

Operations research with Julia and JuMP

Pedro Belin Castellucci

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Mathematical programming

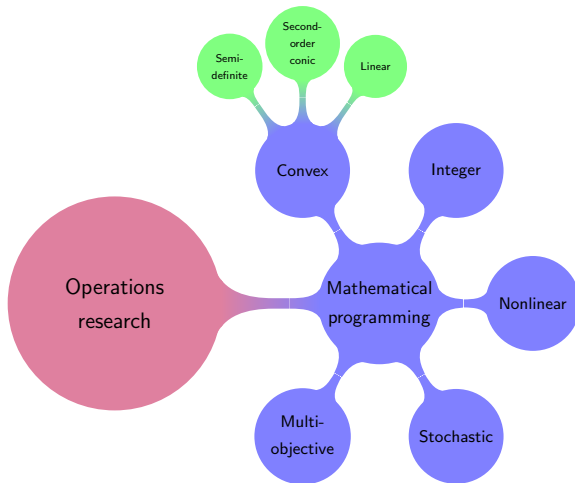


Figure 1: Information from en.wikipedia.org/wiki/Mathematical_optimization.

Which can JuMP handle?

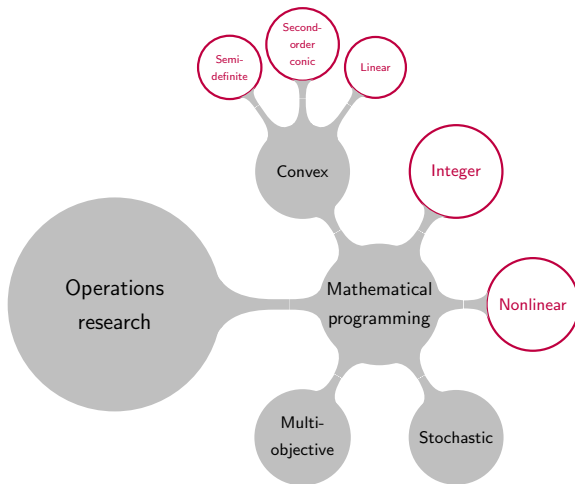


Figure 2: For JuMP documentation check www.juliaopt.org/JuMP.jl/0.15/.

Introduction to Julia programming

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(Did we mention it should be as fast as C?)"

(Jeff Bezanson, Stefan Karpinski, Viral Shah, Alan Edelman (2012))

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How fast is it?

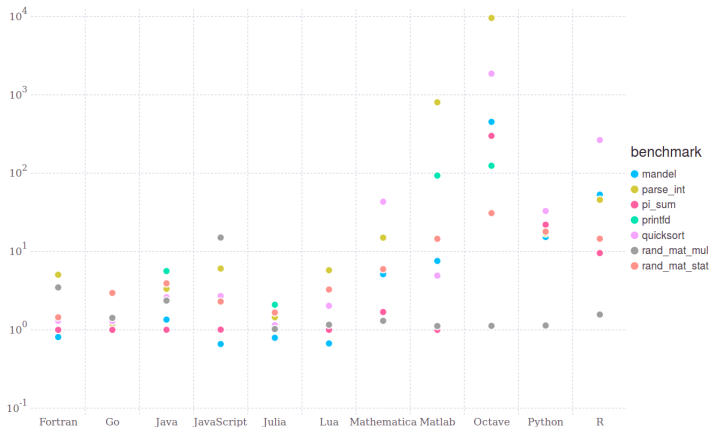


Figure 3: Speed relative to C smaller is better (C performance is 1.0). According to julialang.org/benchmarks/.

Who is using Julia?

Stanford University. Introduction to Multidisciplinary Design Optimization (Prof. Mykel J. Kochenderfer).

MIT. Integer Programming and Combinatorial Optimization (Prof. Juan Pablo Vielma).

MIT. Optimization Methods (Prof. Dimitris Bertsimas and Dr. Phebe Vayanos).

University at Buffalo. Linear Programming (Prof. Changhyun Kwon).

“Sapienza” University of Rome. Operations Research (Giampaolo Liuzzi).

University of South Florida. Nonlinear Optimization and Game Theory (Prof. Changhyun Kwon).

For more check <http://julialang.org/teaching/>.

What can we do with Julia?

Simple Audio IO in Julia	(AudioIO),
A neural network	(BackpropNeuralNet),
Support vector machines	(LIBSVM, LIBLINEAR),
Machine learning	(MachineLearning),
Bioinformatics and Computational Biology	(Bio),
Curve fitting	(CurveFit),
Describe and model financial markets	(FinancialMarkets),
Black-box optimization	(BlackBoxOptim),
Combinatorics	(Combinatorics),
Evolutionary and genetic algorithms	(Evolutionary),

Gurobi, GLPK, CPLEX, Cbc, Clp CoinOptServices, JuMP.

For more check: <http://pkg.julialang.org/>

What will we do?

Basic Julia programming.

Explore JuMP for mixed integer linear problems.

Lesson one

Arrays are indexed starting at one.

¹[www.cs.utexas.edu/users/EWD/transcriptions/EWD08xx/EWD831.](http://www.cs.utexas.edu/users/EWD/transcriptions/EWD08xx/EWD831.html)

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Lesson one

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Why numbering should start at zero?¹

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Why numbering should start at zero?¹ “I don’t know how many of you have ever met Dijkstra, but you probably know that arrogance in computer science is measured in nano-Dijkstras.” (Alan Kay)

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Installing Julia

On Linux/Ubuntu:

```
sudo add-apt-repository ppa:staticfloat/juliareleases
```

```
sudo apt-get update
```

```
sudo apt-get install julia
```

For other platforms check

julialang.org/downloads/platform.html.

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