Teoria dos Grafos e Computabilidade

— Overview —

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Objetivos

Levar o aluno a compreender os principais conceitos relacionados ao projeto e análise de algoritmos; auxiliar o aluno no desenvolvimento das habilidades de desenvolver soluções computacionais para problemas por meio da modelagem usando estratégias de projeto de algoritmo; fornecer subsídios para que os alunos aperfeiçoem suas habilidades de desenvolvimento de sistemas, levando-os a reconhecer a importância da abstração e da redução de problemas

Algorithm Design

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Ementa

Notações para complexidade de algoritmos. Crescimento assintótico de funções e classes de complexidade. Limite inferior para classes de problemas. Análise de algoritmos recursivos. Técnicas de Projeto de Algoritmos: redução, divisão e conquista, programação dinâmica, método guloso, retrocesso e branch and bound. Tratabilidade de problemas. Teoria da Complexidade: classes de problemas P, NP e NP-Completo. Teorema de Cook.

Pedagogical Strategy

- Syncronous lectures
- Slides
- Forum
- Exercises
- Some tools for interactions (like Kahoot)

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Evaluation

- Exams
- Homeworks

Expected knowledge (a priori)

- Data structure
- Graph
- Discrete maths

Sorting

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SOLUTION A permutation (reordering) of the input sequence such $x_{i_1} \leq x_{i_2} \leq \cdots x_{i_n}$

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It's expected to develop methods which are CORRECT and EFFICIENT.

Robot tour optimization

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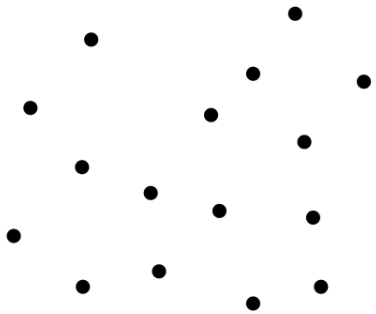

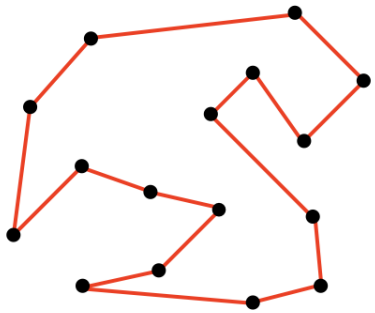

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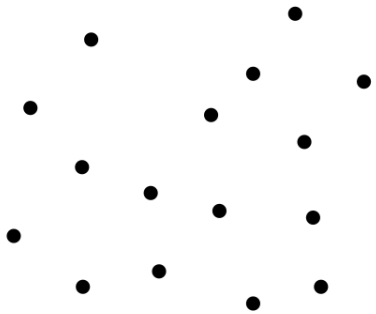

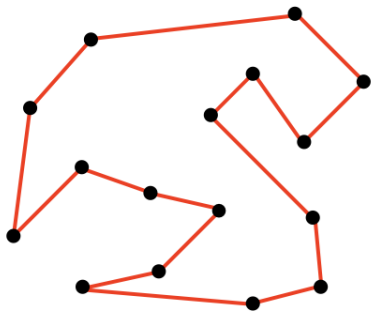

You are given the job to program the robot arm. **Develop algorithms** to find the most efficient tour!

Robot Tour Optimization

Nearest Neighbor Tour *A popular solution starts at some point p_0 and then walks to its nearest neighbor p_1 first, then repeats from p_1, etc. until done.*

Closest Pair Tour *Another idea is to repeatedly connect the closest pair of points whose connection will not cause a cycle or a three-way branch, until all points are in one tour.*

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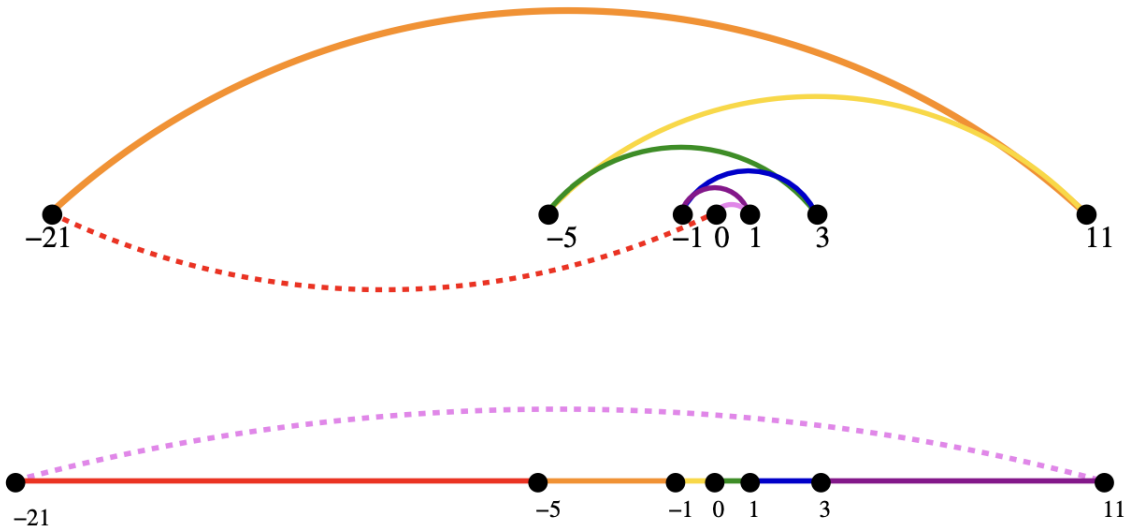

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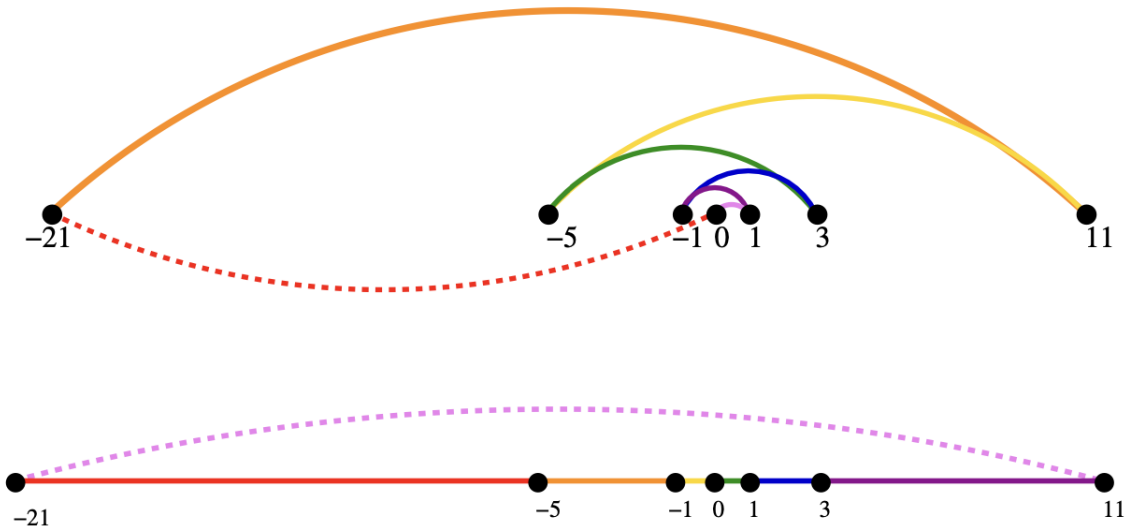

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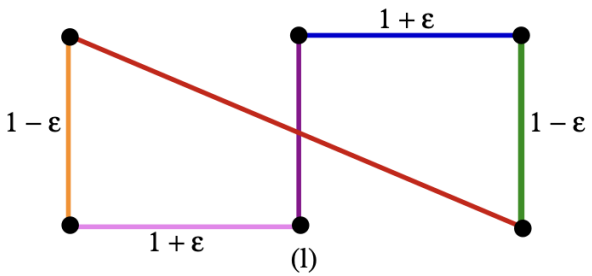

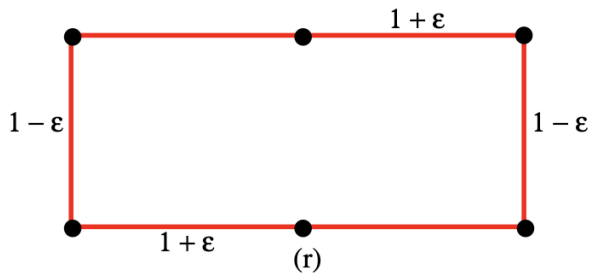

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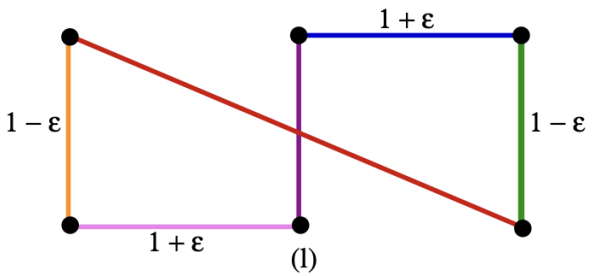

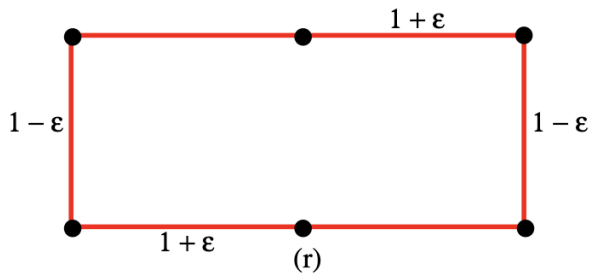

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A Correct Algorithm: Exhaustive Search

*We could try **all possible orderings** of the points, then select the one which minimizes the total length. Since all possible orderings are considered, we are **guaranteed** to end up with the shortest possible tour.*

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9	5	7	1	3	8	2	6

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Solution 2

1. The list is organized as an array $[0, n-1]$
2. Sort L
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How to solve the general problem decreasing the number of instructions?

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