Received: 28 January 2024 | Revised: 31 May 2024 | Accepted: 19 July 2024

DOI: 10.1002/wat2.1756

OVERVIEW

WIREs WATER WILEY

# Optimization methods in water system operation

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**Edited by:** Wendy Jepson, Co-Editor-in-Chief and Jan Seibert, Co-Editor-in-Chief

**Abstract**

Operational water management is a critical global challenge, and decision making can be improved by using mathematical optimization. This paper provides an overview of optimization techniques, both exact and heuristic, used in water management. It focuses on the use of optimization techniques in the short term: operational planning in reservoir management, control of open channels, hydropower scheduling, and operation of polder drainage pumps. Principles of model predictive control, methods for optimization under forecast uncertainty, and approaches for conflict resolution are explained with the help of educational examples and practical cases. Challenges and research questions to be addressed in the future are presented as an outlook.

This article is categorized under:

Engineering Water > Methods
Science of Water > Water and Environmental Change
Water and Life > Conservation, Management, and Awareness

**KEYWORDS**

decision support, hydropower, optimization, scheduling, stochastic optimization

## 1 | INTRODUCTION

Operational water management refers to the planning behind the control of hydraulic structures to meet various human, environmental, and economic needs. Water managers must satisfy a wide range of operational conditions. Polder managers must choose between pumps and sluices for dewatering, taking into account capacity, energy costs, and tides. Dam operators must balance competing goals such as dam safety, water supply, and flood mitigation. In addition, water management must obey a variety of environmental and societal obligations. The importance of water management is further amplified by its relationship to energy: drainage pumps are one of the largest energy *consumers* in low-lying countries. Conversely, water is also used to *generate* energy through hydropower. Multiple objectives, interactions, and sources of uncertainty make operational decisions very complex. Mathematical optimization techniques can help to support decisions and resolve conflicts among competing objectives. In this paper, we introduce common optimization techniques that are applied in operational water management, supplemented by examples that illustrate their application in different operational water management problems.

We use the term optimization to include both exact and heuristic methods. The common denominator of all the optimization techniques reviewed is that they aim to find an operational scheme (decisions taken over time) that best satisfies one or more objectives under given constraints. The flow of water is typically described by differential equations, which constitute some of the constraints. Other constraints can be introduced to limit the actions that can be taken.

This paper focuses on short-term operations with a time horizon ranging from 1 day to a few weeks. We focus on water reservoirs, rivers and canals. Not included are pipe networks (drinking and sewage water), optimization techniques to support water resource planning, and optimization techniques for calibrating water-related models.

Typical challenges in operational water management include:

- a relatively large area of interest with multiple hydraulic structures;
- changing hydrological conditions with extremes (flood, drought);
- uncertainty in hydrological forecasting;
- conflicting operational objectives from different stakeholders;
- seasonal variation: hydrological conditions and operational objectives change with the season;
- linear and nonlinear process equations with continuous and discrete elements;
- time-dependent processes: large dimensions, long channels, long travel times, and storage.

This paper is organized as follows. Section 2 introduces key mathematical concepts. Section 3 describes the most commonly used methods for solving water-related optimization problems. Section 4 explains how conflicts between multiple objectives can be resolved. Model predictive control (MPC), scheduling, and the generation of rule curves and operational rules are presented in Section 5 as different operational management concepts. Section 6 deals with hydrological uncertainty, and Section 7 illustrates various principles from previous sections with practical examples. Finally, Section 8 provides an outlook on future challenges and solutions in the field.

## 2 | OPTIMIZATION PROBLEMS FOR WATER SYSTEMS

In this section, we summarize some fundamental concepts that are critical to applying optimization techniques to water system operations.

### 2.1 | Formulating mathematical optimization models

Mathematical optimization models are mathematical representations of real-world problems that aim to find the best solution among a set of available alternatives. These models consider factors such as hydrological inflows, water demand, and environmental constraints along with the flow equations that apply to a water system. Solving the optimization model yields a reservoir release scheme that best satisfies all purposes under the specified constraints.

In general, an optimization problem can be written as:

$$\text{minimize} f(x) \tag{1}$$

subject to:

$$g_1(x) \leq 0, \tag{2}$$

$$g_2(x) = 0, \tag{3}$$

$$x \in \mathrm{X} \tag{4}$$

where $x$ represents a vector of decision variables or control variables that affect the operation of the water system. The decision variables $x$ might represent pump flow rates, valve settings, reservoir release, or the allocation of water resources over time. $f(x)$ is the objective function, and $g_1(x)$ and $g_2(x)$ are inequality and equality constraints, respectively.

The objective function $f(x)$ represents the operational objectives such as minimizing water spills or minimizing operating costs over the modeling period. Conflict resolution between multiple objectives is discussed in Section 4.

The domain of each decision variable $x$ is defined by the set $X$. For continuous decision variables, $X$ specifies allowable intervals (e.g., water discharge range), whereas for integer or logical decision variables, the set $X$ specifies the set of discrete choices (e.g., gate open or closed). Decisions for $x$ can only be made within the *feasible region*, which is defined as the set of all possible combinations of decision variables that satisfy the problem constraints, that is, Equations (2), (3), and (4).

In operational water management, inequality constraints (2) can specify ranges of feasible system states, such as the capacity limits of pumps and turbines or the upper and lower volume limits of a reservoir. Equality constraints (3) are often represented by system equations, such as a water balance equation. A constraint violation makes the solution infeasible.

The solution to an optimization model provides an optimal operational scheme based on the currently available information, given the boundary conditions, constraints and chosen objectives. Unlike operational rules, this solution is unique to the specified condition. It may, therefore, be sub-optimal or even infeasible in a different scenario.

Usually, water systems can be modeled using partial differential equations (PDEs) as boundary value and initial value problems. To solve such problems, the PDEs must be discretized in time and space, resulting in algebraic or numerical models. To translate the PDEs describing the water flow into constraints, one can use automatic differentiation tools such as CasADi (2024) or the DifferentialEquations and ForwardDiff libraries in Julia (2024). For operational water system models, hydrological inflow is usually the most important boundary condition, see Section 6.

## 2.2 | Problem characteristics

Next, we distinguish problem characteristics that help us understand their fundamental properties and how to solve them.

### 2.2.1 | Linearity and convexity

An optimization problem is linear if both the objective function and all constraints are linear functions of the decision variables. The optimum of a *linear* optimization problem will always be at a corner of the feasible region, where constraints intersect. Consequently, large parts of the solution space can be ignored when solving the problem. There are mathematical methods that can quickly solve a linear optimization problem to optimality (see also Section 2.3.1).

*Convex* functions have a bowl-like shape, with a single minimum and no other local minima, as shown in Figure 1. At any point on a convex function, following the tangent line in the decreasing direction is guaranteed to lead you in the direction of the global minimum. This makes it intuitive to understand why such functions are convenient to minimize.

An optimization problem is convex if (1) the objective function is a convex function, (2) the inequality constraints are convex functions, and (3) the equality constraints are linear equations, or more precisely, affine transformations.

### 2.2.2 | Continuous and discrete problems

A continuous problem is one in which all variables are continuous, that is, the variables can be any real number within a certain range. A discrete problem is one that requires one or more discrete variables, such as integers or Booleans,

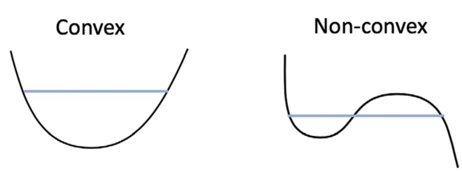



FIGURE 1 Visual representation of a convex function versus a non-convex function, each with a single decision variable.

which can only take a finite number of values. Discrete variables may be needed to account for on/off switching of pumps and turbines. This is discussed in more detail in Section 7.5.

In addition to modeling behavior that is inherently Boolean, another reason to include discrete variables may be to capture nonlinear components with a piecewise linear approximation. When the desired function is non-convex, Boolean variables are needed to keep track of which segment of the linear approximation is active in which part of the solution space.

In principle, discrete problems are harder to solve than their continuous counterparts. Therefore, avoiding or reducing the number of discrete variables whenever possible is good modeling practice. However, there is usually a trade-off between model accuracy and computation time.

## 2.3 | Common optimization problem types

### 2.3.1 | Linear programming

If both the objective and all constraints are linear in the decision variables (Section 2.2.1), and $x$ is continuous, the problem can be solved with linear programming (LP). These types of problems can be solved to global optimality within a reasonable time, and there are even mathematical proofs that state that the runtime grows polynomially, not exponentially, as the problem instance grows. Reservoir optimization (Section 7.2) is a typical application of LP in water management.

To take advantage of the appealing properties of linear optimization, modelers often choose to linearize their model, even if this means that the model becomes a less accurate approximation of the desired nonlinear equations. This linearization can be done by finding a linear approximation to the nonlinear equations/constraints or by implementing a piecewise linear approach. The latter is done, for example, by Can and Houck (1984) and Becker, Ochterbeck, et al. (2023).

Linearization of functions is a particularly common approach in MPC applications (Section 5.1), where the model is frequently re-run with updated measurements. This provides many opportunities to correct for the loss of accuracy associated with the linearization.

### 2.3.2 | Nonlinear problems

If any part of the optimization problem is nonlinear, the problem can be referred to as nonlinear. Note that this can refer to the objective function, the equality constraints, and/or the inequality constraints. The problem can be convex or non-convex and continuous or discrete. In general, nonlinear problems can be difficult to solve, but convex continuous nonlinear problems are easier to solve than non-convex discrete problems.

### 2.3.3 | Integer linear programming

When the objective and constraints are linear in $x$, and $x$ is restricted to a discrete set of values (represented as integers), integer linear programming (ILP) can be used. This technique is sometimes called mixed integer programming (MIP). “Mixed” refers to the fact that the problem is already classified as a MIP if just *some* of the variables are integers; the “linear” aspect of the problem is then omitted from the acronym but is implied.

Many MIP models are found in the context of hydropower scheduling (see Kong et al., 2020). An example of an optimization model solved with MIP is given in Section 7.5. Agha (2006) uses a MIP to compute a pumping schedule. The piecewise linear approximation of nonlinear equations (Section 2.2.2) is usually solved with the help of MIP.

## 3 | SOLVING AN OPTIMIZATION PROBLEM

An optimization problem can be solved in an exact or a heuristic manner. An exact method *provably* finds the best solution in the entire feasible region. Such a solution is called a *global* optimum. Figure 2 contrasts a global optimum with

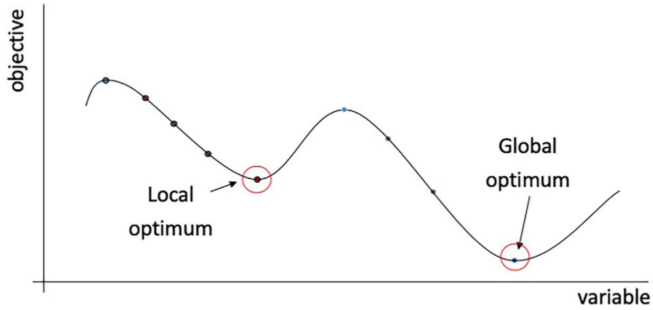



FIGURE 2 Local versus global optima.

a local optimum, which appears optimal when only the local area around it is inspected. Although finding a global optimum is desirable, the ability to do so structurally is limited to specific classes of optimization problems.

When the mathematical structure of a problem makes it challenging to apply exact methods, such as when dealing with non-convex functions or discrete decisions, heuristic methods can be a viable alternative. These heuristic methods prioritize finding a satisfactory, hopefully near-optimal solution quickly over finding an exact optimal solution. In general, heuristic methods are less mathematically demanding, but cannot guarantee that their solutions are globally optimal.

The classification of optimization problems as described in Section 2.3 determines which method can be used to solve them. The following section summarizes the most common optimization methods and techniques used in optimization software.

## 3.1 | Solution methods for exact optimization

Below we introduce selected solution methods. Note that this list is not exhaustive and only covers the most common methods used in water system operation.

### 3.1.1 | Simplex method

The simplex method is usually the default method for solving LPs. The algorithm invokes "pivot" operations to move from one feasible solution to another along the boundaries of the problem (constraints), while iteratively improving the value of the objective function. The process continues until no further improvement in the value of the objective function is possible, indicating that it has found the optimal solution. While the simplex method is efficient in practice, it has exponential worst-case complexity.

### 3.1.2 | Branch and bound

Branch-and-bound algorithms are often applied to MIPs. These algorithms iteratively find bounds for the optimal solution and then systematically divide the problem into smaller sub-problems (branches) to explore better solutions in different directions. Each sub-problem represents a continuous relaxation of a portion of the original problem's solution space. The branching process continues iteratively, creating a tree-like structure of sub-problems. Branch-and-bound algorithms significantly reduce the region of exploration (and the computational time) by cleverly discarding certain branches of the tree that do not contain an optimal solution based on the computed bounds.

Branch-and-bound implementations are usually available in MIP solvers, which also incorporate sophisticated pre- and post-processing techniques and cleverly choose which variable to branch on next.

### 3.1.3 | Interior-point methods

Interior-point methods, also referred to as barrier algorithms, start from a point within the feasible region of the problem and move toward the optimal solution by following a path through the interior of the feasible region. For large problems, this search method is usually more efficient than moving along the boundaries of the problem, as in the simplex method.

Modern implementations of interior-point methods work by transforming the original problem using the concept of *duality*. In optimization, duality refers to the idea that each optimization problem (the primal problem) has a corresponding dual problem, which provides an upper bound on the optimal value of the primal problem. In the context of interior-point methods, the algorithm simultaneously computes sequences of primal and dual feasible points and uses them to find an optimal solution. For a more detailed explanation, see Bertsimas and Tsitsiklis (1997, Ch. 4).

### 3.1.4 | Continuation method and gradient descent

Gradient descent is an iterative method that starts with a feasible solution, computes the gradient of the objective at that point, and takes repeated steps in the opposite (descent) direction along the gradient. This technique is only applicable to continuous differentiable problems.

The method is guaranteed to converge to the global optimum if the problem is convex; in other cases, it will find a local optimum. This local optimum does not have to be unique; it can depend on the starting point (also called the "seed"). The choice of the seed and the step size are important aspects of the method, as they affect the speed of convergence and the quality of the final solution. If they are well chosen, gradient descent can be reasonably fast. However, providing two different starting points can lead to different local optima—even if the starting points are close to one another. This occurs especially in complex nonlinear optimization problems with many optimization variables. It becomes problematic in an operational context when the final solution must be robust to small perturbations.

The continuation method was developed to overcome the limitations of gradient descent. It provides a path-stable solution to the initial starting point. Unlike gradient descent, which solves an optimization problem by adjusting the solution in the direction of the steepest descent, the continuation method solves successive approximations of the original problem. It uses the solution of each approximation as a seed for the next. By avoiding solving the original nonconvex problem in the first step and using the solutions of successive approximations as seeds, this approach reduces the likelihood of getting trapped in local optima.

In the continuation method, the successive approximations of the problem are generated through a continuous (homotopic) deformation of the optimization problem from a simple (linear) problem to the original (nonlinear) one (Baayen et al., 2020). Under certain mathematical conditions (such as convexity of the problem), the solution found by the continuation method is a global optimum (Baayen & Postek, 2022).

### 3.1.5 | Decomposition

An optimization problem can be solved as a whole, or be decomposed into a set of simpler optimization sub-problems that can be solved independently or iteratively to find the optimal solution to the original problem. Decomposition exploits the structure of the problem, such as separability or special constraints, to break it down into simpler subproblems.

Decomposition is particularly useful for large optimization problems that are computationally expensive or impossible to solve as a whole. For instance, if the problem is mixed-integer and nonlinear, it may be difficult to solve directly. In this case, a good option is to solve the nonlinear problem first, assuming all variables are continuous, and then solve the mixed-integer problem sequentially, assuming linear relationships. Whether a decomposition finds the *global* optimum depends on the specific problem and the decomposition method used.

The concept of decomposition is also used in dynamic programming and other specialized solution methods. Dynamic programming techniques were among the first algorithms applied to reservoir optimization (Labadie, 2004; Loucks & van Beek, 2017). Dynamic programming allows only discrete states, making it a less obvious choice for modeling water levels and flows, as Kangrang et al. (2023) also point out. Nevertheless, recent studies have used dynamic programming to derive reservoir operation rules, including Dau et al. (2023) and Brass (2006).

## 3.2 | Heuristics

Unlike the methods described above, heuristics are not concerned with provable global optimality. Heuristics are used to find a satisfactory solution quickly. Heuristic results may depend on where the search starts (seeding), how long the search continues, or even randomness.

Some heuristics fall into the category of *local search*, which takes an initial solution (e.g., a random one) and makes simple changes to it to obtain "neighboring" solutions. If a neighboring solution is better, it is accepted, otherwise the old solution is kept and another neighboring solution is tried. If all neighboring solutions are worse than the current solution, the local search terminates and returns the best-known solution thus far as its result. This explains why local search can get stuck in a local optimum (see Figure 2).

One of the main advantages of heuristics is that they do not impose mathematical requirements on the model (linearity, convexity, affinity) as most exact optimization methods do. Therefore, in principle, any simulation model can be included in a heuristic algorithm. General rules of thumb or feedback control (if-then-else logic) can easily be included if desired. Labadie (2004) finds heuristic optimization methods suitable for parameter tuning. For instance, heuristics have been used to generate rule curves and operational rules (see Section 5.3).

Two popular derivative-free heuristics are:

- Genetic algorithms, inspired by the process of natural selection. A "population" of potential solutions undergoes iterative generations. Each individual in the population represents a potential solution to the problem and is assigned a fitness score. Individuals with higher fitness scores are more likely to become parents in the next generation. A genetic algorithm is used to minimize the cost of flood damage in a river basin by Chiang and Willems (2015).
- Simulated annealing aims to find the global optimum of a given problem by mimicking the cooling and gradual crystallization of a material. Instead of manipulating atoms or molecules, the algorithm explores the solution space of a given problem by iteratively modifying a solution and accepting or rejecting the modifications based on probability. Initially, at higher "temperatures", the algorithm allows for more exploratory moves that may increase the value of the objective function (analogous to higher energy states in metallurgy). As the "temperature" decreases, the algorithm becomes more selective and tends to converge toward better solutions. It has been applied to reservoir systems in Thailand to optimize operating policies (Tospornsampan et al., 2005).

Some heuristics have been found effective in problems with large and complex search spaces, such as discrete non-convex problems. However, in some situations heuristics could be considered less suitable for an operational application than the exact optimization methods described in Section 3.1 for the following reasons:

- *Optimality*: In general, heuristics do not come with proof that their solution is optimal (Mart et al., 2018). While heuristics can be expanded to approach global optimality, that effectively means searching for longer. For practical applications involving heuristics in complex problems, it is common to use early termination criteria, such as time limits or a specified number of iterations without an improvement (Mart et al., 2018).
- *Path-stability*: For operational decision support, it is important that the optimization model produces consistent results. A small change in the input (boundary condition) should not produce a completely different result. This requires a global optimum or at least a path-stable solution, which heuristic methods cannot guarantee. Some heuristics even produce different results when performing multiple runs on the *same* input, see, for example, Mansouri et al. (2022). While this can be used to their advantage (do multiple runs and only show the best result), this variance can be unhelpful when aiming to build trust in an optimization tool for operational decision making.
- *Customization and parameter tuning*: Typically, heuristic methods require significant algorithm customization and fine-tuning, which means that a heuristic that works well on one problem may perform poorly on a similar problem where, for instance, a new constraint or decision variable is included (Labadie, 2004).

Every year, a multitude of metaphor-based heuristics are introduced in the academic literature. However, many of them lack the necessary scientific rigor (Aranha et al., 2022). Consequently, it is crucial to be cautious when selecting heuristics for operational decision making.

## 3.3 | Optimization solvers

In practice, modelers and practitioners in operational water management typically do not implement exact optimization methods from scratch. Instead, they often rely on external solvers.

There are many optimization solvers available for different types of problems. Therefore, identifying the type of optimization problem at hand (as described in Section 2.3) is an important step in selecting the appropriate solver.

While some solvers are free or even open source, others are commercial and offer, at best, a free academic license. Currently, the three leading commercial MIP solvers are the IBM ILOG CPLEX Optimizer (2024), the Gurobi Optimizer (2024), and Xpress (2024). Free alternatives include HiGHS (2024) and CBC (2024). For mixed-integer nonlinear problems, Kronqvist et al. (2019) gives a good overview of available solvers and compares their performance. For nonlinear problems, the solvers Ipopt (Wächter & Biegler, 2006) and Uno (2024) are open-source implementations, and Knitro (2024) is a commercial solver. The NEOS Server (2024) provides free access to many modern solvers, including commercial ones, albeit without the seamless integration with programming languages that dedicated solver packages offer.

Virtually all solvers are sensitive to scaling, that is, they are much more likely to find a good solution quickly when the optimization variables are of the same order of magnitude. If this is not the case, some variables may be reduced to "noise." It is good modeling practice to formulate a well-scaled problem before feeding it to an optimization solver, which often has built-in algorithms to further improve scaling.

## 3.4 | Optimization software for operational water management

Examples of off-the-shelf software solutions for water-related optimization in operational contexts include RiverWare (CADSWES, Clement & Zagona, 2020), RTC-Tools, (2024) (Deltares), Kisters RTO (Kisters, 2024), SHOP (SINTEF, 2024), WEB.BM (Ilich, 2022), and ModSim (Colorado State University, Labadie, 2006). These software solutions provide a generic framework to formulate the optimization problem for a water system and use exact optimization methods to find the solution.

For heuristics, there are not as many distinct off-the-shelf software products. However, Talsim (SydroConsult, 2024) is an example of software with a heuristic method.

# 4 | CONFLICT RESOLUTION BETWEEN MULTIPLE OBJECTIVES

Operational water management often requires meeting multiple objectives. These may be regulatory constraints or objectives related to making the best use of available resources. Choices must be made regarding how to deal with multiple objectives, especially when these objectives conflict.

## 4.1 | Weighted penalty functions

Perhaps the simplest way to deal with multiple objectives is to define one all-encompassing objective function that is a weighted combination of the individual objectives. By multiplying the more important objectives with a larger penalty, the optimal solution should prioritize them. One difficulty with this approach is that different objectives may have different scales of measurement. A simple solution would be to assign a weight to a high-priority objective that is *many* orders of magnitude larger than the weight assigned to objectives with lower priority. However, this can lead to numerical issues in the optimization algorithm and make the optimization process insensitive to changes in the smaller-weighted objective. As a result, the optimization process can become slow and inefficient, and the true trade-offs between objectives may not be adequately captured.

## 4.2 | Pareto optimization

Pareto optimization is a generalization of the weighted penalty approach. A Pareto optimal solution is one in which no objective can be improved without making another one worse, and the Pareto front is the set of all Pareto optimal solutions. Finding one point on the frontier is an optimization problem in itself, and finding the entire set involves solving many versions of the same problem (Buber et al., 2019). It is therefore important to choose where on the Pareto front the operator wants the solution to be chosen, and this is influenced by the choice of penalty values in the objective function. Our practical experience is that it can be difficult to set the penalties so that the desired solution is achieved, so it may be necessary to tune these penalties. A different set of penalties may cause the solution to move along the Pareto front and produce a different solution for the operator.

### 4.3 | Goal programming

In so-called lexicographic goal programming, also known as preemptive goal programming, objectives are formulated as desired values over time, ordered according to their priority, and solved in consecutive optimization problems (Can & Houck, 1984; Eschenbach et al., 2001; Loganathan & Bhattacharya, 1990). In each optimization, the objective function contains a subset of all goals, with each goal represented by one or more objective function terms. In the subsequent optimization problem, constraints ensure that the goal achievement in the previous optimization is not changed for the worse. An example is given in Figure 7 in Section 7.2.These constraints can be formulated in two different ways:

1. Deviations from a target are interpreted as a fixed time series. If at any time step the solution was within the required range, the original range is retained as a constraint. If the solution was outside this range, the solution value becomes the new constraint.
2. Deviations from a target are interpreted in an aggregated fashion. Sum the deviations over the planning horizon and formulate a constraint that limits the aggregated deviation to that amount. This allows the deviation to be shifted in time.

Practical difficulties one may encounter are related to floating-point numbers and solver accuracy. By taking the result of one iteration and adding it as a constraint in the next iteration, the solver may report the optimization problem as infeasible. Such cases can be resolved by slightly relaxing the constraint.

When multiple objectives have the same priority, it is possible to combine goal programming with Pareto optimality (Xevi & Khan, 2005).

## 5 | OPERATIONAL MANAGEMENT CONCEPTS

After having introduced mathematical optimization concepts and methods in the previous chapter, this chapter introduces several ways in which optimization models can be used in operational water management.

### 5.1 | Model Predictive Control

The basic idea of Model Predictive Control (MPC) is to carry out a run with an optimization model that generates an operational scheme for the entire forecast horizon, yet to apply only the first portion of this scheme—until a new inflow forecast is available. With the new inflow forecast and the updated system state, a new model run is performed. As time passes, the forecast horizon reaches further into the future. By repeating the procedure, the operational scheme gradually adapts to the best available information about the current state and the expected future. Because of its iterative adaptive procedure, MPC is also referred to as a moving horizon, receding horizon, or rolling horizon approach.

MPC originates from industrial control systems (Camacho et al., 2007; Morari et al., 1988). In most cases, a continuous linear model is used, even if it is an approximation of reality, as explained in Section 2.2.1. The often implicit idea behind this is that even if the real processes are not linear, they can be considered to be approximately linear over a small operating range, and the feedback mechanism of the MPC can compensate to some extent for a mismatch between the model result and the real system behavior.

However, water management requires anticipating extreme events several hours or even days in advance. This may require an accurate representation of the physics in the optimization model with nonlinear equations for the entire forecasting horizon. This becomes even more important when operational limits and objectives have a legal status or the consequences of violating them are very severe. Two such examples are (1) floods, where the consequences of decisions can have a huge impact on society, and (2) environmental obligations that operators must meet before other uses of water, such as hydropower production, can be served.

Consequently, the research of MPC for operational water management contains both optimization models with linear flow equations (Clement & Zagona, 2020; Hashemy et al., 2013; van Overloop et al., 2010; Wahlin & Zimbelman, 2018) as well as optimization models with nonlinear process equations (Baayen et al., 2020; Horváth, Smoorenburg, et al., 2022; Horváth, van Esch, et al., 2022; Montero et al., 2013; Rötz & Theobald, 2019; Schwanenberg et al., 2010; Schwanenberg et al., 2012; Schwanenberg et al., 2014; Schwanenberg et al., 2015; Talsma et al., 2014; Xu, 2013). Castelletti et al. (2023) found that many MPC papers for water management provide little detail about the optimization method.

## 5.2 | Scheduling

Scheduling means making an operational plan for the future, usually involving discrete decisions (e.g., hours when turbines are in on/off state). Scheduling does not have the distinct feedback mechanism that we saw in MPC. A schedule provides the framework for control actions and related decisions that are executed on a detailed time scale.

Short-term hydropower scheduling aims to generate an optimal dispatching schedule for hydropower plants or individual generation units. The so-called unit commitment problem aims to determine the on/off status of the units for each period (Ponrajah et al., 1997; Witherspoon & Ponrajah, 1997). Unit commitment usually requires mixed-integer logic. In contrast, the unit load dispatch problem aims to determine the respective dispatch of the committed units (Ponrajah et al., 1997). If the committed units are known, this can be formulated as a continuous problem. The two problems can also be combined and unified as a hydro unit commitment problem. In addition to being used as a framework for operating hydropower plants, the hydropower schedule is also used to prepare activities in the energy market.

Typical optimization objectives include maximizing total profit (Kong et al., 2020), minimizing total unit discharge for a given load (Kong et al., 2020), and a system-wide generation target (load balance, Becker, Ochterbeck, et al., 2023; Schwanenberg et al., 2014, 2015). Section 7.4 presents an example of a load balance optimization for a reservoir cascade.

Hydropower generation may be subject to operational limitations from other use functions. For instance, fish spill obligations (Clement & Zagona, 2020; Schwanenberg et al., 2014, 2015) aim to divert a certain amount of water around turbines to reduce fish mortality. Other typical environmental obligations are a minimum flow or water level range (Schwanenberg et al., 2014, 2015).

## 5.3 | Reservoir rule curves

Reservoir management often involves specifying a maximum or minimum water level—or the corresponding reservoir volume—over the days or months of a year. These are known as *rule curves*, and they are connected to optimization in two ways: (1) optimization models can be used to derive them, and (2) they subsequently form objectives in operational optimization.

While traditionally, rule curves were developed by using graphical methods to analyze historical reservoir inflow data (Maniak, 2016), nowadays optimization techniques are commonly used to generate rule curves based on a large set of inflow scenarios (Cuvelier et al., 2018; Kangrang et al., 2023; Lohr, 2007; Marth, 2021; Marth & Becker, 2023). There are two main approaches to this. The first is to optimize the reservoir release with the objective of minimizing or maximizing the volume of water in the reservoir for the lower and upper rule curves, respectively. The output of the optimization is a set of period-of-record solutions that are then post-processed to rule curves using statistical methods such as the envelope function or fractiles (Cuvelier et al., 2018; Dau et al., 2023; Marth, 2021; Marth & Becker, 2023; Uysal, Schwanenberg, et al., 2018).

The second approach incorporates the rule curves into a reservoir model. The optimization variables are not the control variables (reservoir release), but the parameters that describe the rule curves (Kangrang et al., 2023; Lohr, 2001; van der Vat, 2015). Consequently, operational rules that determine the reservoir release in dependence of the rule curves must be specified. This includes rules for when the reservoir level is above the upper rule curve (flood situation, often a pass-inflow policy applies here), below the lower rule curve (water scarcity, hedging rules often apply here) and for normal conditions.

### 5.4 | The role of simulation models for optimization

Simulation models and optimization models are fundamentally different. Simulation models can only represent predetermined decision rules that are evaluated sequentially at each time step, rather than finding optimal decisions that consider the entire planning horizon. Therefore, solutions obtained from simulation models are short-sighted and usually suboptimal. However, by simulating the operational problem step by step, rather than optimizing all decisions at once, simulation models are less computationally demanding and do not impose special requirements on the underlying mathematical structures as their counterpart optimization models do (see Section 3). For this reason, simulation models can represent system nonlinearities more accurately than optimization models.

A combination of both optimization and simulation techniques can be a viable, or even preferred, approach for effective decision support in scenarios where optimization cannot fully capture the intricacies of the system. In this combined approach, the optimization model first establishes the operational scheme using a simplified model (e.g., linear). This scheme is then applied to a so-called simulation companion model. This simulation model provides a more accurate representation of the flow processes, higher spatial resolution, or additional operational rules. A simulation model can also be used to compare the optimal solution to traditional feedback rules. Another way of combining them is to incorporate a simulation model into a heuristic method that treats it as a black-box element of the optimization problem.

## 6 | OPTIMIZATION UNDER HYDROLOGICAL UNCERTAINTY

Water management decisions can be affected by many sources of uncertainty, including water and energy demand, electricity prices, failures of infrastructure or equipment, and hydrological forecasts.

For the sake of brevity, the examples in this section focus mainly on hydrological uncertainty due to the strong dependence between water management decisions and hydrological inflows. This is because changes in hydrological conditions can directly influence decisions related to water storage, distribution, and usage. However, the techniques for dealing with uncertainty that we describe next can be applied to all of the sources of uncertainty mentioned above.

### 6.1 | Ensemble forecast

A common way to convey the uncertainty of future inflows is through ensemble forecasts. These ensembles represent a collection of possible future scenarios, each of which can be viewed as an independent pathway or *ensemble member*. To generate ensemble forecasts, weather organizations run multiple numerical weather prediction models. These model runs correspond to different initial conditions to account for uncertainty in the model parameters (Leutbecher & Palmer, 2008). Typically, each member is considered equally likely to occur, although in optimization models it is possible to assign different probabilities to individual members. The weather ensemble forecast must then be transformed into an *inflow* ensemble forecast. This step requires hydrological models.

The operational decision should make the best use of the inflow ensemble forecast. Optimizing for one individual member of the ensemble forecast neglects the uncertainty of the inflow forecast, and a decision that is optimal for one ensemble member will be not optimal for other ensemble members.

In stochastic programming, uncertain problem parameters are characterized by known probability distributions. Since the ensemble members represent a finite number of predetermined scenarios, each of which is assumed to occur with a given probability, the associated optimization problem can be formulated as a "stochastic program." Within stochastic programming, several approaches are available. Selected approaches are discussed in the following.

### 6.2 | Cross-scenario optimization

This approach, also referred to as holistic ensemble optimization (Becker, Kim, et al., 2023), involves the pursuit of a single solution (e.g., release strategy) that satisfies all ensemble members. In the optimization model, this is achieved by imposing the same decision variables on all ensemble members. In order to represent all ensemble members simultaneously, the objective function is defined as a weighted sum of the outcomes of the ensemble members (where the weights correspond to their probabilities), and the equality and inequality constraints (Equations 2 and 3) of each ensemble member are specified as part of the optimization problem.

Since, in this approach, a single optimal solution must satisfy the constraints of all ensemble members or scenarios, this solution tends to be driven by the most extreme ensemble members. For example, in flood prevention, the recommended discharge will be dictated by the combination of ensemble members that produces the highest inflow. Consequently, one solution for all ensemble members does not fully represent the fact that decisions can adapt over time in each scenario and does not take advantage of the richer information available from the individual members. To overcome this limitation, one can use a tree-based method, which we describe below.

### 6.3 | Tree-based ensemble optimization

Operational decisions can usually be framed as multi-stage stochastic optimization problems, where decisions are made in a sequence over multiple stages under incomplete information about the future rather than all at once. This approach allows for flexibility and adaptability, as decisions can be revised at each stage based on observed outcomes and updated predictions (Dantzig, 1959).

Tree-based ensemble optimization is a classic approach to multi-stage stochastic optimization (Kall et al., 1994). When applying this approach, the process begins by transforming the ensemble forecasts into an event tree, which serves as an equivalent representation of the uncertain parameters. This transformation is achieved by segmenting the time horizon into distinct intervals, or stages, and grouping ensemble members with similar characteristics within each stage. The start of each stage represents a point in time when the uncertain parameters up to that point are revealed. As a new stage begins, the branches of the scenario tree split to represent subgroups of ensemble members. This event tree forms the basis of an optimization problem, with groups of decision variables attached to each branch of the tree, and with the objective of determining optimal actions that are consistent within each branch across all stages.

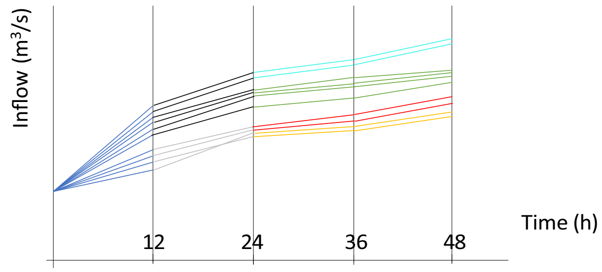
For example, consider the ensemble in Figure 3a and imagine that we need to determine optimal values for a single control (e.g., discharge) that may change every hour. For the first 12 h, there are 12 decision variables, one for each hour. However, for hours 12–24, there are 24 decision variables, half of which reflect the optimal decisions in the gray scenarios, and the other half reflect the optimal decisions in the black scenarios. The idea behind this is that by hour 12, some uncertainty is revealed: the user has learned whether the current inflows are in a wet (black) or dry (gray) scenario, and at that point will decide which half of the 24 decision variables to proceed with. From time 24, there are even more variables: four per hour. An optimization model is solved that encompasses the variables for all groups and all time steps together. A result may look like Figure 3b, where it is shown that there is only one solution for each group. This approach can be seen as a middle ground between finding actions for each ensemble member individually and finding actions that should work for all ensemble members simultaneously (i.e., cross-scenario). More details on the application of tree-based ensemble optimization for operational water management can be found in Raso et al. (2014) and Becker, Kim, et al. (2023). Although the tree-based approach has received considerable attention in the literature, it has not yet been widely applied to real-world operations.

### 6.4 | Robust optimization

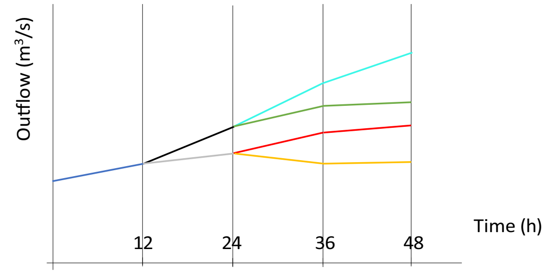
When the distribution of the uncertain parameter(s) is unknown, one can use robust optimization, which assumes that the parameters are bounded within a feasibility set. Robust optimization inherently takes a worst-case perspective, seeking the solution that performs best when all uncertain parameters take their worst-case values. For example, robust optimization has been applied to the unit commitment problem in power systems because of the effect of uncertainty in wind power (Bertsimas et al., 2012; Jiang et al., 2011; Xiong et al., 2016).

## 7 | APPLICATIONS

This section explains various aspects of water system operations using optimization techniques. Through examples, we show the functional principle of selected aspects from the previous chapters.




(a) Input: 10 ensemble members, divided into groups at predefined breaching points (times 12 and 24).




(b) Output: the resulting control variables are unique within each group.

**FIGURE 3** Illustrative example of input and output for tree-based ensemble optimization. The colors represent different groups.

## 7.1 | Basic example: Optimizing reservoir release

We first present a basic example to help understand the concept of optimization-supported operational decisions.

A reservoir is to be managed under a given 2-h forecast of inflows given in 1-h increments. At the start of the time horizon ($t = 0$), the reservoir is already full, and a large inflow is anticipated. The expected inflows are 3.5 $m^3/s$ in the first hour, and 8.0 $m^3/s$ in the second hour. The operator wants to store as much of the inflow as possible, but also prevent the reservoir from overflowing at any time.

### 7.1.1 | Basic model

The relationship between the water volume $V$ in the reservoir and the inflows and outflows, $Q_{in}$ and $Q_{out}$, respectively, is given by the storage equation:

$$\frac{\partial V}{\partial t} = Q_{in} - Q_{out}. \tag{5}$$

Considering the hourly forecasts, Equation (5) can be discretized over the planning period $T = \{1, 2\}$, and defined as an equality constraint (see Equation 3) in the optimization model:

$$V(t) - V(t-1) = Q_{in}(t) - Q_{out}(t), \quad \text{for} \quad t \in \{1, 2\}. \tag{6}$$

where $Q_{in}(t)$, $Q_{out}(t)$ are the discretized hourly flows for each time period, and $V(t)$ is the resulting water volume at time $t$. As the operator can decide on the reservoir release, $Q_{out}$ is defined in the model as a *decision variable* or *control*, and $V(t)$ as a *state variable*. Since the inflow values are known, $Q_{in}(t)$ is treated as a forcing or boundary condition.

To ensure that as much water is stored in the reservoir as possible, the objective function minimizes the reservoir release accumulated over all time steps $t$ from 1 to 2:

$$\text{minimize} \sum_{t=1}^{2} Q_{out}(t). \tag{7}$$

Next, we add inequality constraints to ensure the volume stays within the desired range of the reservoir. Suppose the full supply level is 420,000 $m^3$ and the dead storage is 380,000 $m^3$:

$$380,000 \leq V(t) \leq 420,000 \quad \text{for} \quad t \in \{1, 2\}. \tag{8}$$

Since the reservoir was initially full, we represent this initial condition by a constraint:

$$V(0) = 420{,}000. \quad (9)$$

The outflow cannot be negative at any time. Moreover, suppose we want to bound the outflow by 6 $m^3/s$ = 21,600-$m^3/h$ in order to prevent the river section downstream of the reservoir from flooding. That reads as constraints:

$$0 \leq Q_{out}(t) \leq 21{,}600 \quad \text{for} \quad t \in \{1,2\}. \quad (10)$$

### 7.1.2 | Solution

The optimization model anticipates the inflow peak that occurs at $t = 2$. Figure 4 shows that during the first hour, a release of 5.5 $m^3/s$ is planned, such that the volume temporarily drops to 412,800 $m^3$. This ensures that during the second hour the release does not exceed the maximum of 6 $m^3/s$ and that the maximum storage capacity is not exceeded in the last time step.

This solution is optimal, but not necessarily unique: it would also have been possible to release 6 $m^3/s$ at time 1, followed by 5.5 $m^3/s$ at time 2. In fact, any variation between these numbers, as long as both stay within the maximum discharge of 6 $m^3/s$ (Equation 8) and sum up to 11.5 $m^3/s$ would be equally good as they achieve the same objective value in Equation (7).

If one wishes to further specify the desired behavior, it is possible to add more constraints or add terms to the objective function. For example, to prevent sudden large changes in flow, it is quite common to specify the difference in flow between two consecutive time periods (and either put a limit on such a difference as a constraint, or add it to the objective to be minimized). Adding such a term to the objective in the example above would lead to a unique optimal solution that releases 5.75 $m^3/s$ at both time steps. We present more realistic optimization cases with multiple conflicting objectives and more constraints in the following sections.

## 7.2 | A reservoir model under flood conditions with goal programming

As shown in the previous example, operators can anticipate a flood event by pre-releasing water from the reservoir to create space to catch the flood wave in the reservoir. This example shows how optimization models can assist in deciding the timing and amount of the reservoir release.

Our example is inspired by O'Connell and Harou (2012). The model has been used for several lectures and trainings (Becker, 2022, 2023; CARECECO, 2021a, 2021b). An RTC-Tools implementation of this model is available on request. The model is also available as an interactive web application on the Viktor platform (Deltares, 2024b).

The modeling area and model schematic are shown in Figure 5. The Trout Lake reservoir impounds a river. Downstream of the reservoir, a tributary flows into the river (location "Alder"). A "River City" is located at the downstream end of the water system. At this location, a minimum flow and a maximum discharge must be maintained whenever possible.

As in the previous example, the basic equation is the reservoir equation (Equation 5). This equation is implemented as an equality constraint. Inequality constraints are also applied to the reservoir volume and the reservoir release to represent the dead storage and the physical maximum volume as well as the capacity of the bottom outlet and spillway, respectively. In addition to the hydrological inflow to the reservoir $Q_{in}$, the model contains another boundary condition, the tributary inflow. The discharge at River City is computed as the sum of the reservoir outflow $Q_{out}$ and the tributary inflow.

The model has multiple operational objectives for reservoir management as given in Table 1. An outer operational range for the dam operations is given, with a minimum operational volume and the volume that corresponds to the surcharge level as the operational maximum for flood conditions. Since these objectives address dam safety and water supply security, they have the highest priority in this example. The discharge objectives at River City consider minimum flow rates to ensure sufficient water for cargo shipping, a stable water supply for public, agricultural and industrial uses, and so on. They also take into account water quality and ecology. The discharge maximum usually reflects the flood

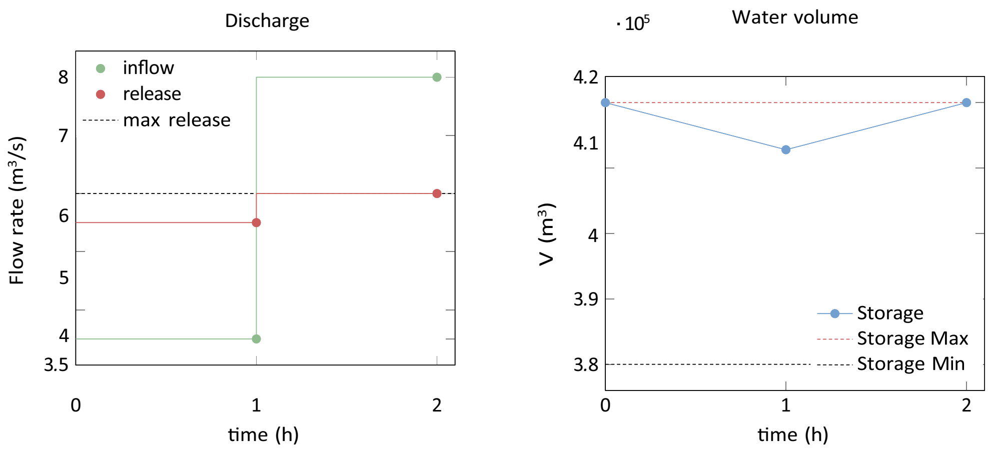



**FIGURE 4** Example of water volume in a reservoir (right), predicted inflow and optimized corresponding release (left). During the first hour, an inflow of 3.5 $m^3$/s and an outflow of 5.5 $m^3$/s (left) imply that the volume drops by (5.5–3.5) $\times$ 60 $\times$ 60 = 7200 $m^3$, thus decreasing the water volume from 420,000 $m^3$ at $t = 0$ to 412,800 $m^3$ at time $t = 1$ (right).

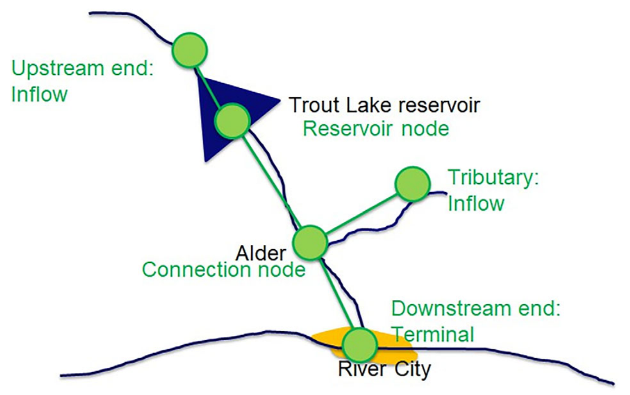



**FIGURE 5** A schematic of the Blue River system and the Trout Lake reservoir. Example inspired by O'Connell and Harou (2012).

stage and is intended to prevent flood damage. The discharge range for River City has priority 2. The third objective is the preferred operational range of the reservoir volume.

The storage equation (Equation 5), the physical minimum and maximum reservoir volume constraints (not shown here), and the objectives specified in Table 1 form a linear optimization problem.

Figure 6 shows an optimization result for reservoir operations in a flood scenario. The optimization results are discrete, but plotted as a continuous time series for better visualization in this example and all subsequent examples. The reservoir inflow time series (Trout Lake $Q_{\text{in}}$) has a flood peak on January 6, when the discharge is much higher than under normal conditions. The tributary flow (Alder $Q_{\text{in}}$) also has a flood peak. The peak is small compared to the reservoir inflow, but the peak discharge exceeds the maximum discharge at River City.

To create flood storage for the upcoming flood wave in the reservoir, the optimization model immediately increases the reservoir release (Trout Lake $Q_{\text{out}}$), but in a way that the discharge at River City does not exceed the maximum flow. The reservoir release takes into account the tributary inflow: if the tributary flow increases, the reservoir release is decreased accordingly. The maximum discharge at River City is only exceeded when the tributary inflow is higher than the maximum discharge at River City. The tributary inflow cannot be controlled. Reducing the reservoir release to zero during this period is the best way to limit the exceedance of the maximum discharge as much as possible.

The early release reduces the reservoir volume, which allows a large portion of the flood water to be stored in the reservoir without exceeding the maximum reservoir volume. The maximum volume is reached after the flood peak. Immediately after the flood peak, the reservoir release can be reduced. Initially, the reservoir is already filled above the upper operational range objective and, under the given inflow conditions, the operational range is not met.

**TABLE 1** Priorities of operational objectives.

| Priority | Objective | Parameter |
|---|---|---|
| 1 | Minimum and maximum | Volume |
| 2 | Discharge at River City | Discharge |
| 3 | Operational range | Volume |

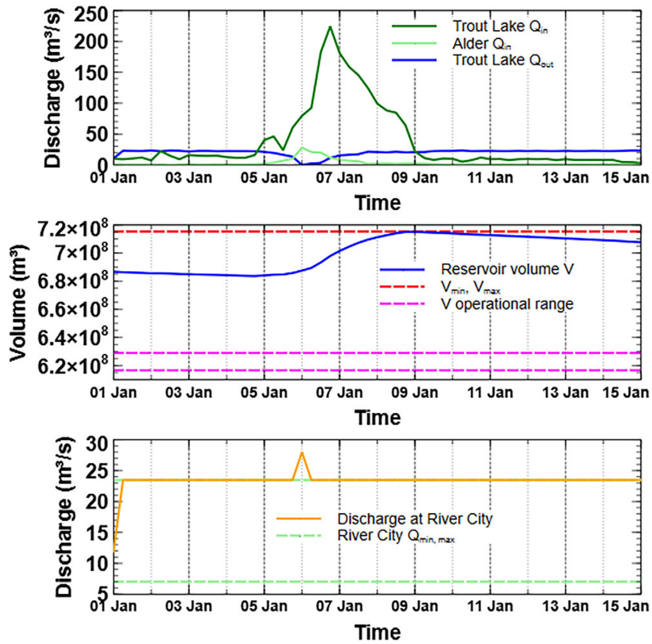
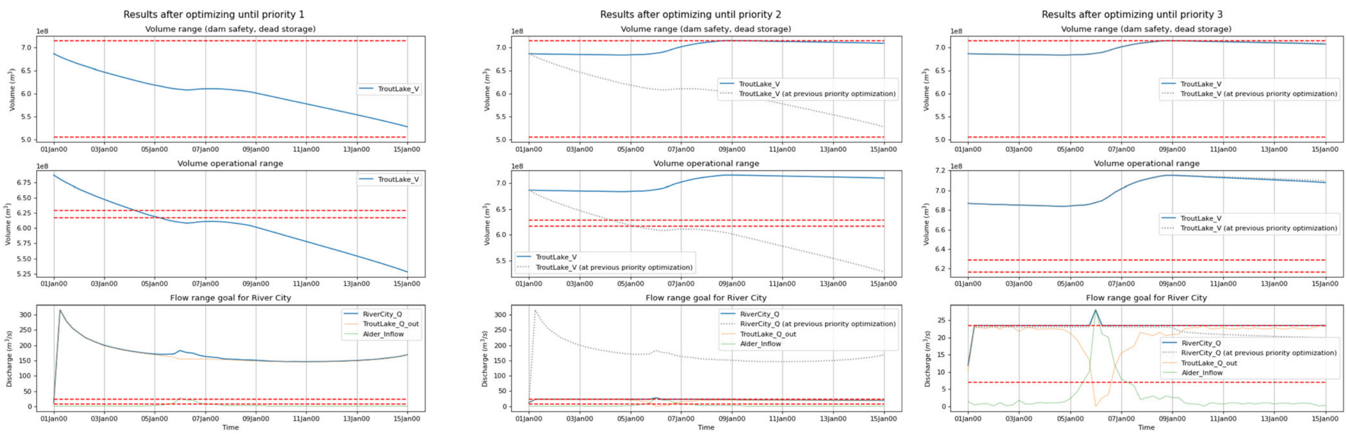



**FIGURE 6** Optimization of reservoir release for a flood scenario.




(a) Result after solving priority 1 (b) Result after solving priority 2 (c) Result after solving priority 3

**FIGURE 7** Development of the solution with goal programming.

Goal programming (Section 4.3) was applied to this example to resolve the conflicts between the objectives. Figure 7 shows the development of the solution whose result is shown in Figure 6. In the first optimization, only the objective with the highest priority in Table 1 is active. All other objectives are not yet active, so they are not taken into account in the optimization and are therefore not obeyed. At River City, the discharge reaches high values that are much higher than the operational range (Figure 7a). In the second optimization, the discharge range at River City (objective 2 in

Table 1) is added to the optimization. This objective affects the reservoir release. The optimization model uses the remaining solution space: the minimum and maximum volumes in priority 1 are still met, but the result is closer to the upper operational limit for the reservoir volume. The third optimization aims to bring the volume into the operational range. Objective 3 cannot be achieved, but the addition of this objective brings the reservoir volume a little closer to the operational range. However, the constraints from the previous objectives don't allow the solution to get closer to this goal. To achieve this, the release at River City remains at its maximum.

## 7.3 | MPC for reservoir operations

Figure 8 illustrates the concept of MPC (Section 5.1) with the help of another single reservoir optimization model. Multiple optimization runs of an optimization model are carried out consecutively. Seven time steps of the forecasting period are known to the model. According to the moving horizon concept, the first part of each optimized operational scheme is applied. In the current case, the reservoir volume at the first optimized time step is taken as initial condition for the next optimization run. With each consecutive run, the model looks one time step further ahead.

In Figure 8a,c, each optimization run is represented by a different color. Each optimization run aims to make the best use of the reservoir by releasing as much water as necessary to not exceed the maximum reservoir volume (Figure 8c,d), but no more than necessary.

At time step 13, the maximum volume is reached for the first time, but not exceeded. With each repetition, the model looks further ahead and adapts the release scheme to the realized volume and updated inflow forecast (Figure 8b). Figure 8b,d show the first optimized reservoir release and the corresponding volume over time as the resulting operational scheme. A different forecast length produces a different result—the longer the forecast horizon, the earlier the model can adapt to future events (Becker et al., 2014; Becker, Kim, et al., 2023; Talsma et al., 2013).

## 7.4 | Hydropower optimization with a linearization approach and continuation method

It is common to operate reservoirs for power generation. Hydropower generation $P$ is a function of the turbine flow and the head difference:

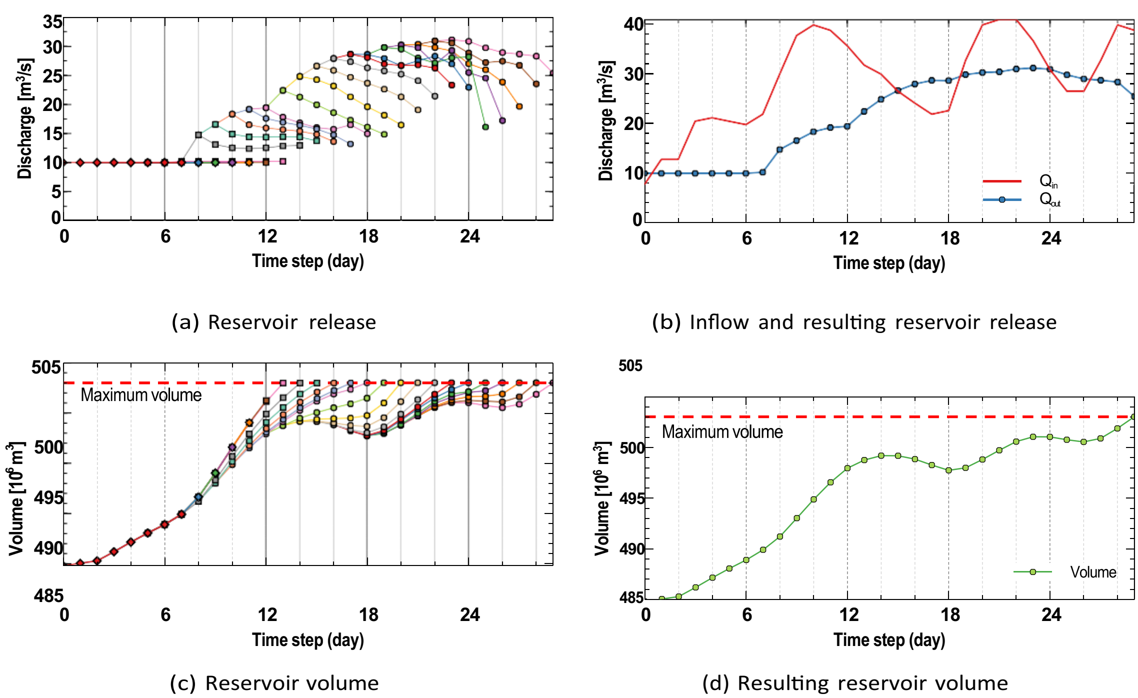



**FIGURE 8** Consecutive optimization results in a moving horizon setting (Becker, Kim, et al., 2023).

$$P = \Delta h \cdot Q_{\text{turbine}} \cdot \eta \cdot \rho \cdot g \tag{11}$$

where $\eta$ is the turbine efficiency, $\rho$ is the density of water, and g is the gravity constant. The head difference $\Delta h$ is defined as:

$$\Delta h = H - h_{\text{tailwater}} \tag{12}$$

where $H$ is the headwater elevation and $h_{\text{tailwater}}$ is the tailwater elevation. If the headwater is a reservoir lake, usually a rating function that relates the volume of water in the reservoir to the reservoir water level $H$ is used. This function takes into account the shape of the basin:

$$H = f(V). \tag{13}$$

The tailwater level $h_{\text{tailwater}}$ is expressed as a function of the reservoir outflow $Q_{\text{out}}$:

$$h_{\text{tailwater}} = f(Q_{\text{out}}). \tag{14}$$

Clearly, the hydropower equations make the reservoir optimization problem nonlinear due to:

- the relationship between water level and reservoir volume (Equation 13).
- the tailwater equation (Equation 14).
- the hydropower equation (Equation 11) with the product of two optimization variables $Q_{\text{turbine}}$ and $\Delta h$.
- the turbine efficiency $\eta$, which is a function of $Q_{\text{turbine}}$ and $\Delta h$.

In particular, if the power production function (Equation 11) is defined as an equality constraint, the optimization problem becomes non-convex and thus hard to solve (see Section 2.2.1). This optimization problem, which considers the dynamics of the head difference and the nonlinear relationship between reservoir volume and reservoir water level, can be solved using the *continuation method* (Section 3.1.4). However, the solution to this problem is not necessarily a global optimum due to the non-convexity of the problem.

*Piecewise linearization* of the power equation is another approach to approximating nonlinear equations in a linear optimization problem. When capturing the non-convexities of an optimization problem, a piecewise linear approximation usually involves binary variables and a MIP formulation (Section 2.3.3), which may introduce undesired jumps in the optimization result, but captures the dynamic behavior of the head difference to some extent (Becker, Ochterbeck, et al., 2023).

When the problem objectives incentivize the power production (for instance, maximixing the power generation or the revenue generated from it), the power production function (Equation 11) can be approximated by *hyperplanes* using linear inequality constraints. This approach makes the optimization problem linear and thus easy to solve (Rodríguez et al., 2018, 2021).

A simpler approach to linearize the problem is to assume a *constant head difference* $\Delta h$ (Equation 12) and to approximate the water volume-level relationship (Equation 13) with linear equations. This simplified problem can be solved efficiently as a linear program (Section 2.3.1), but it neglects the dynamics of the head difference and the nonlinear relationships of the problem: power generation now depends only on discharge with no incentive to keep the head water level high. When the results derived from this linear model are recalculated using the original non-linear equations, discrepancies are inevitable.

To illustrate the effect of model simplifications, in Figures 9 and 10 we compare the results between two approaches: a detailed *nonlinear model* solved with the continuation method, and a simplified *constant head difference* model solved as a linear program. For this example, we consider a cascade reservoir with three generating plants, where a short-term generation schedule needs to be determined to meet a power generation target. In Figure 9 results suggest using one plant for base load, that is, generation is constant over time, while the other two plants reflect the variations of the generation target in the schedule. Since the linear model uses a constant head difference, the actual power generation (Equations 11–14) corresponding to its optimized discharge and the resulting head difference reveals the consequence of this simplification: the power generation is below the generation target (Figure 9).

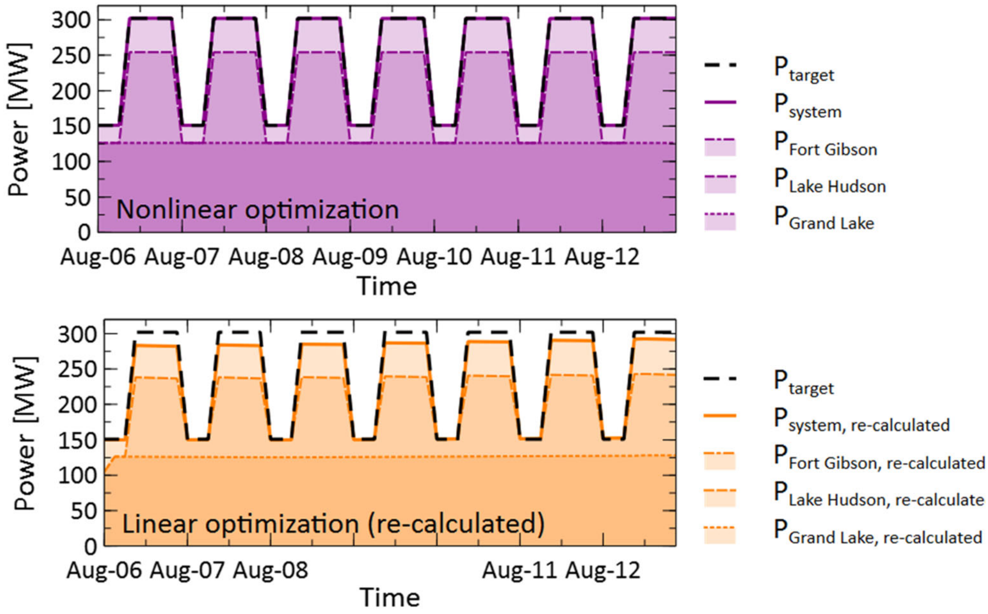



**FIGURE 9** System-wide power generation, power generation target, and power generation of plants in the reservoir cascade from a nonlinear and a linear optimization (modified from Becker, Ochterbeck, et al., 2023).

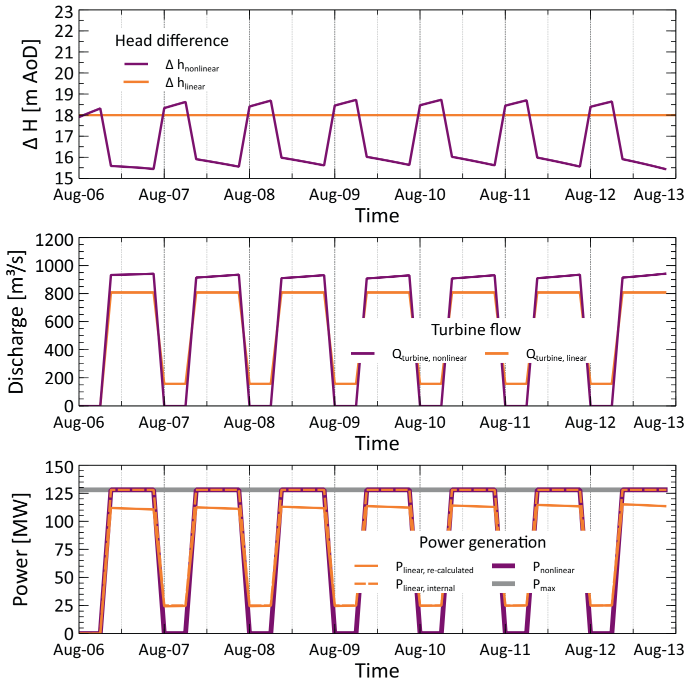



**FIGURE 10** Head difference, turbine flow, power generation of one plant, and the generator limit from a nonlinear and a linear optimization (modified from Becker, Ochterbeck, et al., 2023).

Figure 10 shows the head difference, turbine flow, and power generation of one of the plants that are not generating base load energy. The nonlinear optimization model compensates for the decreasing head during heavy load hours by increasing the turbine flow to generate power at the generator limit during heavy load hours. In contrast, the simplified linear model would lead to operating the generators below their capacity when the power generation corresponding to its advised discharge is re-computed using the nonlinear equations.

Thus, if the water system contains nonlinear equations, a trade-off must be made: linear equations can be solved to global optimality. This means that the solution is stable and consistent, which is relevant for an operational context (see Section 3.2). A solution of a nonlinear and non-convex optimization problem with the continuation method allows incorporating the desired equations, but does not necessarily provide a global optimum. The solution is still path-stable, though (see Section 3.1.4). Another drawback of the continuation method is that discrete logic can not be incorporated. Finally, linear optimization models are easier to solve and thus run faster; computational speed is also important for the operational application of optimization models.

## 7.5 | A mixed-integer optimization model for pumped storage hydropower scheduling

Pumped storage hydropower (PSH) is used in hydroelectric power generation to help balance the power grid and aid in generating more power when prices are low or demand is high. PSH plants can have reversible pump turbines. At a single point in time, the pump-turbine unit is either off, acting as a pump (moving water into a storage reservoir), or acting as a turbine (generating power while releasing water from storage) (Ma et al. 2022). To ensure that a pump-turbine unit does not pump and generate at the same time in an optimization model, discrete logic must be introduced. Thus, models that optimize the operations of PSH are often mixed-integer problems (Daadaa et al., 2021; Garcia-Gonzalez et al., 2008).

So-called Big-M constraints can be used to implement the logic so that the PSH units operate in discrete settings (Wen et al., 2019; Zhu et al., 2021). Figure 11 shows the result of a MIP optimization model for a simple PSH (Mitchell & Becker, 2023). The model schematization contains an upper and a lower reservoir with a spillway, and one reversible unit. The lower reservoir has an inflow boundary condition, and electricity prices vary throughout the day. Goal programming is applied with the following objectives, in order of priority:

1. Meet power generation target.
2. Minimize spill flow.
3. Maximize system revenue.
4. Minimize use of PSH unit.

The operating ranges of the storage and lower reservoir are imposed as inequality constraints. The third priority objective, maximizing system revenue, implies that pumping should occur when costs are low, as shown in the figure.

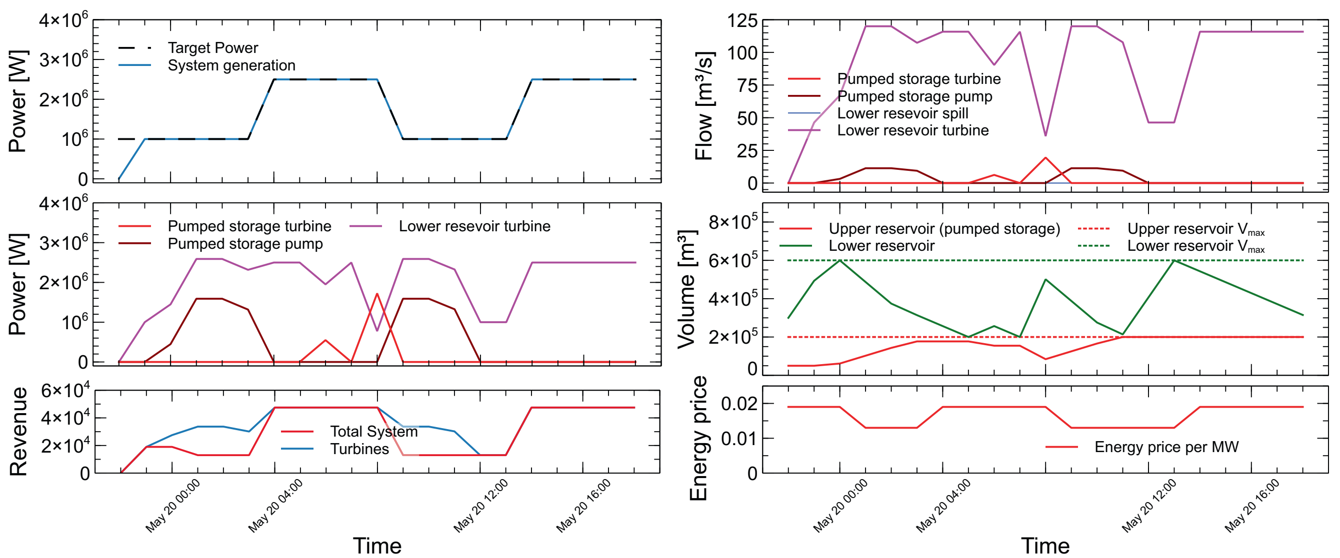



**FIGURE 11** Result of optimization for load balance in a PSH system. PSH units are reversible, so they cannot pump and generate simultaneously, and pumping occurs when prices are low. (This example is based on Deltares, 2024a).

The last priority indicates that there is a preference to generate using the turbine of the lower reservoir over the PSH unit, except for when capacity is needed in the upper (storage) reservoir.

## 7.6 | An optimization model for pump optimization in a decision support system for a Dutch polder system

For polder systems, the primary objective is to maintain water levels within safe limits, with excess water conducted to the sea using pumps and sluices. Other objectives to be considered are water quality (ensuring a minimum discharge in the canals for flushing to uphold water quality standards), ecology (avoiding pumping during fish migration periods, minimum discharge for fish ladders), and economy (preferring sluices to pumps for drainage and pumping during low tide). Operational decisions are driven by rainfall, sea level, and wind speed.

Figure 12 shows a decision support system (DSS) for a Dutch polder system. The DSS helps operators in their day-to-day operations by collecting and processing data, running models, processing model results, presenting results to users, reporting, archiving data, preparing warnings for dissemination (flood, drought), and supporting sending operational decisions to Supervisory Control and Data Acquisition (SCADA) systems, automatically or manually.

Figure 13 depicts diagrams in the DSS showing the optimization results. Under given precipitation conditions and sea level, the optimization model produces an optimal schedule where water is discharged with gravity flow through sluices if possible. This is, however, only possible during three time windows when the sea level is below the inland water level. To keep the inland water levels within the operational range, pumps support the drainage. This comes with energy costs and is most efficient at low tide. To account for the change in pump efficiency as a function of head and discharge, pump efficiency curves have been transformed into hyperplanes. This allows us to formulate a pump cost minimization objective that can be solved using Mixed ILP (Section 2.3.3).

Operators use the results of the optimization model as a guide. In extreme situations (flood and drought management), operators compare multiple runs with different settings before deciding on their preferred control action. Operators may also choose to apply the results of the optimization model to a simulation companion model (Section 5.4), which represents the flow in the water system with higher accuracy (Saint-Venant equations) and higher spatial resolution. This allows the operators to verify the solution or see where the schedule of pump and sluice operations needs fine-tuning.

The final schedule is transferred to the local SCADA of each individual structure. Deviations between the optimal scheme and the actual operation are taken into account by updating the initial state for the next optimization run with measurement data (MPC, Section 5.1). The DSS automatically collects the corresponding observations and processes them to model input data.

The DSS is not limited to control actions: flood and drought management includes issuing early warnings and making decisions to activate disaster response measures at different levels, ranging from following emergency protocols to evacuating neighborhoods.

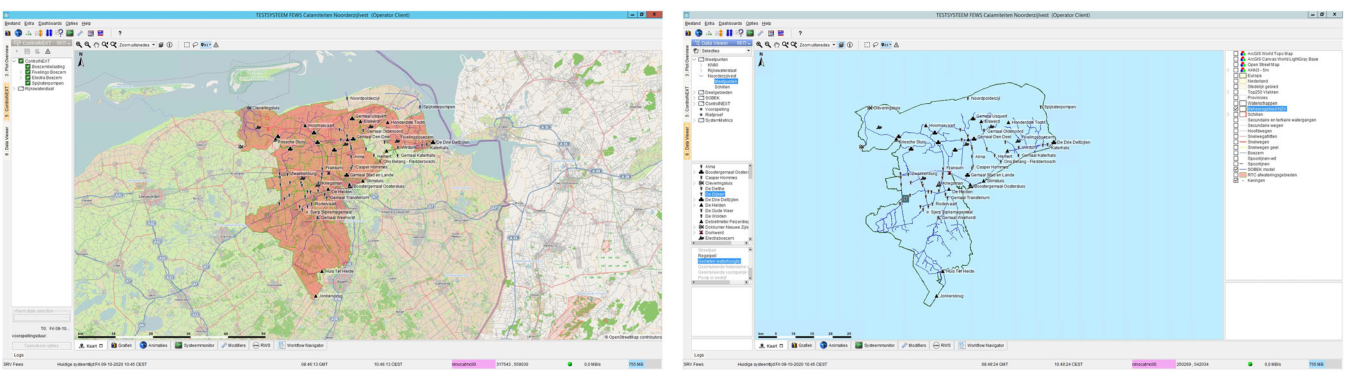


FIGURE 12 Main screen of a DSS for a Dutch polder system configured in Delft-FEWS (Werner et al., 2013) with a map view showing the canal system and pumping stations. On the left with a map background, on the right only the water system (Becker et al., 2021).

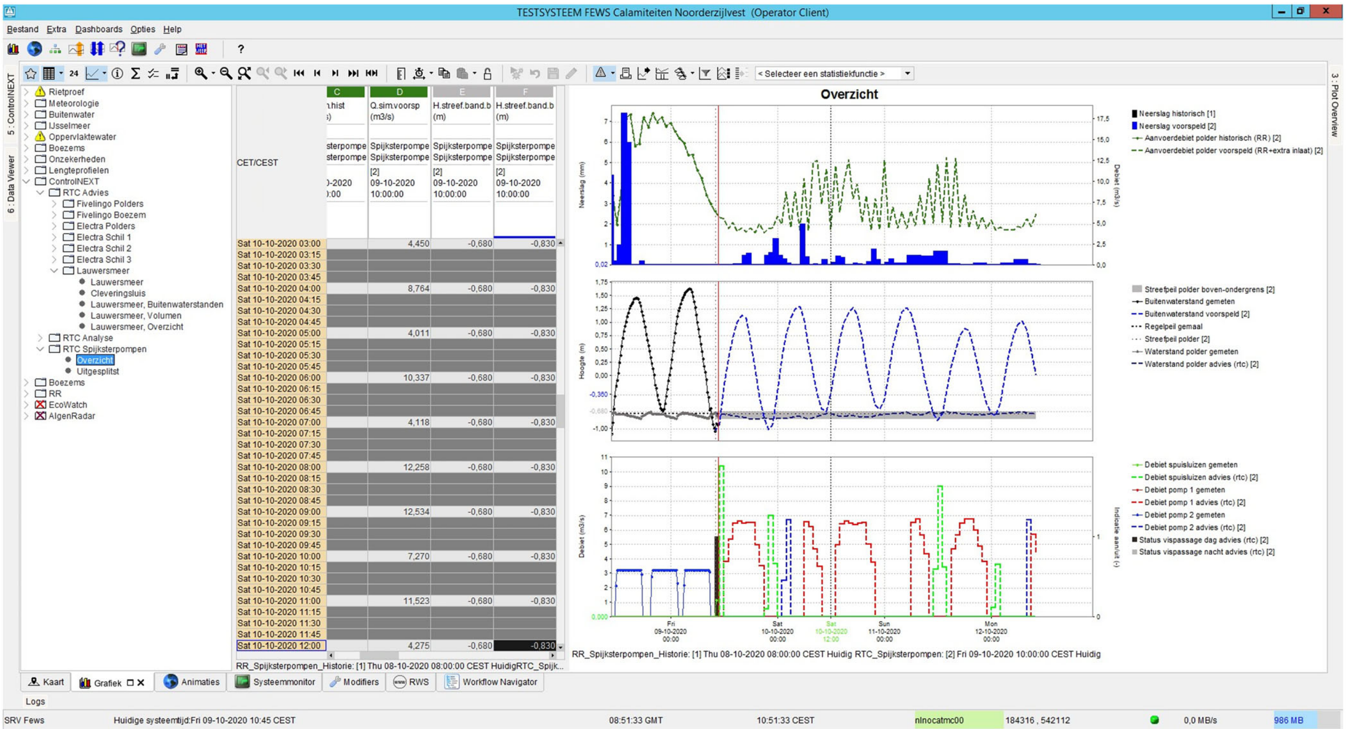



**FIGURE 13** Display of optimization results in a DSS for a Dutch polder system (Becker et al., 2021). Top diagram: precipitation (blue bars) and inflow (dashed green line) to the polder system; middle diagram: inland water level (dashed black line), inland water level operational range (gray) and sea level (dashed blue line); bottom diagram: discharge through pumps (blue and red) and sluices (green line).

## 8 | OUTLOOK: PUTTING OPTIMIZATION INTO PRACTICE

The added value of mathematical optimization in water management compared to conventional operations has been demonstrated in various studies (Castelletti et al., 2023). In particular, reservoir management has been studied intensively (Labadie, 2004). However, many of the studies in the literature are limited to demonstrating the *potential* of optimization techniques; putting optimization techniques into practice remains a challenge. The majority of hydraulic infrastructure is still operated in a conventional manner, that is, without optimization techniques and, in many cases, without modeling support.

Climate change, population growth, and increased economic activity increase the pressure on water systems. This means that in the future they must be operated in a better way than they are today. Against that background, we expect mathematical optimization to become more important.

Recent developments are promising: optimization techniques have found their way into off-the-shelf software solutions for operational management, making it easier to develop robust optimization models and understand the results. The number of use cases where optimization models support operators in their day-to-day duties is growing.

To establish optimization techniques in practice, optimization models must produce results that are accepted by operators. This means that the optimization model must represent the relevant processes with sufficient accuracy. Given the mathematical restrictions (Section 3), this is in particular challenging when nonlinear equations or discrete logic must be incorporated in the optimization model. Development should focus on making existing techniques like the piecewise-linear approximation easier to apply and on the development of new methods to linearize nonlinear equations or tackle nonlinear optimization.

The optimization model must produce an operational scheme that can be applied in practice. For example, hydropower companies are unlikely to approve a turbine dispatch plan that exhibits "spiky" behavior (abrupt and frequent changes in turbine flow). Similarly, operators would avoid schedules with excessive cycling (on/off switching) of turbines or pumps. Secondary operational preferences to account for wear and tear of machinery, preferred operating hours, or system states to be avoided are typically captured in optimization models by goals and constraints. These concepts need to be further developed to be accepted by practitioners.

Furthermore, the optimization model must be reliable under a variety of conditions, including extreme situations. During a flood, water management decisions are intertwined with emergency response decisions, such as preparing for evacuation. Droughts require balancing multiple water uses with different beneficiaries and priorities. In winter conditions, machinery must be protected from ice, while ice layers on canals require a stable water level to prevent unprotected banks from being damaged by breaking ice. In the Netherlands, where ice skating is very popular, operators are encouraged to operate in a way that supports the formation of a thick ice layer on the canals.

Fast optimization techniques and fast solvers are important prerequisites for using optimization models for operational decision support. Commercial solvers are often very expensive, and licensing costs are only affordable in exceptional cases. Open-source solvers are an interesting alternative, and promising open-source solvers for optimization problems have recently been released (HiGHS, 2024; Uno, 2024).

For those water systems that are already operated with model-based DSSs it should be considered to move toward automated control. Automated control is already common for individual hydraulic structures, but on the system level automated control is not yet very common in operational water management. Now that many studies have shown the potential of system-wide model-based control, emphasis should be placed on linking MPC to the hardware, that is, the actuators of the hydraulic structures.

The use of forecast ensembles is common practice in flood forecasting to represent the uncertainty of hydrological forecasts (Cloke & Pappenberger, 2009). Although the forecast ensembles of weather services can be translated into a hydrological forecast ensemble and the potential of ensemble forecast optimization has been shown in several studies (Becker, Kim, et al., 2023; Fan et al., 2016; Horváth, Smoorenburg, et al., 2022; Horváth, van Esch, et al., 2022; Naumann et al., 2015; Raso et al., 2014; Schwanenberg et al., 2014; Uysal, Alvarado Montero, et al., 2018), ensemble optimization is not yet common practice. The challenge here is to provide insight into the methods and show the added value for day-to-day operational support.

Mathematical optimization needs a place in the academic education of those who build the models and those who apply them. This includes understanding the practical needs of operators and how to translate them into optimization models for operations. It is also important that operators understand the mathematical limitations of the optimization models they use.

## AUTHOR CONTRIBUTIONS

**Bernhard Peter Josef Becker:** Writing – original draft (lead); writing – review and editing (lead). **Caroline Jeanne Jagtenberg:** Writing – original draft (lead); writing – review and editing (lead). **Klaudia Horváth:** Writing – original draft (equal); writing – review and editing (equal). **Ailbhe Mitchell:** Writing – original draft (equal); writing – review and editing (equal). **Jesús Andrés Rodríguez-Sarasty:** Writing – original draft (equal); writing – review and editing (equal).

## CONFLICT OF INTEREST STATEMENT

The authors declare no conflict of interest.

## DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

## RELATED WIREs ARTICLES

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**How to cite this article:** Becker, B. P. J., Jagtenberg, C. J., Horváth, K., Mitchell, A., & Rodríguez-Sarasty, J. A. (2024). Optimization methods in water system operation. *WIREs Water*, e1756. https://doi.org/10.1002/wat2.1756