Multi-criteria ordered clustering with stochastic parameters: Application to the Human Development Index

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Abstract

Development indices have been proposed as a mean to encourage nations to focus on people, economic and sustainability capabilities to develop a country. Usually, an index computation allows to build a ranking which in turns is used to segment the set of elements into homogeneous groups. Several studies have proposed non-compensatory multi-criteria approaches to compute these indices, mainly because it could lead to a better ranking and segmentation. Approaches are based on clustering techniques which intend to deal with a main difficulty: how to build a completely ordered partitioning of countries being ranked. However, there is not approaches dealing with uncertain parameters and imprecise data, a main issue in development indices analysis. In this paper, a non-compensatory ordered clustering approach is proposed which uses the rationale of PROMETHEE II multi-criteria method for ranking countries and a K-means-based algorithm to find clusters, which integrates robustness analysis by considering uncertain parameters. An application to the Human Development Index is performed. We show that this is a promising approach in contexts of stochastic information. Limitations are discussed and future research is proposed.

Keywords: HDI, clustering, K-means , outranking relation

1. Introduction

Development indices are broadly used in business, policy-making, research, development-political debates, allocation of development aid, international climate accord designs, and economics (Wolff et al., 2011). However, these have been object of main criticisms (Noorbakhsh, 1998; Berenger and Chouchane, 2007; Klugman et al., 2011; Martinez, 2013): (1) the

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compensatory effect; (2) the underlying concepts and available information to define and compute the dimensions used to build an index; (3) the categorization process used to segment objects of analysis. For instance, the Human Development Index, released by United Nations Development Programme (UNDP), evaluates 189 United Nations countries on the basis of three criteria: the life expectancy, the years of education and the Gross National Income (GNI). Its computation considers that a country exhibiting a bad performance in one criteria, can be compensated with a good performance in another. However, this is not desirable since, for instance, bad health indices can be hidden by good GNI, which can exacerbe inequality in access to health systems.

Non-compensatory methods have been proposed as an alternative to the development indices calculation methods. For instance, in the case of HDI, Natoli and Zuhair (2011) analyze three aggregation techniques designed for benchmarking and ranking of countries according to aggregated dimensions: additive methods, geometric aggregations, and non-compensatory methods. They focus on the Condorcet approach as a non-compensatory aggregation technique for progress measures. Lozano and Gutiérrez (2009) propose to use the minimum of the component indexes in HDI instead of the arithmetic average. Mazziotta and Pareto (2015) compare two non-compensatory composite indices for measuring multidimensional phenomena and monitoring their changes over time: a non-linear composite index and a two-parameter function, which is an intermediate case between a compensatory and a full non-compensatory index. The authors found that the non-linear composite seems to be less compensatory that the second one.

Usually, afterwards the computation of development indices, a ranking process is performed, followed by a segmentation step which is used to identify groups of interests. However, several scholars have proposed that ordered clustering strategies could be better adapted to that purpose. These are based on non-compensatory multi-criteria decision analysis (MCDA) approaches (De Smet, 2014). Chen et al. (2018) propose a PROMETHEE-based algorithm, which integrates the K-means rationale, to cluster countries in the HDI Report. They found that the ordered clustering is highly consistent with the HDI ranks. De Smet et al. (2012) propose an exact multi-criteria algorithm to find ordered categories, based on valued preference degrees. Boujelben and De Smet (2016) propose an approach that integrates PROMETHEE and K-means, a popular greedy algorithm for partitioning a set of elements into a pre-defined number of clusters so as to minimize the distances to the cluster centroids (the cluster centers). The underlying idea of their approach is based on the notion of belief distance between the alternatives and the centroids. Monteiro et al. (2018) propose a supervised classification approach based on the ELECTRE TRI method in which categories are defined *a priori* by using fixed category boundaries. Different from other proposals using the outranking-based models, these authors consider that there is no imprecise data in the information sources. Usually, these methods proceed in three step: 1) each pair of candidates is compared in terms of preferences; 2) clusters are built, partially based on the preferences information; 3) clusters are ordered by eliminating inconsistencies (Fernandez et al., 2010). The last step could be time consuming if the whole space of possibilities should be explored to detect inconsistencies. Leiva and Vidal (2013) proposed an interesting algorithm called Warped-K-means that could be used to solve this problem by first ranking the set of candidates and then applying an $O(n)$ process to find clusters. Although this method is not specific for multi-criteria problems, we propose an adaptation to be used in our approach.

Computation of development indices also reveals an issue related to the information available for evaluating some of the relevant dimensions. Several types of imperfect information have been highlighted by authors analyzing the quality of information used in the development indices calculation. Thus, the weighting process assigning equal importance levels to dimensions in HDI has been criticized and methods to deal with have been proposed (Zheng and Zheng, 2015). In addition, the importance of considering imprecise data in constructing composite indicators has been remarked (Cherchye et al., 2011). There are multiple indicators used for different kind of indexes, for instance, the Environmental Performance Index, the Internal Market Index, the Human Development Index, or the Technology Achievement Index. However, for some indicators only interval information is available, within what a true value is believed to lie. Wolff et al. (2011) analyzed three sources of data error in HDI calculation and classification: measurement error due to data revisions, data error due to formula updating and misclassification due to inconsistent categories cut-off values. Actually, measurement error from data updating impacts the rough data, the indices normalization and the HDI calculation. They found that from 11% up to 34% of all countries could be interpreted as misclassified in the development categories. These authors also found that on average the expected absolute deviation is nine rank positions. These results mean that interpretation of HDI ranks must be very careful. Moreover, methods applied to HDI, which are based on the outranking relation, as for instance those using the PROMETHEE or the ELECTRE TRI methods, consider stable parameters. However, this choice seems to be inadequate because of imprecision introduced by data updating and inconsistent categories cut-off values. Thus, parameter values could be supposed to be uncertain, something that has not been yet considered in the ordered clustering literature, as applied to development indices.

In this article, a stochastic approach for ordered clustering is proposed, such that: (1) actions (e.g., countries) are first ranked by using the PROMETHEE II method rationale; (2) ordered clusters are built by using the Warped-K-means approach; (3) uncertain thresholds and criteria weights are considered such that the approach integrates Monte Carlo Simulation and robustness analysis is performed. An application to the HDI problem is performed.

The article is organized as follows. In Section 2 basics of the clustering approach are presented. Section 2.3 presents the clustering approach. In Section

3, countries included in the HDI 2018 report are clustered by following the proposed approach. Results are presented and discussed. Limitations of this approach and final conclusions are described in Section 4.

2. Materials and methods

2.1. PROMETHEE II rationale

Multi-criteria clustering approaches have been developed that are applied to use the global outranking relation to model imprecise information regarding the evaluation of alternatives. Outranking was early proposed in Elimination Et Choix Traidusant la Realite (ELECTRE) methods (Figueira et al., 2010) and further became an essential feature in Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE) methods Brans and Vincke (1985). The outranking relation helps to model global assertions from kind "the alternative b is at least as good as the alternative a", based on information collected at the level of each criteria. Non-compensatory methods based on the outranking relation could lead to higher computational complexity. However, problems exist in which compensation among criteria is not allowable.

Let $A = \{a_1, a_2, \ldots, a_n\}$ be a set of actions (e.g., countries); let $F = \{g_1, g_2, \ldots, g_m\}$ be a coherent family of criteria used to evaluate each action and $W = \{w_j \mid 0 \leq w_j \leq 1, \sum_{j=1}^{m} w_j = 1\}$ the criteria weights. The vector of performances of an action $(g_1(a), g_2(a), \ldots, g_m(a))$ is called the *evaluation profile*. For any pair $b, a \in A$, the deviation of evaluations on criterion j, $= g_j(b) - g_j(a)$, can be calculated. PROMETHEE uses this deviation to compute a preference function $p_j(b, a)$, which allows to evaluate how preferred is an action over another, on the criterion j (Brans and Vincke, 1985). Here, we use a modified version of the standard preference function

$$c_j(b,a) = \begin{cases} 0 & \text{if } g_j(a) - g_j(b) > p_j, \\ 1 & \text{if } g_j(a) - g_j(b) \leq q_j, \\ \dfrac{g_j(b) - g_j(a) + p_j}{p_j - q_j} & \text{otherwise,} \end{cases} \tag{1}$$

In (1), p_j, q_j are called the strict-preference and the indifference thresholds, respectively. Based on the $c_j(b, a)$ ($j = 1, \ldots, m$) functions, the following indices can be computed,

$$\begin{cases} C(b,a) = \sum_{j=1}^{m} w_j c_j(b,a), \\ C(a,b) = \sum_{j=1}^{m} w_j c_j(b,a), \end{cases} \tag{2}$$

where $C(b, a)$ expresses with which degree b is preferred to a, over all the criteria. Next, the level in which an element b is preferred to, or it is outranked by, the others elements in A can be expressed in terms of two values:

$$\begin{cases} \psi^+(b) = \frac{1}{n-1} \sum_{a \in A, a \neq b} C(b,a), \\ \psi^-(b) = \frac{1}{n-1} \sum_{a \in A, a \neq b} C(a,b), \end{cases} \tag{3}$$

where $\psi^+(b)$ is called the *positive outranking flow* of b and $\psi^-(b)$ is called the *negative outranking flow*. In PROMETHEE II, a net outranking flow is calculated by the following expression

$$\psi(b) = \psi^+(b) - \psi^-(b). \tag{4}$$

This function satisfies $-1 \leq \psi(b) \leq 1$. When $\psi(b) > 0$, b outranks all other alternatives in all criteria more than it is outranked. When $\psi(b) < 0$, b is outranked by all other alternatives in all criteria more than it outranks them. This function can be used to build a complete pre-order (ranking) among actions. In Appendix A, we show that $p_j(b, a) = 1 - c_j(a, b)$. Thus, it can be shown that the ranking induced by $\psi(b)$ is the same than the ranking induced by the standard PROMETHEE II.

2.2. Cluster centroids and the indifference relation

Let $C_h, (h = 1, \ldots, K; K \geq 2)$ be a set of ordered categories such that $C_1 \succ C_2 \succ \ldots \succ C_K$, where the relation $\succ$ means that for any C_i, C_j such that $i > j$, an element $a_i \in C_i$ is not worse than any element $a_j \in C_j$. Conversely, a_j is worse than a_i, in terms of preferences. A special set of actions $B = \{b_1, \ldots, b_K\}$ is defined, in which b_h is a fictitious action called the *centroid*, a central reference action being representative of C_h.

Now, let us define a trapezoidal fuzzy number (TrFN) (Ban et al., 2011), as represented in Figure 1.

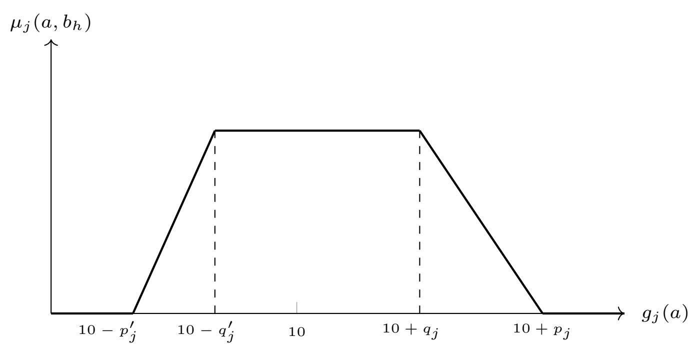

Figure 1: Trapezoid function built from $c_j(b, a), c_j(a, b)$

The TrFn $\mu_j(a, b_h)$ can be constructed from two outranking relations (Perny and Roy, 1992). Actually, let us define

$$c_j^{inv}(a, b) = \begin{cases} 0 & \text{if } g_j(b) - g_j(a) > p'_j, \\ 1 & \text{if } g_j(b) - g_j(a) \leq q'_j, \\ \dfrac{g_j(a) - g_j(b) + p'_j}{p'_j - q'_j} & \text{otherwise.} \end{cases} \tag{5}$$

which we call the *inverse preference function*, where inverse thresholds p'_j, q'_j are introduced (Roy et al., 2012), as shown in Figure 1.

Any alternative $a \in A$ can be evaluated as whether it belongs to a class C_h or not, whenever a criterion j is considered, by using the membership function $\mu_j(a, b_h)$. Let us define

$$i(a, b_h) = min\{C(b_h, a), C(a, b_h)\} \tag{6}$$

as an *indifference function* evaluating in which degree a belongs to C_h. Note that $0 \leq i(a, b_h) \leq 1$. If $i(a, b_h) = 0$ means that a should not be assigned into the cluster C_h. On the contrary, $i(a, b_h) = 1$ means that a has to be assigned into that group. Thus, the function evaluates with which degree an action belongs to a cluster.

2.3. P-WKM-E: a novel clustering process

Let us assume that A is ordered in terms of preferences, by using the PROMETHEE II method described in Section 2.1. Thus, a permutation $A^{(p)} = \{a_{i_1}, a_{i_2}, \ldots, a_{i_n}\}$ is obtained in which, for all $v \leq u$ it follows $a_{i_v} \preceq a_{i_u}$, i.e. a_{i_u} is preferred or indifferent to a_{i_v}. Let $A^{(p)}$ be partitioned into groups $C_1 \succ C_2 \succ \ldots \succ C_K$ with centroids $b_1, b_2, \ldots, b_K$, respectively.

Essentially, MCDA clustering processes integrating K-means lie in two features (De Smet and Eppe, 2009; Lolli et al., 2014; Panapakidis and Christoforidis, 2018; Chen et al., 2018). First, the definition of an indifference-based metric that measures how indifferent is an alternative to a central alternative, or centroid, representing a category. Thus, an alternative is assigned to the cluster where the most indifferent centroid is found. Second, once all the alternatives have been assigned, the centroids are updated, using the information regarding the alternatives in their respective groups. This process continues until a stop condition is satisfied.

The Warped-K-means (WKM) is an approach based on K-means that uses the following procedure (Leiva and Vidal, 2013): 1) building a permutation $A^{(p)}$ from the set A of actions; 2) finding initial groups and centroids by defining boundaries of clusters; 3) moving boundaries and re-calculate the centroids, until the set of centroids is stable. In Figure 2 an example of permutation is represented, with three clusters, boundaries L_h and centroids b_h ($h = 0, 1, 2$). The left boundary of a cluster is shown which coincides with the first element in each group. Actions in the set $[L_h, L_{h+1}[$ are assigned into C_h, and centroids could be calculated as usual in the K-means algorithm (i.e. computing the average profile). Note the action a in this figure. If $i(a, b_0) < i(a, b_1)$ then a should be better assigned into C_1. Thus, re-assignment of this action means that L_1 should be moved to the left, centroids should be updated and the process repeated. Several rounds could be applied, in ascending and descending directions, up to the centroids changes are minimal or a number of iterations is reached. Leiva and Vidal (2013) provides a detailed explanation of this method.

WKM was originally proposed for problems in which each action profile is modeled as a real vector, and distances between actions and centroids are computed by metric distances, e.g. the Euclidean one. In this article, we propose to adapt it to cases in which similarity between an action and a centroid is replaced by the indifference relation presented in Section 6. In addition, WKM bases the improvement of the K-means global similarity function as a corner step of its process. We propose do not use such a function and use instead two assignment rules.

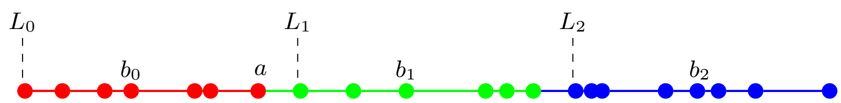

Figure 2: Example of three clusters boundaries L_h and centroids b_h ($h = 0, 1, 2$)

Therefore, let us define the following process:

1. Building a a permutation $A^{(p)}$ from the set A, by using the PROMETHEE II method (Section 2.1).

2. Compute initial $It = 0$, L_h, b_h ($h = 0, \ldots, K$).

3. Repeat

   (a) *Descending Rule*. Decrease h from $K - 1$ until 1:
   For $a \in \underleftarrow{C_h}$:

   - if $\min\{\sigma_I(a, b_h), \sigma_D(b_h, a)\} > \min\{\sigma_I(a, b_{h+1}), \sigma_D(b_{h+1}, a)\}$ then:
     - Assign a to C_h; otherwise, assign a to C_{h+1}.
     - Update L_h, b_h ($h = 0, \ldots, K$).

   (b) *Ascending Rule*. Increase h from 1 until the first K:
   For $a \in \underrightarrow{C_h}$:

   - if $\min\{\sigma_I(a, b_h), \sigma_D(b_h, a)\} > \min\{\sigma_I(a, b_{h-1}), \sigma_D(b_{h-1}, a)\}$ then:
     - Assign a to C_h; otherwise, assign a to C_{h-1}.
     - Update L_h, b_h ($h = 0, \ldots, K$).

   (c) $It = It + 1$.

   Until $It = MaxIt$

Expression $\underleftarrow{C_h}$ means that this set must be run from right to left. Similarly, $\underrightarrow{C_h}$ means that it is run from left to right. The descending and ascending rules are adapted from the ELECTRE TRI-C procedure introduced by Almeida et al. (2008) in the context of multi-criteria sorting processes. The Repeat-Until loop mimics the iterative K-means algorithm. We propose to control it by a maximum number of iterations, $MaxIt$.

Because this approach integrates features coming from PROMETHEE, Warped K-means and ELECTRE TRI-C, we call this the *P-WKM-E clustering method*.

Below, the procedure described is applied to the HDI problem. Actions are identified and criteria defined. Initially, we consider that thresholds are stochastic. Next, thresholds are fixed and weights are considered stochastic. The whole process lead to a comparison between results produced by the approach proposed and the segmentation outcomes published in the 2018 HDI Report.

3. Clustering HDI countries

Since 1990, the United Nations Development Programme (UNDP) releases the Human Development Index (HDI) to emphasize that people and their capabilities should be the ultimate criteria for assessing the development of a country, not only economic growth. Thus, the 2018 HDI ranking process evaluates 189 United Nations countries on the basis of three criteria aggregated from nine dimensions. In addition, based on the HDI quartiles, four ordered groups of countries are segmented: very high human development, high human development, medium human development, and low human development.

In this article, data published in the 2018 HDI Report is retrieved to perform the analysis. Thus, there are 189 countries to be clustered. The family of criteria to be considered consists of the three indices: life expectancy (g_1), years of schooling (g_2) and Gross National Income, GNI (g_3). These indices are computed using the formulae defined in the HDI Technical Notes (UNDP, 2019).

3.1. Initial settings

In order to have an idea about the number of clusters that should be considered in our approach, the K-means Elbow Method was roughly run on indices computed from HDI data. The Elbow Method is used to determine the optimal number of clusters in K-means clustering (Ketchen and Shook, 1996). This gives four clusters, i.e. $K = 4$, which parallels the number of groups defined in the Human Development Report. In order to define an initial set of centroids, the K-means algorithm is run. The following initial centroids are found:

$$
\begin{aligned}
b_1 &= (0.27, 0.25, 0.43), \\
b_2 &= (0.51, 0.39, 0.64), \\
b_3 &= (0.70, 0.53, 0.83), \\
b_4 &= (0.90, 0.71, 0.93).
\end{aligned}
$$

For each criterion j, the minimum distance between consecutive centroids $m_j = \min_h\{g_j(b_h) - g_j(b_{h-1})\}$ is calculated, then $m_1 = 0.19, m_2 = 0.14, m_3 = 0.10$. In order to guarantee a separability condition, the following restriction is defined

$$p'_j + p_j \leq m_j \, (j = 1, 2, 3). \quad (7)$$

3.2. Stochastic thresholds

Let us assume that p_j, q_j, p'_j, q'_j are uncertain. We may assume that stochastic variables $\xi_{p_j}, \xi_{q_j}, \xi_{p'_j}, \xi_{q'_j}$ correctly model each parameter. We assume that each stochastic variable is uniformly distributed in an interval $[\xi_m, \xi_M]$. If a stochastic interval is defined for each variable, as $[m_j - 0.1, m_j + 0.1] \, (j = 1, 2, 3)$, then the following linear equations system may be defined:

$$\begin{aligned}
p_1, p'_1 &\in [0.090, 0.290], \\
p_2, p'_2 &\in [0.040, 0.240], \\
p_3, p'_3 &\in [0.000, 0.200], \\
q_1, q'_1 &\in [0.045, 0.145], \\
q_2, q'_2 &\in [0.020, 0.120], \\
q_3, q'_3 &\in [0.000, 0.100], \\
p_1 + p'_1 &\leq 0.19, \\
p_2 + p'_2 &\leq 0.14, \\
p_3 + p'_3 &\leq 0.10.
\end{aligned}$$

A set of 1000 random values is generated for each parameter which means that the clustering process is performed 1000 times. The number of iterations (see Section 2.3) is defined as $MaxIt = 30$ because preliminary simulations show that this is enough to reach stable centroids. Weights are considered to be equal: $w_j = 1/3$ for $j = 1, 2, 3$. The approach is implemented in Python and it is run in an Apple Mac 2.3 Intel i5 computer.

Results of clustering are shown in Table 1, where four groups of alternatives are presented, as originally defined in the HDI ranking. The codes of clusters are as follows: C_4, VERY HIGH DEVELOPMENT; C_3, HIGH DEVELOPMENT; C_2, MEDIUM DEVELOPMENT; C_1, LOW DEVELOPMENT. Observe, for instance, that countries ranging from 1 (Norway) up to 62 (Seychelles) belong to the group of very high developed countries. Clusters built by the proposed approach are coded as C_1, C_2, C_3, C_4 and satisfy $C_1 \prec C_2 \prec C_3 \prec C_4$.

Given a row in a tabular structure, values indicate how frequently an alternative (country) is assigned into a cluster, regarding the 1000 stochastic simulations. For instance, the alternative 1 in the group VERY HIGH DEVELOPMENT, 100% of times is assigned into C_4. Instead, country 49 is assigned into C_4 88% of times and into C_3 12% of simulations. Colors help to recognize these frequencies: the more intense a color, the higher the frequency an alternative reaches in a cluster.

The following results can be highlighted:

Table 1: Ordered clustering vs HDI development groups: stochastic thresholds and fixed weights

VERY HIGH DEVELOPMENT

Country	C_4	C_3	C_2	C_1
1	1.00	0.00	0.00	0.00
2	1.00	0.00	0.00	0.00
3	1.00	0.00	0.00	0.00
4	1.00	0.00	0.00	0.00
5	1.00	0.00	0.00	0.00
6	1.00	0.00	0.00	0.00
7	1.00	0.00	0.00	0.00
8	1.00	0.00	0.00	0.00
9	1.00	0.00	0.00	0.00
10	1.00	0.00	0.00	0.00
11	1.00	0.00	0.00	0.00
12	1.00	0.00	0.00	0.00
13	1.00	0.00	0.00	0.00
14	1.00	0.00	0.00	0.00
15	1.00	0.00	0.00	0.00
16	1.00	0.00	0.00	0.00
17	1.00	0.00	0.00	0.00
18	1.00	0.00	0.00	0.00
19	1.00	0.00	0.00	0.00
20	1.00	0.00	0.00	0.00
21	1.00	0.00	0.00	0.00
22	1.00	0.00	0.00	0.00
23	1.00	0.00	0.00	0.00
24	1.00	0.00	0.00	0.00
25	1.00	0.00	0.00	0.00
26	1.00	0.00	0.00	0.00
27	1.00	0.00	0.00	0.00
28	1.00	0.00	0.00	0.00
29	1.00	0.00	0.00	0.00
30	1.00	0.00	0.00	0.00
31	1.00	0.00	0.00	0.00
32	1.00	0.00	0.00	0.00
33	1.00	0.00	0.00	0.00
34	1.00	0.00	0.00	0.00
35	1.00	0.00	0.00	0.00
36	1.00	0.00	0.00	0.00
37	1.00	0.00	0.00	0.00
38	1.00	0.00	0.00	0.00
39	1.00	0.00	0.00	0.00
40	1.00	0.00	0.00	0.00
41	1.00	0.00	0.00	0.00
42	1.00	0.00	0.00	0.00
43	1.00	0.00	0.00	0.00
44	1.00	0.00	0.00	0.00
45	1.00	0.00	0.00	0.00
46	1.00	0.00	0.00	0.00
47	1.00	0.00	0.00	0.00
48	1.00	0.00	0.00	0.00
49	0.88	0.12	0.00	0.00
50	0.48	0.52	0.00	0.00
51	0.26	0.74	0.00	0.00
52	0.15	0.85	0.00	0.00
53	0.00	1.00	0.00	0.00
54	0.00	1.00	0.00	0.00
55	0.00	1.00	0.00	0.00
56	0.00	1.00	0.00	0.00
57	0.00	1.00	0.00	0.00
58	0.00	1.00	0.00	0.00
59	0.00	1.00	0.00	0.00
60	0.00	1.00	0.00	0.00
61	0.00	1.00	0.00	0.00
62	0.00	1.00	0.00	0.00

HIGH DEVELOPMENT

Country	C_4	C_3	C_2	C_1
63	0.00	1.00	0.00	0.00
64	0.00	1.00	0.00	0.00
65	0.00	1.00	0.00	0.00
66	0.00	1.00	0.00	0.00
67	0.00	1.00	0.00	0.00
68	0.00	1.00	0.00	0.00
69	0.00	1.00	0.00	0.00
70	0.00	1.00	0.00	0.00
71	0.00	1.00	0.00	0.00
72	0.00	1.00	0.00	0.00
73	0.00	1.00	0.00	0.00
74	0.00	1.00	0.00	0.00
75	0.00	1.00	0.00	0.00
76	0.00	0.99	0.01	0.00
77	0.00	0.99	0.01	0.00
78	0.00	0.98	0.02	0.00
79	0.00	0.95	0.05	0.00
80	0.00	0.92	0.08	0.00
81	0.00	0.87	0.13	0.00
82	0.00	0.85	0.15	0.00
83	0.00	0.66	0.34	0.00
84	0.00	0.63	0.37	0.00
85	0.00	0.53	0.47	0.00
86	0.00	0.49	0.51	0.00
87	0.00	0.20	0.80	0.00
88	0.00	0.18	0.82	0.00
89	0.00	0.08	0.92	0.00
90	0.00	0.06	0.94	0.00
91	0.00	0.04	0.96	0.00
92	0.00	0.03	0.97	0.00
93	0.00	0.02	0.98	0.00
94	0.00	0.01	0.99	0.00
95	0.00	0.00	1.00	0.00
96	0.00	0.00	1.00	0.00
97	0.00	0.00	1.00	0.00
98	0.00	0.00	1.00	0.00
99	0.00	0.00	1.00	0.00
100	0.00	0.00	1.00	0.00
101	0.00	0.00	1.00	0.00
102	0.00	0.00	1.00	0.00
103	0.00	0.00	1.00	0.00
104	0.00	0.00	1.00	0.00
105	0.00	0.00	1.00	0.00
106	0.00	0.00	1.00	0.00
107	0.00	0.00	1.00	0.00
108	0.00	0.00	1.00	0.00
109	0.00	0.00	1.00	0.00
110	0.00	0.00	1.00	0.00
111	0.00	0.00	1.00	0.00
112	0.00	0.00	1.00	0.00
113	0.00	0.00	1.00	0.00
114	0.00	0.00	1.00	0.00
115	0.00	0.00	1.00	0.00
116	0.00	0.00	1.00	0.00

MEDIUM DEVELOPMENT

Country	C_4	C_3	C_2	C_1
117	0.00	0.00	1.00	0.00
118	0.00	0.00	1.00	0.00
119	0.00	0.00	1.00	0.00
120	0.00	0.00	1.00	0.00
121	0.00	0.00	1.00	0.00
122	0.00	0.00	1.00	0.00
123	0.00	0.00	1.00	0.00
124	0.00	0.00	1.00	0.00
125	0.00	0.00	1.00	0.00
126	0.00	0.00	1.00	0.00
127	0.00	0.00	1.00	0.00
128	0.00	0.00	1.00	0.00
129	0.00	0.00	1.00	0.00
130	0.00	0.00	1.00	0.00
131	0.00	0.00	1.00	0.00
132	0.00	0.00	1.00	0.00
133	0.00	0.00	1.00	0.00
134	0.00	0.00	1.00	0.00
135	0.00	0.00	1.00	0.00
136	0.00	0.00	0.99	0.01
137	0.00	0.00	0.95	0.05
138	0.00	0.00	0.94	0.06
139	0.00	0.00	0.94	0.06
140	0.00	0.00	0.91	0.09
141	0.00	0.00	0.81	0.18
142	0.00	0.00	0.71	0.28
143	0.00	0.00	0.57	0.43
144	0.00	0.00	0.51	0.49
145	0.00	0.00	0.41	0.59
146	0.00	0.00	0.40	0.60
147	0.00	0.00	0.35	0.65
148	0.00	0.00	0.22	0.78
149	0.00	0.00	0.19	0.81
150	0.00	0.00	0.17	0.83
151	0.00	0.00	0.08	0.91
152	0.00	0.00	0.05	0.95
153	0.00	0.00	0.02	0.98

LOW DEVELOPMENT

Country	C_4	C_3	C_2	C_1
154	0.00	0.00	0.01	0.99
155	0.00	0.00	0.00	1.00
156	0.00	0.00	0.00	1.00
157	0.00	0.00	0.00	1.00
158	0.00	0.00	0.00	1.00
159	0.00	0.00	0.00	1.00
160	0.00	0.00	0.00	1.00
161	0.00	0.00	0.00	1.00
162	0.00	0.00	0.00	1.00
163	0.00	0.00	0.00	1.00
164	0.00	0.00	0.00	1.00
165	0.00	0.00	0.00	1.00
166	0.00	0.00	0.00	1.00
167	0.00	0.00	0.00	1.00
168	0.00	0.00	0.00	1.00
169	0.00	0.00	0.00	1.00
170	0.00	0.00	0.00	1.00
171	0.00	0.00	0.00	1.00
172	0.00	0.00	0.00	1.00
173	0.00	0.00	0.00	1.00
174	0.00	0.00	0.00	1.00
175	0.00	0.00	0.00	1.00
176	0.00	0.00	0.00	1.00
177	0.00	0.00	0.00	1.00
178	0.00	0.00	0.00	1.00
179	0.00	0.00	0.00	1.00
180	0.00	0.00	0.00	1.00
181	0.00	0.00	0.00	1.00
182	0.00	0.00	0.00	1.00
183	0.00	0.00	0.00	1.00
184	0.00	0.00	0.00	1.00
185	0.00	0.00	0.00	1.00
186	0.00	0.00	0.00	1.00
187	0.00	0.00	0.00	1.00
188	0.00	0.00	0.00	1.00
189	0.00	0.00	0.00	1.00

- As observed in the VERY HIGH DEVELOPMENT group, the alternatives 1 up to 48 reach 100% frequency in C_4. The alternatives 49 up to 52 have some ambiguous arguments to be allocated into C_4 or C_3. However, countries 53-62 are 100% of times assigned into C_3 (high development countries).
- The group named HIGH DEVELOPMENT has three well defined zones: 1) from country 63 up to 75, assignment to C_3 is crisp; 2) Countries 76-94 present ambiguity in their assignments; 3) From 95 up to 116, countries are allocated into C_2.
- The group named MEDIUM DEVELOPMENT has two zones: 1) A crisp set of countries allocated to C_2, 100% times; 2). An ambiguos set covering countries 136-153.
- The group named LOW DEVELOPMENT is almost free of ambiguity (exception for country 154).

Other approaches for clustering based on the outranking relation have not relieved in the uncertain nature of parameters, but only in the imprecise information available to evaluate countries, thus using deterministic thresholds (De Smet et al., 2012; De Smet, 2014; Fernandez et al., 2010; Chen et al., 2018). However, there is evidence showing that imprecision is uncertain, which depends on multiple factors, changing on time. Wolff et al. (2011) found that the higher the development status of a country, the more precise are the reported data. This seems to encompass our results: countries in the upper part of very developed countries are strongly assigned to C_4. These authors also found that when many countries are close to the group cut-off thresholds (quartiles), up to 45% of the developing countries are misclassified. Table 1 shows this phenomenon. For instance, 40% countries in the lower part of the VERY HIGH DEVELOPMENT group are classified in C_3.

Garcia et al. (2010) studied different sources of uncertainty both in data sources and methodological choices in the HDI calculation. They found that main sources of uncertainty resulted in non biased HDI rankings. A main finding was that shifts in ranking were minor in the upper part of the VERY HIGH DEVELOPMENT and the lower part of the LOW DEVELOPMENT groups. However, the absolute shift in ranking seemed to be minor in all levels of development, which did not compromise the robustness of the HDI, regardless of the development level. As a consequence, the HDI ranking remains robust. Our results somehow contradicts such a statement. There are several countries in which the frequency of each of two consecutive classes are around 25% and 55% (e.g., countries 37, 40, 93, 101, 124, 137, 169). This means that ranks of those countries will be different from the respective HDI ranks, because these can be assigned into categories different from the HDI development groups.

3.3. Stochastic weights

Garcia et al. (2010) studied the distribution of the rankings derived from applying different weights, normalizations and an alternative functional form of life expectancy. They found that uncertainty in weights produced shift in rankings, but without significant bias regarding the HDI ranking. In order to explore the impact of uncertain weights, we implement Monte Carlo simulations by considering $w_j \sim U(0,1)\,(j = 1,2,3)$. In Table 3 results obtained when uncertain weights and deterministic thresholds are considered: $p_1 = p'_1 = 0.20; p_2 = p'_2 = 0.14; p_3 = p'_3 = 0.10; q_j = q'_j = p_j/2\,(j = 1,2,3)$.

In this case, ambiguity of assignments increased. For instance, in the VERY HIGH DEVELOPMENT group there are now countries from 30 up to 62 that show less than perfect frequency in C_4. The whole HIGH DEVELOPMENT group shows some level of ambiguity. In the MEDIUM DEVELOPMENT group, even if countries 117-135 have down their level, they still have a high frequency to be assigned into C_2. The LOW DEVELOPMENT countries seem to be already having strong arguments to be placed into C_1.

This outcomes can be interpreted as a partially confirmatory evidence in favor of the Garcia et al. (2010)'s result regarding shift in rankings. Although we may affirm that four groups of development exist, we did not found that the groups reported in the HDI Report and those built by our approach totally coincide. However, simulations reveal that uncertainty has an impact on outcomes.

Let us define a minimal threshold of 80% to accept that a country effectively belongs to a cluster. In Table 2, countries assignments obtained with the original HDI Report and the two stochastic runs we have performed are shown. Ambiguos assignments are presented in columns $C_4 - C_3$, $C_3 - C_2$ and $C_2 - C_1$. This allows to identify the set of countries that could be more confidently be allocated into specific groups. For instance, it could be plausible that countries 1-47 belong to C_4 (VERY HIGH DEVELOPMENT).

Table 2: Assignments of HDI countries in three cases

Method	C_4	$C_4 - C_3$	C_3	$C_3 - C_2$	C_2	$C_2 - C_1$	C_1
HDI Report	1-62		63-116		117-153		154-189
P-WKM-E (thresholds)	1-49	50-51	52-82	83-86	87-141	142-148	149-189
P-WKM-E (weights)	1-47	48-58	59-79	80-96	97-142	143-153	154-189

The approach proposed has two components. Firstly, an ordered clustering is obtained because actions are first ordered and limits of clusters are re-located without alter the order. Whenever the ascending or the descending rule are applied, clusters are initially "frozen" which means that actions inspected to re-assignment are taken as if these were not allocated yet. This is the strategy that ELECTRE TRI-C sorting method applies (Almeida et al., 2008). Thus, moving boundaries occurs after any rule is used. Secondly, in the special case of development indices, this approach seems to be well-adapted because, according to the reviewed literature, most of data available for evaluation is uncertain. However, we did not take into account, for instance, incomplete or vague information. This kind of poor information needs to be considered in a future research.

4. Conclusions

In this paper, a non-compensatory and stochastic multi-criteria clustering approach is proposed and it is applied to the group of countries included in the 2018 Human Development Index Report. Results obtained with this approach are compared to the deterministic 2018 HDI ranking. Findings show that an ordered set of clusters is built. In addition, uncertainty shapes results, but it depends on the variable in which randomness is considered. We have found that four groups of countries seem to be adequate, as proposed by the original HDI Report, but uncertainty makes ambiguity zones to appear which changes countries from clusters proposed by the HDI.

Therefore, we have found that considering uncertainty in clustering process can impact the classification of countries in the 2018 HDI Report. However, we may conclude that uncertainty in parameters used in outranking-based approaches and uncertainty in weights do not impact in the same way. Our approach is based on the outranking relation, which uses the following preferential parameters: the indifference and the preference thresholds, and the weights of criteria. We have found that uncertainty in weights leads to more ambiguity in assignment of countries to clusters than uncertainty in thresholds.

A limitation of our approach is the usage of K-means like algorithm because results are dependent on the initial set of cluster centers. To mitigate this drawback, we have used some heuristics to find the number and initial centroids. Anyway, future research is needed to consider this feature in contexts of stochastic variables.

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Table 3: Ordered clustering vs HDI development groups: fixed thresholds and stochastic weights

VERY HIGH DEVELOPMENT

Country	C_4	C_3	C_2	C_1
1	1.00	0.00	0.00	0.00
2	1.00	0.00	0.00	0.00
3	1.00	0.00	0.00	0.00
4	1.00	0.00	0.00	0.00
5	1.00	0.00	0.00	0.00
6	1.00	0.00	0.00	0.00
7	1.00	0.00	0.00	0.00
8	1.00	0.00	0.00	0.00
9	1.00	0.00	0.00	0.00
10	1.00	0.00	0.00	0.00
11	1.00	0.00	0.00	0.00
12	1.00	0.00	0.00	0.00
13	1.00	0.00	0.00	0.00
14	1.00	0.00	0.00	0.00
15	1.00	0.00	0.00	0.00
16	1.00	0.00	0.00	0.00
17	1.00	0.00	0.00	0.00
18	1.00	0.00	0.00	0.00
19	1.00	0.00	0.00	0.00
20	1.00	0.00	0.00	0.00
21	1.00	0.00	0.00	0.00
22	1.00	0.00	0.00	0.00
23	1.00	0.00	0.00	0.00
24	1.00	0.00	0.00	0.00
25	1.00	0.00	0.00	0.00
26	1.00	0.00	0.00	0.00
27	1.00	0.00	0.00	0.00
28	1.00	0.00	0.00	0.00
29	1.00	0.00	0.00	0.00
30	0.99	0.01	0.00	0.00
31	0.99	0.01	0.00	0.00
32	0.99	0.01	0.00	0.00
33	0.98	0.02	0.00	0.00
34	0.99	0.01	0.00	0.00
35	0.98	0.02	0.00	0.00
36	0.98	0.02	0.00	0.00
37	0.98	0.02	0.00	0.00
38	0.98	0.02	0.00	0.00
39	0.96	0.04	0.00	0.00
40	0.96	0.04	0.00	0.00
41	0.94	0.06	0.00	0.00
42	0.93	0.07	0.00	0.00
43	0.94	0.06	0.00	0.00
44	0.92	0.08	0.00	0.00
45	0.92	0.08	0.00	0.00
46	0.88	0.12	0.00	0.00
47	0.83	0.17	0.00	0.00
48	0.78	0.22	0.00	0.00
49	0.72	0.28	0.00	0.00
50	0.67	0.33	0.00	0.00
51	0.57	0.43	0.00	0.00
52	0.52	0.47	0.00	0.00
53	0.45	0.55	0.00	0.00
54	0.36	0.64	0.00	0.00
55	0.31	0.68	0.00	0.00
56	0.22	0.77	0.00	0.00
57	0.22	0.78	0.00	0.00
58	0.21	0.79	0.00	0.00
59	0.18	0.82	0.00	0.00
60	0.19	0.81	0.01	0.00
61	0.13	0.87	0.00	0.00
62	0.14	0.85	0.00	0.00

HIGH DEVELOPMENT

Country	C_4	C_3	C_2	C_1
63	0.11	0.88	0.01	0.00
64	0.10	0.89	0.02	0.00
65	0.09	0.90	0.01	0.00
66	0.06	0.91	0.02	0.00
67	0.06	0.93	0.01	0.00
68	0.04	0.94	0.02	0.00
69	0.04	0.92	0.04	0.00
70	0.02	0.95	0.04	0.00
71	0.02	0.94	0.05	0.00
72	0.01	0.93	0.06	0.00
73	0.01	0.94	0.06	0.00
74	0.01	0.91	0.08	0.00
75	0.01	0.86	0.13	0.00
76	0.01	0.89	0.10	0.00
77	0.01	0.87	0.13	0.00
78	0.01	0.86	0.14	0.00
79	0.00	0.82	0.17	0.00
80	0.00	0.79	0.21	0.00
81	0.00	0.76	0.24	0.00
82	0.00	0.74	0.26	0.00
83	0.00	0.72	0.28	0.00
84	0.00	0.69	0.31	0.00
85	0.00	0.66	0.34	0.00
86	0.00	0.62	0.38	0.00
87	0.00	0.58	0.42	0.00
88	0.00	0.55	0.45	0.00
89	0.00	0.52	0.48	0.00
90	0.00	0.48	0.52	0.00
91	0.00	0.40	0.60	0.00
92	0.00	0.38	0.62	0.00
93	0.00	0.36	0.64	0.00
94	0.00	0.32	0.68	0.00
95	0.00	0.24	0.76	0.00
96	0.00	0.20	0.79	0.00
97	0.00	0.18	0.82	0.00
98	0.00	0.16	0.84	0.00
99	0.00	0.16	0.84	0.00
100	0.00	0.13	0.87	0.00
101	0.00	0.12	0.88	0.00
102	0.00	0.12	0.88	0.00
103	0.00	0.12	0.88	0.00
104	0.00	0.10	0.90	0.00
105	0.00	0.08	0.92	0.00
106	0.00	0.08	0.92	0.00
107	0.00	0.07	0.93	0.00
108	0.00	0.07	0.92	0.00
109	0.00	0.07	0.93	0.00
110	0.00	0.05	0.95	0.00
111	0.00	0.05	0.94	0.00
112	0.00	0.05	0.94	0.00
113	0.00	0.05	0.95	0.00
114	0.00	0.04	0.95	0.01
115	0.00	0.04	0.96	0.00
116	0.00	0.04	0.96	0.00

MEDIUM DEVELOPMENT

Country	C_4	C_3	C_2	C_1
117	0.00	0.03	0.96	0.01
118	0.00	0.03	0.97	0.01
119	0.00	0.02	0.98	0.00
120	0.00	0.03	0.97	0.00
121	0.00	0.03	0.96	0.01
122	0.00	0.02	0.97	0.01
123	0.00	0.02	0.97	0.01
124	0.00	0.02	0.96	0.02
125	0.00	0.01	0.98	0.02
126	0.00	0.01	0.97	0.02
127	0.00	0.01	0.97	0.02
128	0.00	0.01	0.97	0.02
129	0.00	0.01	0.96	0.03
130	0.00	0.01	0.96	0.04
131	0.00	0.01	0.95	0.04
132	0.00	0.01	0.94	0.05
133	0.00	0.01	0.94	0.05
134	0.00	0.01	0.93	0.06
135	0.00	0.01	0.91	0.08
136	0.00	0.00	0.91	0.08
137	0.00	0.00	0.89	0.11
138	0.00	0.00	0.88	0.12
139	0.00	0.00	0.86	0.14
140	0.00	0.00	0.85	0.15
141	0.00	0.00	0.83	0.17
142	0.00	0.00	0.80	0.20
143	0.00	0.00	0.77	0.22
144	0.00	0.00	0.74	0.25
145	0.00	0.00	0.70	0.30
146	0.00	0.00	0.64	0.36
147	0.00	0.00	0.55	0.45
148	0.00	0.00	0.53	0.47
149	0.00	0.00	0.47	0.53
150	0.00	0.00	0.42	0.58
151	0.00	0.00	0.34	0.66
152	0.00	0.00	0.28	0.72
153	0.00	0.00	0.24	0.75

LOW DEVELOPMENT

Country	C_4	C_3	C_2	C_1
154	0.00	0.00	0.19	0.81
155	0.00	0.00	0.20	0.80
156	0.00	0.00	0.16	0.84
157	0.00	0.00	0.11	0.89
158	0.00	0.00	0.09	0.91
159	0.00	0.00	0.07	0.93
160	0.00	0.00	0.06	0.94
161	0.00	0.00	0.03	0.96
162	0.00	0.00	0.04	0.96
163	0.00	0.00	0.02	0.98
164	0.00	0.00	0.03	0.97
165	0.00	0.00	0.01	0.98
166	0.00	0.00	0.01	0.99
167	0.00	0.00	0.01	0.99
168	0.00	0.00	0.01	0.99
169	0.00	0.00	0.00	0.99
170	0.00	0.00	0.00	1.00
171	0.00	0.00	0.00	1.00
172	0.00	0.00	0.00	1.00
173	0.00	0.00	0.00	1.00
174	0.00	0.00	0.00	1.00
175	0.00	0.00	0.00	1.00
176	0.00	0.00	0.00	1.00
177	0.00	0.00	0.00	1.00
178	0.00	0.00	0.00	1.00
179	0.00	0.00	0.00	1.00
180	0.00	0.00	0.00	1.00
181	0.00	0.00	0.00	1.00
182	0.00	0.00	0.00	1.00
183	0.00	0.00	0.00	1.00
184	0.00	0.00	0.00	1.00
185	0.00	0.00	0.00	1.00
186	0.00	0.00	0.00	1.00
187	0.00	0.00	0.00	1.00
188	0.00	0.00	0.00	1.00
189	0.00	0.00	0.00	1.00

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Appendix A. PROMETHEE II with a concordance preference function

In the original PROMETHEE method the preference function is defined as follows:

$$p_j(b,a) = \begin{cases} 0 & \text{if } g_j(b) - g_j(a) > p_j, \\ 1 & \text{if } g_j(b) - g_j(a) \leq q_j, \\ \dfrac{g_j(b) - g_j(a) - q_j}{p_j - q_j} & \text{otherwise,} \end{cases} \tag{A.1}$$

In ELECTRE methods, (1) is called a partial concordance function (Figueira et al., 2010), such that

$$c_j(a,b) = \begin{cases} 0 & \text{if } g_j(b) - g_j(a) > p_j, \\ 1 & \text{if } g_j(b) - g_j(a) \leq q_j, \\ \dfrac{g_j(a) - g_j(b) + p_j}{p_j - q_j} & \text{otherwise.} \end{cases} \tag{A.2}$$

Thus, it is straight to show that $p_j(b,a) = 1 - c_j(a,b)$. From this, it follows that

$$\begin{aligned} \Pi(a,b) &= \sum_j w_j(1 - c_j(b,a)), && \text{(A.3)} \\ &= 1 - \sum_j w_j c_j(b,a), && \text{(A.4)} \\ &= 1 - C(b,a). && \text{(A.5)} \end{aligned}$$

In PROMETHEE, negative and positive flows are represented by functions ϕ^-, ϕ^+, such that

$$\begin{cases} \phi^+(b) = \frac{1}{n-1} \sum_{a \in A, a \neq b} \pi(b,a), \\ \phi^-(b) = \frac{1}{n-1} \sum_{a \in A, a \neq b} \pi(a,b), \end{cases} \tag{A.6}$$

which means that

$$\begin{aligned} \phi^+(b) - \phi^-(b) &= \frac{1}{n-1} \sum_{a \in A, a \neq b} \left((1 - C(a,b)) - (1 - C(b,a)) \right), && \text{(A.7)} \\ &= \frac{1}{n-1} \sum_{a \in A, a \neq b} (C(b,a) - C(a,b)) && \text{(A.8)} \\ &= \psi^+(b) - \psi^-(b) && \text{(A.9)} \end{aligned}$$

where $\psi^+(b), \psi^-(b)$ may be interpreted as the positive and negative concordance flows. This also means that the same preorder induced by the PROMETHEE II could be also obtained by using the function (1), introduced in the Section 2.1.