	7.082	2.474	0.841	7.5800	4.224	0.624	6.4458	4.6897	0.309
n = 75	10.0	0.0	-	10.0	0.0	-	5.248	4.8391	0.0645
n = 90	10.0	0.0	-	10.0	0.0	-	5.3693	4.8544	0.0704
n = 100	10.0	0.0	-	10.0	0.0	-	5.1432	4.8671	0.039
n = 0-100	6.9605	1.3511	1.451	7.5541	3.3448	0.7636	4.8916	4.8916	-0.022

Notes: Values presented in the table are simple averages of all iterations and are in \$100.00 monetary units. Simulations have 100 people in the audience. Estimates obtained by 100000 random games. Standard variation equals zero implies that the player does not change the strategy's best decision.
[1.] Sharpe ratio (SR) with no data implies that expected return of strategy does not change in all iterations.

8 Conclusion

The final summary of strategies is an evident trade-off between simplicity and precision. The simpler strategies S_1 and S_2 are affected by the variance for small samples. Therefore, if the player follows the strategies using the threshold values defined by equations 9 and 15, the player does not need to perform any calculation. The individual glances at the percentage of votes for the highest voted answer and makes a decision.

In the overlapped strategies, the player has to do these steps:

1. Calculate the variance of the both alternatives;

2. Calculate the lower and upper bounds for both final alternatives using preferably one standard deviation;

3. Check if the intervals overlap;

4. If they do not overlap, calculate the amplitude an distance between the intervals;

5. Calculate the Distance-to-Amplitude Ratio;

6. Make decision based on DAR value.

Although not impossible, the player has to know the preferred strategy *a priori* and have good mathematical abilities to perform these calculations under time restriction. Therefore, the choice of which strategy to use is dependent in the characteristics of the player.

The simulation results show that S_1 is the most simple among all strategies. It does not need calculation and extra assumptions. Nevertheless, it requires that an alternative has 84% of the total votes to guarantee, in all samples, the best outcome. On the other hand, S_2^0 is the strategy that presents the greater likelihood of the best outcome, however it needs the extra assumptions and it is relatively more complex to calculate than the other strategies.

This paper has provided an insight on a particular but interesting problem. Although it may seem like a improbable situation for most people to find themselves in, it serves as a *rationale* in case it happens. An advantage of this approach is that it can be used in every situation in which the player still has the two life-lines available[10]. However, a problem of which this paper has not found a solution is when there are three concurrent alternatives, fluctuating in the 30% range of the votes, after the audience is asked. Even if this is an infrequent event, it will probably require more assumptions and this analyses escapes the initial intentions of this paper.

As for future improvements, theorists should seek to introduce microeconomic theory in the analysis, especially the field of Choice under Uncertainty, and relax on some assumptions, allowing, for example, a small correlation of answers between a certain group on the audience.

References

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[10]The threshold value given by equations 9 an 15 must be recalculated for different outcome values. See Appendix

Appendix

A.1 Procedures for estimation

The concept of evaluating the mean probability of a sample of independent answers with different probabilities derives from the maximization problem of the probability mass function (PMF) of the binomial distribution. Given that the probabilities are know but the size of the groups are unknown, it is possible to invert problem of the strategies in such a way that n can be estimated.

In this sense, the problem becomes: Given an audience of size **N** and the highest voted answer with **k** votes, what is the probability that maximizes the likelihood of the outcome of **k** votes of **N** answers? The short answer is $p^* = \frac{k}{N}$. The derivation of this result follows:

Given the PMF of the (binomial) distribution of answers, where X is the number of votes for the highest voted alternative, the optimization problem becomes:

$$\begin{aligned} \max_{p} \quad & P(X=k) = \binom{N}{k} p^k (1-p)^{N-k} \\ \text{s.t.} \quad & p \in (0.5, 1) \\ & N, k > 0 \end{aligned} \tag{17}$$

The restrictions only determine the intervals of the parameters, therefore, the maximization of equation (21) is:

$$\frac{\partial P(X)}{\partial p} = \frac{N!\left(kx^{k-1}(1-x)^{N-k} - x^k(N-k)(1-x)^{N-k-1}\right)}{k!\,(N-k)\,!}$$

$$\frac{\partial P(X)}{\partial p} = 0 \implies p^* = \frac{k}{N} \tag{18}$$

To evaluate if p^* maximizes the PMF of the distribution, we must check the second order conditions. The result found is:

$$\text{If} \quad p^* \in (0,1) \implies \frac{\partial P(X)}{\partial p} < 0$$

Therefore, **p*** is maximum point of $P(X)$. Using this result, we find that the sum of all the answers in the audience is the sum of probabilities of each individual. Using S_1 as an example, then:

$$S_n = X_1 + ..X_n + X_{n+1} + ... + X_N = \sum_{i=1}^{n} p_i + \sum_{j=n+1}^{N} p_j \tag{19}$$

$$S_n = \sum_{i=1}^{n} p_i + \sum_{j=n+1}^{N} p_j = \frac{N+n}{2} \tag{20}$$

The mean probability of the audience answering the question correctly is given by:

$$\overline{p} = \frac{S_n}{N} \tag{21}$$

If we substitute S_n in the mean probability formula, we finally end up with the values:

$$\overline{p} = 0.5 + \frac{n}{2N} \tag{22}$$

Finally, using equations (20) and (22), we have:

$$k = S_n = E[X], \quad \text{from } S_1 \tag{23}$$

The same process follows for S_2. The mean probability of the audience answering the question correctly according to the strategy S_2 is:

$$\overline{p} = 0.25 + \frac{3n}{4N} \tag{24}$$

A.2 Other results

Table 5: Share of votes and audience needed for each question - Strategy S_1

Question	Value ($)	Save Heaven?	Share of votes (%)	Share of audience (%)
1	100	No	0	50.0
2	200	No	50.0	75.0
3	300	No	66.7	83.3
4	500	No	60.0	80.0
5	1000	Yes	50.0	75.0
6	2000	No	0	50.0
7	4000	No	33.4	66.7
8	8000	No	42.25	0.71
9	16000	No	47.13	0.73
10	32000	Yes	48.1	0.74
11	64000	No	0	50.0
12	125000	No	33.4	66.7
13	250000	No	42.20	71.25
14	500000	No	46.53	73.82
15	1000000	No	48.41	74.17