Universiteit van Amsterdam

HAET

Hierarchical Attention Erwin Transolver for Large Scale Mesh Processing

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Motivation and Problem

Motivation and Problem

Point Clouds and Meshes

Point Clouds/Meshes are essential for **Physical Simulations**

↓

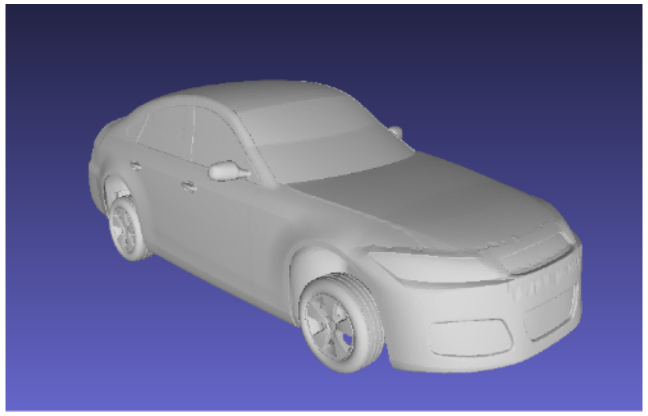
Real-world simulations involve **millions of irregular points**

↓

Processing must **preserve geometric structure and physical fields**

↓

Current PDE solvers **struggle** with such large inputs



Example from the
DrivAerNet benchmark

Motivation and Problem
Computational Challenges

- Transformer scale poorly: $O(N^2)$ **attention is impractical** at large N
- Possible solution: **Downsampling** = leads to **loss of physical detail** since points are strongly dependent on each other
- Models must:
  - Preserve local features
  - Capture long-range dependencies
  - Scale efficiently

Existing Solutions

Existing Solutions
General

Better efficiency in attention via geometric ball trees

↑

Erwin

Transolver++

↓

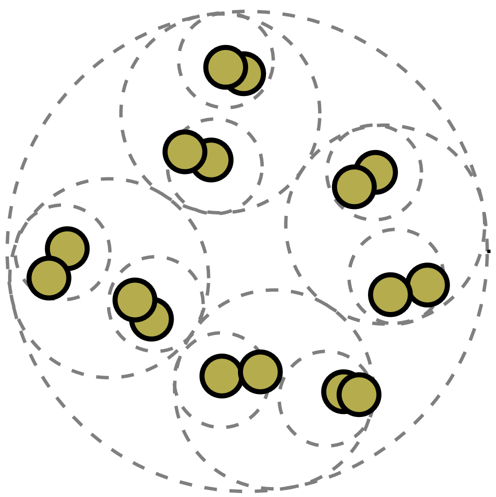
Physics aware tokenization of the input to reduce its dimension

Each solves part of the problem, **but not both**

Existing Solutions

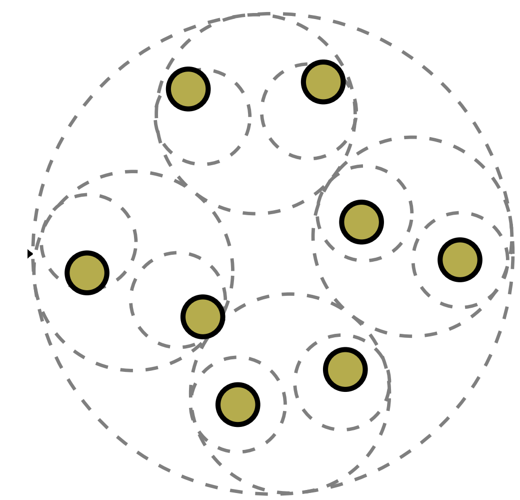
Erwin

Tree Structure



O(N) Complexity

Hierarchical Ball Attention



Erwin

[Point Cloud] → [Ball Groups] → [Hierarchical Processing]

Limitations

- Large-scale Challenges
- Geometric Context Loss
- Memory Management

Advantages

- Linear Scaling
- Efficient Processing
- Multi-scale Analysis

Existing Solutions
Erwin

Physical Aware slices

Million-point Processing

Adaptative Temperature

Transolver++

[Input] → [Slice] → [Attention] → [Deslice] → [Output]

Limitations

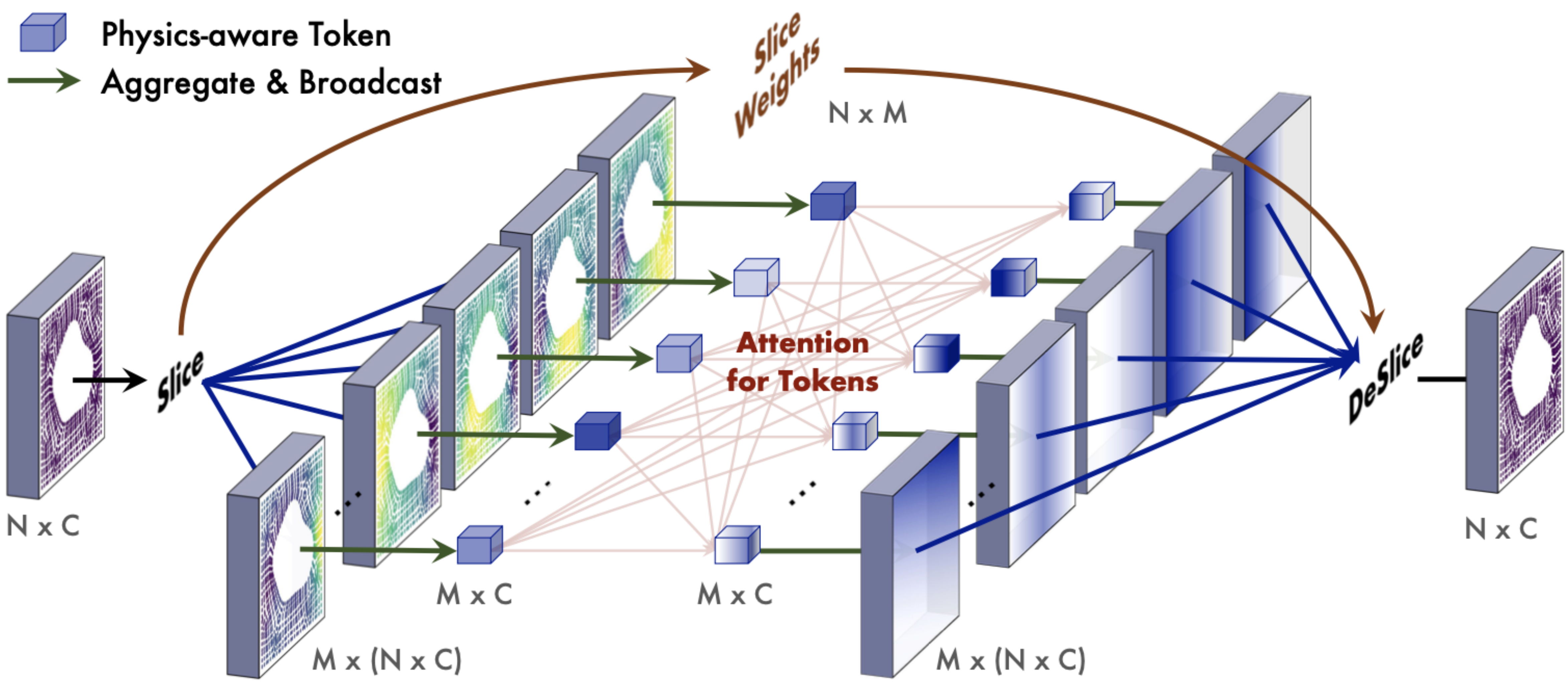
- Quadratic Scaling in attention
- Slice Limitations
- Memory Requirements

Advantages

- Large Scale Processing
- Physical Context
- Adaptive Learning

Existing Solutions

Transolver & Transolver++



Can we design a model that scales to millions of points with fewer bottlenecks, while remaining aware of the underlying physics?

Key Contributions

Key Contributions

Solve the Scaling Problem

ErwinFlash → **Speeding Up Erwin**

HAET* → **Hybrid Architecture**

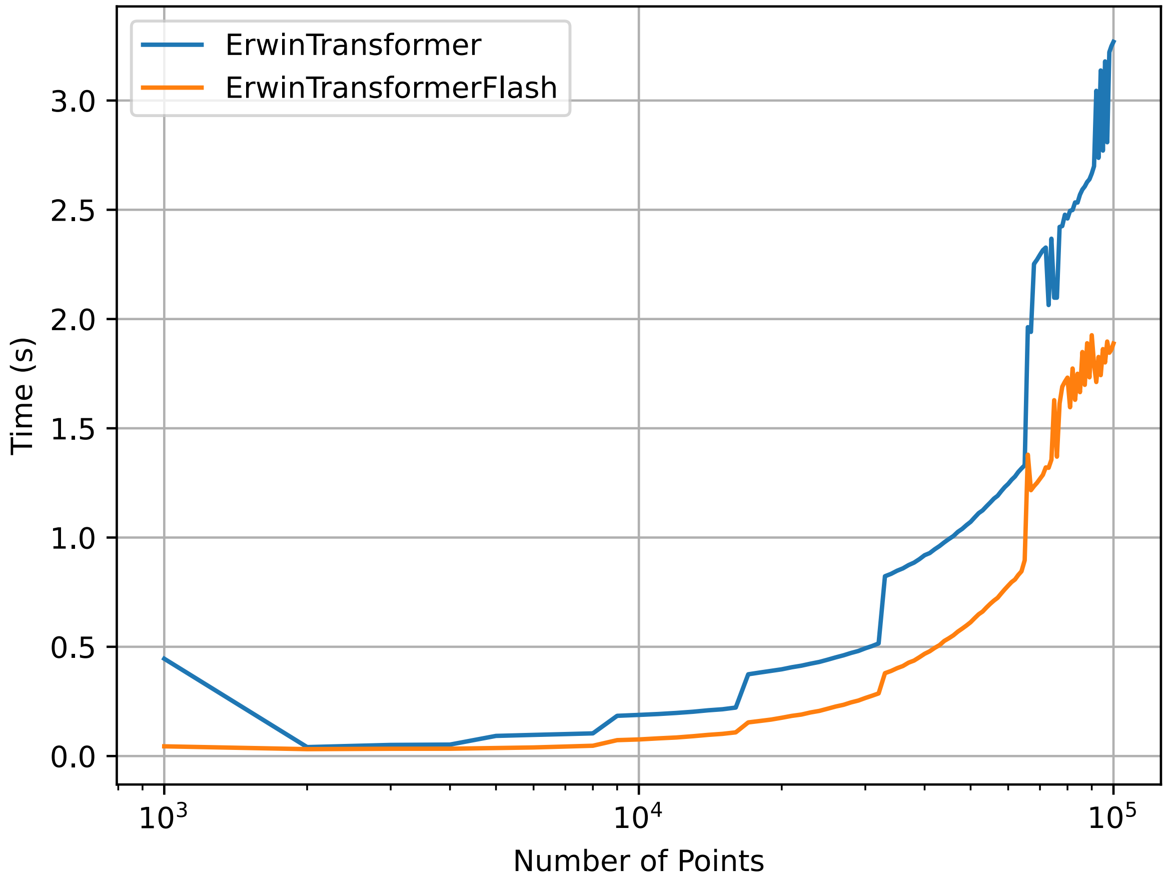
*Hierarchical Attention Erwin Transolver

ErwinFlash

ErwinFlash

Description and Results

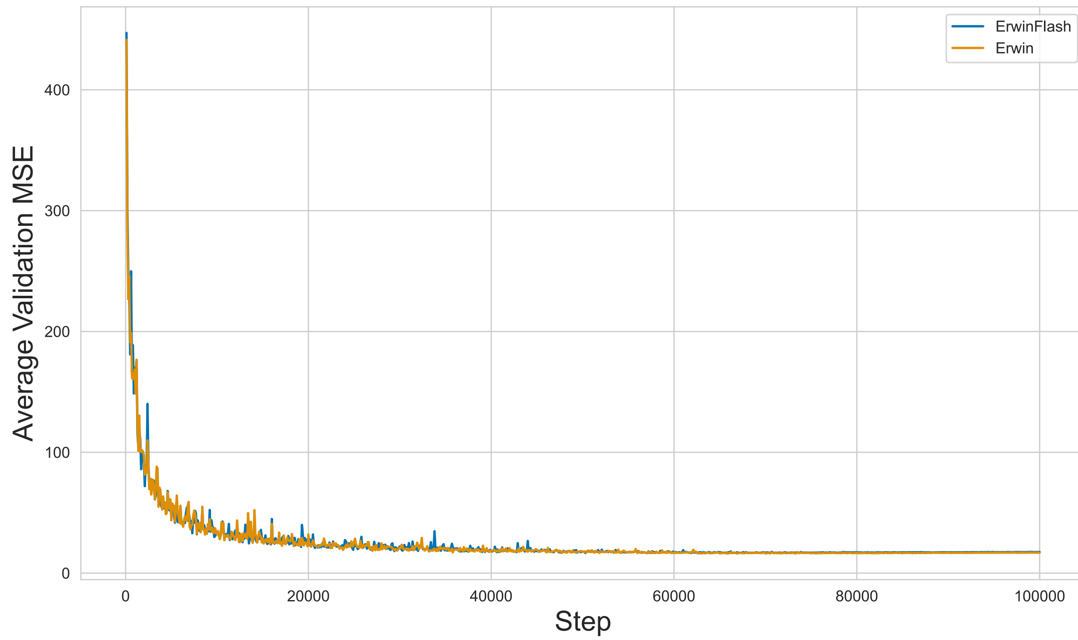
- **Key Optimizations:**
  - FlashAttention
  - Mixed-Precision Training
  - Fused CUDA Operations
- **Improvements:**
  - Speed: 1.9x over Erwin
  - Lower GPU memory usage
  - Identical training/validation loss curves
  - More stable GPU performance

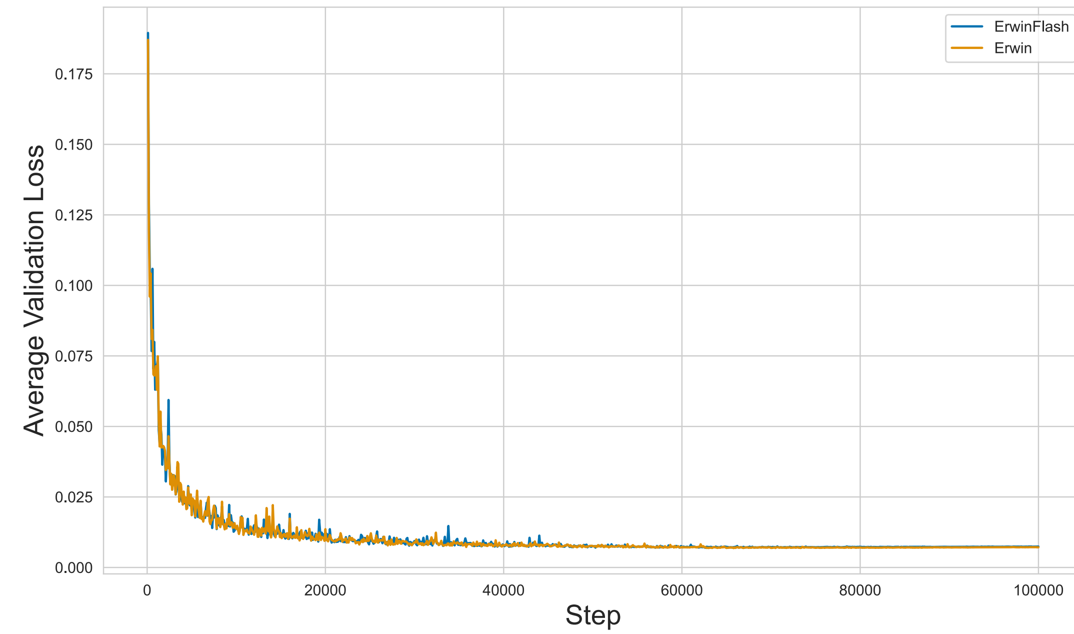


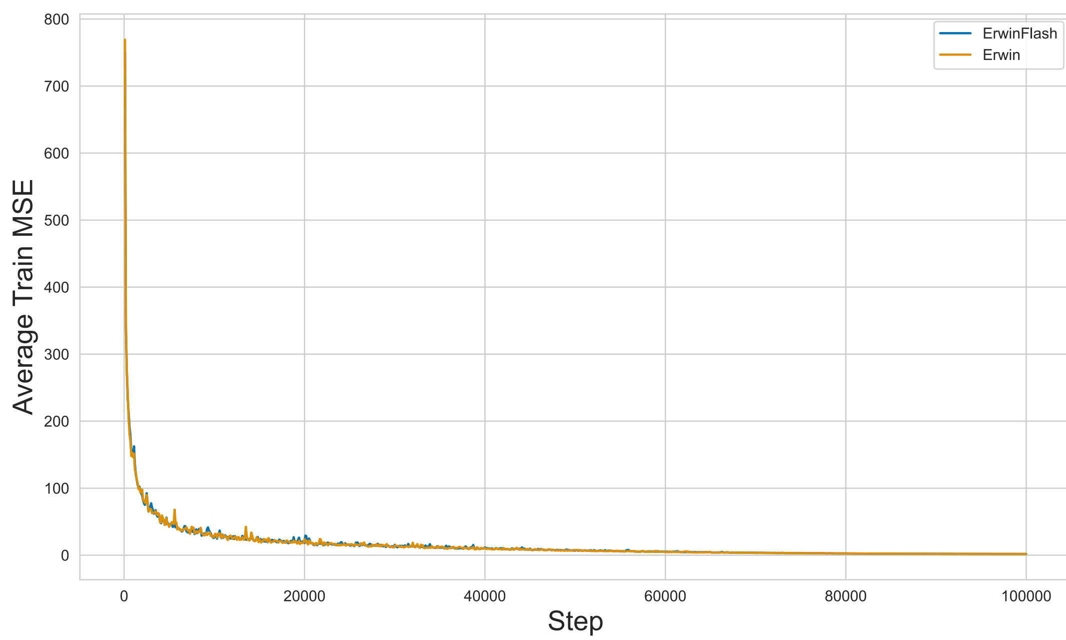
ErwinFlash Speed Improvement Results

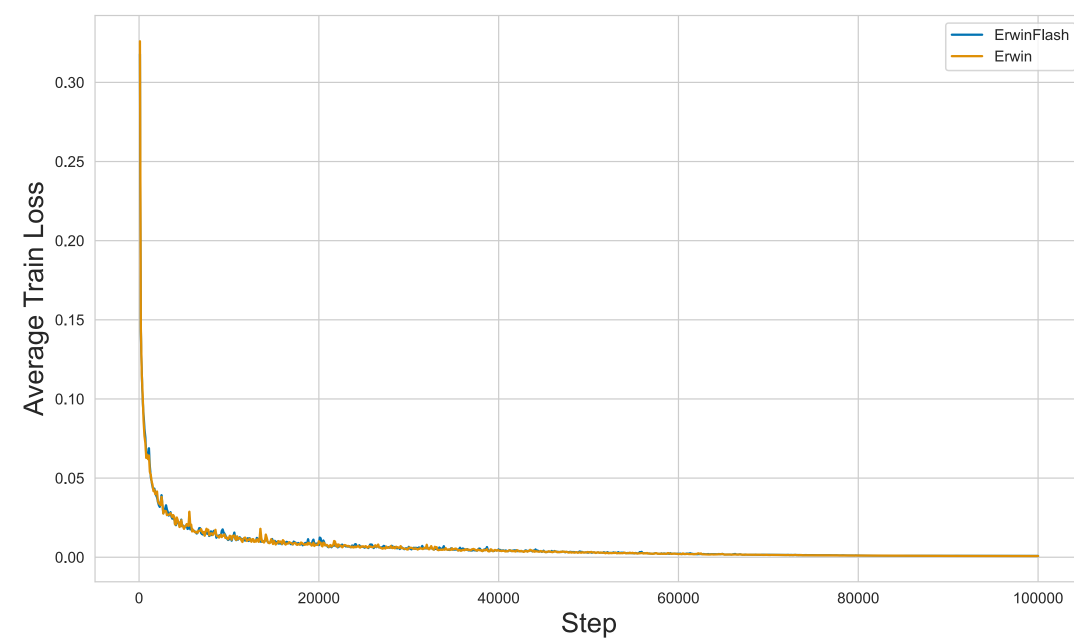
ErwinFlash

ShapeNet Car Results



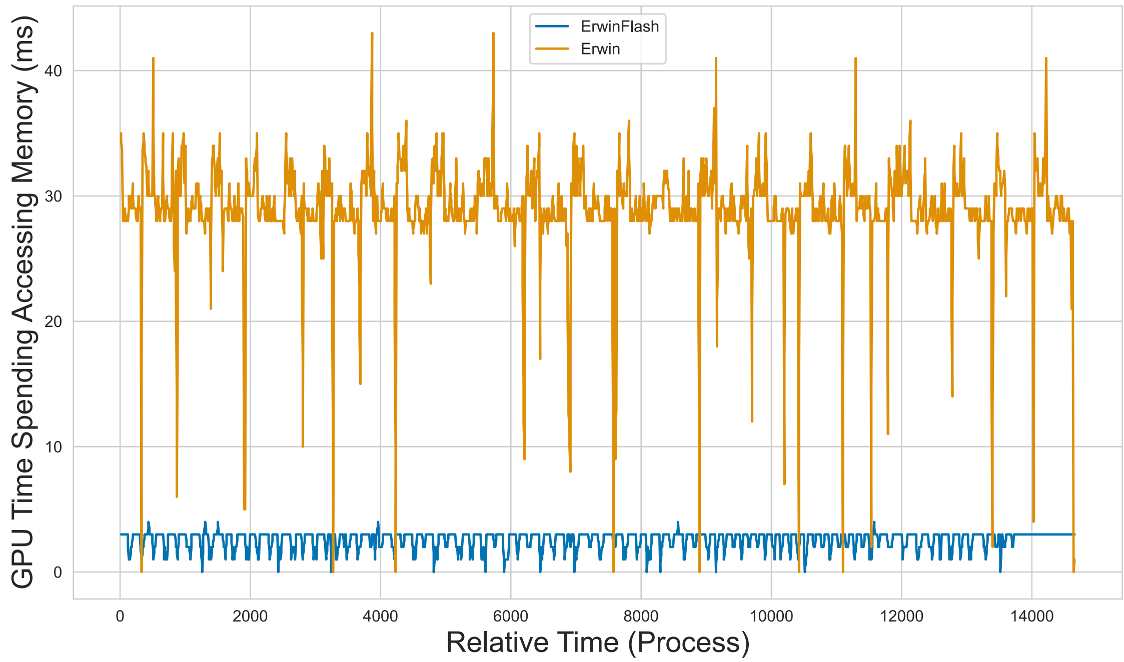


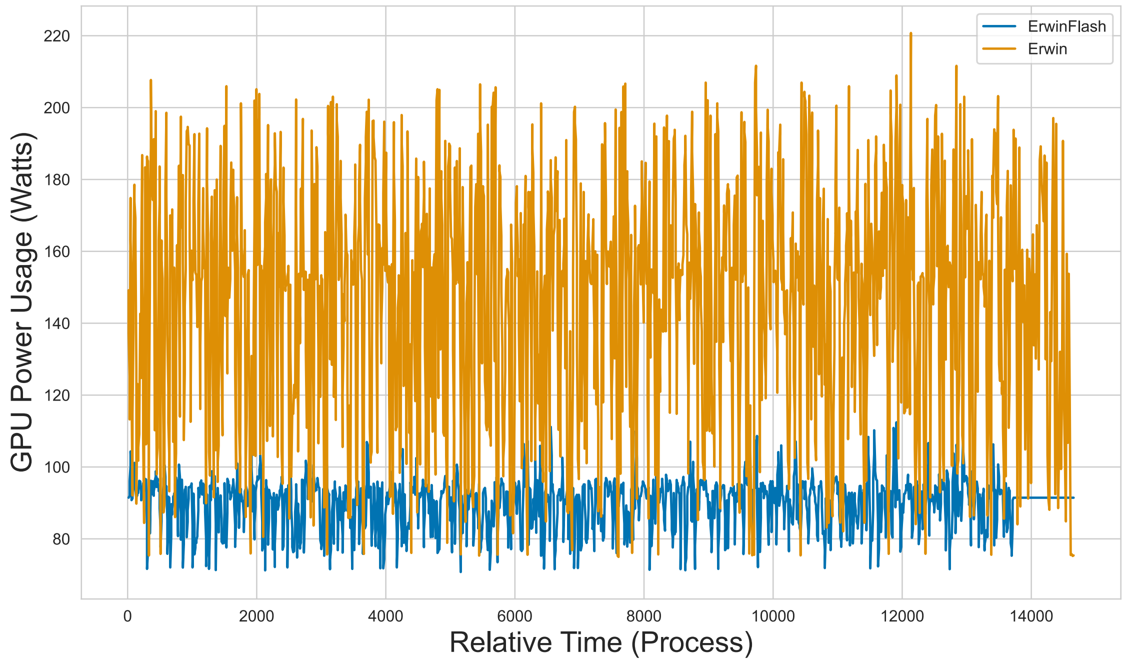


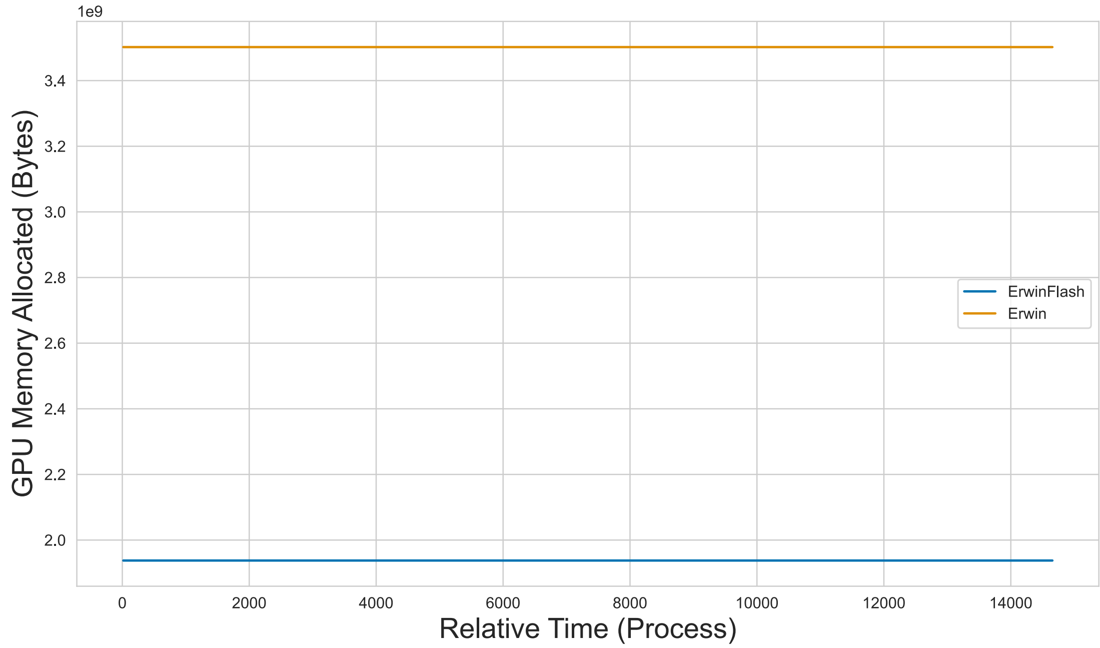


ErwinFlash

ShapeNet Car Results







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Hierarchical Attention Erwin Transolver

	Pros	Cons
Erwin	Scales Linearly	Cannot handle millions of input points
Transolver++	Can handle millions of input points “Downsampling” is physics-based	Attention between slices is still $O(N^2)$ Needs small number of slices
HAET	Can handle millions of points Hierarchical Attention between slices scales linearly Number of slices is not a bottleneck	Hierarchical attention within localized groups, i.e., Attention is not done among all slices

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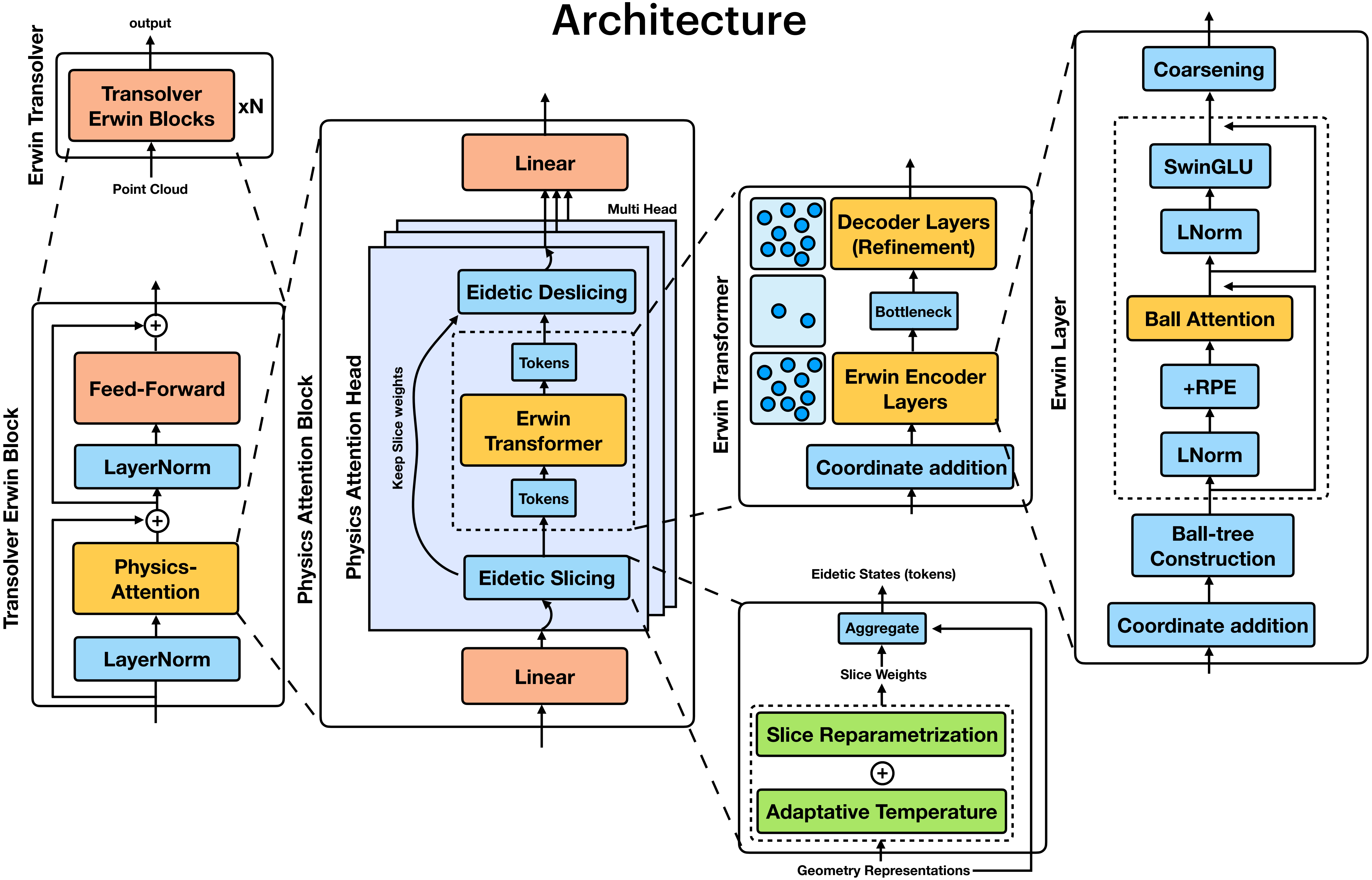
Architecture



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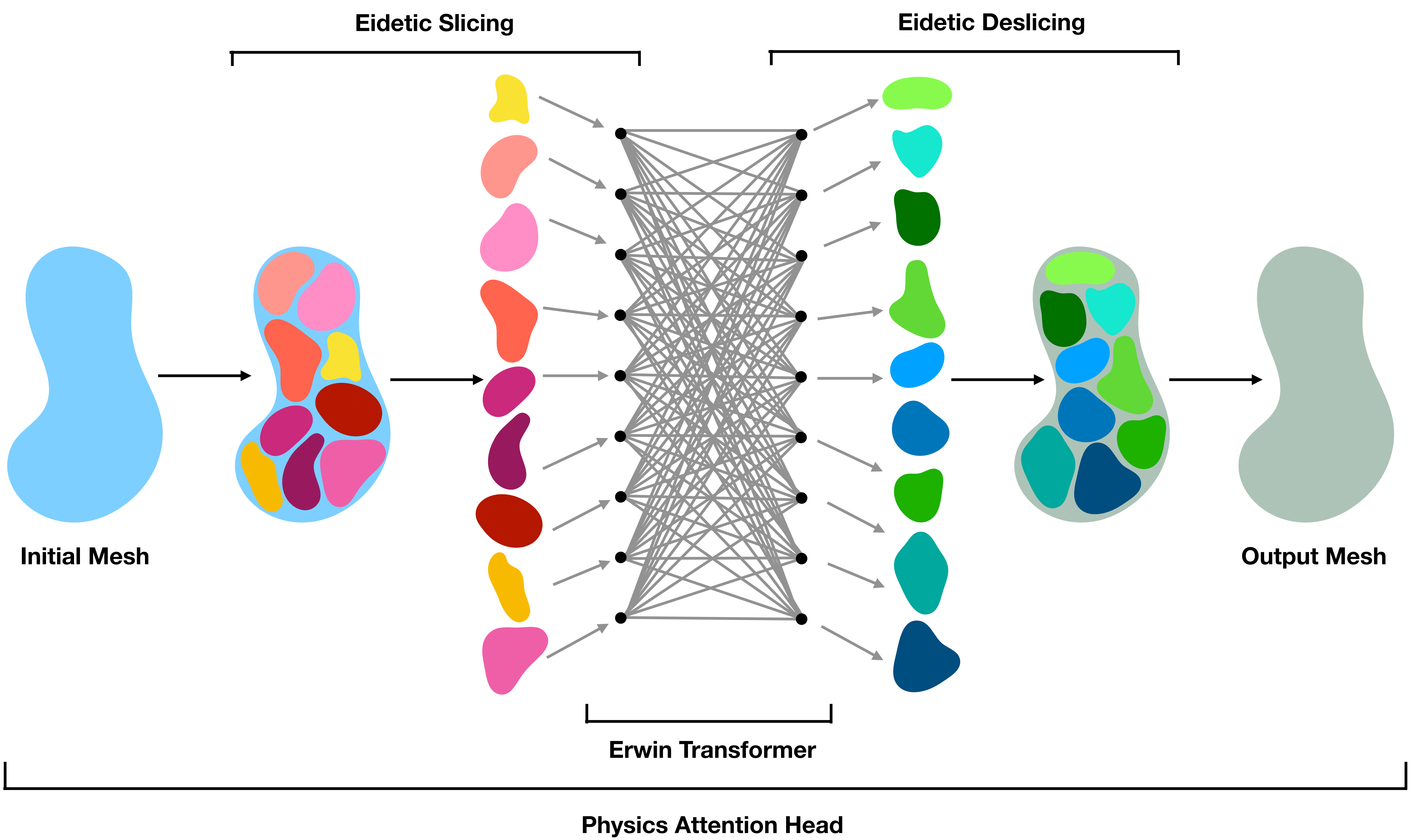
Physics Attention Head



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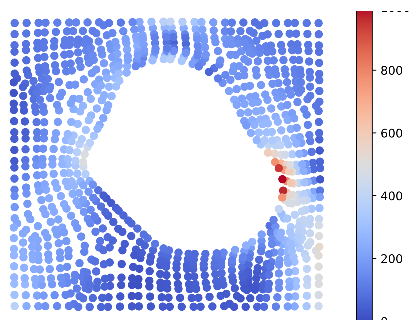
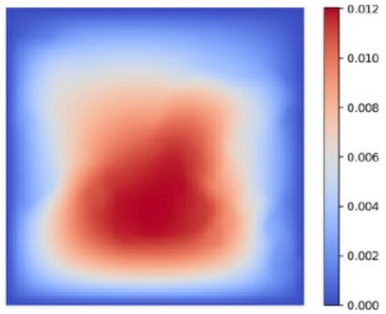
Standard Benchmarks

Elasticity



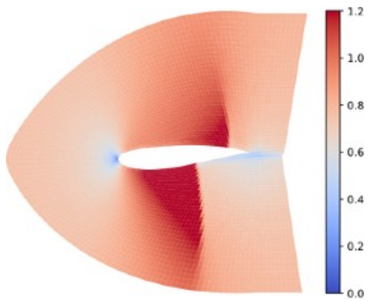
- Point Cloud
- 2D
- 972 Mesh Points

Darcy



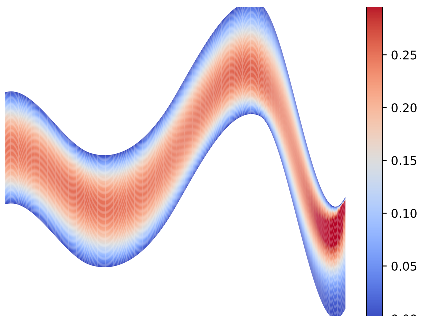
- Regular Grid
- 2D
- 7225 Mesh Points

Airfoil



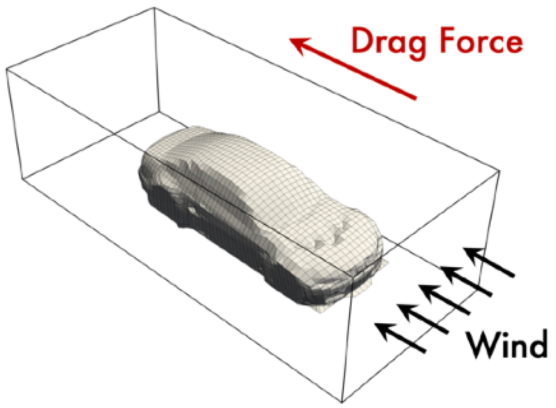
- Structured Mesh
- 2D
- 11271 Mesh

Pipe



- Structured Mesh
- 2D
- 16641 Mesh

ShapeNet Car Design



- Unstructured Mesh
- 3D
- 32186 Mesh Points

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Results

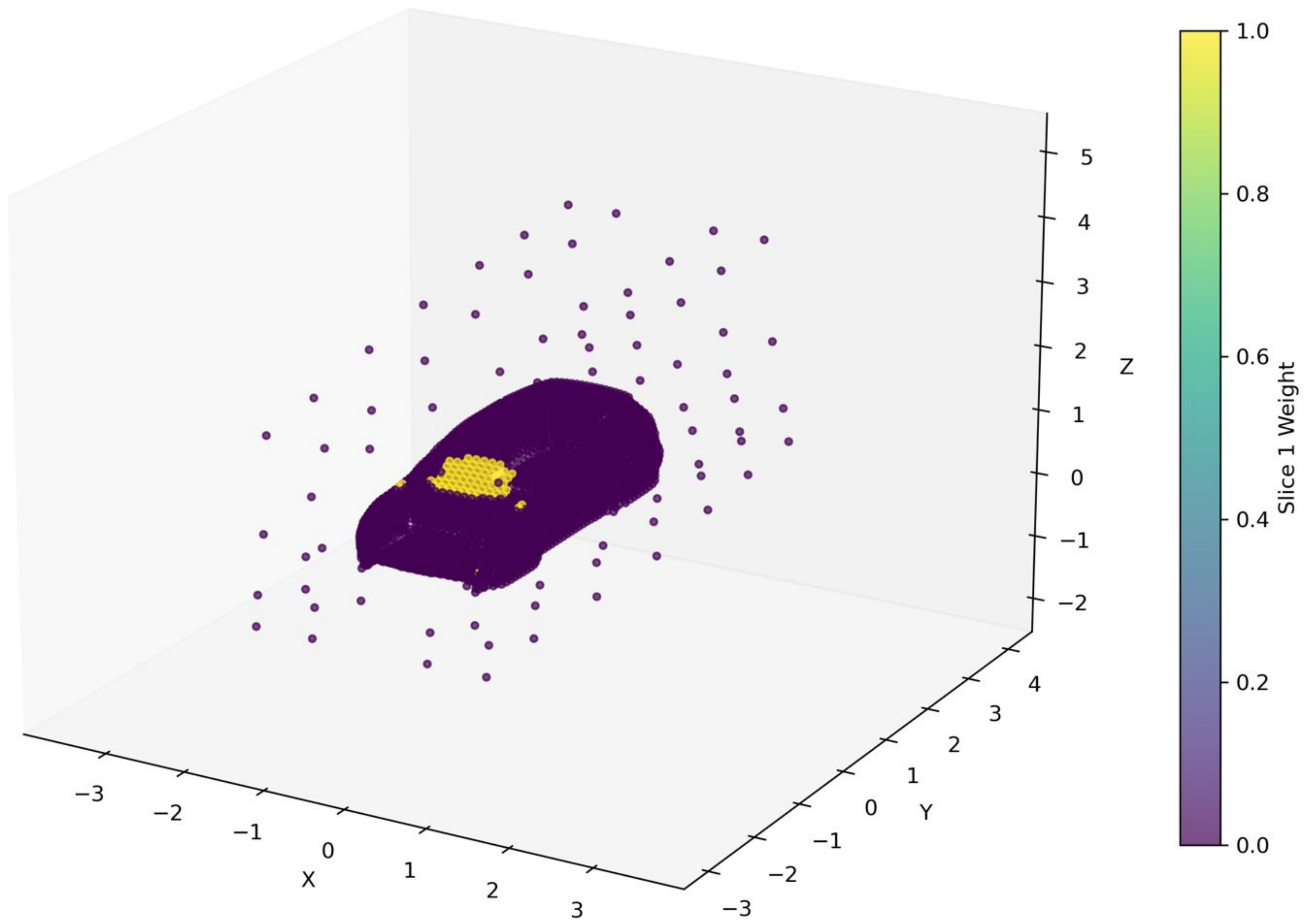
Model	Point Cloud	Structured Mesh			Regular Grid	
	Elasticity	Plasticity	Airfoil	Pipe	Navier–Stokes	Darcy
FNO	/	/	/	/	0.1556	0.0108
WMT	0.0359	0.0076	0.0075	0.0077	0.1541	0.0082
U-FNO	0.0239	0.0039	0.0269	0.0056	0.2231	0.0183
geo-FNO	0.0229	0.0074	0.0138	0.0067	0.1556	0.0108
U-NO	0.0258	0.0034	0.0078	0.0100	0.1713	0.0113
F-FNO	0.0263	0.0047	0.0078	0.0070	0.2322	0.0077
LSM	0.0218	0.0025	<u>0.0059</u>	0.0050	0.1535	0.0065
Galerkin	0.0240	0.0120	0.0118	0.0098	0.1401	0.0084
HT-Net	/	0.0333	0.0065	0.0059	0.1847	0.0079
OFormer	0.0183	<u>0.0017</u>	0.0183	0.0168	0.1705	0.0124
GNOT	<u>0.0086</u>	0.0336	0.0076	<u>0.0047</u>	0.1380	0.0105
FactFormer	/	0.0312	0.0071	0.0060	0.1214	0.0109
ONO	0.0118	0.0048	0.0061	0.0052	<u>0.1195</u>	0.0076
Transolver	**0.0064**	**0.0012**	**0.0053**	**0.0033**	**0.0900**	<u>0.0057</u>
HAET (Ours)	0.108	/	0.0085	0.0050	/	**0.0053**

Model	ShapeNet Car			
	Volume ↓	Surf↓	C_D ↓	ρ_D ↑
Simple MLP	0.0512	0.1304	0.0307	0.9496
GraphSAGE	0.0461	0.1050	0.0270	0.9695
PointNet	0.0494	0.1104	0.0298	0.9583
Graph U-Net	0.0471	0.1102	0.0226	0.9725
MeshGraphNet	0.0354	0.0781	0.0168	0.9840
GNO	0.0383	0.0815	0.0172	0.9834
Galerkin	0.0339	0.0878	0.0179	0.9764
Geo-FNO	0.1670	0.2378	0.0664	0.8280
GNOT	0.0329	0.0798	0.0178	0.9833
GINO	0.0386	0.0810	0.0184	0.9826
3D-GEOCA	0.0319	<u>0.0779</u>	<u>0.0159</u>	0.9842
Transolver	**0.0207**	**0.0745**	**0.0103**	**0.9935**
Erwin	0.0766	0.1335	0.1006	0.7681
HAET (Ours)	<u>0.0257</u>	0.0914	0.0174	<u>0.9865</u>

* PDE Benchmarks metrics: Relative L2

* ShapeNet metrics: Relative L2 of the surrounding (Volume) and surface (Surf) physics fields, drag coefficient, and their Spearman's rank correlations.

HAET
Results



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Computational Efficiency

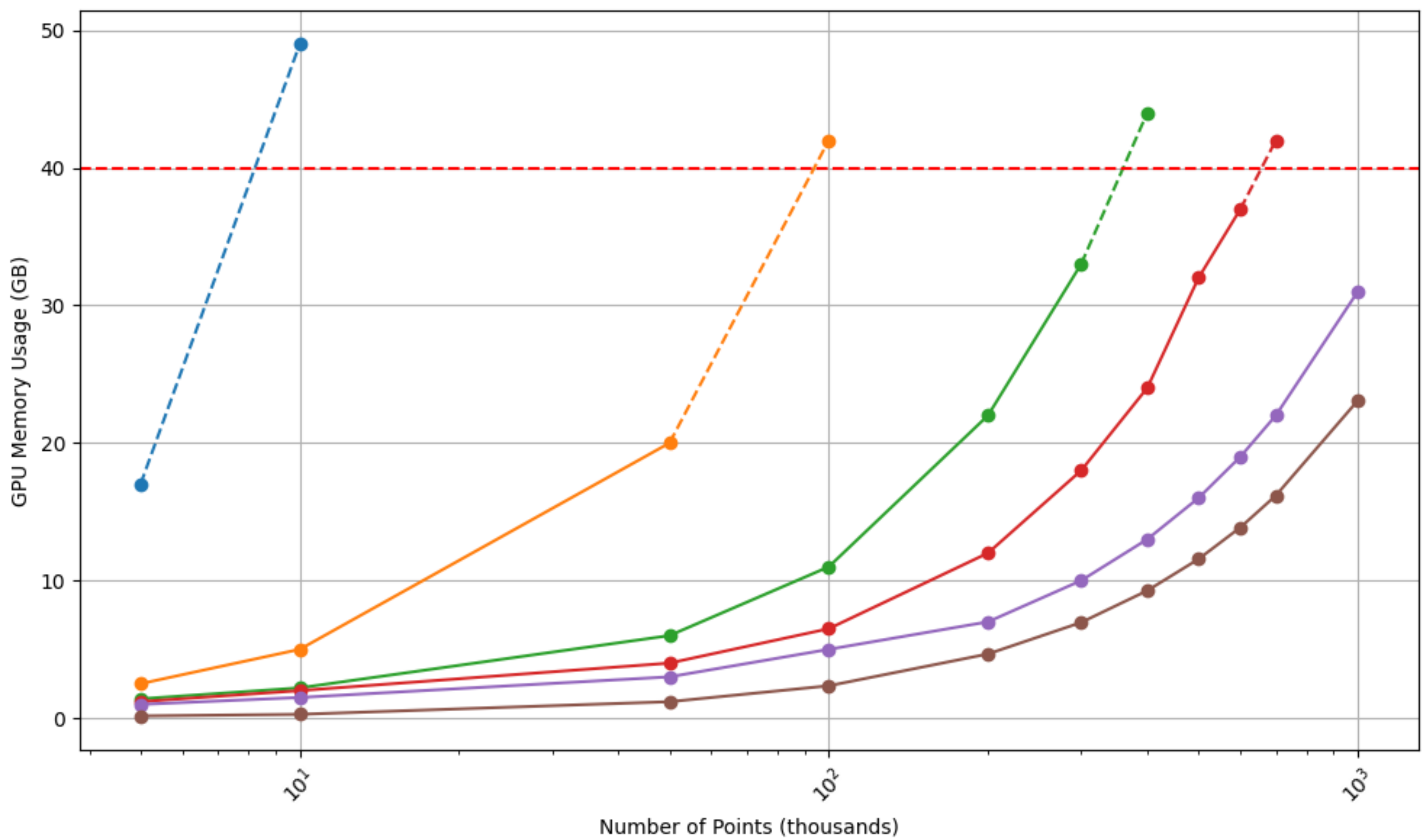

Figure 4: GPU memory usage across models as a function of input size. Models are color-coded as follows: —— **VanillaAttention**, —— **Galerkin**, —— **GNoT**, —— **Transolver**, —— **Transolver++**, —— **HAET**. Transolver++ and HAET demonstrate superior memory efficiency, remaining below the 40 GB threshold up to 1M points.

Conclusion and Future Work

- More efficient: less compute time, less memory
- Performance: close or better than SOTA models

- Future Work
  - Training on larger datasets that would require sharding both the model and the datasets
  - Trying a different task: Classification (e.g. Reduced-8 Dataset)
  - More ambitious: Same concept but using Diffusion instead of Attention

Thank You!

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Algorithm 1 Parallel Physics-Attention with Eidetic States

Input: Input features $\mathbf{x}^{(k)} \in \mathbb{R}^{N_k \times C}$ on the k-th GPU.
Output: Updated output features $\mathbf{x}'^{(k)} \in \mathbb{R}^{N_k \times C}$.
// drop $\mathbf{f}$ to save 50% memory.
Compute $\cancel{\mathbf{f}^{(k)}}, \mathbf{x}^{(k)} \leftarrow \text{Project}(\mathbf{x}^{(k)})$
Compute $\tau^{(k)} \leftarrow \tau_0 + \text{Ada-Temp}(\mathbf{x}^{(k)})$
Compute weights $\mathbf{w}^{(k)} \leftarrow \text{Rep-Slice}(\mathbf{x}^{(k)}, \tau^{(k)})$
Compute weights norm $\mathbf{w}_{\text{norm}}^{(k)} \leftarrow \sum_{i=1}^{N_k} \mathbf{w}_i^{(k)}$
Reduce slice norm $\mathbf{w}_{\text{norm}} \leftarrow \text{AllReduce}(\mathbf{w}_{\text{norm}}^{(k)})$ $\mathcal{O}(M)$
Compute eidetic states $\mathbf{s}^{(k)} \leftarrow \frac{\mathbf{w}^{(k)\top}\mathbf{x}^{(k)}\cancel{\mathbf{f}^{(k)}}}{\mathbf{w}_{\text{norm}}}$
Reduce eidetic states $\mathbf{s} \leftarrow \text{AllReduce}(\mathbf{s}^{(k)})$ $\mathcal{O}(MC)$
Update eidetic states $\mathbf{s}' \leftarrow \text{Attention}(\mathbf{s})$
Deslice back to $\mathbf{x}'^{(k)} \leftarrow \text{Deslice}(\mathbf{s}', \mathbf{w}^{(k)})$
Return $\mathbf{x}'^{(k)}$