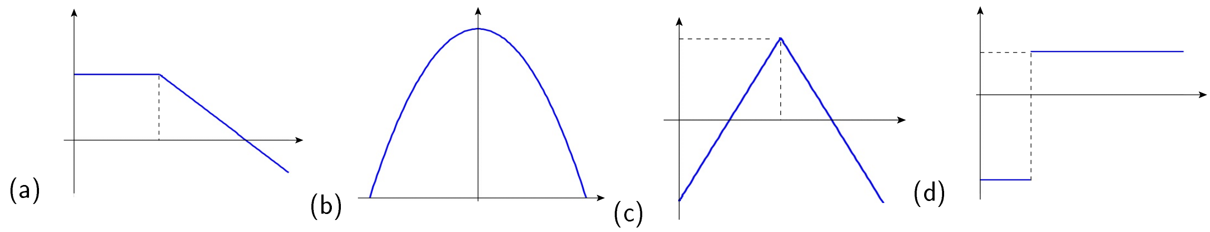
Cálculo

folha 7 — 2017'18

Primitivas.

1. Considere, em cada alínea, a função $f : I :\longrightarrow \mathbb{R}$, I um intervalo, representada graficamente por



Esboce, caso exista, uma função F, primitiva de f em I, sabendo que:

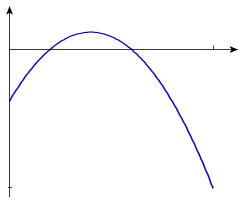
(a) $I = [0, 5]$

(b) $I = [-1, 1]$, $f(x) = 1 - x^2$ e $F(0) = 0$

(c) $I = [0, 4]$ e $F(0) = -2$

(d) $I = [0, 4]$ e $F(0) = 1$

2. Seja $f : [0, 5] \longrightarrow \mathbb{R}$ representada graficamente na figura ao lado. Considere uma função primitiva de f, $F : [0, 5] \longrightarrow \mathbb{R}$.



(a) Encontre os pontos críticos de F.

(b) Classifique os pontos críticos de F.

3. Encontre F, uma antiderivada da função f, sabendo que $F(1) = 0$. A solução encontrada é única?

(a) $f(x) = \operatorname{sen} x \, \cos x$

(b) $f(x) = \operatorname{sen}(2x) \, \cos x$

(c) $f(x) = \operatorname{sen}^2 x$

(d) $f(x) = \dfrac{1}{x}, x > 0$

4. Calcule os seguintes integrais indefinidos

(a) $\displaystyle\int (3x^2 - 2x^5)\, dx$

(b) $\displaystyle\int (\sqrt{x} + 2)^2\, dx$

(c) $\displaystyle\int (2\theta + 10)^{20}\, d\theta$

(d) $\displaystyle\int x^4 (x^5 + 10)^9\, dx$

(e) $\displaystyle\int y^2 e^{y^3}\, dy$

(f) $\displaystyle\int \sqrt{2x + 1}\, dx$

(g) $\displaystyle\int \frac{2x + 1}{x^2 + x + 3}\, dx$

(h) $\displaystyle\int \frac{t}{3 - t^2}\, dt$

(i) $\displaystyle\int \frac{1}{4 - 3x}\, dx$

(j) $\displaystyle\int \operatorname{th} x\, dx$

(k) $\displaystyle\int \frac{1}{e^{3x}}\, dx$

(l) $\displaystyle\int \frac{-7}{\sqrt{1 - 5x}}\, dx$

(m) $\displaystyle\int \frac{\sqrt{1 + 3 \ln a}}{a}\, da$

(n) $\displaystyle\int z \, \operatorname{sen} z^2\, dz$

(o) $\displaystyle\int \frac{1}{x(\ln^2 x + 1)}\, dx$

(p) $\displaystyle\int \left(\frac{2}{x} - 3\right)^2 \frac{1}{x^2}\, dx$

(q) $\displaystyle\int \operatorname{sen}(\pi - 2x)\, dx.$

5. Usando primitivação por partes calcule:

(a) $\displaystyle\int \ln x\,dx$

(b) $\displaystyle\int x\,\mathrm{sen}(2x)\,dx$

(c) $\displaystyle\int \mathrm{arctg}\,x\,dx$

(d) $\displaystyle\int x\cos x\,dx$

(e) $\displaystyle\int \ln(1-x)\,dx$

(f) $\displaystyle\int x\,\ln x\,dx$

(g) $\displaystyle\int x^2\,\mathrm{sen}\,x\,dx$

(h) $\displaystyle\int x\,\mathrm{sen}\,x\,\cos x\,dx$

(i) $\displaystyle\int \ln^2 x\,dx$

(j) $\displaystyle\int e^x\cos x\,dx$

(k) $\displaystyle\int \mathrm{arcsen}\,x\,dx$

(l) $\displaystyle\int e^{\mathrm{sen}\,x}\,\mathrm{sen}\,x\,\cos x\,dx$

(m) $\displaystyle\int \frac{\mathrm{arcsen}\,\sqrt{x}}{\sqrt{x}}\,dx$

(n) $\displaystyle\int x\,\mathrm{arctg}\,x\,dx$

(o) $\displaystyle\int x^2\,\ln x\,dx$

(p) $\displaystyle\int \mathrm{sen}(\ln x)\,dx$

(q) $\displaystyle\int \mathrm{ch}\,x\,\mathrm{sen}(3x)\,dx$

(r) $\displaystyle\int x^3\,e^{x^2}\,dx.$

6. Calcule os seguinte integrais indefinidos.

(a) $\displaystyle\int \frac{3x^2-4x-1}{(x^2-1)(x-2)}\,dx$

(b) $\displaystyle\int \frac{2x^2+x+1}{(x-1)(x+1)^2}\,dx$

(c) $\displaystyle\int \frac{4x^2+x+1}{x^3-x}\,dx$

(d) $\displaystyle\int \frac{27}{x^4-3\,x^3}\,dx$

(e) $\displaystyle\int \frac{x^4-8}{x^3-2\,x^2}\,dx$

(f) $\displaystyle\int \frac{x+3}{(x-2)\,(x^2-2\,x+5)}\,dx$

7. Calcule as seguintes primitivas usando a substituição indicada.

(a) $\displaystyle\int x\sqrt{x-1}\,dx, \quad x=t^2+1$

(b) $\displaystyle\int \sqrt{1-x^2}\,dx, \quad x=\mathrm{sen}\,t$

(c) $\displaystyle\int \frac{e^{2x}}{1+e^x}\,dx, \quad x=\ln t$

(d) $\displaystyle\int \sqrt{1+x^2}\,dx, \quad x=\mathrm{sh}\,t$

8. Calcule os seguintes integrais indefinidos

(a) $\displaystyle\int \frac{x}{x^2-1}\,dx$

(b) $\displaystyle\int \frac{x}{\sqrt{x^2-1}}\,dx$

(c) $\displaystyle\int \frac{1}{x}\,\mathrm{sen}(\ln x)\,dx$

(d) $\displaystyle\int \frac{-3}{x\,(\ln x)^3}\,dx$

(e) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

(f) $\displaystyle\int \frac{e^x}{1-2e^x}\,dx$

(g) $\displaystyle\int \frac{1}{\cos^2(7x)}\,dx$

(h) $\displaystyle\int \left(\sqrt{2x-1}-\sqrt{1+3x}\right)\,dx$

(i) $\displaystyle\int \frac{1}{x}\,\left(1+\ln^2 x\right)\,dx$

(j) $\displaystyle\int \frac{2+\sqrt{\mathrm{arctg}(2x)}}{1+4x^2}\,dx$

(k) $\displaystyle\int \frac{e^{\mathrm{arctg}\,x}}{1+x^2}\,dx$

(l) $\displaystyle\int \frac{\mathrm{sen}\,x}{\sqrt{1+\cos x}}\,dx$

(m) $\displaystyle\int \frac{1}{(2+\sqrt{x})^7\,\sqrt{x}}\,dx$

(n) $\displaystyle\int \mathrm{tg}^2\,x\,dx$

(o) $\displaystyle\int \frac{x+[\mathrm{arcsen}(3x)]^4}{\sqrt{1-9\,x^2}}\,dx$

(p) $\displaystyle\int \frac{x\,e^{\sqrt{1-x^2}}}{\sqrt{1-x^2}}\,dx$

(q) $\displaystyle\int \frac{1}{\cos^2 x\,\mathrm{sen}^2 x}\,dx$

(r) $\displaystyle\int \cos^2 x\,\mathrm{sen}^2 x\,dx$

(s) $\displaystyle\int \frac{1}{1+e^x}\,dx$

(t) $\displaystyle\int \frac{1}{x\sqrt{x^2-1}}\,dx$

(u) $\displaystyle\int \frac{1}{x^2\,\sqrt{4-x^2}}\,dx.$

9. Em cada alínea, determine a única função $f:\mathbb{R}\longrightarrow\mathbb{R}$, duas vezes derivável, tal que:

(a) $f''(x)=4x-1, \quad x\in\mathbb{R}, \quad f(1)=3 \quad \text{e} \quad f'(2)=-2$

(b) $f''(x)=\mathrm{sen}\,x\cos x, \quad x\in\mathbb{R}, \quad f(0)=0 \quad \text{e} \quad f'(0)=1.$