Circuit Theory and Electronics Fundamentals

T2

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Contents

1 Introduction

The goal of this laboratory assignment is to study and analyse a RC circuit composed by a dependent current source, a capacitor, resistors, and lastly, one dependent and one independent voltage source to find the natural and forced response and frequency analysis of the said circuit. A comparison will be done between the NgSpice simulation and the theoretical analysis of the circuit.

The main objective will be to further learn about both methods of analysis, learning about their similarities, differences and which positive and negative sides each of them have.

The circuit is represented with resort to *LibreOffice Draw* and can be viewed in figure 1.

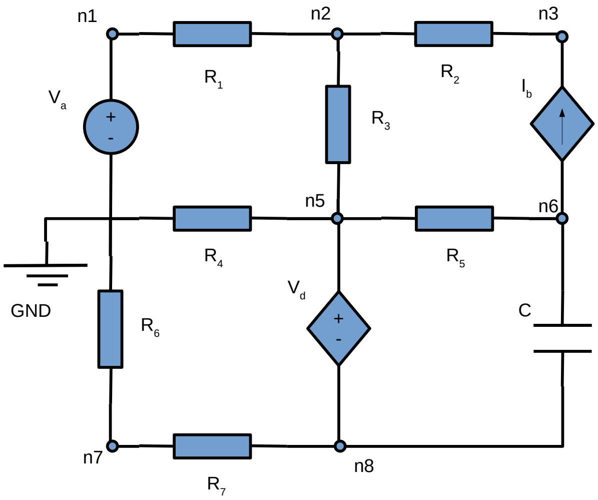

Figure 1: Studied Circuit

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically. The known values in $kOhm$, V, mS, μF can be checked in the table below.

R_1	1.02362892933
R_2	2.08640382129
R_3	3.09996108706
R_4	4.08183334334
R_5	3.04169575790
R_6	2.04156679366
R_7	1.04156790057
C	1.04908336809
K_b	7.29571922963
K_d	8.16840649221

Table 1: Known Data

2.1 For $t < 0$

For $t < 0$, $v_s = V_s$, according to the given equation. Futhermore the capacitor acts like an open circuit since it is fully charged and no current goes through it. Therefore, $I_c = 0$.

Applying the node method, it is possible to solve the circuit with the information upwards, since all the other information is given in 1. The system used to solve this circuit is shown below.

$$\begin{cases} V_1 = V_s \\ V_d = V_5 - V_8 \\ V_b = V_2 - V_5 \\ \frac{V_7 K_d}{R_6} = V_d \\ \frac{V_1 - V_2}{R_1} + \frac{V_3 - V_2}{R_2} - \frac{V_5 - V_2}{R_3} = 0 \\ \frac{V_2 - V_3}{R_2} - K_b V_b = 0 \\ \frac{-V_7}{R_6} + -\frac{V_8 - V_7}{R_7} = 0 \\ \frac{V_7 - V_8}{R_7} - \frac{V_5}{R_4} + \frac{V_6 - V_5}{R_5} + \frac{V_2 - V_5}{R_3} = 0 \\ \frac{V_5 - V_6}{R_5} + K_b V_b = 0 \end{cases}$$

Using Octave software it was possible to collect the data shown in table 2 and 3.

V_1	5.178860
V_2	4.933049e+00
V_3	4.408850e+00
V_5	4.967487e+00
V_6	5.731699e+00
V_7	-1.994280e+00
V_8	-3.011723e+00
V_b	3.443737e-02
V_d	-7.979210e+00
V_s	5.178860e+00

Table 2: Voltages in all nodes in V

I_1	-0.240136
I_2	2.512454e-01
I_3	1.110897e-02
I_4	1.216974e+00
I_5	2.512454e-01
I_6	-9.768380e-01
I_7	-9.768380e-01
I_b	2.512454e-01
I_d	-9.768380e-01
I_s	-2.401364e-01

Table 3: Currents in all branches in mA

The voltages and currents were then calculated.

2.2 For $t = 0$

In order to solve the second exercise, firstly it is necessary to replace the capacitor by a voltage source Vx, which will impose a voltage of exactly what was determined for the capacitor in the first exercise. This is the first step to compute de Req as seen from the capacitor because now it is possible to run a new nodal analysis with vs set to 0 in order to determine I_x, which is the current that goes through the capacitor.

Solving the matrix below it is possible to determine the values in 2.2 and thus determine R_{eq}.

Compute R_{eq} is fundamental because this allows to determine the natural solution of the system through the RC constant. This application will be demenstrated in the next exercise.

$$\begin{cases} V_1 = V_s \\ V_d = V_5 - V_8 \\ V_b = V_2 - V_5 \\ V_x = V_6 - V_8 \\ \frac{V_7 K_d}{R_6} = V_d \\ \frac{V_1 - V_2}{R_1} + \frac{V_3 - V_2}{R_2} - \frac{V_5 - V_2}{R_3} = 0 \\ \frac{V_2 - V_3}{R_2} - K_b V_b = 0 \\ \frac{-V_7}{R_6} - \frac{V_8 - V_7}{R_7} = 0 \\ \frac{V_7 - V_8}{R_7} - \frac{V_5}{R_4} + \frac{V_6 - V_5}{R_5} + \frac{V_2 - V_5}{R_3} - \frac{V_x}{R_{eq}} = 0 \\ \frac{V_5 - V_6}{R_5} + K_b V_b + \frac{V_x}{R_{eq}} = 0 \end{cases}$$

Through Octave it was possible to collect the data shown in table 4 and 5.

V_1	0.000000
V_2	-6.049490e-16
V_3	-1.440730e-15
V_5	-1.195182e-15
V_6	8.743422e+00
V_7	1.804316e-15
V_8	2.724843e-15
V_b	5.490679e-17
V_d	3.920025e-15
V_s	0.000000e+00
V_x	8.743422e+00

Table 4: Voltages in all nodes in V

I_1	-0.000000
I_2	7.525891e-18
I_3	-1.904001e-16
I_4	-2.928052e-16
I_5	2.874522e+00
I_6	4.799008e-16
I_7	4.799008e-16
I_b	4.005845e-16
I_d	4.799008e-16
I_s	-5.909846e-16
I_x	2.874522e+00

Table 5: Currents in all branches in mA

The value for V_x was estabilished as $V_6 - V_8$ being V_6 and V_8 determined in the previous section.

2.3 Natural Solution

Now the task was to determine the natural solution for V_6. Using V_x, determined in the previous exercise, as the initial condition, it was possible, applying 1, since this is a RC circuit, to determine the natural solution, plotted in figure 2.

$$V_{6n} = Ae^{\frac{-t}{\tau}} \qquad \tau = R_{eq}C \tag{1}$$

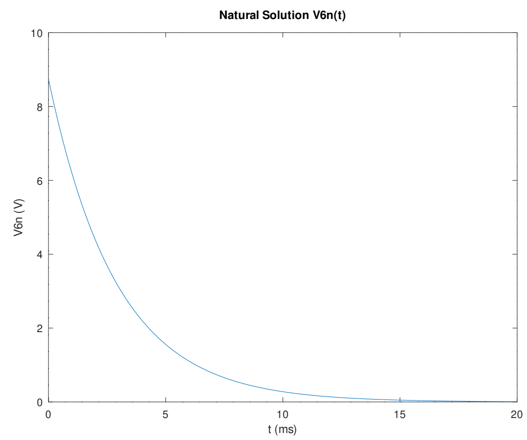

Figure 2: Natural Solution

2.4 Forced Solution

For this section of the analysis, one was asked to find the forced solution $V_6f(t)$ for a frequency of 1000Hz. In order to simplify the calculations, and as suggested, a phasor with constant V_s of 1V was used. A similar node analysis was ran, using V_s as the voltage source, and replacing the capacitance of capacitor C with its impendance Z. Like so, the following nodal equations were derived:

$$Z_c = \frac{1}{j\omega C} \qquad \omega = 2\pi f \tag{2}$$

$$\begin{cases} V_1 = V_s \\ V_d = V_5 - V_8 \\ V_b = V_2 - V_5 \\ \frac{V_7 K_d}{R_6} = V_d \\ \frac{V_1-V_2}{R_1} + \frac{V_3-V_2}{R_2} - \frac{V_5-V_2}{R_3} = 0 \\ \frac{V_2-V_3}{R_2} - K_b V_b = 0 \\ \frac{-V_7}{R_6} - \frac{V_8-V_7}{R_7} = 0 \\ \frac{V_7-V_8}{R_7} - \frac{V_5}{R_4} + \frac{V_6-V_5}{R_5} + \frac{V_2-V_5}{R_3} - \frac{V_6-V_8}{Z_c} = 0 \\ \frac{V_5-V_6}{R_5} + K_b V_b + \frac{V_8-V_6}{Z_c} = 0 \end{cases}$$

Spliting the complex numbers and determining the angles and the modules, the two following rables were made with the following equations:

$$V_{complex_i} = V_i e^{-jphase(i)} \tag{3}$$

With angle and abs commands in octave we were able to calculate the following values.

V_1	1.000000e+00
V_2	9.531327e-01
V_3	8.616507e-01
V_5	9.471228e-01
V_6	5.742283e-01
V_7	3.802382e-01
V_8	5.742283e-01

Table 6: Complex Amplitudes

Ph_1	0.000000e+00
Ph_2	-3.749681e-21
Ph_3	-1.224400e-20
Ph_5	-4.257358e-21
Ph_6	3.141434e+00
Ph_7	3.141593e+00
Ph_8	3.141593e+00

Table 7: Phases

Therefore, we can state that:

$$V_6 = 5.742283e - 01e^{-j3.141434e+00} \tag{4}$$

2.5 Natural and Forced Superimposed

By converting the phasor to real time functions, a function to evaluate V_6, for $t > 0$, was found by simply adding the natural and forced solutions, previously calculated for a frequency of 1000Hz. On the other hand, the equation for $V_s(t)$ for $t > 0$ was already implicit on the initial circuit diagram. Both equations are written below.

$$V_{ifinal} = V_{in} + V_{if} \tag{5}$$

So, calculating V_{6final}:

$$V_6(t) = e^{\frac{-t}{R_{eq}C}} + Ae^{-jphase(i)} \tag{6}$$

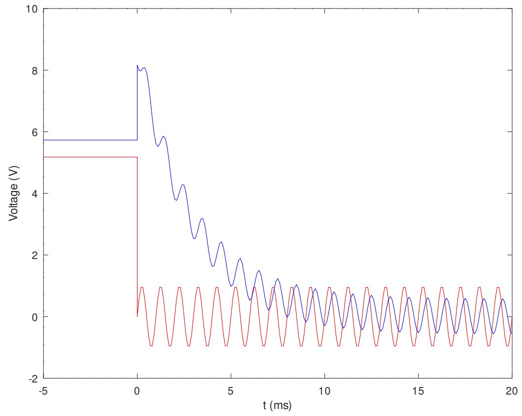

Figure 3: Natural and Forced Superimposed

2.6 Frequency Response

The graphs for phase and magnitude were plotted. The magnitude in dB is calculated with the help of the function abs and in a logarthmic scale, multiplied by the 20. The phase in degrees is calculated with the function angle and there has to be a conversion from rad to degrees.

$$Z = \frac{1}{j2\pi fC} \tag{7}$$

with f being a logharitmic scale vector from -6 to 6 with 200 entries.

The system used was:

$$\begin{cases} V_1 = V_s \\ V_d = V_5 - V_8 \\ V_b = V_2 - V_5 \\ \frac{V_7 K_d}{R_6} = V_d \\ \frac{V_1 - V_2}{R_1} + \frac{V_3 - V_2}{R_2} - \frac{V_5 - V_2}{R_3} = 0 \\ \frac{V_2 - V_3}{R_2} - K_b V_b = 0 \\ \frac{-V_7}{R_6} - \frac{V_8 - V_7}{R_7} = 0 \\ \frac{V_7 - V_8}{R_7} - \frac{V_5}{R_4} + \frac{V_6 - V_5}{R_5} + \frac{V_2 - V_5}{R_3} - \frac{V_6 - V_8}{Z} = 0 \\ \frac{V_5 - V_6}{R_5} + K_b V_b + \frac{V_8 - V_6}{Z} = 0 \end{cases}$$

The complex variables $Vsfre(k)$, $Vxfre(k)$ and $V6fre(k)$ were assigned and equalled to the just calculated V_1, $V_6 - V_8$ and V_6, respectively.

The two following graphics were plotted using a base 10 logarithmic scale for frequencies, the logarithmic value of the abs of the variables stated above and the angle of these complex variables, converted to degrees.

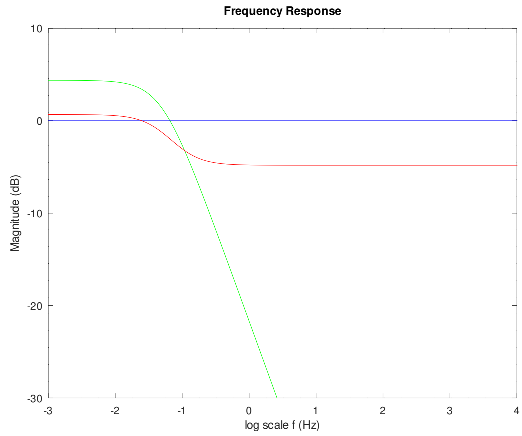

Figure 4: Magnitude (dB) / Frequency (Hz)

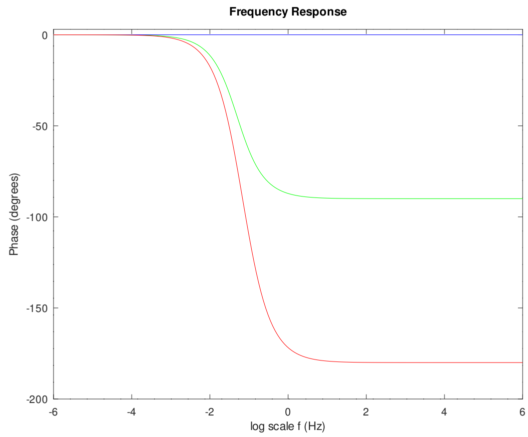

Figure 5: Phase (degrees) / Frequency (Hz)

3 Simulation Analysis

3.1 For $t < 0$

The first simulation using ngspice, was an equivalent simulation to the circuit solved theoretical in the first exercise. Notice that an extra voltage source was introduce with only the purpose to server as an ameter, controlling the current for the dependent voltage source. The results are shown in the table 8 that will serve to compare with the results obtained.

Name	Value [A or V]
@c1[i]	0.000000e+00
@g1[i]	-2.51245e-04
@r1[i]	-2.40136e-04
@r2[i]	-2.51245e-04
@r3[i]	-1.11090e-05
@r4[i]	1.216975e-03
@r5[i]	2.512453e-04
@r6[i]	-9.76838e-04
@r7[i]	-9.76838e-04
n1	5.178860e+00
n2	4.933049e+00
n3	4.408850e+00
n5	4.967487e+00
n6	5.731699e+00
n7	-1.99428e+00
n8	-3.01172e+00
n9	0.000000e+00

Table 8: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.2 For $t = 0$

The second simulation is conducted with the goal to determine I_x. This is made by setting v_s to 0 and turning the capacitor into a voltage source V_x which imposes the same potencial between n6 and n8 as the capacitor and operating at $t = 0$.

The results below are the results obtained in the simulation. This simulation is not only needed as essencial because with V_x and I_x we will be able to determine the natural response of the sistem in the next steps.

Name	Value [A or V]
@g1[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	2.874522e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
n1	0.000000e+00
n2	0.000000e+00
n3	0.000000e+00
n5	0.000000e+00
n6	8.743422e+00
n7	0.000000e+00
n8	0.000000e+00
n9	0.000000e+00

Table 9: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.3 Natural Solution

In the third simulation we use the simulator to perform a transiet analysis. For this some initial conditions were given to the software which then realized the analysis. The graphics shown represent the transient analysis for the node n6 between [0,20]ms.

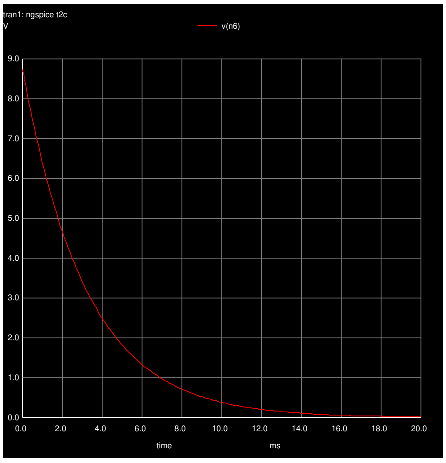

Figure 6: Natural Solution

3.4 Total Solution

The simulation conducted next tried to simulate the total response of the node n6. Imposing the conditions given in the exercise, a report showing the response and the stimulus (which was a analysis in n1) is represented in 7.

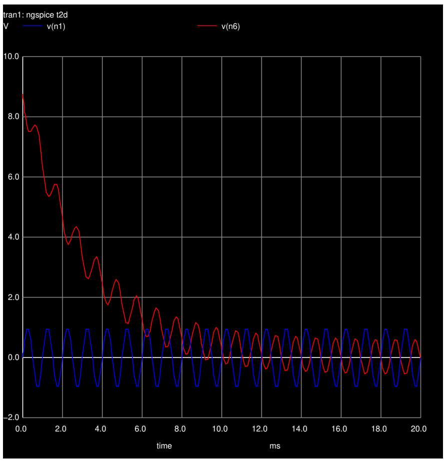

Figure 7: Total Solution

3.5 Frequency Response

At last, a frequency analysis was made in order to compare the frequency response between v_s and n6. The following graphics were obtained.

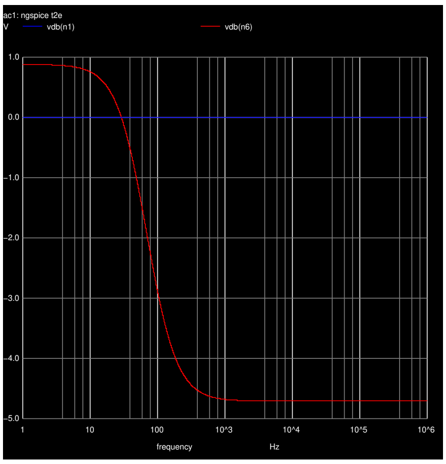

Figure 8: Magnitude

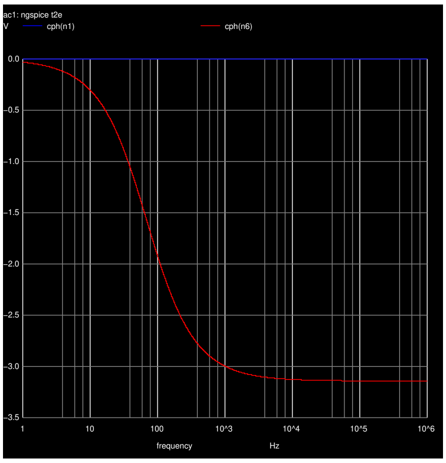

Figure 9: Phase

We noticed a significant difference bettween them . The frequency response in V_s is null in opposite to the one seen in n6. This is due to the fact that V_s changes ccording to the frequency, thus remaining constant. V_6 on the other hand changes its value according to V_s showing a frequency analysis that changes through time.

4 Conclusion

As for the first lab experiment, the values for current in branches, voltage in nodes and the equivalent resistance calculated through the simulation made with Ngspice and the theorical analysis with nodal method in Octave tools were in perfect harmony. The differences between the obtained values are negletible. This is due to the fact that the circuit is also linear, because the capacitor is a linear element.

We can agree that the assignment was completed with success.