UNIVERSITY OF AMSTERDAM

Faculty of Science

Exam

Machine Learning 1

Midterm Exam
Date: 25 September 2015
Time: 15:00 - 16:30

Number of pages: 4 (including front page)
Number of questions: 5
Maximum number of points to earn: 60
At each question is indicated how many points it is worth.

BEFORE YOU START

- Please **wait** until you are instructed to open the booklet.
- Check if your version of the exam is complete.
- Write down **your name, student ID number**, and if applicable the **version number** on **each sheet** that you hand in. Also **number the pages**.
- Your **mobile phone** has to be switched off and in the coat or bag. Your **coat and bag** must be under your table.
- **Tools allowed**: Nothing.

PRACTICAL MATTERS

- The first 30 minutes and the last 15 minutes you are not allowed to leave the room, not even to visit the toilet.
- You are obliged to identify yourself at the request of the examiner (or his representative) with a proof of your enrollment or a valid ID.
- During the examination it is not permitted to visit the toilet, unless the proctor gives permission to do so.
- 15 minutes before the end, you will be warned that the time to hand in is approaching.
- If applicable, please fill out the evaluation form at the end of the exam.

Good luck!

Questions

1. In Amsterdam, scooters are allowed to ride on the bike paths if they have a blue license plate, but are not allowed on the roads. The reverse is true for scooters with yellow license plates. The city estimates that at any given time, anywhere in the city, that a scooter on a bike path is yellow 5% of the time (i.e. the vast majority of the scooters stick to where they belong).

   One evening there is a hit-and-run accident between a scooter and a cyclist on a bike path. A witness tells police that the scooter had a yellow license plate. The police want to assess the reliability of the witness by testing him with different scooters under the same conditions the evening of the accident. The witness correctly identifies the colour of a license plate 8/10 times. In other words, if the police test the witness with a blue bike, the witness will claim they saw blue 8 of 10 tests with blue; the same is true for testing and claiming a yellow bike.

   We introduce a discrete random variable C for license plate colour that can take values y or b (yellow or blue). We are interested in the probability of the colour of a scooter's license plate *on the bike path.* We also introduce a discrete random variable W for the color that a witness claims to see that can take on values y and b (yellow or blue).

   Given this information, answer the following questions:

   (a) What is $P(C = b)$ and $P(C = y)$ on a bike path? **/1**

   (b) What is the probability that the accident was caused by a blue licensed scooter, if the witness claims it was blue? I.e. what is $P(C = b|W = b)$? **/3**

   (c) If there was no witness, what would be the probability that the accident was caused by a yellow plate? **/2**

2. Your friend is working on a research project and has been given a small set of training data, but has not been given the test set. Instead your friend's supervisor keeps the test set, but allows the student to send models (with trained model weights) and receive the test error back. Your friend is very frustrated because he sent two sets of weights to be tested, weights $\mathbf{w}_A$ and $\mathbf{w}_B$. For model A, on the training set, your friend computed a mean-squared-error of 0.01, but received back an error of 0.67 from his supervisor. For model B, on the training set, your friend had an error of 0.71 and on the test 0.69. Your friend explains that for model A he used a penalty of $\lambda = 0.001$, and after receiving the test results, tried model B where he used $\lambda = 10$.

   With this information, answer the following questions:

   (a) Which model is overfitting and which model is underfitting? **/1**

   (b) You explain a procedure for selecting λ to your friend that will try to avoid overfitting and underfitting and only requires the training set. What is the procedure called? What is the algorithm? Why does it work? Be clear but brief. **/6**

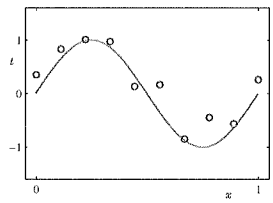
   (c) You apply the procedure to $\lambda \in \{0.001, 0.1, 1, 2, 5, 10\}$. Draw a graph that includes the error values in the question along with the results of the procedure (the values of $\log \lambda$ along the x-axis (equally spaced is ok), the values

of the error along the y-axis). You can decide how to interpolate the error values, they just need to be a plausible outcome of the procedure relative to the error values in the problem statement. Label plots and axes accordingly. Remember you have run $\lambda \in \{0.001, 10\}$ on the train and test, but not on your procedure. /4

(d) Indicate on the graph which regions are overfitting and which are underfitting. /1

(e) Indicate on the graph the value of λ your friend should select. /1

(f) Your friend is still confused about what is going on. You explain the bias-variance error decomposition to him. Reproduce the figure below 2 times. In one, plot the solution ($y(x, \mathcal{D})$) of a model with high-variance and low-bias and in the other plot, a model with low-variance and high-bias (both trained on the data set $\mathcal{D}$ shown as circles). Note the true regression function $h(\mathbf{x})$ is shown as a solid line. /2



(g) Consider the following error terms found in the expected loss:

i. $\int \{h(\mathbf{x}) - t\}^2 p(\mathbf{x}, t) d\mathbf{x} dt$

ii. $\int \{\mathbb{E}_{\mathcal{D}}[y(\mathbf{x}, \mathcal{D})] - h(\mathbf{x})\}^2 p(\mathbf{x}) d\mathbf{x}$

iii. $\int \{\mathbb{E}_{\mathcal{D}}[y(\mathbf{x}, \mathcal{D})] - y(\mathbf{x}, \mathcal{D})\}^2 p(\mathbf{x}) d\mathbf{x}$

Label the error terms as bias, variance, or noise. As modelers, which term(s) do we have control over? /3

3. Assume a classification problem with two classes $\mathcal{C}_0$ and $\mathcal{C}_1$. We observe the following data pairs: $\{t_n, x_n\} = \{(0, 1), (0, 1.5), (0, 2.0), (1, 2.5), (1, 3.0), (1, 3.5)\}$. Assume that if $t_n = 1$ the pair belongs to $\mathcal{C}_1$, otherwise to $\mathcal{C}_0$.

(a) Write down the prediction function $y_0(x, w_0, w_{00})$ and $y_1(x, w_1, w_{10})$ for a linear least-squares classifier. /2

(b) Write down the prediction function $y(x, w, w_0)$ for a logistic-regression classifier. /1

(c) What probability does the logistic-regression prediction function correspond to? /1

(d) Make a graph of the data and plot the prediction functions for the two classifiers. /2

(e) Imagine you now receive 3 more data pairs: $\{(1, 10), (1, 11), (1, 12)\}$. Draw another graph including the new and original data and prediction functions for both classifiers based on all the data. /2

(f) Explain why "too correct" data (the new pairs) affect the models differently by addressing the modeling assumptions and/or objective functions made by least-squares and logistic regression. /4

4. Consider the following general set-up. You have a data set of input-output pairs $\{\mathbf{t}, \mathbf{X}\}$, where $\mathbf{t}$ is an N by 1 vector of target values and $\mathbf{X}$ is an N by D matrix of input data. Assume that for model m there are parameters $\boldsymbol{\theta}_m$ and model hyper-parameters $\boldsymbol{\gamma}_m$. The likelihood function for the nth data pair is $p(t_n|\mathbf{x}_n, \boldsymbol{\theta}_m, \boldsymbol{\gamma}_m)$ and the prior distribution for $\boldsymbol{\theta}_m$ is $p(\boldsymbol{\theta}_m|\boldsymbol{\gamma}_m)$. E.g. if model m was the linear regression model studied in class, then $\boldsymbol{\gamma}_m = \{\alpha, \beta\}$ (the precisions of the prior and likelihood functions) and $\boldsymbol{\theta}_m = \mathbf{w}$, the regression weights. For the questions below, consider the general case, not the linear regression example.

   (a) Write down the exact form of **maximum log-likelihood** learning for this model. Write your answer in the form $\boldsymbol{\theta}_m = \arg\max_{\boldsymbol{\theta}_m} O(\boldsymbol{\theta}_m, \boldsymbol{\gamma}_m, \mathbf{t}, \mathbf{X})$, but you fill in the details of $O(\boldsymbol{\theta}_m, \boldsymbol{\gamma}_m, \mathbf{t}, \mathbf{X})$ using the definitions in the problem statement. **/2**

   (b) Do the same for **maximum a-posteriori log-likelihood** learning. **/2**

   (c) Write down the expression for the **evidence** for model m using the product and sum rule. **/2**

   (d) Write down the expression for the **posterior distribution** of $\boldsymbol{\theta}_m$, using the general probability densities defined in the problem statement. Label the likelihood, prior, evidence terms. Ensure that all the conditioning statements are correct. **/3**

   (e) Describe one way that the posterior distribution can be used to make predictions for new input vectors $\mathbf{x}^\star$. You can use words or write the expression. **/3**

   (f) Assume that there is an analytic solution to the evidence computation in part (c) above, for both model m and also for another model s with hyperparameters $\boldsymbol{\gamma}_s$. How can we use these analytic solutions to select the best model? **/2**

5. Imagine you have written some computer vision software for a flying drone. Your software will predict—10 times per second—whether the drone will hit a tree in the next second or whether there is no tree. In other words, can compute $P(C_0 = \text{tree}|\mathbf{x})$ (which implies $P(C_1 = \text{no tree}|\mathbf{x}) = 1 - P(C_0 = \text{tree}|\mathbf{x})$). The action associated with predicting "tree" is to quickly move (swerve) perpendicular to the drone's current direction (a_0: action="swerve"). The action associated with predicting "no tree" is to continue following the current direction (a_1: action = "continue"). You estimate a loss of 100 if the drone hits a tree, and a loss of 1 every time the drone unnecessarily swerves to avoid a non-existent tree. With this information answer the following questions:

   (a) Write down the loss matrix associated with this problem. Make sure the rows and columns are labeled. **/3**

   (b) At one moment the drone predicts $P(C_0|\mathbf{x}) = 0.15$. The drone needs to make a decision. Compute the expected losses for each possible decision/action. What action will the drone take? **/5**

   (c) What value of the prediction $P(C_0|\mathbf{x})$ will cause the drone to be unable to make a decision? **/2**