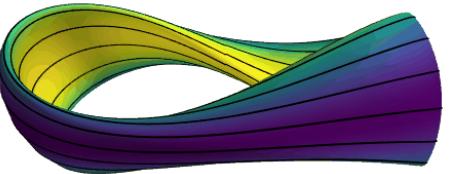


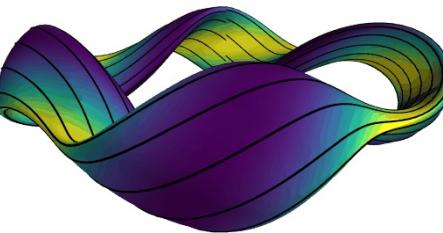
The Direct Optimization Framework III Stellarator Design: Transport and Turbulence Optimization

What quantities can we now directly optimize for?

How to combine several of these quantities together?



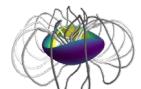
Rogerio Jorge¹



EUROfusion Enabling Research Team, Lisbon¹, A. Goodman², M. Landreman³, S. Buller³, A. Giuliani⁴, F. Wechsung⁴, P. Kim³, W. Sengupta⁵, N. R. Mandell⁵, T. Qian⁵, W. Dorland^{3,5}, M. Padidar⁶, the SIMSOPT team

¹ IST, University of Lisbon, ² Max Planck Institute for Plasma Physics, ³ University of Maryland, ⁴ New York University,

⁵ Princeton Plasma Physics Laboratory, ⁶Cornell University



PART I

Introducing stellarators

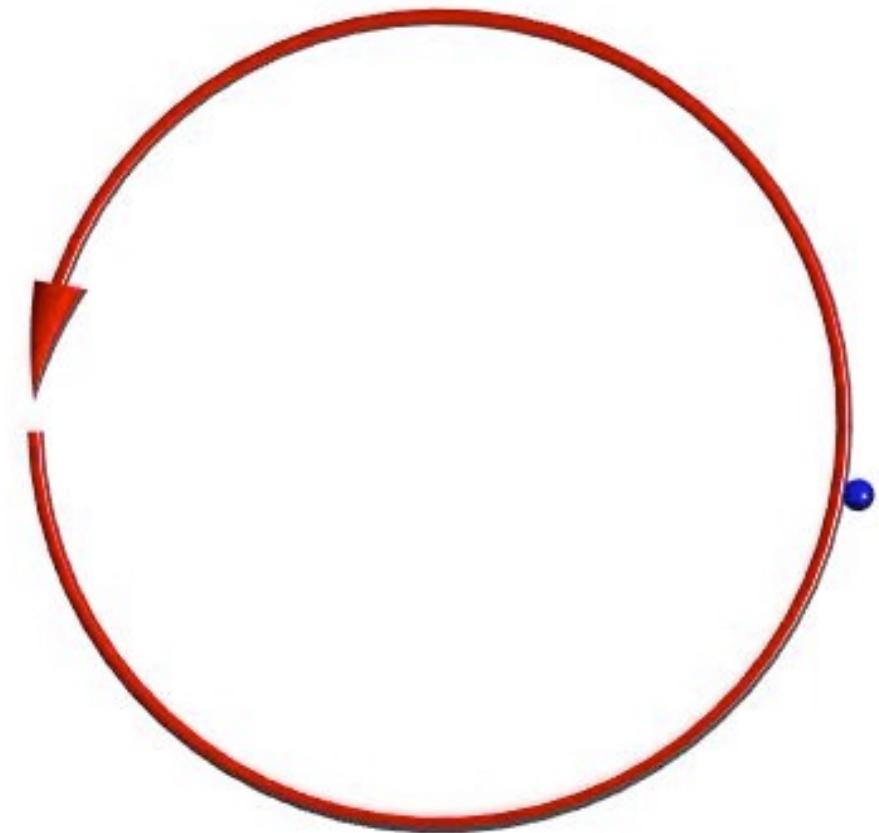
How to use magnetic fields to confine plasmas?

Losses at the ends

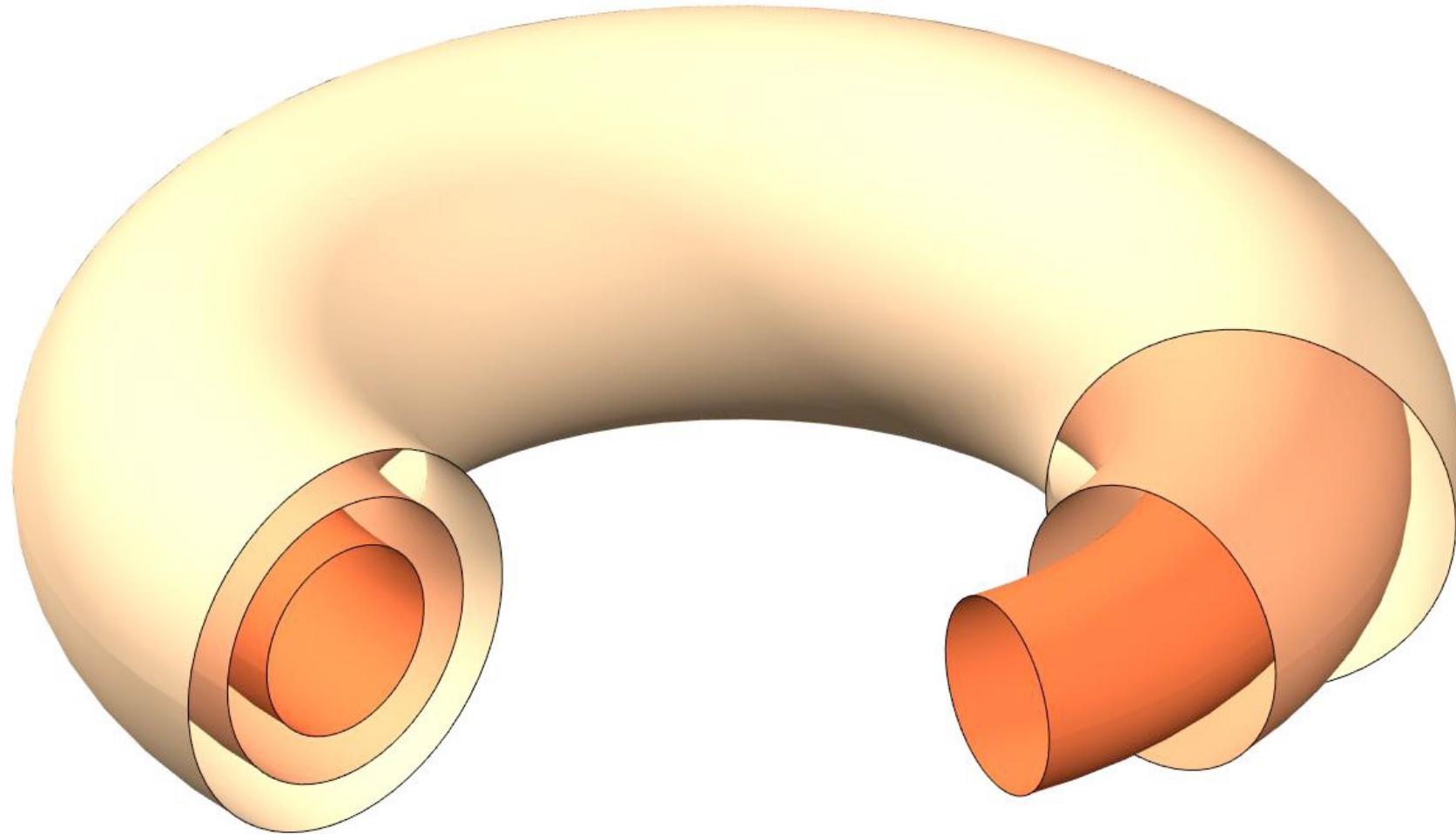


Vertical losses

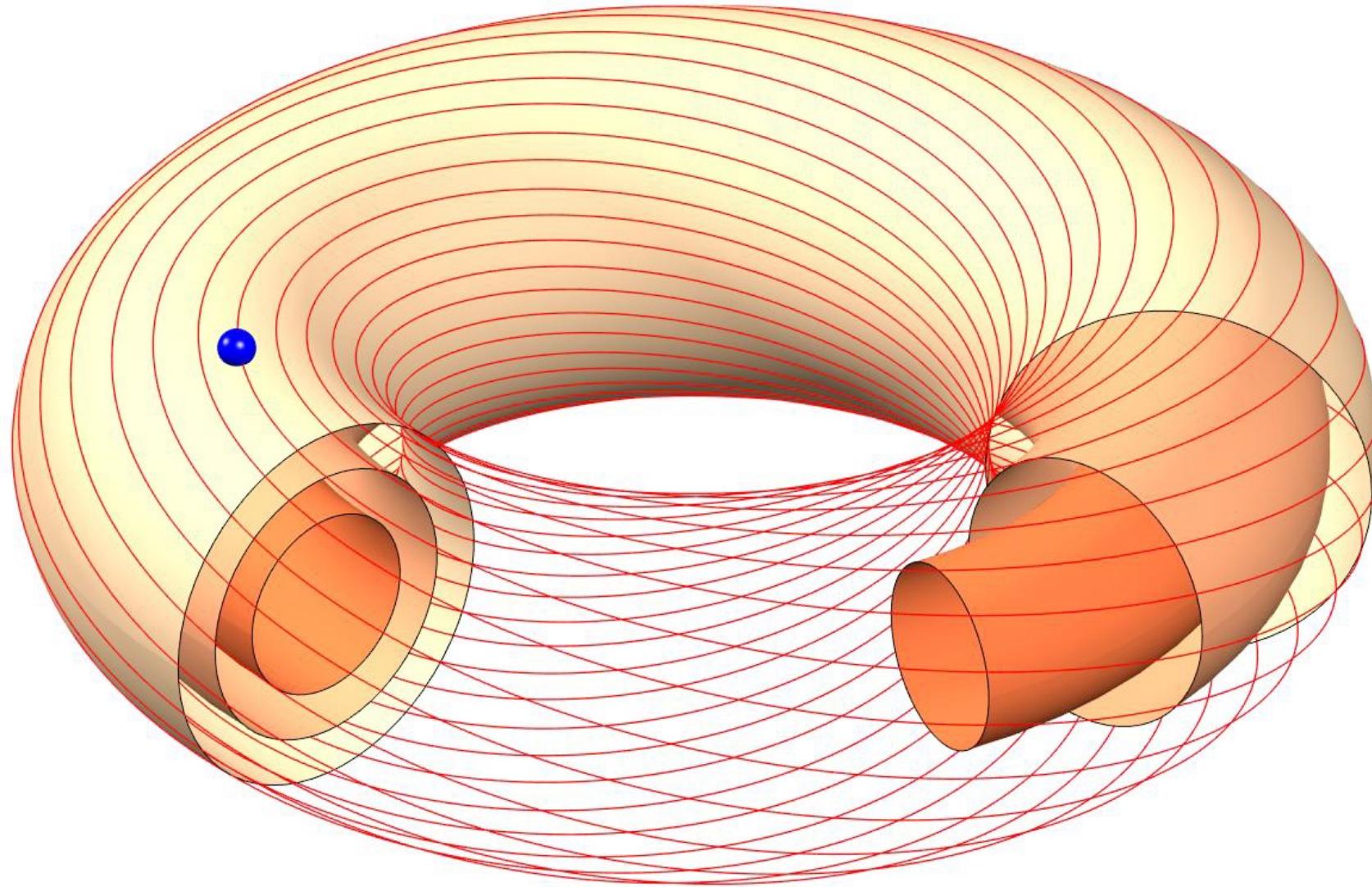
(the so-called drifts)



Solution: wrap field lines around a toroidal surface

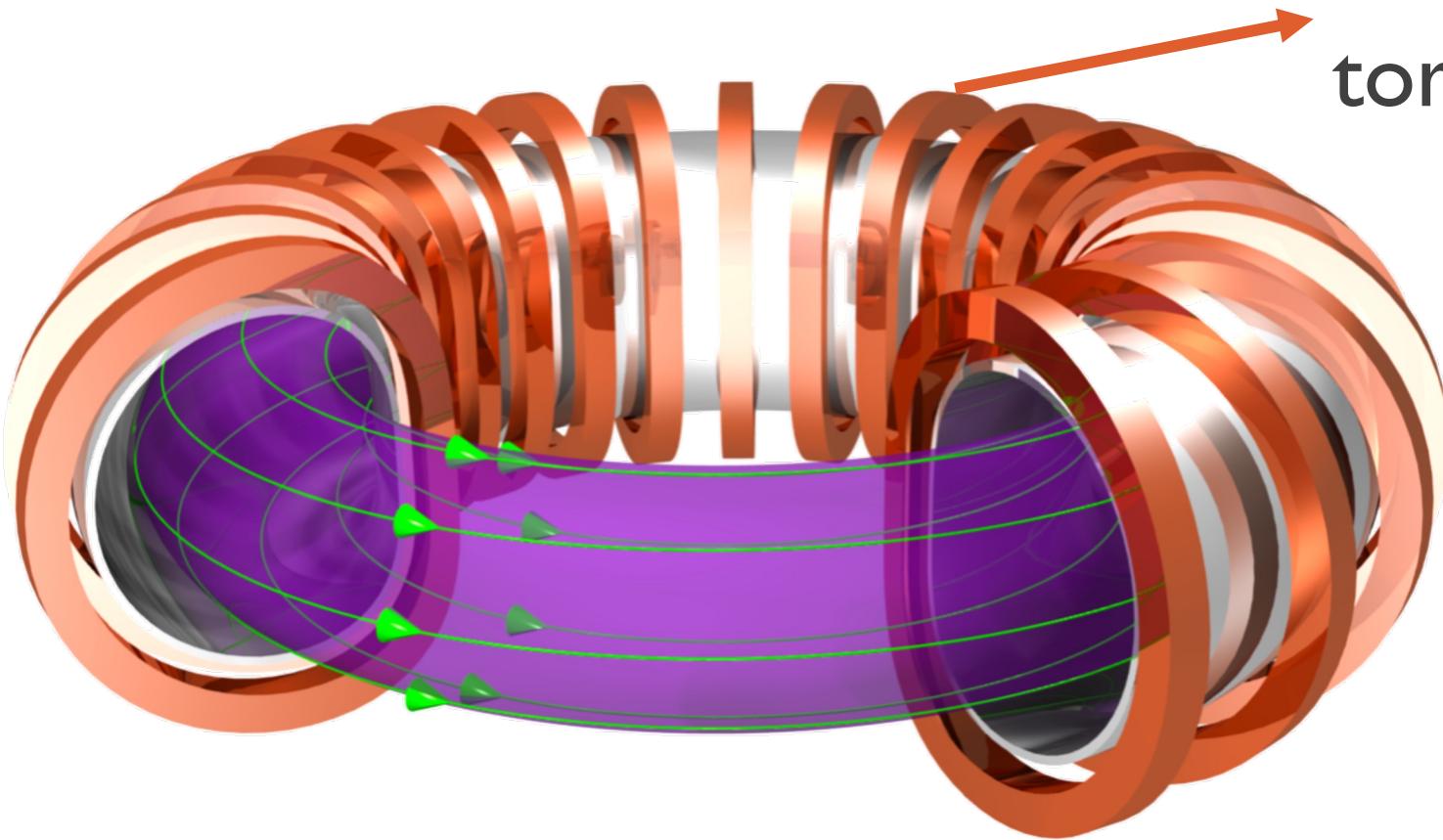


Particles are now confined



Two ways of creating nested magnetic surfaces

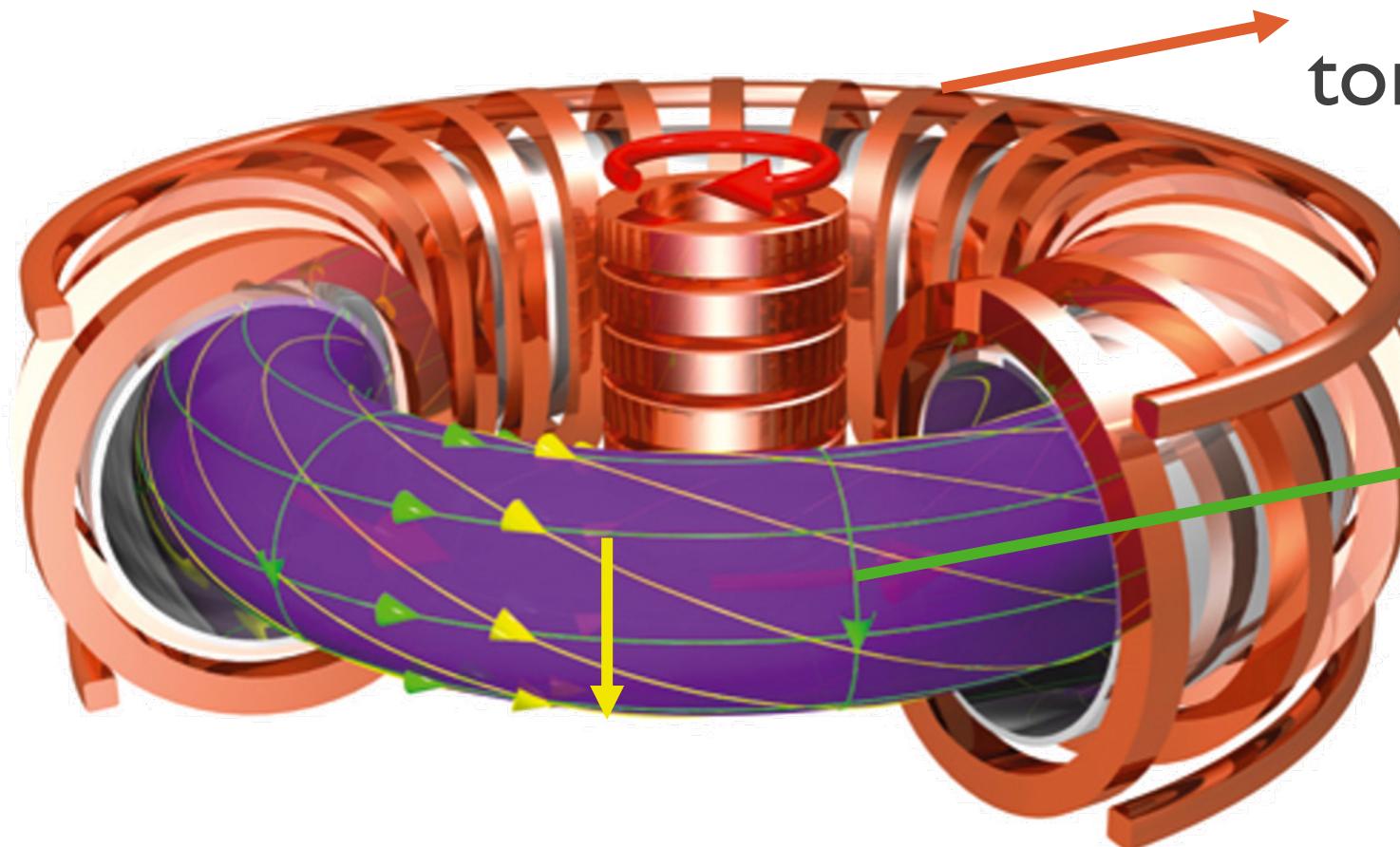
I. Tokamaks



Use coils to create a toroidal magnetic field

Two ways of creating nested magnetic surfaces

I. Tokamaks



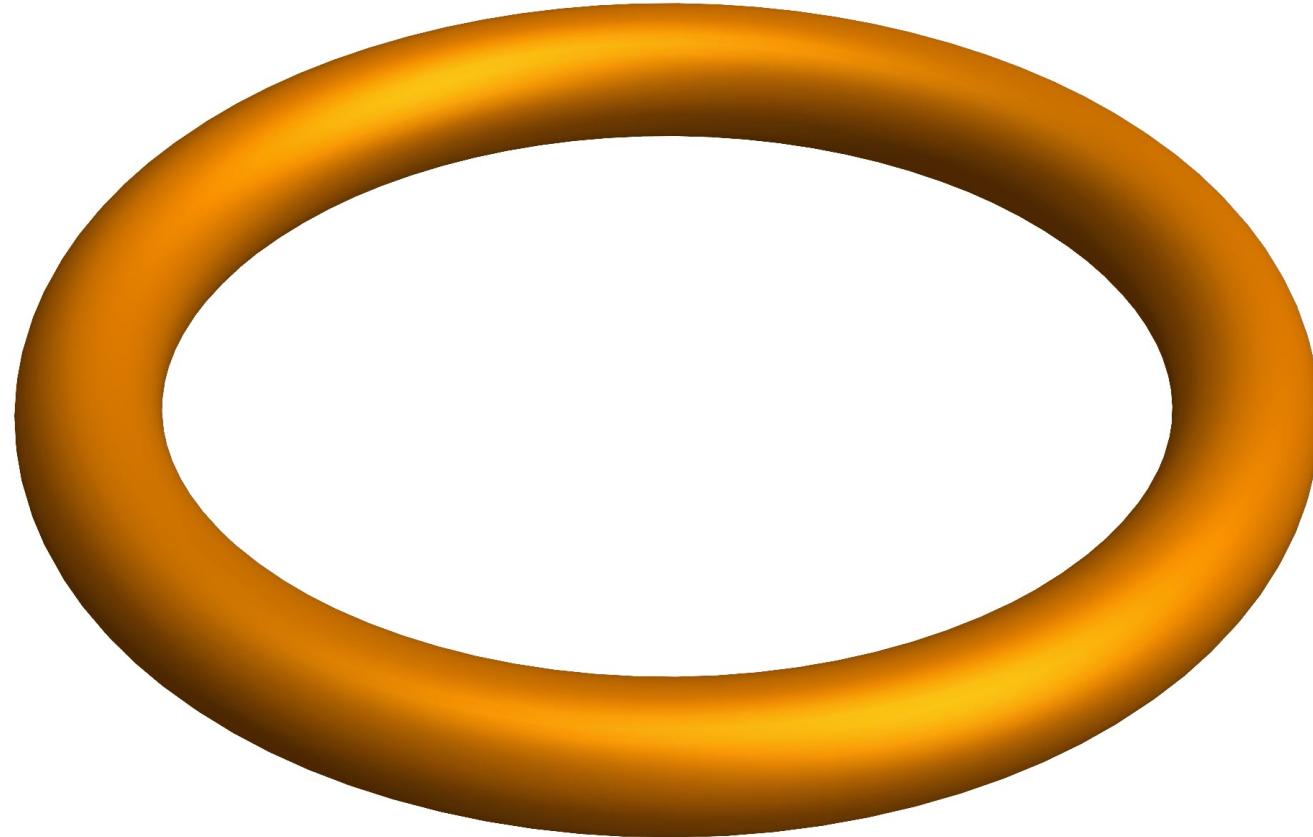
Use coils to create a toroidal magnetic field

Create extra poloidal field using a plasma current

To have nested surfaces, tokamaks need an internal current that can provide free energy for large scale instabilities

Two ways of creating nested magnetic surfaces

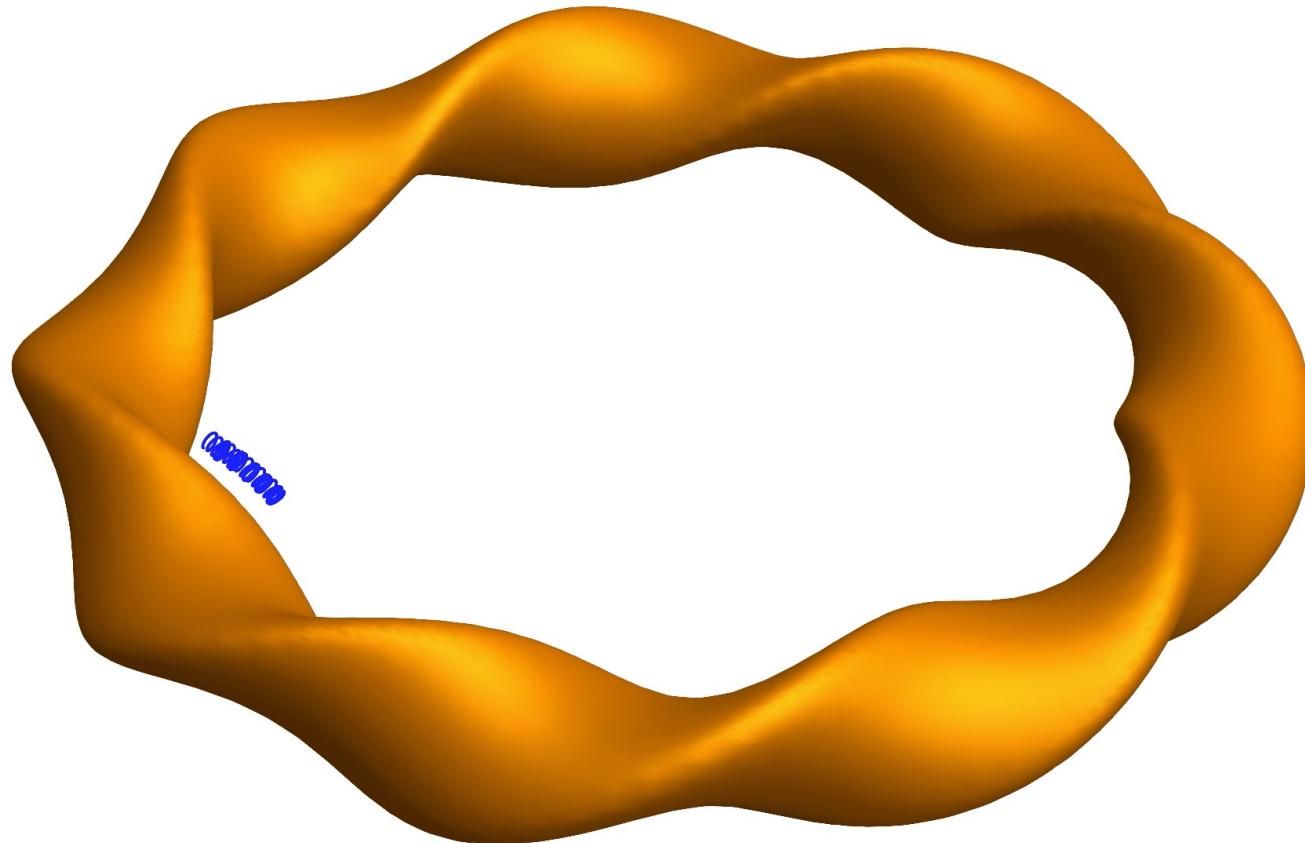
2. Stellarators



Break toroidal
symmetry

Two ways of creating nested magnetic surfaces

2. Stellarators

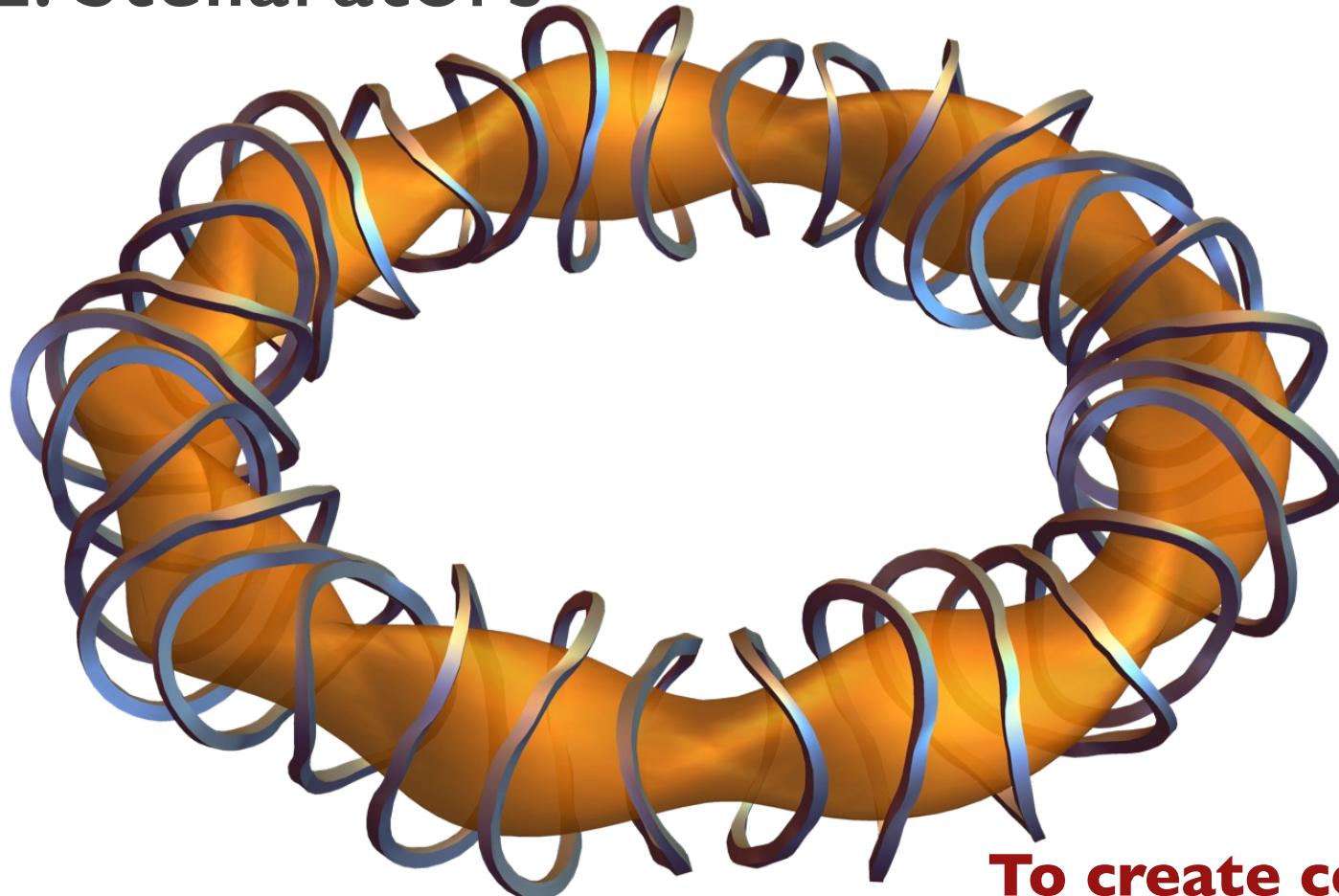


Break toroidal
symmetry

Use extra freedom to
confine particles

Two ways of creating nested magnetic surfaces

2. Stellarators



Break toroidal
symmetry

Use extra freedom to
confine particles

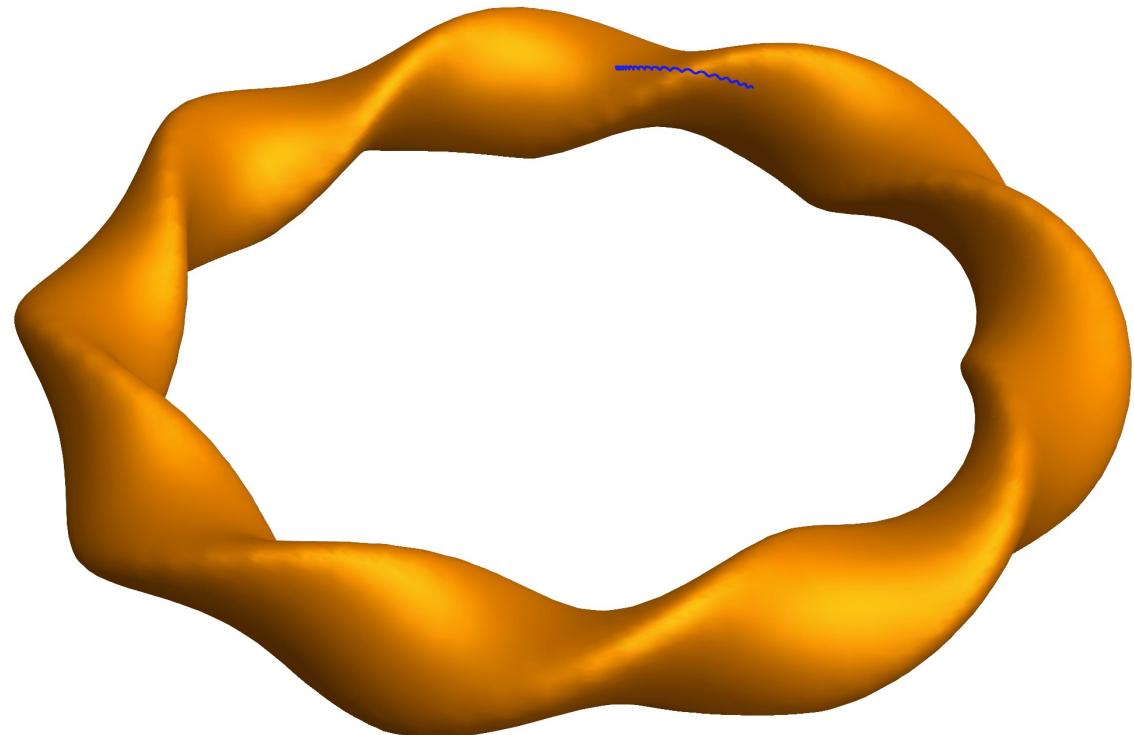
To create complex shapes

stellarators require complex

Stellarator optimization

Radial Displacement

Turn this

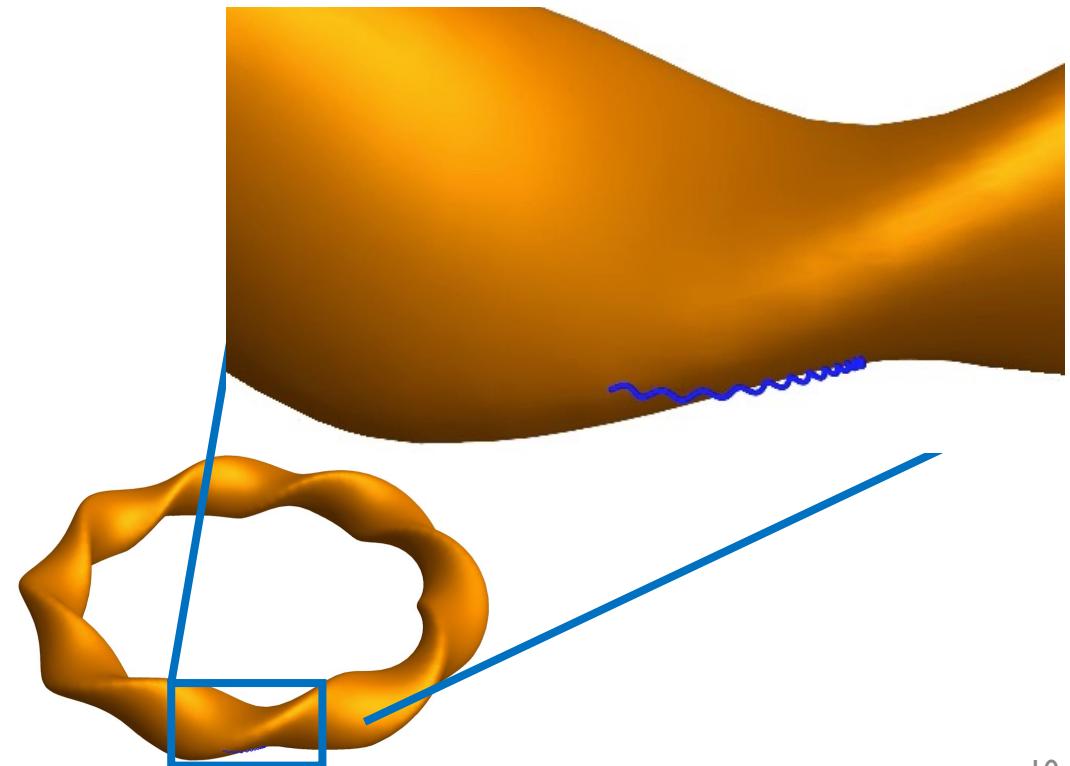


$$\Delta\psi = \oint dt \underbrace{\mathbf{v}_d \cdot \nabla\psi}_{\text{Guiding center drift}} = 0 \quad (\text{averages to zero})$$

Radial directions

Guiding center drift

Into this



How does the tokamak confine all particles?

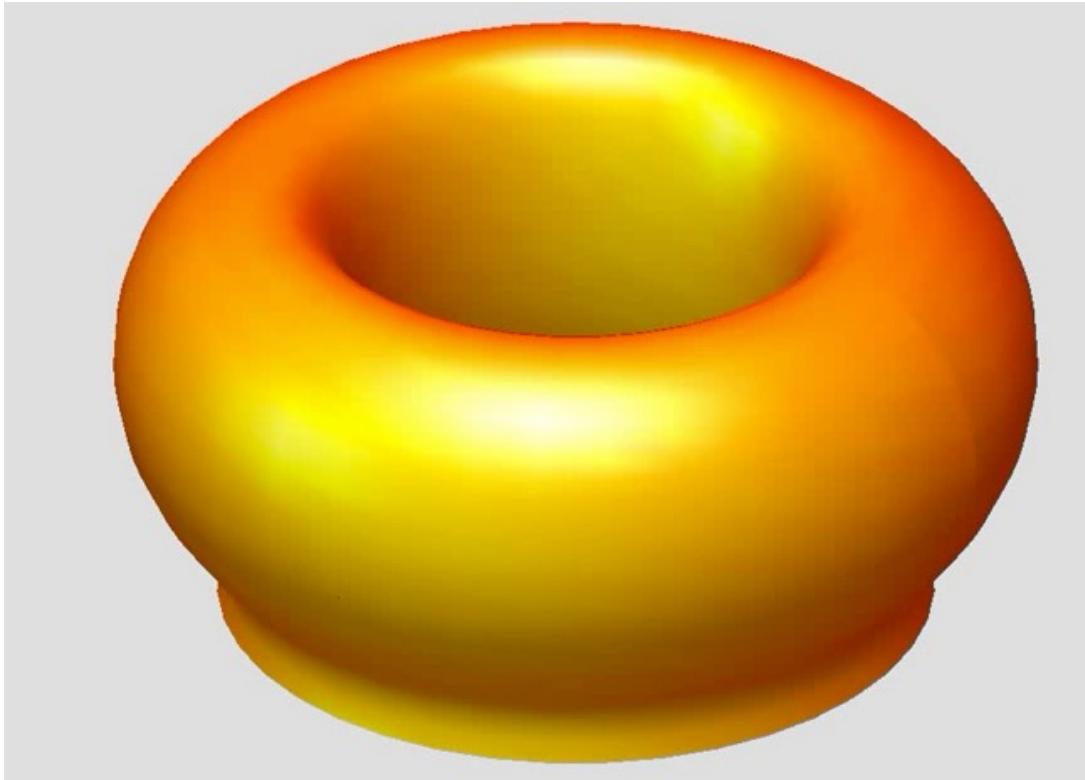
Tokamak B field is
axisymmetric



Conserved canonical
angular momentum



Confined trajectories



Can stellarators do the same?

Find a hidden symmetry in the Lagrangian

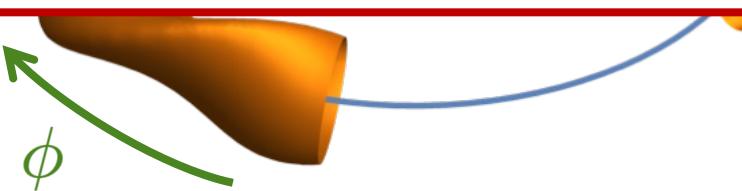
Lagrangian for a charged particle

$$L = \frac{m|\dot{\mathbf{x}}|^2}{2} + q\mathbf{A} \cdot \dot{\mathbf{x}}$$

Simplify L by removing fast gyromotion

L depends only on the angles θ and ϕ via $B = |\mathbf{B}|$

If $\partial B / \partial \phi = 0$, the canonical angular momentum $p_\phi = \partial L / \partial \dot{\phi}$ is conserved!



This symmetry idea was the basis of the Simons collaboration

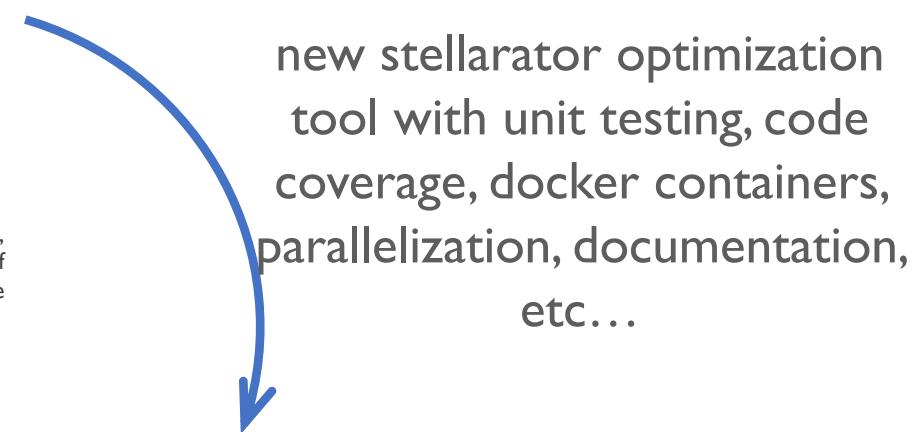


Drs. Jim and Marilyn Simons

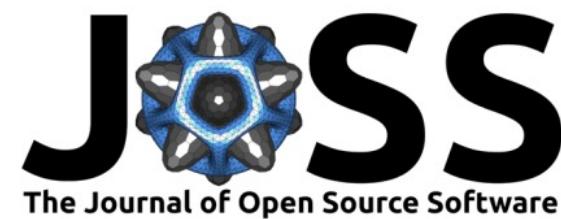
Hidden Symmetries and Fusion Energy Collaboration

Founding PIs - USA, Australia, Germany, Switzerland, England

University of Maryland, Princeton Plasma Physics Laboratory, Cornell University, Columbia University, New York University, University of Texas-Austin, University of Wisconsin-Madison, University of Colorado-Boulder, EPFL, Max-Planck Institute, The Australian National University



new stellarator optimization tool with unit testing, code coverage, docker containers, parallelization, documentation, etc...



Editor: [Dan Foreman-Mackey](#)

Reviewers:

- [@ZedThree](#)
- [@StanczakDominik](#)

Submitted: 21 June 2021

Published: 10 September 2021

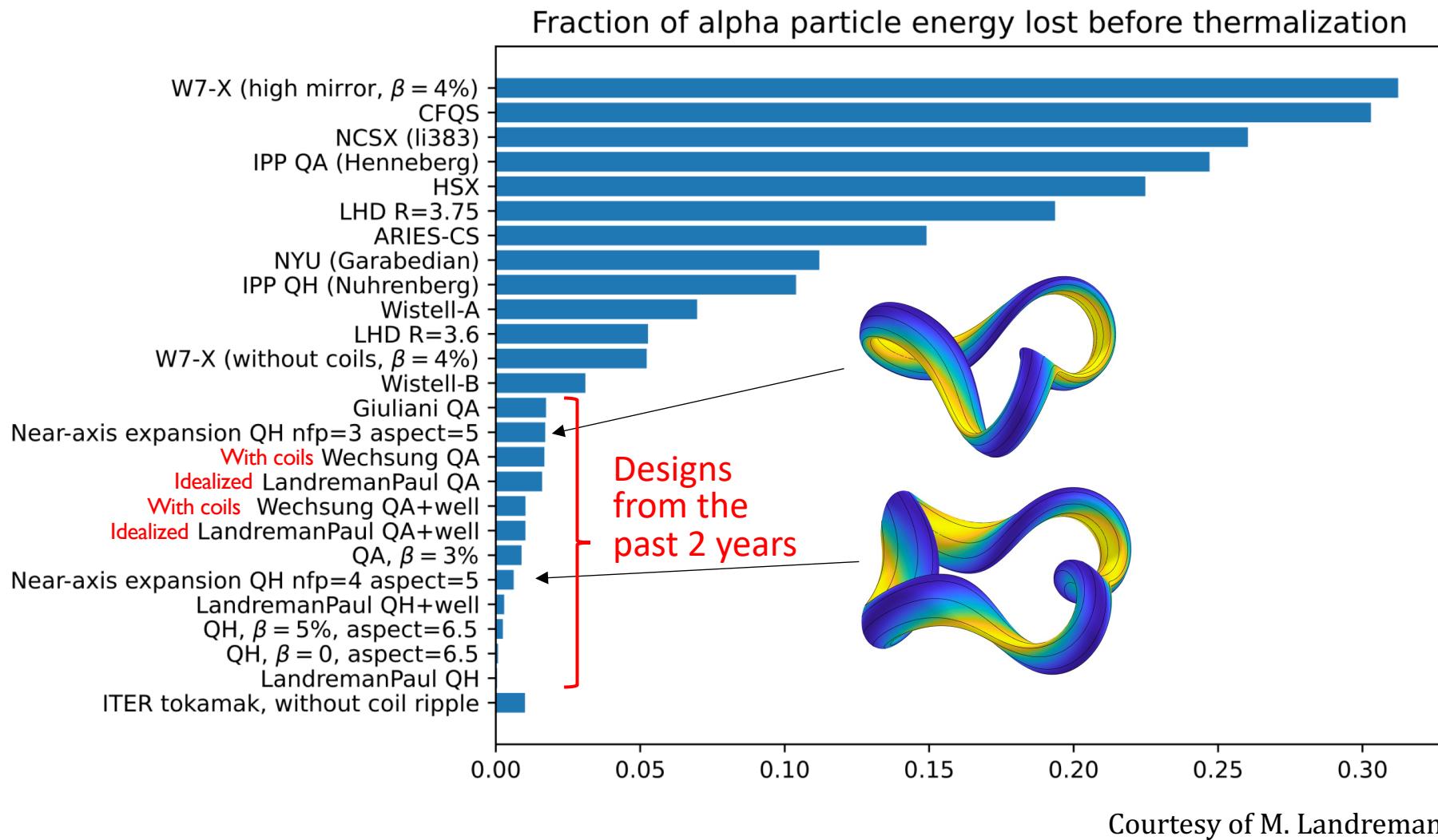
[hiddenSymmetries / simsopt](#) Public

SIMSOPT: A flexible framework for stellarator optimization

Matt Landreman¹, Bharat Medasani², Florian Wechsung³, Andrew Giuliani³, Rogerio Jorge¹, and Caoxiang Zhu²

¹ Institute for Research in Electronics and Applied Physics, University of Maryland, College Park 2 Princeton Plasma Physics Laboratory 3 Courant Institute of Mathematical Sciences, New York University

Which led to tremendous progress in the stellarator world



PHYSICAL REVIEW LETTERS

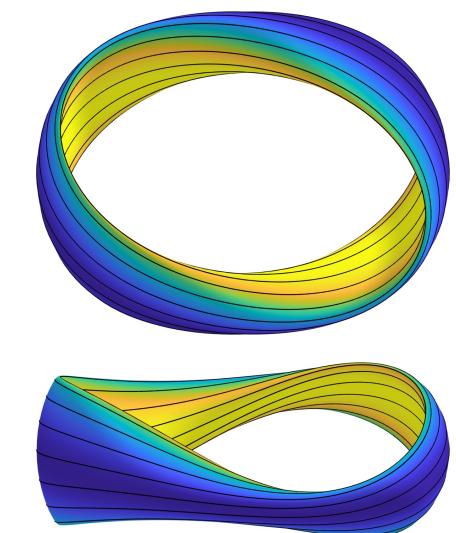
Featured in Physics Edition Suggestion

Magnetic Fields with Precise Quasisymmetry for Plasma Confinement

Matt Landreman and Elizabeth Paul
Phys. Rev. Lett. **128**, 035001 – Published 18 January 2022

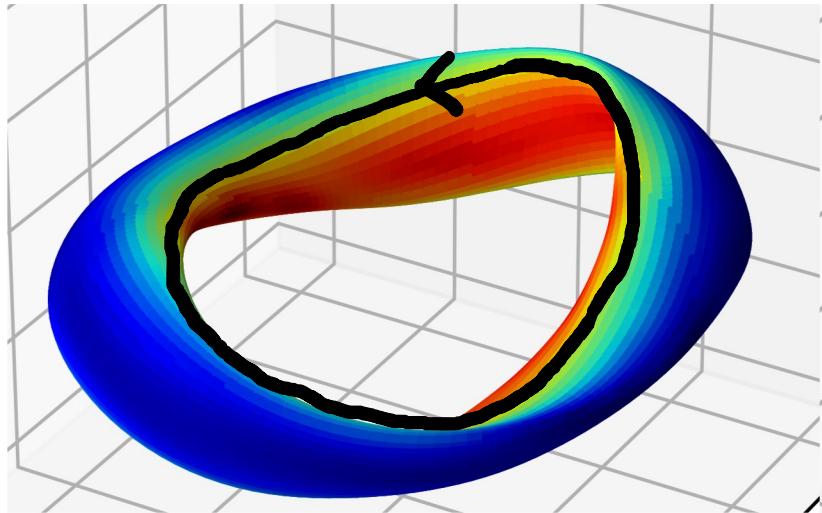
Physics See Viewpoint: Quasisymmetric Stellarators

Article References Citing Articles (17) Supplemental Material PDF HTML

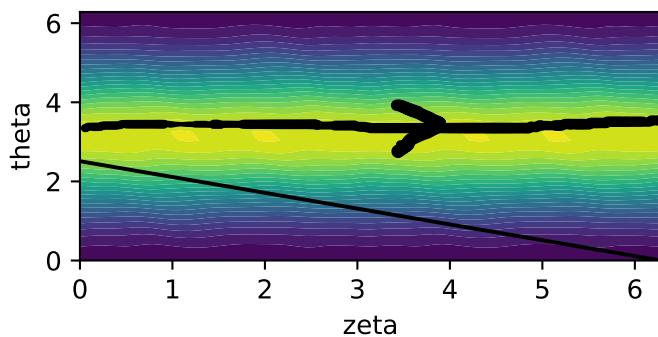


Stellarator optimization – optimize $|B|$

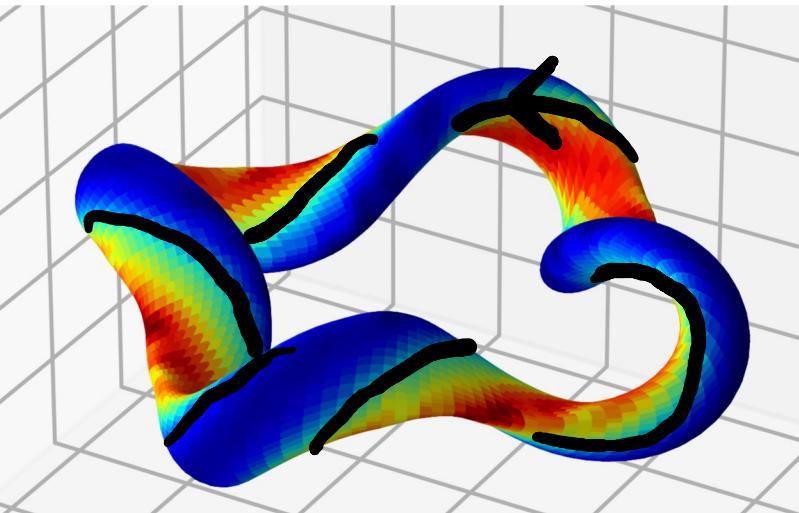
Quasi-axisymmetry



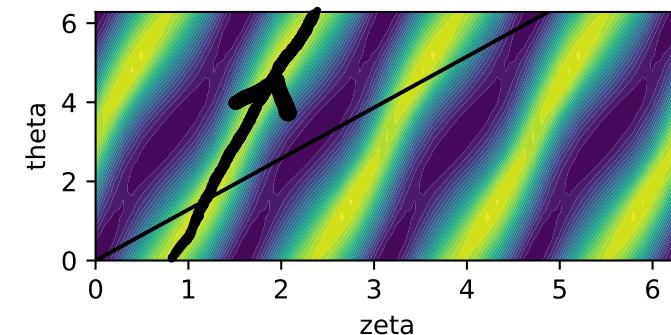
$|B|$ at half radius
1-based index=64



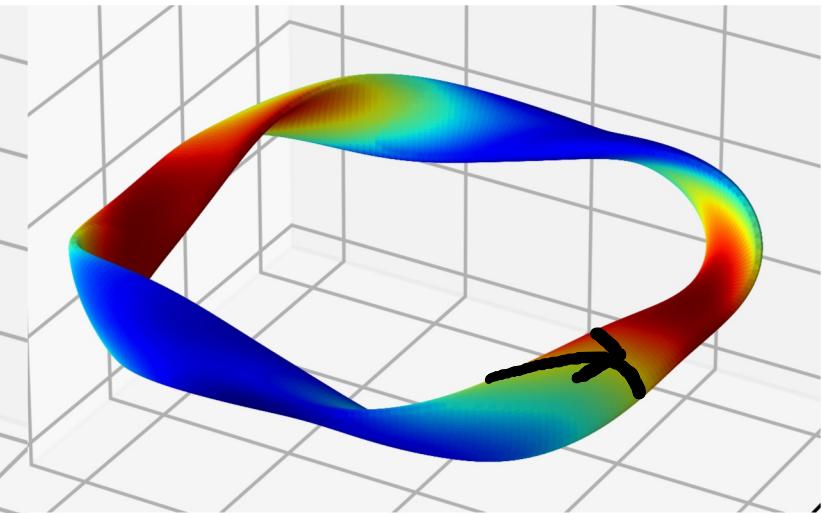
Quasi-helical symmetry



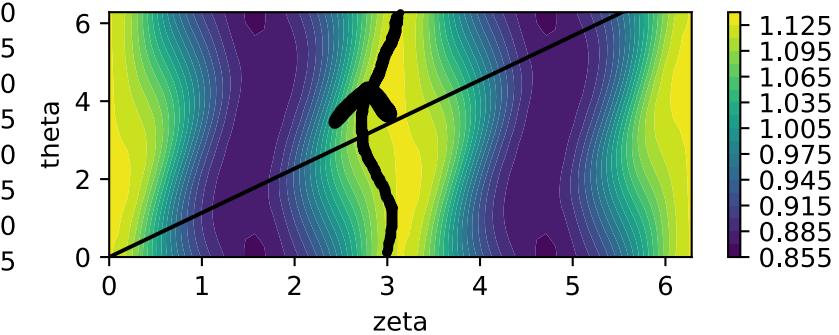
$|B|$ at half radius
1-based index=64



Quasi-isodynamic

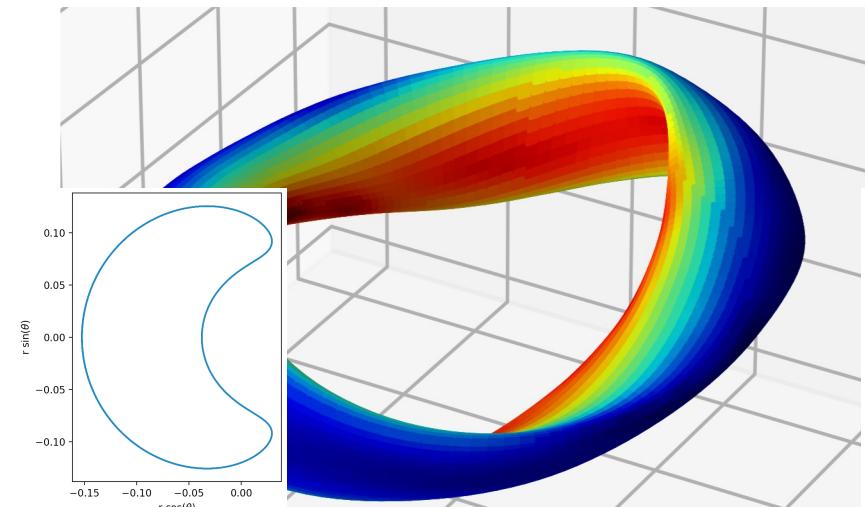


$|B|$ at half radius
1-based index=39

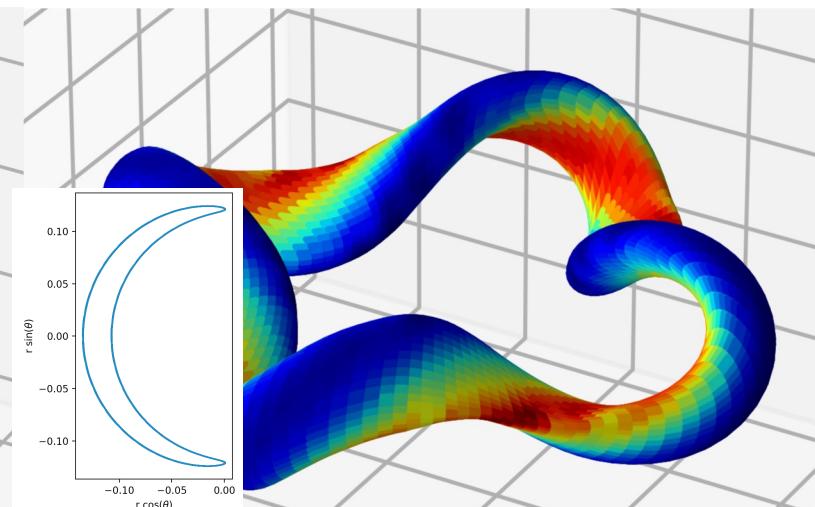


Stellarator optimization – optimize $|B|$

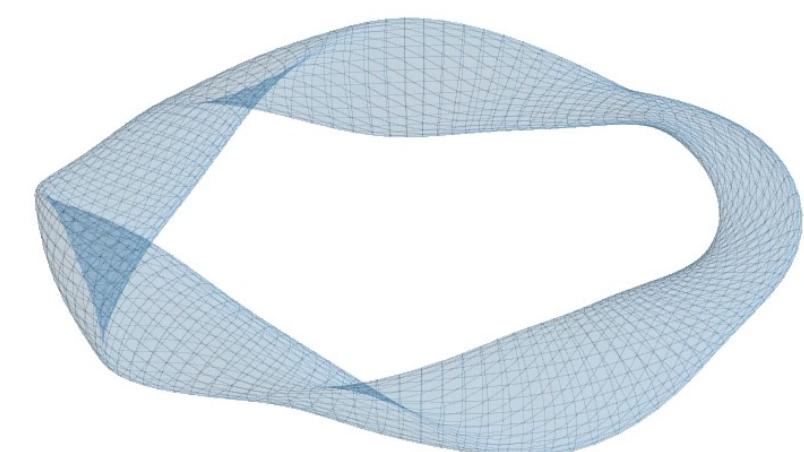
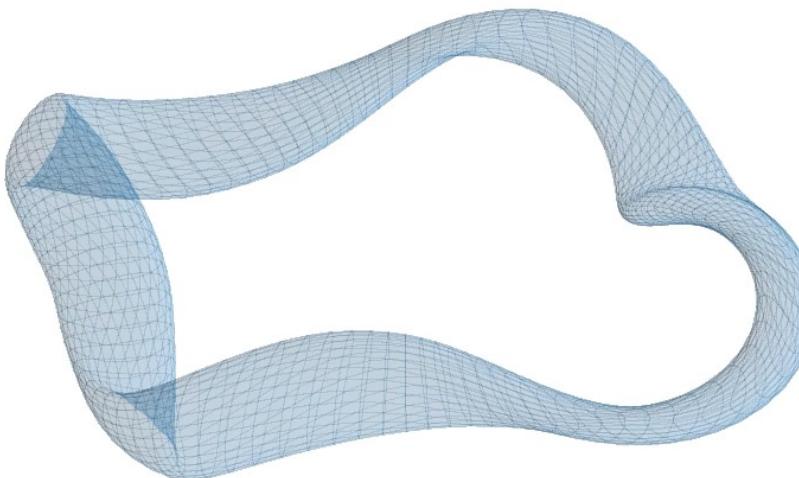
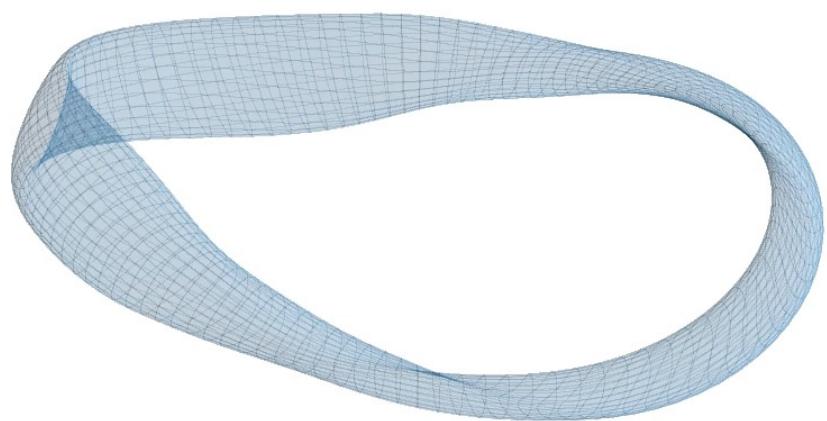
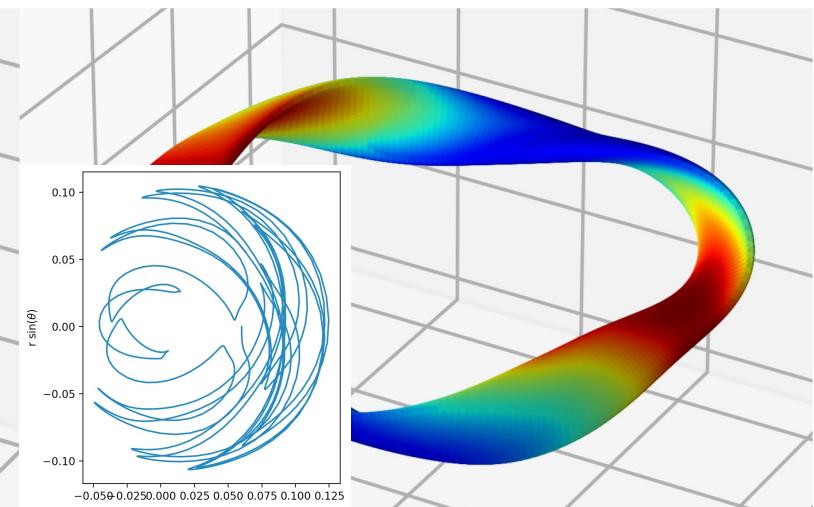
Quasi-axisymmetry



Quasi-helical symmetry



Quasi-isodynamic



PART 2

Recent breakthroughs in stellarators

A new idea...

Direct stellarator optimization

Directly minimize the quantity you want to minimize

It may or may not
be a good idea...



Compare with proxies

What is the ultimate cost function? And initial conditions?

Direct quantities

Fast particle confinement
Low neoclassical transport

Coil complexity

Turbulent transport

Plasma stability

Best possible stellarator
(robustness, global minimum)



Proxies

Quasisymmetry, Quasi-isodynamic...

Shape complexity, $L_{\nabla B}$...

Effective radius of curvature, quasilinear...

Mercier criterion, shape complexity...

Near-axis, good initial points...

What shall we do? Direct or proxy optimization?

Direct approaches covered today

- Direct constructions using the near-axis expansion
- Direct fast particle optimization
- Direct turbulence optimization: quasi-linear and nonlinear approaches
- Direct coil optimization: single-stage approaches

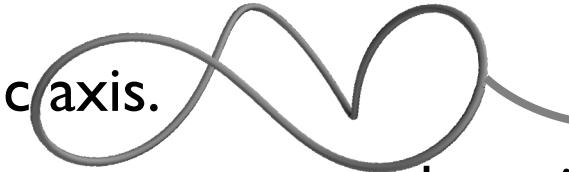
Direct approaches covered today

- Direct constructions using the near-axis expansion
- Direct fast particle optimization
- Direct turbulence optimization: quasi-linear and nonlinear approaches
- Direct coil optimization: single-stage approaches

What is the near-axis expansion?

- Inputs:

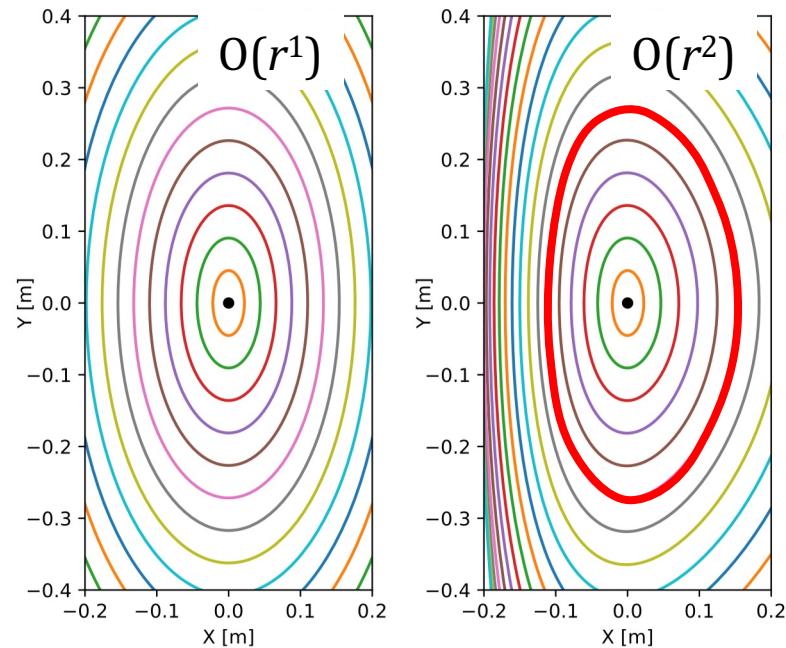
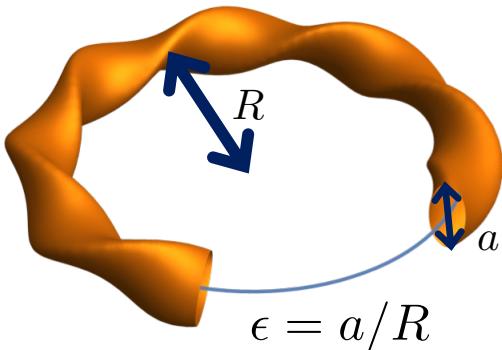
- Shape of the magnetic axis.
- 3-5 other numbers (e.g., current on the axis).



$$\mathbf{r} = \mathbf{r}_0 + \sqrt{s} \left[\frac{\bar{\eta}}{\kappa} \cos \theta \mathbf{n} + \frac{\kappa}{\bar{\eta}} (\sin \theta + \boxed{\sigma} \cos \theta) \mathbf{b} \right]$$

- Outputs:

- Shape of the surfaces around the axis.
- Rotational transform on axis.
- ...

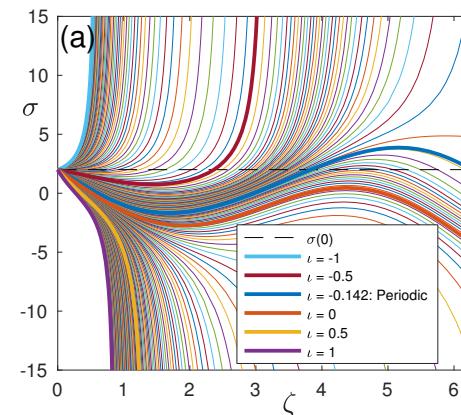


- Quasisymmetry guaranteed in a neighborhood of axis.
- Can pick any surface to pass to traditional MHD solver

The ideal MHD equation at $\mathcal{O}(\epsilon)$ yields

$$\mathbf{J} \times \mathbf{B} = \nabla P$$

$$\frac{d\sigma}{d\varphi} + (\iota_0 - N) \left(\frac{\bar{\eta}^4}{\kappa^4} + 1 + \sigma^2 \right) - \left(\frac{I_2}{B_0} - \tau \right) \frac{L\bar{\eta}^2}{\pi\kappa^2} = 0$$



Now able to compute higher order solutions fast

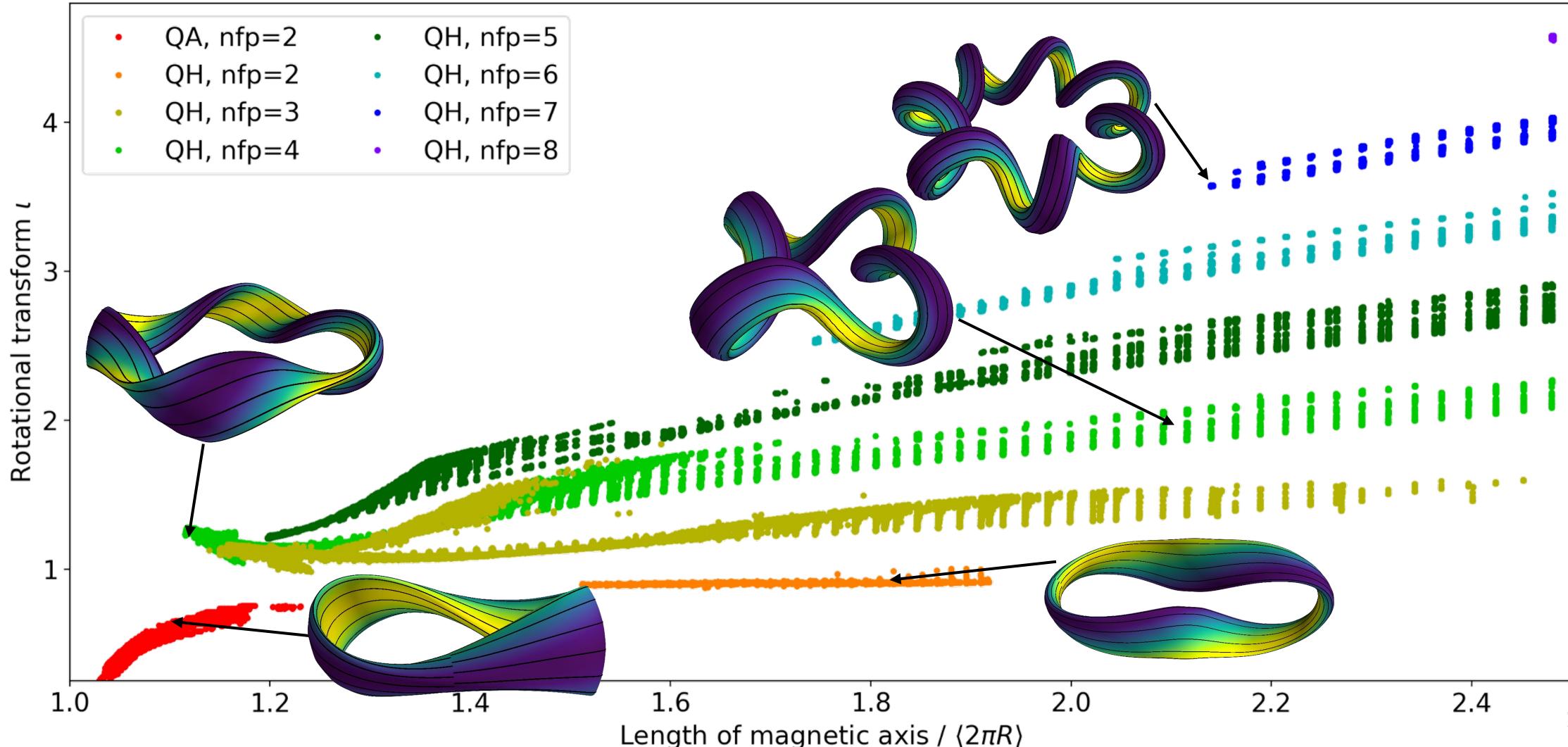
M. Landreman, Journal of Plasma Physics , Volume 88 , Issue 6 , December 2022 , 905880616

landreman / qsc Public



3.1×10^5 optimized stellarators shown
 5.1×10^6 optimizations computed
 6.0×10^{10} equilibria computed

The “rainbow” plot



How does it compare with traditional optimization?

R. Jorge & M. Landreman, Plasma
Phys. Control. Fusion 63, 1 (2020)

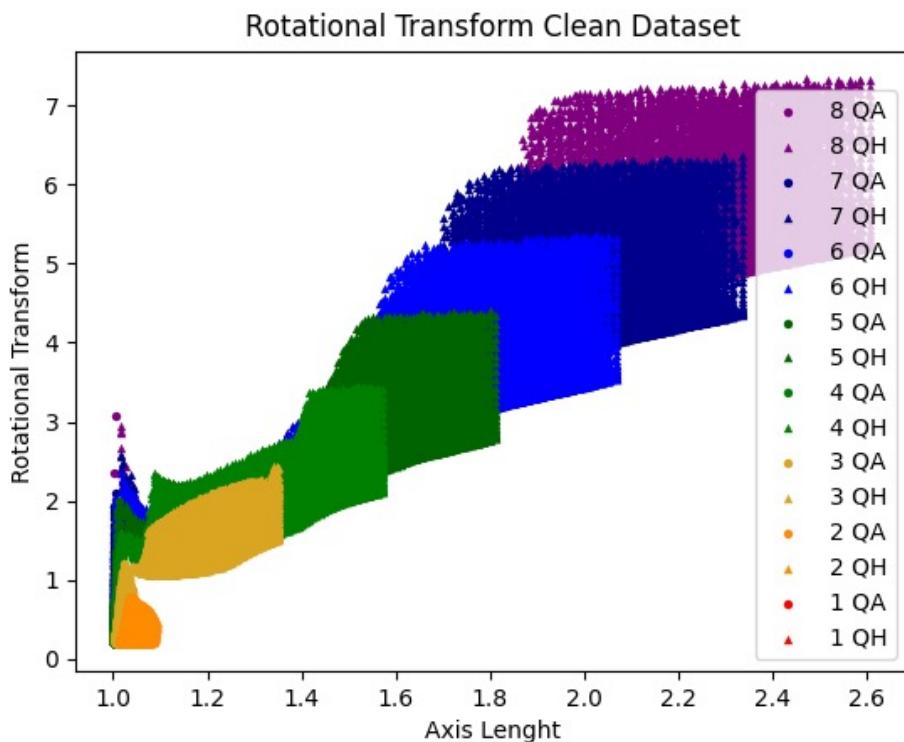
Take $|B|$ from VMEC and fit to near-axis $|B|$

Configuration	Aspect ratio A	Field periods n_{fp}	Plasma pressure β	ι from optimized configuration	ι from construction	Best-fit $\bar{\eta}$ [m $^{-1}$]	B_0 [T]
NZ1988	12	6	0%	1.42	1.42	0.157	0.205
HSX	10	4	0%	1.05	1.06	1.28	1.00
KuQHS48	8.1	4	4%	1.29	1.27	0.147	1.20
WISTELL-A	6.7	4	3%	1.09	1.03	0.791	2.54
Drevlak	8.6	5	4%	1.50	1.50	0.0899	3.97
NCSX	4.4	3	4%	0.392	0.409	0.408	1.55
ARIES-CS	4.5	3	4%	0.412	0.498	0.0740	5.69
QAS2	2.6	2	3%	0.260	0.267	0.347	1.79
ESTELL	5.3	2	0%	0.202	0.202	0.570	1.00
CFQS	4.3	2	0%	0.382	0.515	0.586	0.933
Henneberg	3.4	2	3%	0.317	0.314	0.302	2.41

Table 1: Quasisymmetric configurations considered in this study.

Use the database to train a **neural network** (J. Candido, IST Lisbon)

Take a dataset with both moderate and low elongation stellarators

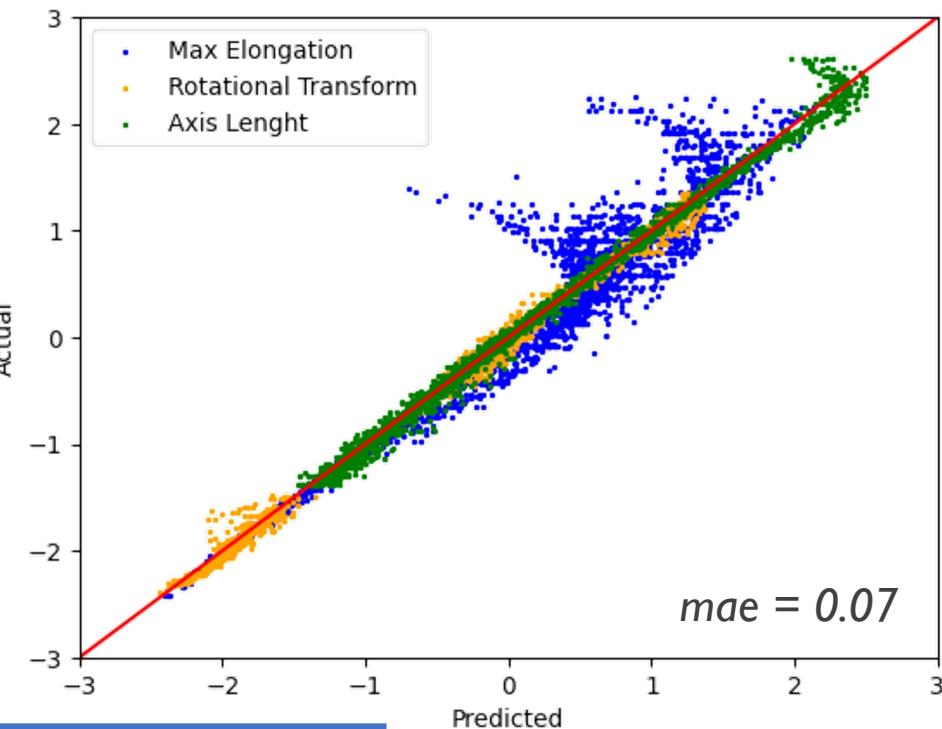


Train a NN on the scaled inverse problem

$y = (\text{axis length}, \iota, \text{elongation})$

$x = (\text{axis shape}, \bar{\eta})$

Tanh activation
Hidden layer sizes = (35, 35, 35)
Solver = Adam



This allows us to find stellarator shapes with a particular shape instead of finding them via optimization



JoaoAGCandido / PICNeuralNetworkQuasisymmetricStellarator Public

How do particles behave in near-axis fields? (P. Figueiredo, IST Lisbon)

- Ongoing work: Enabling Research grant EUROfusion (ENR.IST.MOD.01)
- Developed a new code: **NEAT** (python)

Integration with
DESC underway

Based on a new open-source library **gyronimo**
Solve guiding-center equations of motion in arbitrary coordinates

::gyronimo:: - gyromotion for the people, by the people -

An object-oriented library for gyromotion applications in plasma physics.

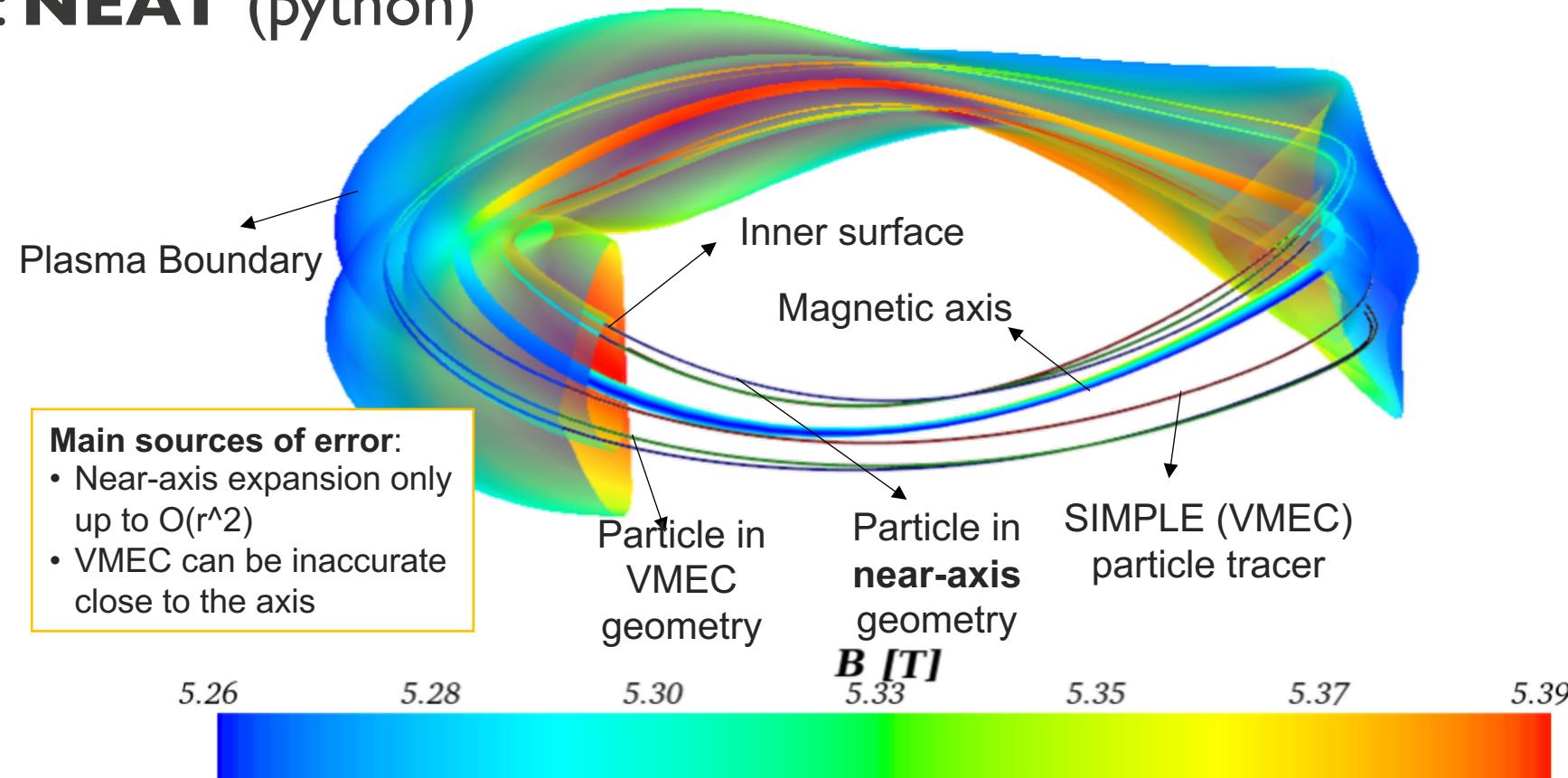
Philosophy and purpose:

Have you ever had a bright and promising idea about gyromotion in plasmas that just faded away the moment you realised the amount of non-trivial, tedious, unrewarding, non-physics details you would have to implement before you could get a simple glimpse over the results?



Search or jump to...

prodrigs / gyronimo Public



New code: NEAT

NEAT  [rogeriojorge/NEAT](#) Public 

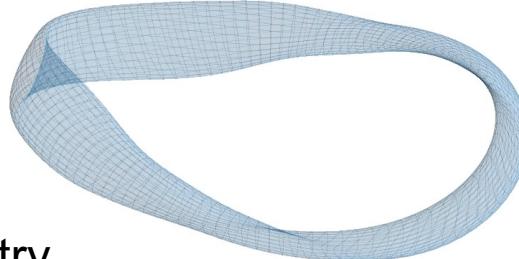
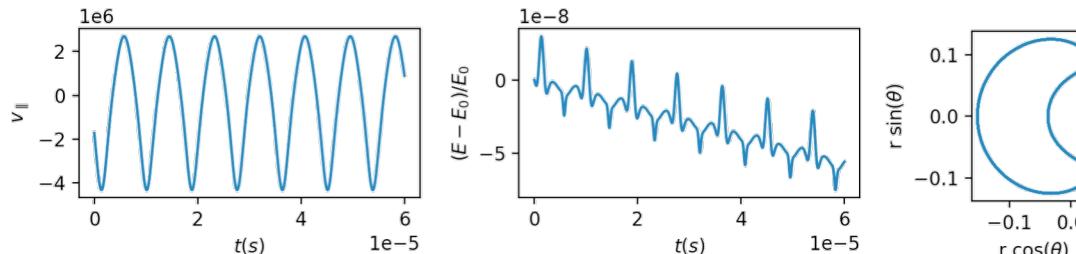
NEar-Axis opTimation

license [GPL-3.0](#) build [passing](#) docs [passing](#)  [codecov](#) 61%

User-friendly example in near-axis geometry

```
1 from neat.fields import StellnaQS
2 from neat.tracing import ChargedParticle, ParticleOrbit
3 g_field = StellnaQS.from_paper(1)
4 g_particle = ChargedParticle()
5 g_orbit = ParticleOrbit(g_particle, g_field tfinal=1e-4)
6 g_orbit.plot_orbit()
```

Similarly for VMEC



Open-source and automatic testing with GitHub actions

The screenshot shows the GitHub Actions interface. At the top, there are tabs for 'Actions' (which is selected), 'Projects', 'Wiki', 'Security', 'Insights', and 'Settings'. Below this, the 'Workflows' section is shown with a 'New workflow' button and a list of workflows: 'All workflows' (selected), 'CI', 'Codestyle', 'Linting', and 'Publish Python distribution t...'. A search bar 'Filter workflow runs' is present. To the right, a table shows '865 workflow runs' with a single entry: 'Now added save_loss_fraction function' by 'rogeriojorge' at 'simopt_optimization' with a status of 'success' and a duration of '5 hours ago 1m 6s'.

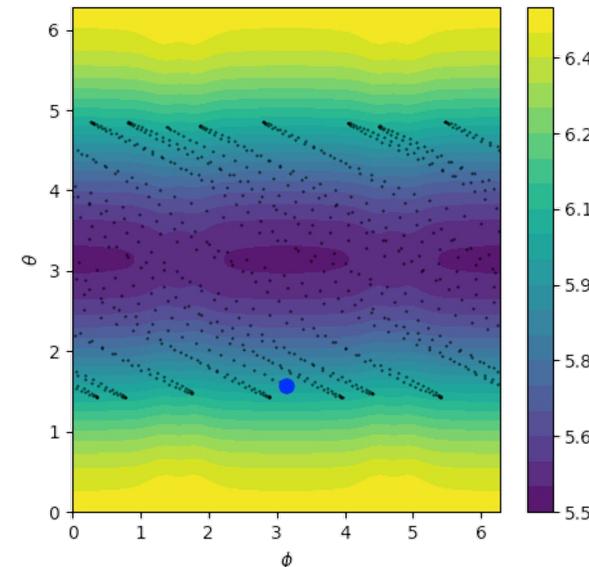
Already being used by the stellarator optimization community

Critical gradient turbulence optimization toward a compact stellarator reactor concept

G. T. Roberg-Clark,* G. G. Plunk, P. Xanthopoulos, C. Nührenberg, S. A. Henneberg, and H. M. Smith
Max-Planck-Institut für Plasmaphysik, D-17491, Greifswald, Germany
(Dated: January 18, 2023)

Integrating turbulence into stellarator optimization is shown by targeting the onset for the ion-temperature-gradient mode, highlighting effects of parallel connection length, local magnetic shear, and flux surface expansion. The result is a compact quasi-symmetric stellarator configuration, admitting a set of uncomplicated coils, with significantly reduced turbulent heat fluxes compared to a known stellarator. The new configuration combines low values of neoclassical transport, good alpha particle confinement, and Mercier stability at a plasma beta of almost 2%.

```
print("Creating B contour plot")
g_orbit.plot_orbit_contourB()
```



arXiv:2301.06773v1 [physics.plasm-ph] 17 Jan 2023

when rescaled to an ARIES-CS-equivalent [36] minor radius and volume averaged magnetic field strength, using the NEAT code [37, 38] (Fig. 2). Increased neoclassical

Direct approaches covered today

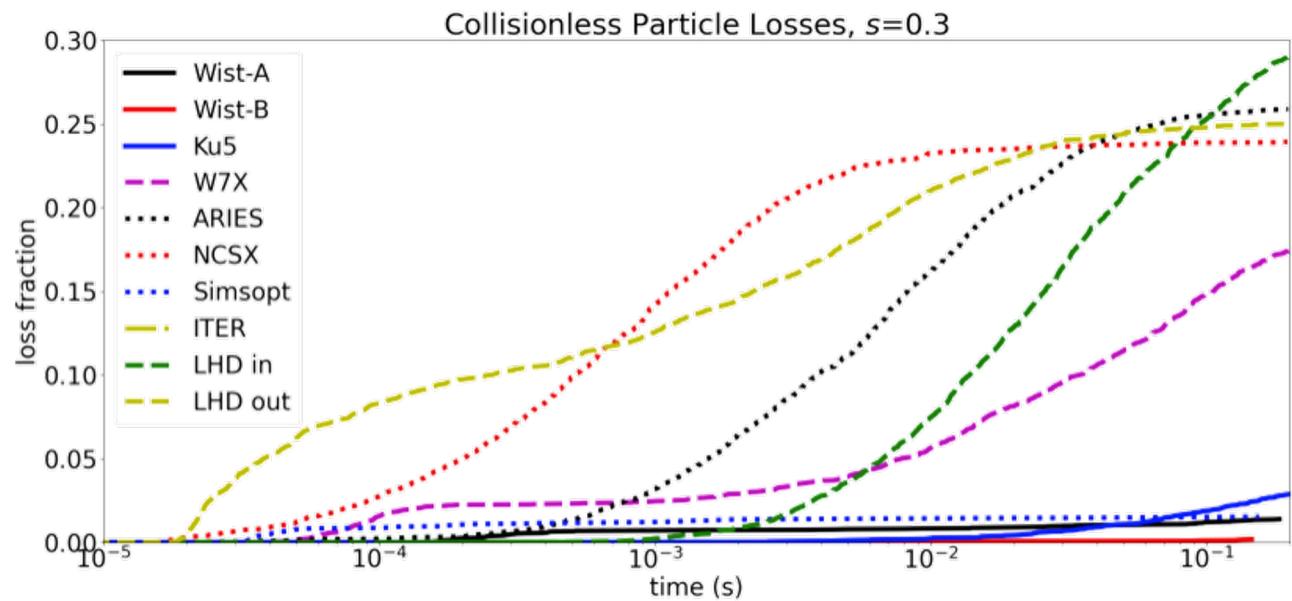
- Direct constructions using the near-axis expansion
- Direct fast particle optimization
- Direct turbulence optimization: quasi-linear and nonlinear approaches
- Direct coil optimization: single-stage approaches

Fast particle optimization

Typical proxies

- Quasisymmetry
- Quasi-isodynamic
- Nemov's Γ_c
- Γ_α

J. L. Velasco et al., Nuclear Fusion, Volume 61, Number 11, October 2021, 116059



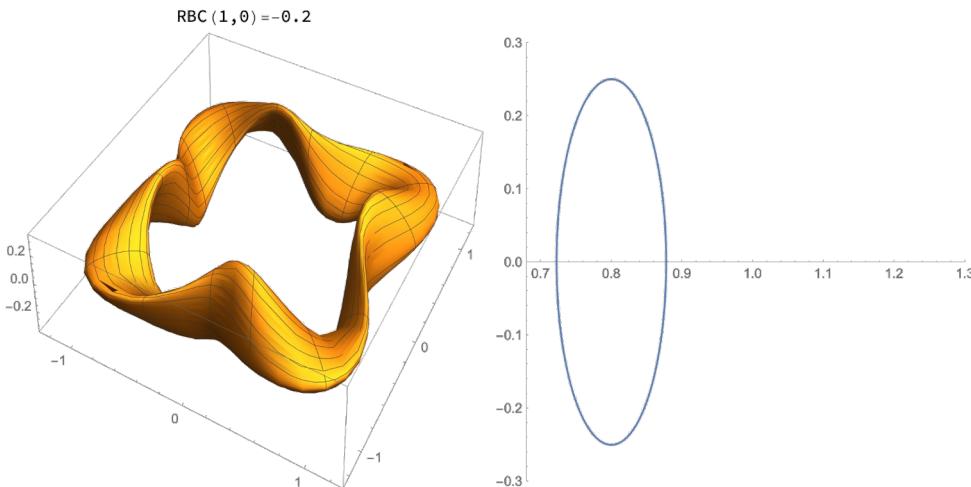
A. Bader et al., Nuclear Fusion, Volume 61 , Number 11, October 2021, 116060

What if we directly minimize particle losses?

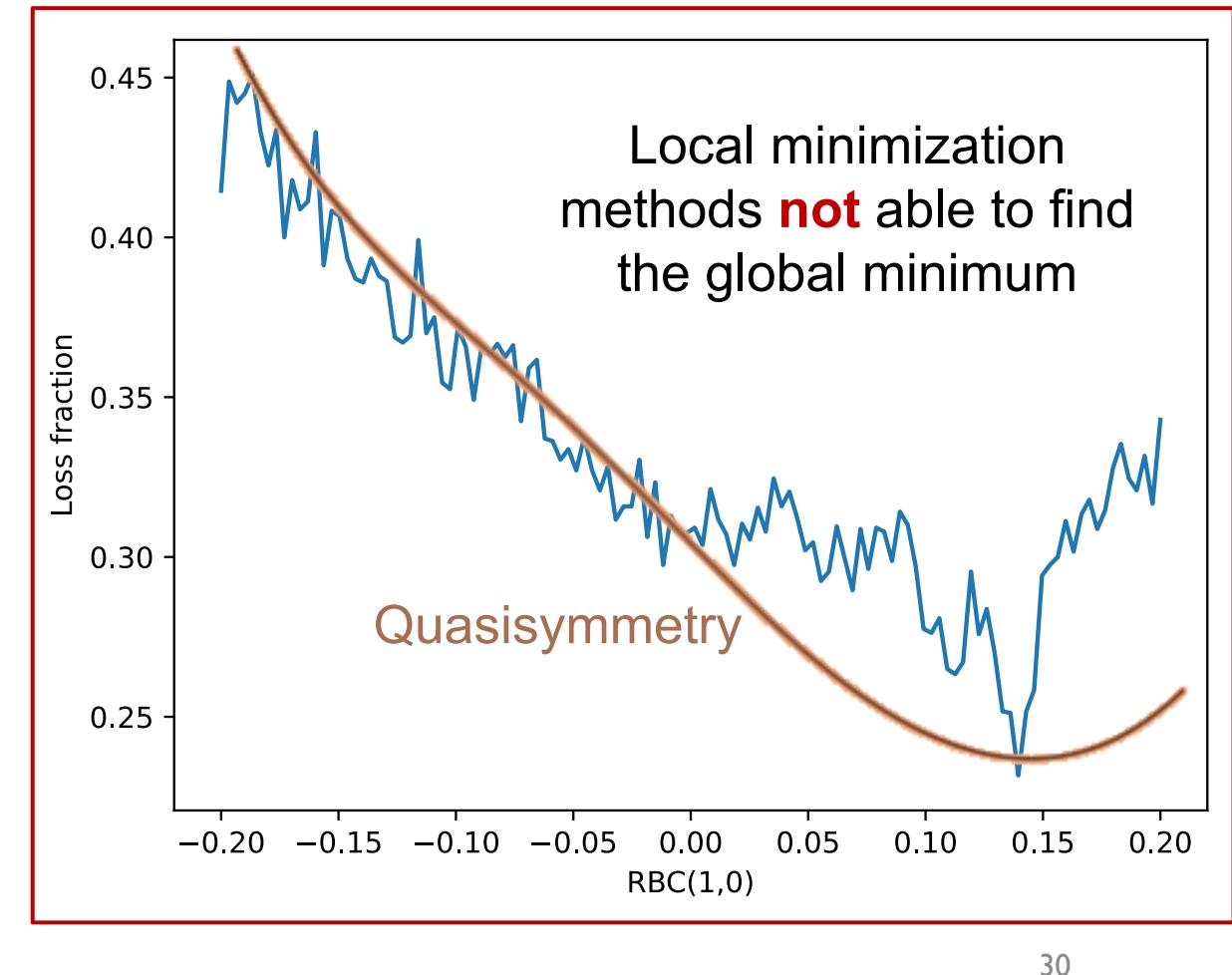
Direct fast particle optimization using NEAT

Minimal benchmark problem

- Trace 2400 particles for 5×10^{-4} s with the SIMPLE code
- Scale the minor radius and magnetic field to half of the ARIES-CS reactor
- Save the fraction of loss particles in an array for each RBC(-1,1)
- Each point takes ~1 second on a laptop



Minimize particle loss fraction to the wall



Direct fast particle optimization using NEAT

Stochastic optimization – generalized dual annealing

Initial condition:

- Circular torus without current
- 100% particle losses

Parameter space – 9 parameters:

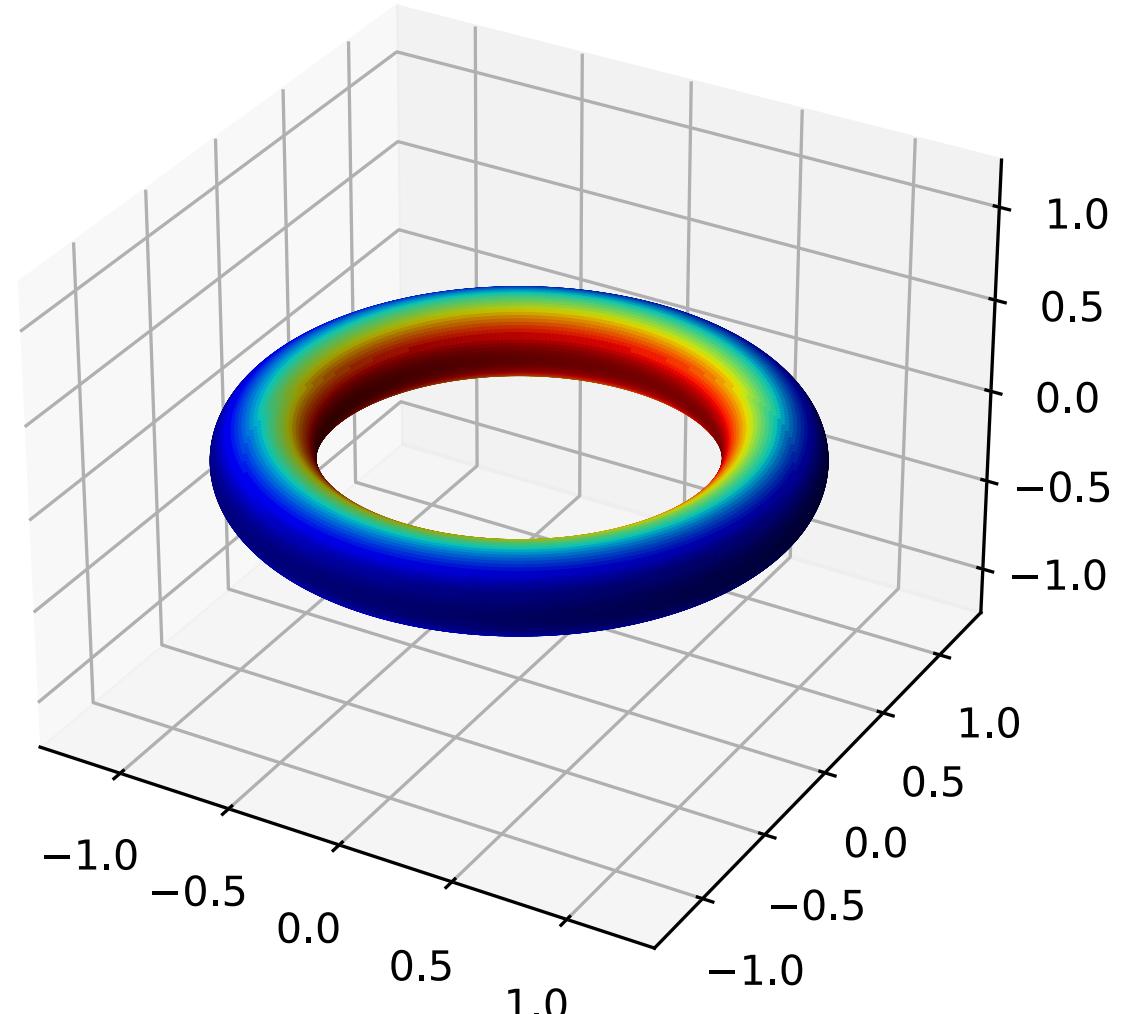
- RBC(1,0), RBC(1,1), RBC(0,1), RBC(-1,1)
- ZBS(1,0), ZBS(1,1), ZBS(0,1), ZBS(-1,1)

100% losses



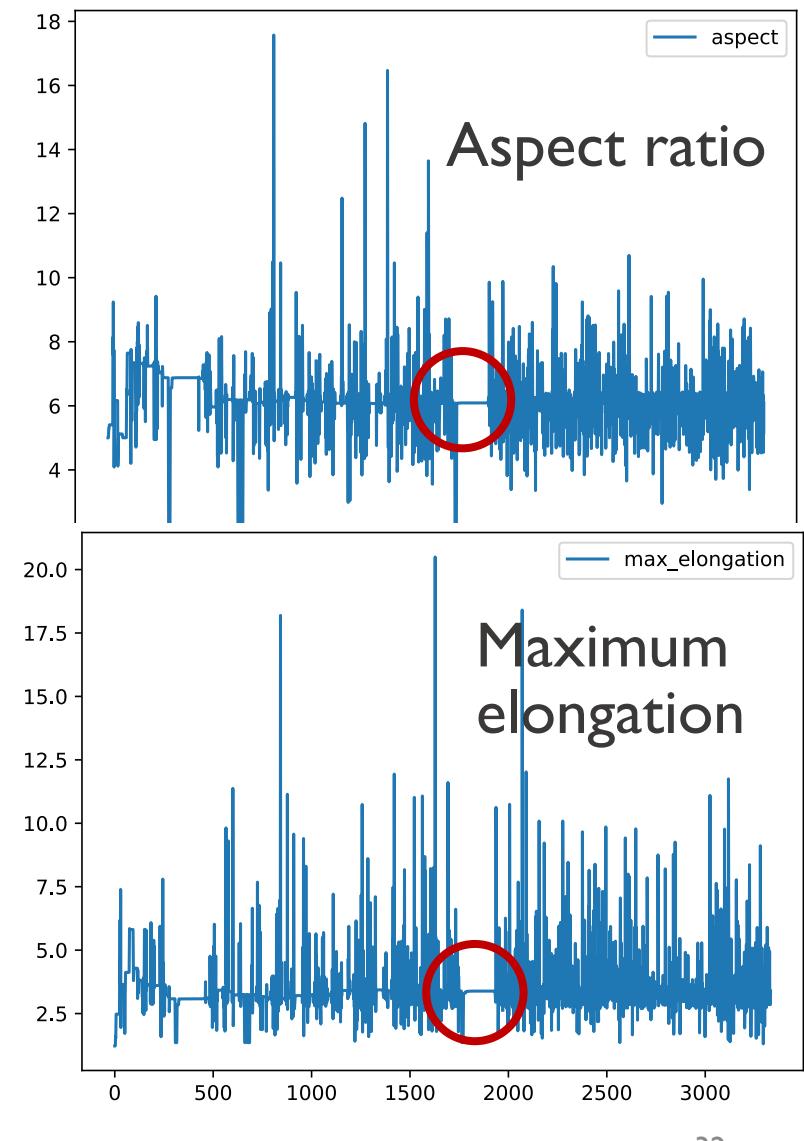
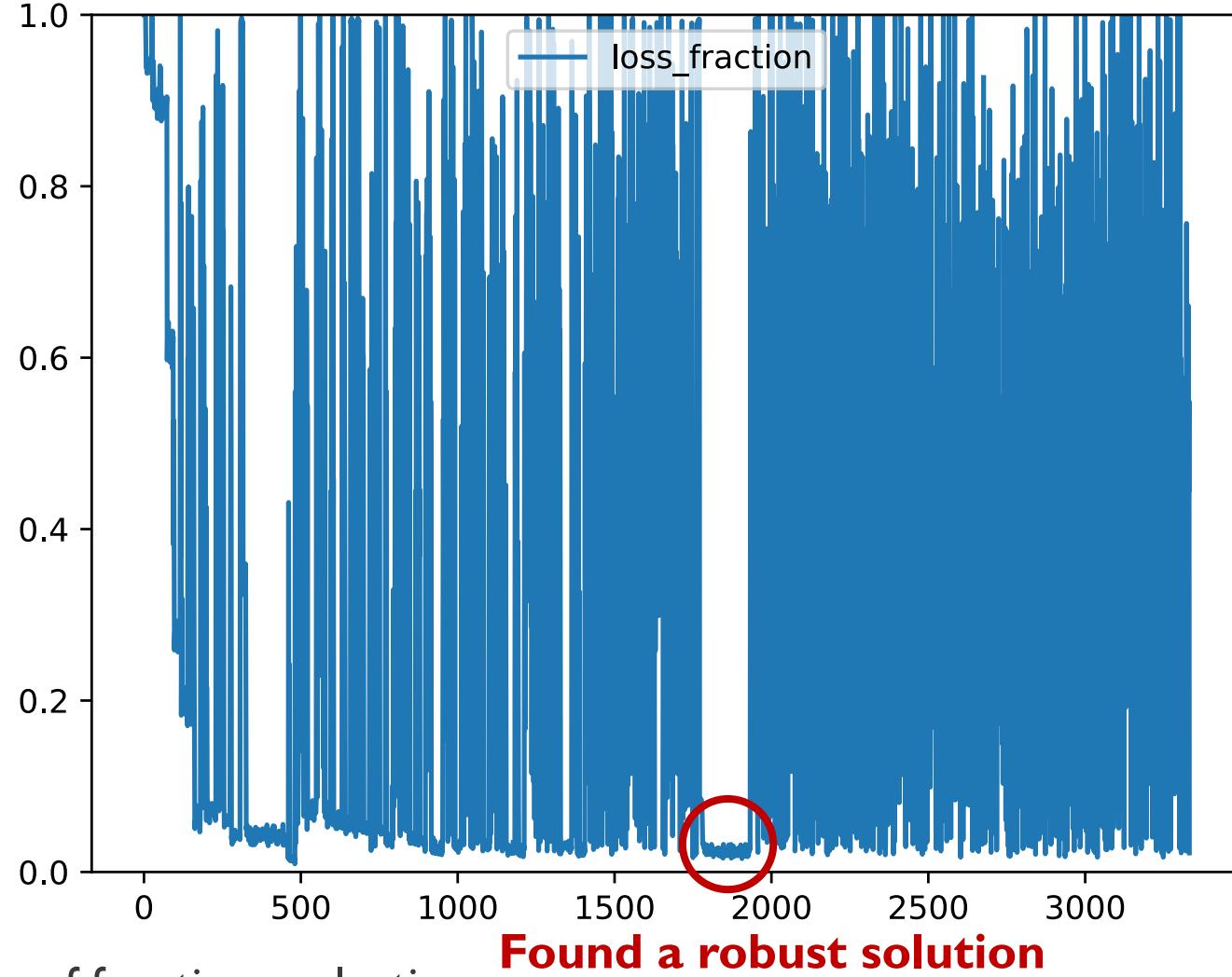
scipy.optimize.dual_annealing

```
scipy.optimize.dual_annealing(func, bounds, args=(), maxiter=1000,
minimizer_kwarg... None, initial_temp=5230.0, restart_temp_ratio=2e-05, visit=2.62
```



Direct fast particle optimization using NEAT

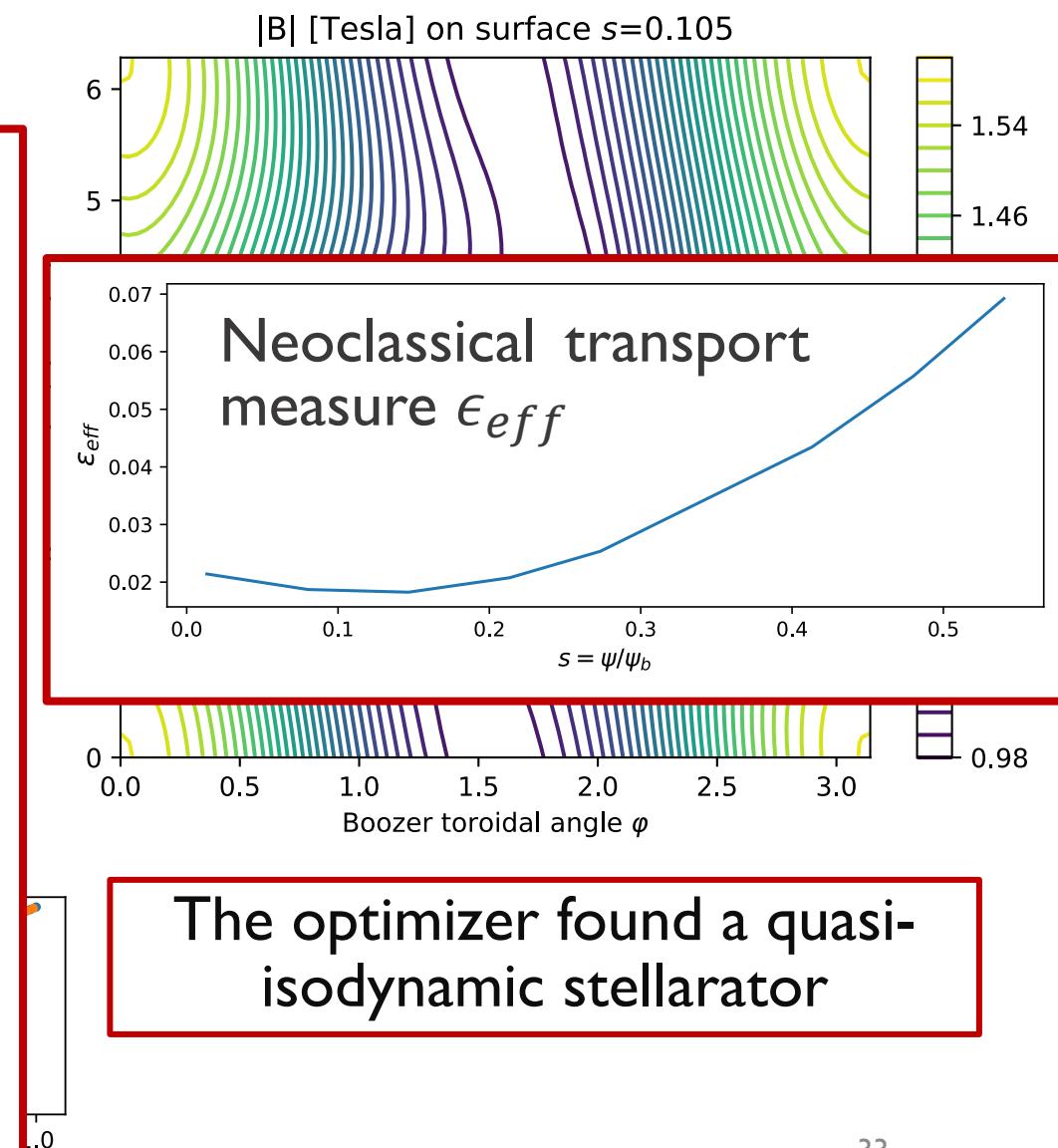
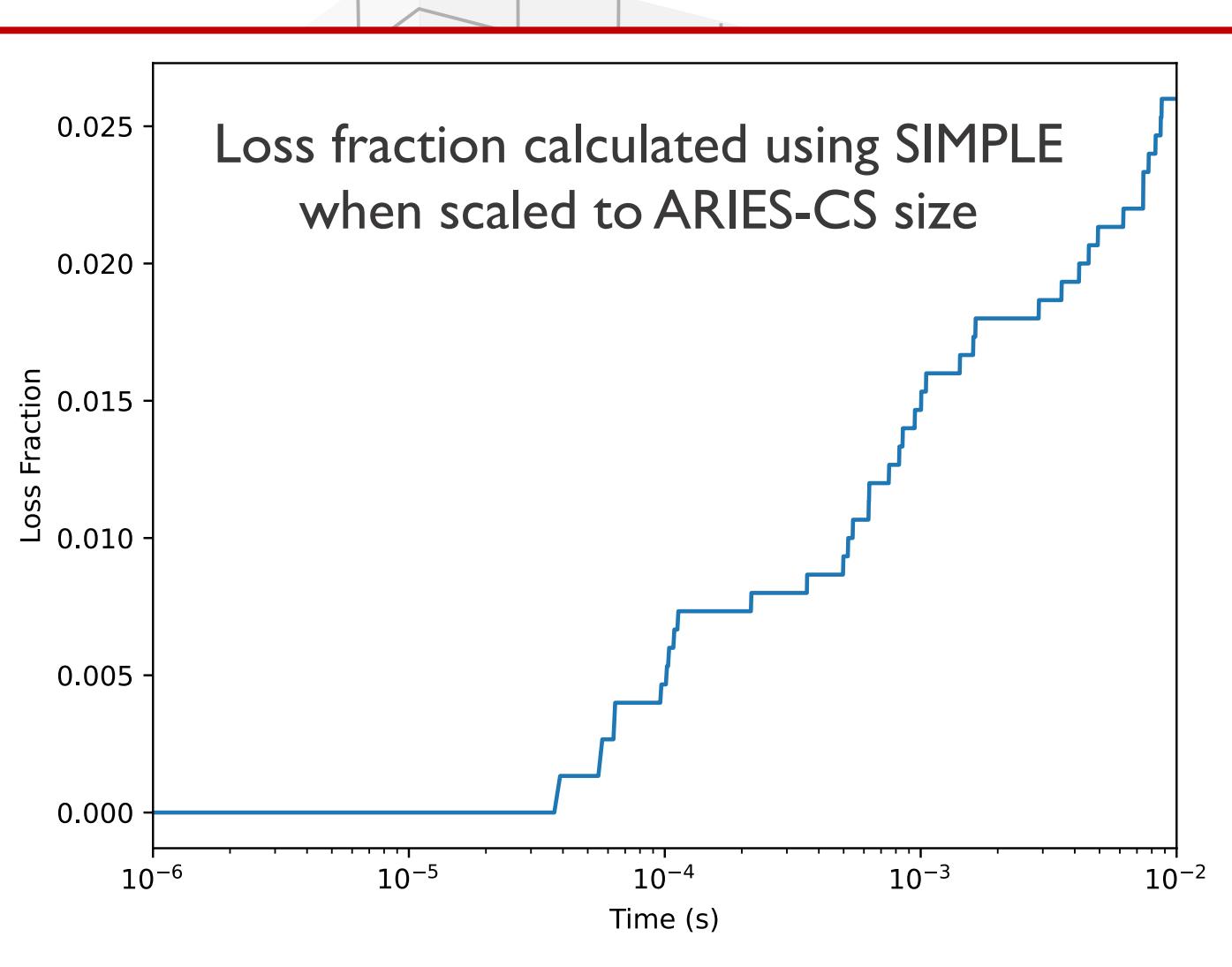
Optimization process



Direct fast particle optimization using NEAT

Solution found

rogeriojorge / EOptimization



Include collisions and realistic birth profiles – Bindel, Landreman, Padidar (2023)

arXiv > physics > arXiv:2302.11369

Physics > Plasma Physics

[Submitted on 22 Feb 2023]

Direct Optimization of Fast-Ion Confinement in Stellarators

David Bindel, Matt Landreman, Misha Padidar

Optimize for small lost energy

$$\underset{\mathbf{w} \in \mathbb{R}^{n_w}}{\text{minimize}} \quad \mathcal{J}(\mathbf{w}) := \mathbb{E}_{\mathbf{x}, v_{\parallel}} [\mathcal{J}_{\text{energy}}(\mathbf{x}, v_{\parallel}, \mathbf{w})]$$
$$B_-^* \leq B(\mathbf{x}, \mathbf{w}) \leq B_+^* \quad \forall \mathbf{x} \in \mathcal{P}$$

Restrict mirror ratio

Radial birth distribution of particles

$$f_s(s) \propto (1 - s^5)^2 (1 - s)^{-2/3} \exp(-19.94(12(1 - s))^{-1/3}).$$

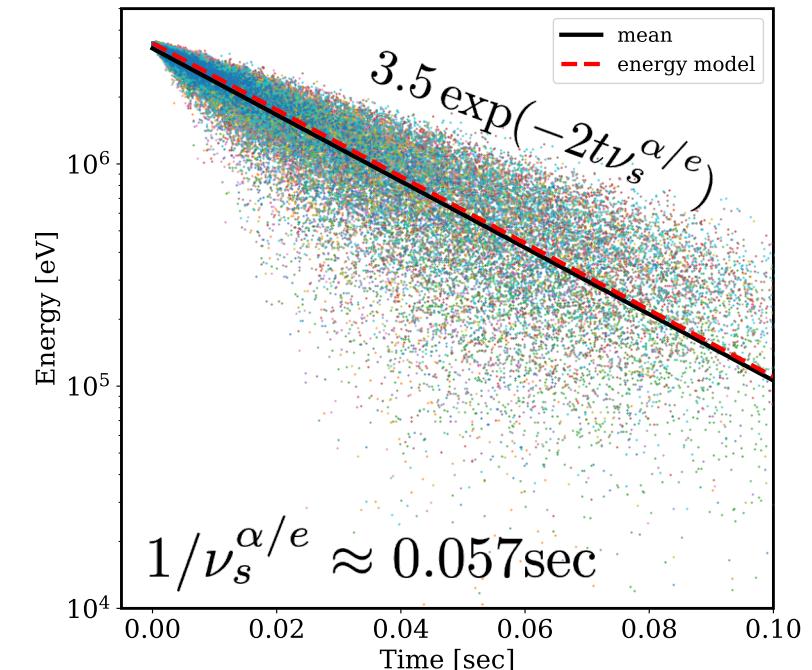
Particles uniformly distributed over flux surfaces

$$f_{\theta, \zeta}(\theta, \zeta | s) \propto |\sqrt{g}|.$$

Energy loss model

$$\mathcal{J}_{\text{energy}}(\mathbf{x}, v_{\parallel}, \mathbf{w}) = 3.5 e^{-2\mathcal{T}(\mathbf{x}, v_{\parallel}, \mathbf{w})/t_{\max}}$$
$$\mathcal{T} = \min\{t, t_{\max}\}$$

Obtained using a fit to 200 000 tracings



Particle tracing simulations with collisions using the ANTS code

Include collisions and realistic birth profiles – Bindel, Landreman, Padidar (2023)

arXiv > physics > arXiv:2302.11369

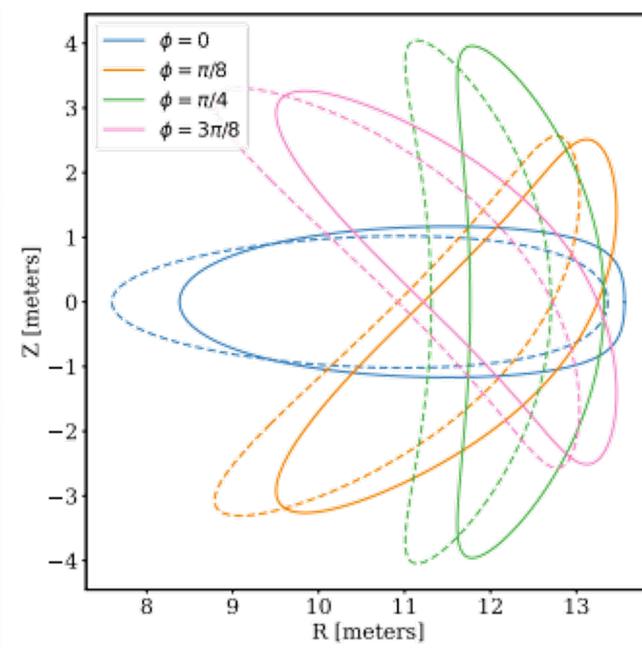
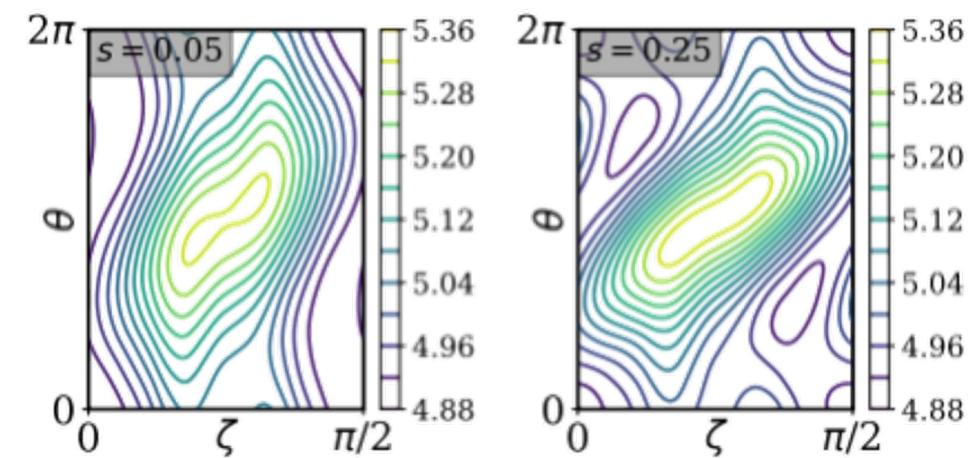
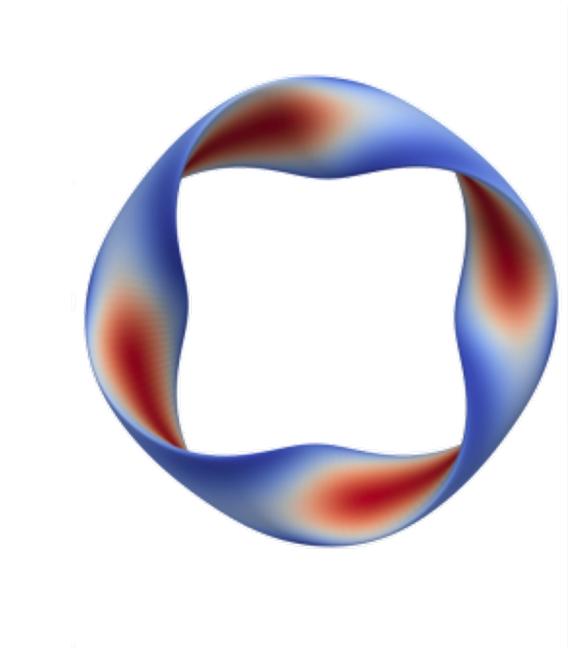
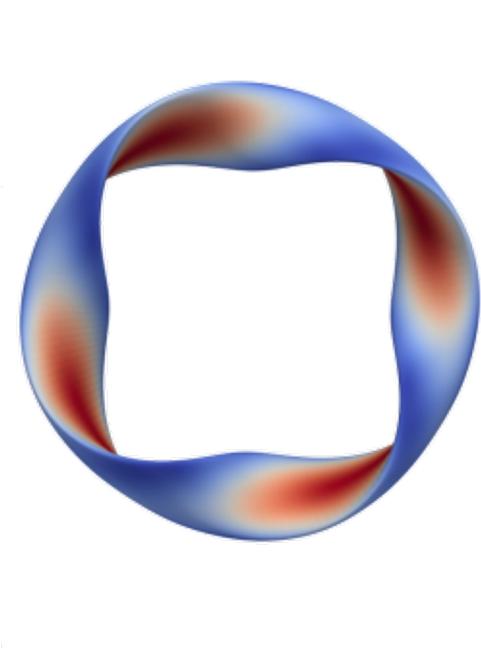
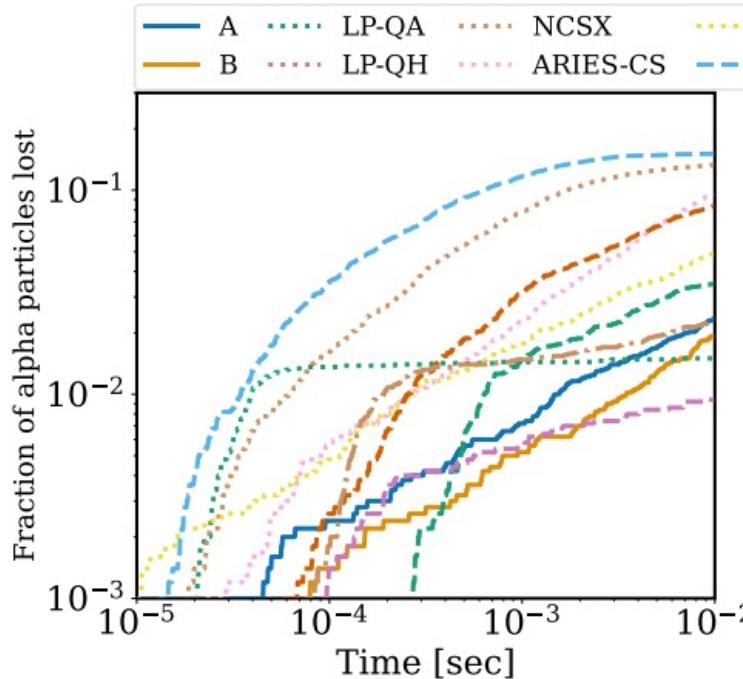
Physics > Plasma Physics

[Submitted on 22 Feb 2023]

Direct Optimization of Fast-Ion Confinement in Stellarators

David Bindel, Matt Landreman, Misha Padidar

Resulting stellarators **far** from omnigenity



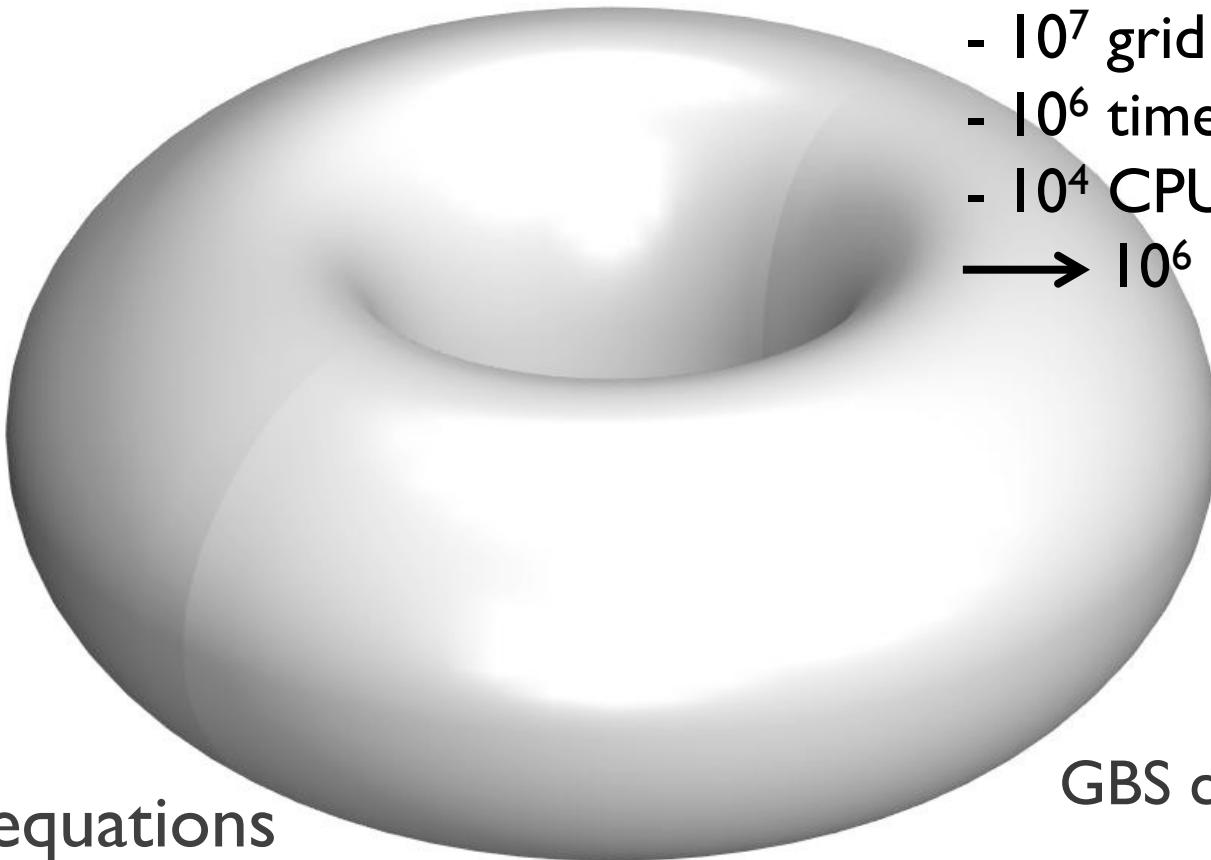
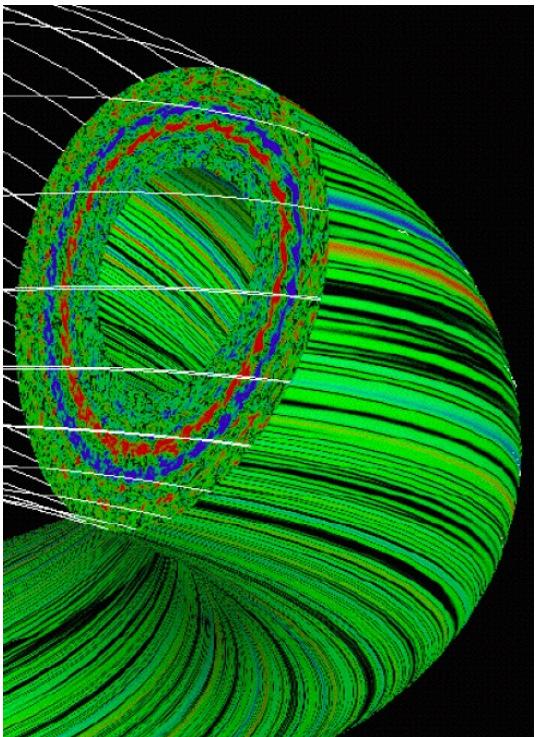
Direct approaches covered today

- Direct constructions using the near-axis expansion
- Direct fast particle optimization
- Direct turbulence optimization: quasi-linear and nonlinear approaches
- Direct coil optimization: single-stage approaches

Direct stability optimization

The gyrokinetic equation

$$\frac{\partial F}{\partial t} + \dot{\mathbf{R}} \cdot \nabla F + \dot{v}_{\parallel} \frac{\partial F}{\partial v_{\parallel}} = \langle C(F) \rangle$$



- 10^7 grid points
 - 10^6 time steps
 - 10^4 CPUs
- 10^6 CPU hours

Fluid equations

$$\frac{\partial n}{\partial t} = - \frac{[\phi, n]}{B} + \frac{2}{eB} [nC(T_e) + T_eC(n) - enC(\phi)] - n(\mathbf{b} \cdot \nabla z) \frac{\partial V_{\parallel e}}{\partial z} - V_{\parallel e}(\mathbf{b} \cdot \nabla z) \frac{\partial n}{\partial z} + S_n,$$
$$m_i n \frac{\partial V_{\parallel i}}{\partial t} = - \frac{m_i n}{B} [\phi, V_{\parallel i}] - m_i n V_{\parallel i} (\mathbf{b} \cdot \nabla z) \frac{\partial V_{\parallel i}}{\partial z} - \frac{2}{3} (\mathbf{b} \cdot \nabla z) \frac{\partial G_i}{\partial z} - (\mathbf{b} \cdot \nabla z) \frac{\partial [n(T_e + T_i)]}{\partial z}$$

GBS code (EPFL)

Direct stability optimization



Physics > Plasma Physics

[Submitted on 20 Aug 2020]

The Use of Near-Axis Magnetic Fields for Stellarator Turbulence Simulations

R. Jorge, M. Landreman

$$\frac{\partial n}{\partial t} = - \frac{[\phi, n]}{B} + \frac{2}{eB} [nC(T_e) + T_eC(n) - enC(\phi)] - n(\mathbf{b} \cdot \nabla z) \frac{\partial V_{\parallel e}}{\partial z} - V_{\parallel e}(\mathbf{b} \cdot \nabla z) \frac{\partial n}{\partial z} + S_n,$$
$$m_i n \frac{\partial V_{\parallel i}}{\partial t} = - \frac{m_i n}{B} [\phi, V_{\parallel i}] - m_i n V_{\parallel i} (\mathbf{b} \cdot \nabla z) \frac{\partial V_{\parallel i}}{\partial z} - \frac{2}{3} (\mathbf{b} \cdot \nabla z) \frac{\partial G_i}{\partial z} - (\mathbf{b} \cdot \nabla z) \frac{\partial [n(T_e + T_i)]}{\partial z}$$

List of all coefficients needed to solve the gyrokinetic and fluid equations

$$\mathbf{Q} = \left\{ B, \mathbf{b} \cdot \nabla z, |\nabla \psi|^2, |\nabla \alpha|^2, \nabla \psi \cdot \nabla \alpha, (\mathbf{b} \times \nabla B) \cdot \nabla \alpha, (\mathbf{b} \times \nabla B) \cdot \nabla \psi, (\mathbf{b} \times \boldsymbol{\kappa}) \cdot \nabla \alpha \right\}.$$

The gyrokinetic equation

$$\frac{\partial F}{\partial t} + \dot{\mathbf{R}} \cdot \nabla F + \dot{v}_{\parallel} \frac{\partial F}{\partial v_{\parallel}} = \langle C(F) \rangle$$

Fluid equations

How to directly optimize?

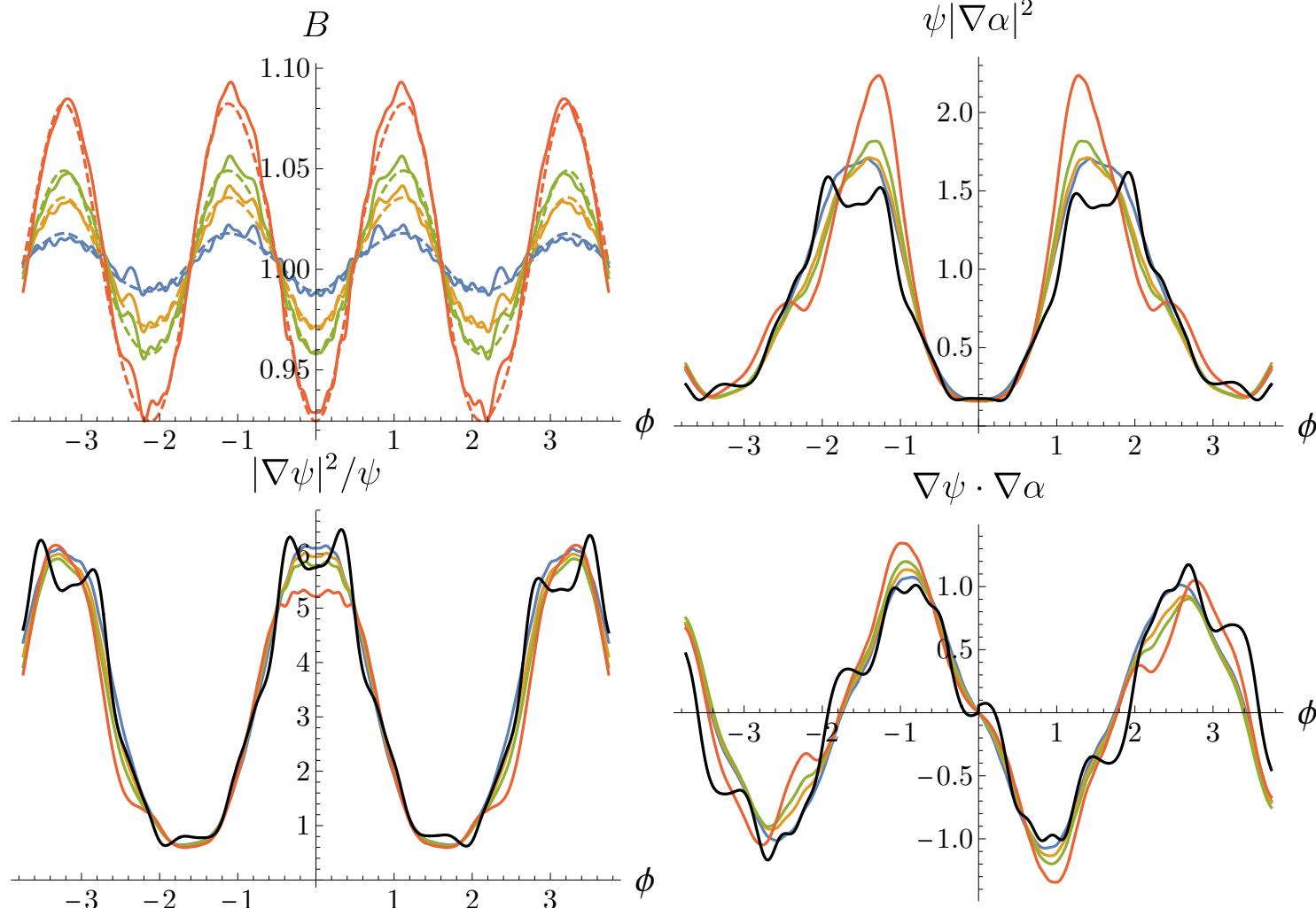
- Use VMEC/DESC/SPEC/GVEC or use the near-axis expansion
- Calculate coefficients at each step of the optimization

Direct stability optimization using the near-axis expansion

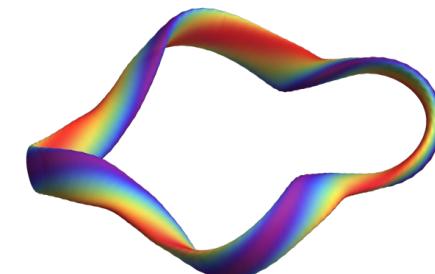
R Jorge and M Landreman 2021 Plasma
Phys. Control. Fusion 63 014001

rogeriojorge / NearAxisGyrokinetics Public

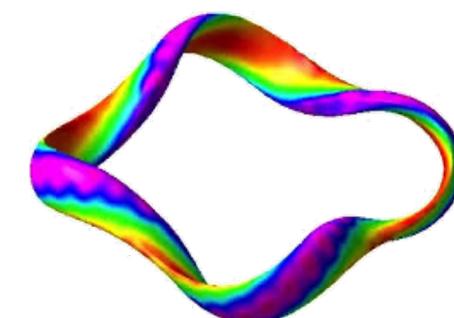
Comparison of coefficients between
HSX and near-axis equivalent



HSX Near-axis



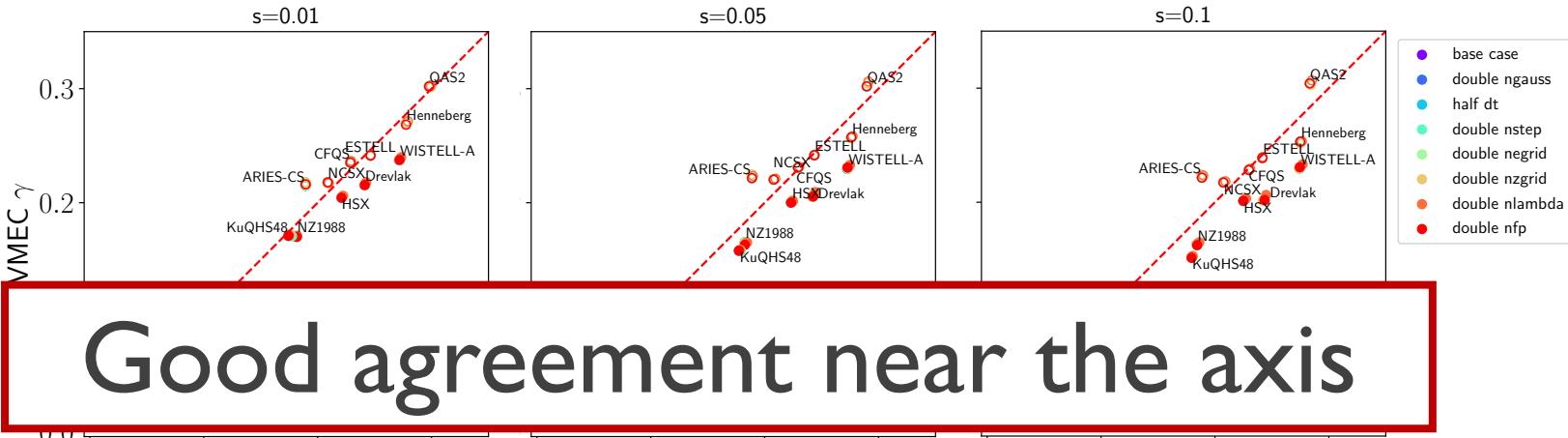
HSX VMEC



Direct stability optimization using the near-axis expansion

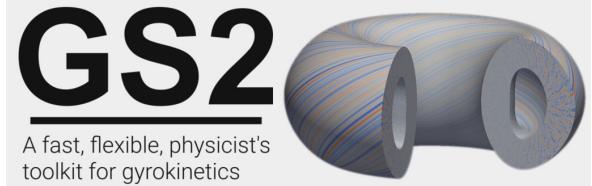
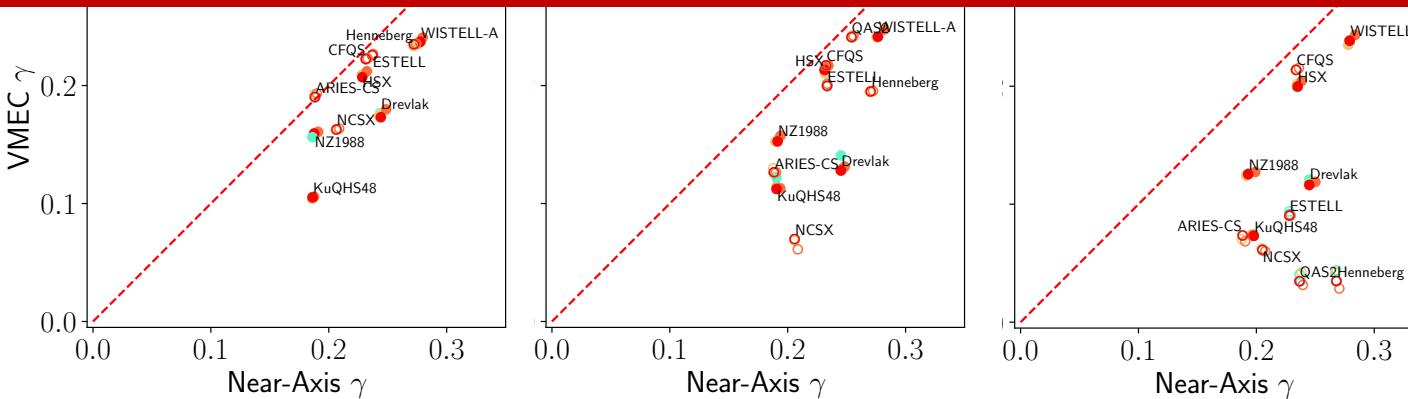
R Jorge and M Landreman 2021 Plasma
Phys. Control. Fusion 63 074002

$$k_y \rho = 1 \quad a/L_T = 5 \quad a/L_n = 2$$

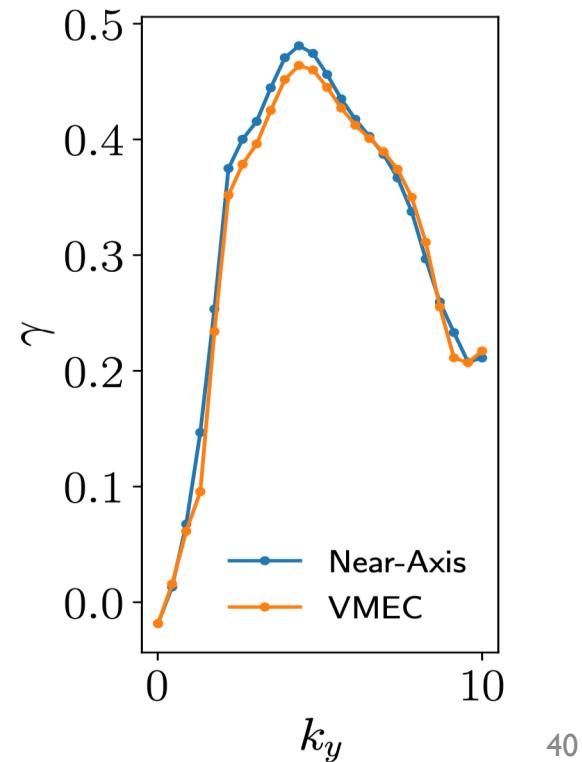


Good agreement near the axis

Overestimate away from the axis



ITG stability at HSX
 $s = 0.01, a/L_T = 6, a/L_n = 4$



Direct microstability optimization – R. Jorge et al.

arXiv > physics > arXiv:2301.09356

Physics > Plasma Physics

[Submitted on 23 Jan 2023]

Direct Microstability Optimization of Stellarator Devices

R. Jorge, W. Dorland, P. Kim, M. Landreman, N. R. Mandell, G. Merlo, T. Qian

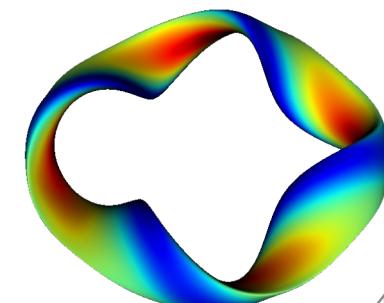
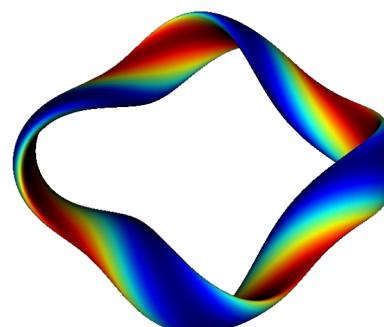
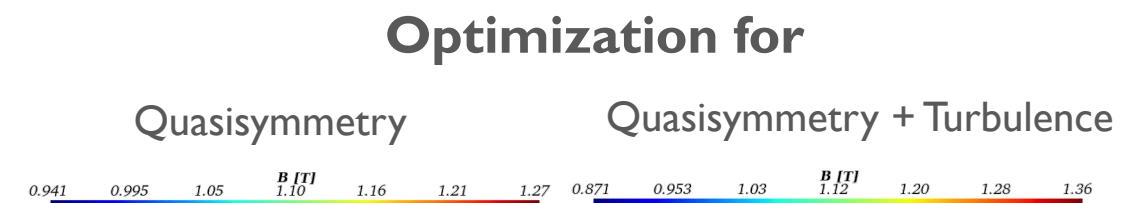
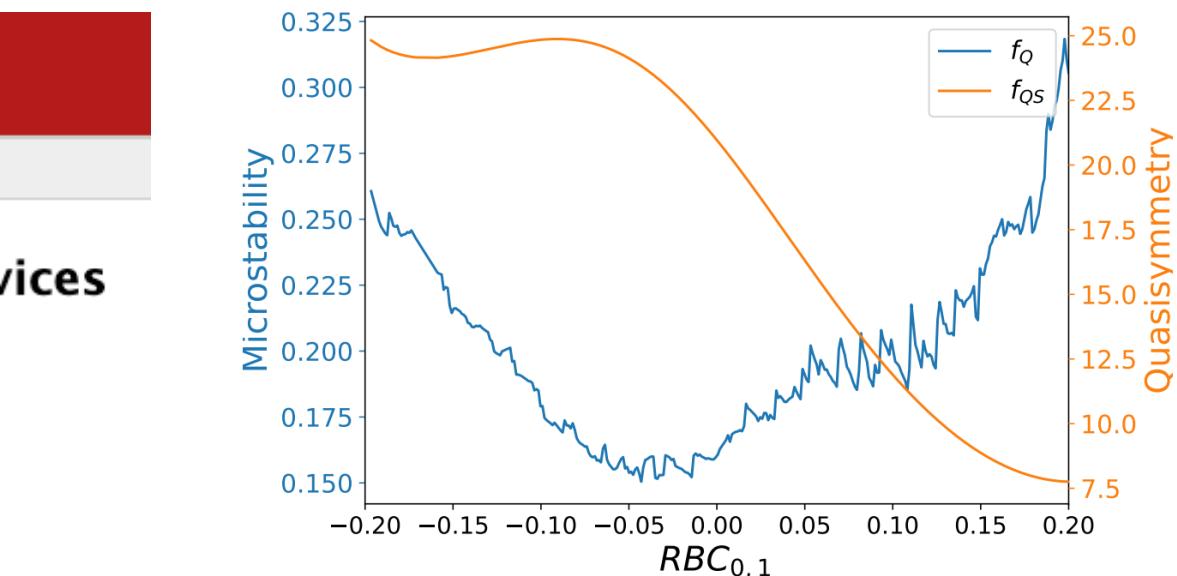
rogeriojorge / single_stage_optimization Public GitHub

$$J = \omega_{f_Q} f_Q + f_{QS}$$

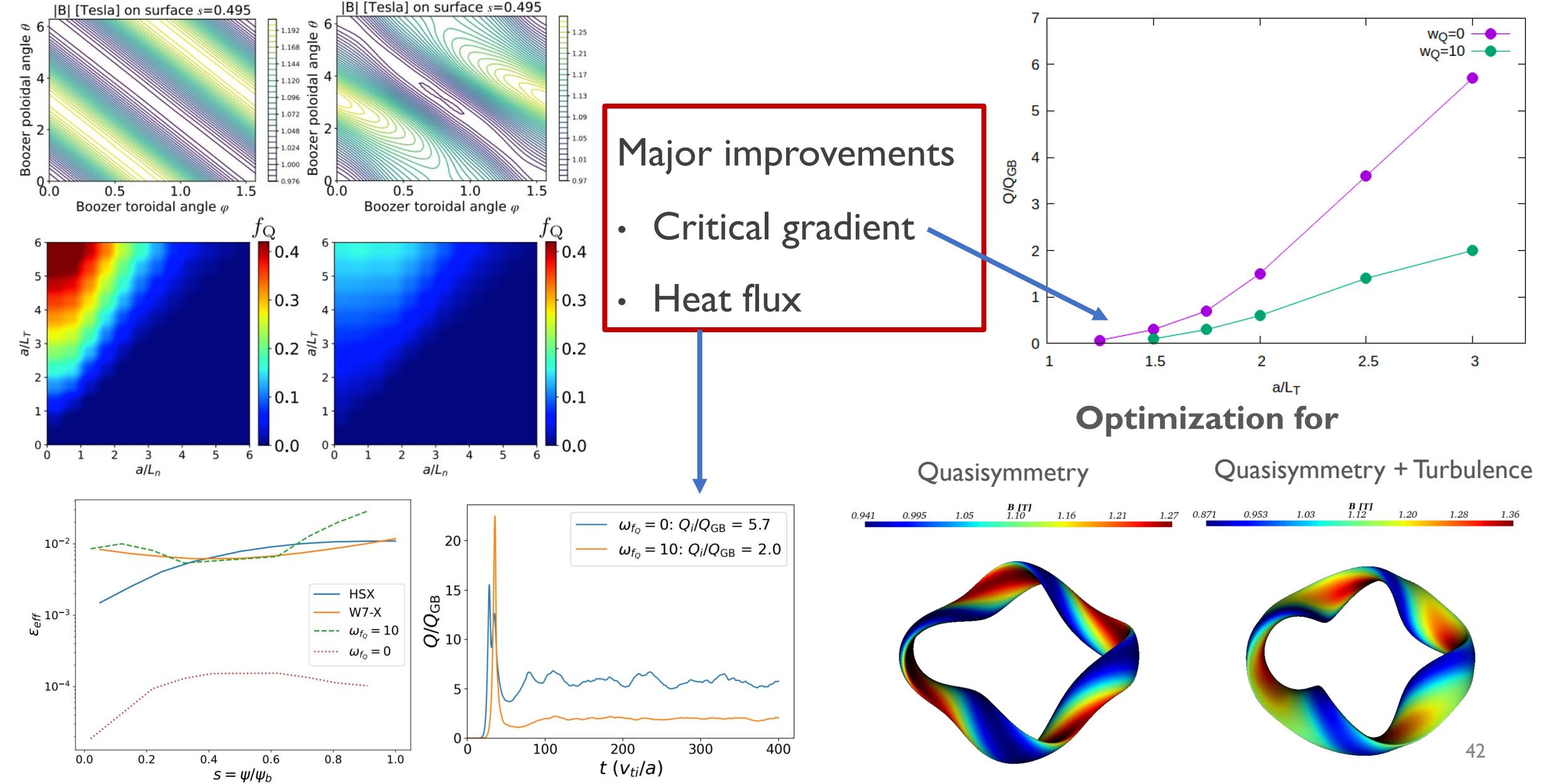
Quasilinear estimate
for the heat flux

Quasisymmetry
cost function

$$f_Q = \sum_{k_y} \frac{\gamma(k_y)}{\langle k_\perp^2 \rangle (k_y)}$$



Direct microstability optimization – R. Jorge et al.

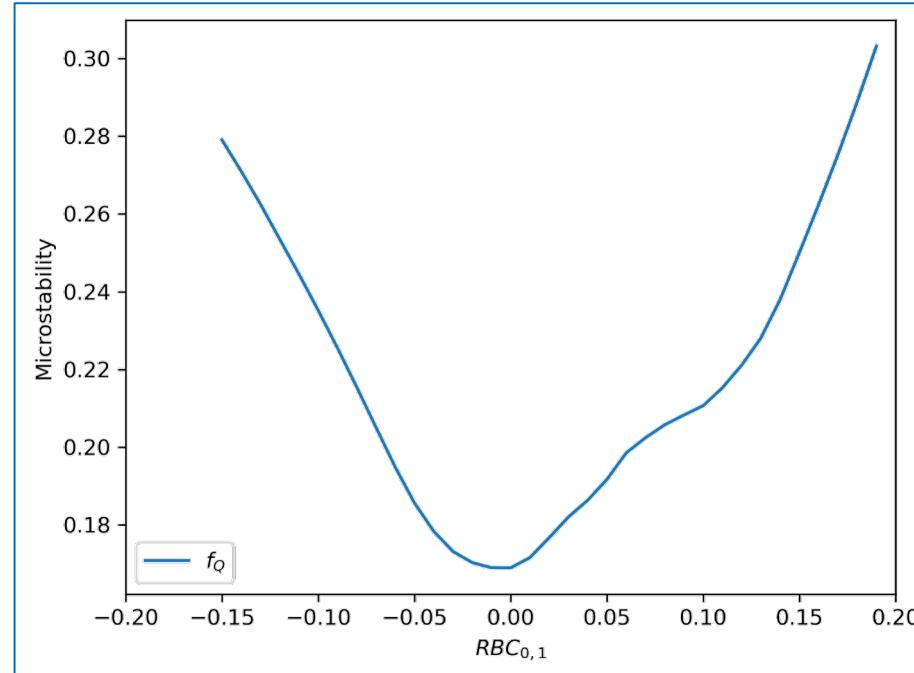


How good is a quasilinear estimate? – S. Buller (Sherwood 2023)

Similar exercise but using:



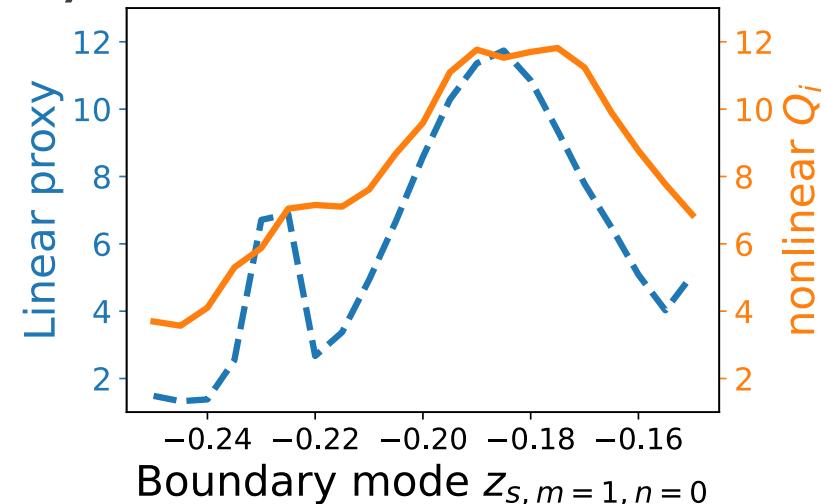
Gyrokinetic *stella* code [stellaGK/stella](#) Public



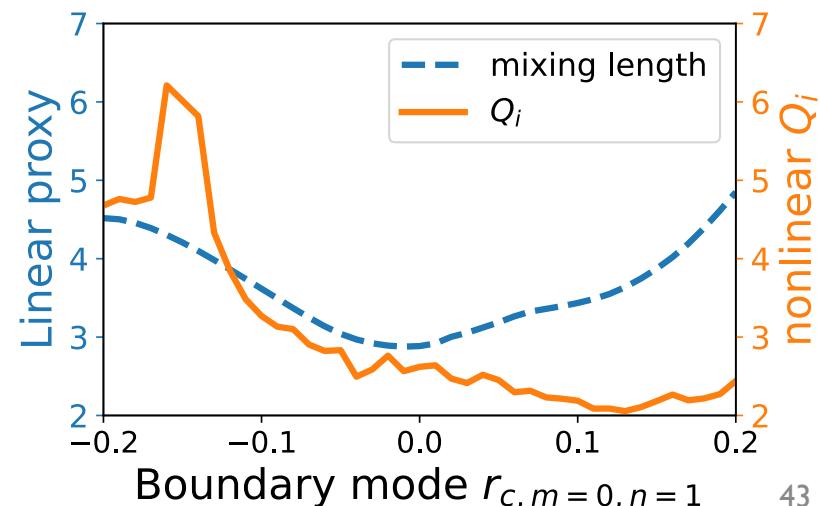
Noise-free objective function!

Microstability reduced configurations
can be found much more efficiently

Quasi-axisymmetric case



Quasi-isodynamic case



Direct turbulence optimization – Kim, Mandell, Qian, et al.



arXiv > physics > arXiv:2209.06731
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Physics > Plasma Physics

[Submitted on 14 Sep 2022 (v1), last revised 19 Oct 2022 (this version, v3)]

GX: a GPU-native gyrokinetic turbulence code for tokamak and stellarator design

N. R. Mandell, W. Dorland, I. Abel, R. Gaur, P. Kim, M. Martin, T. Qian

\mathcal{L}	\mathcal{M}	Q_i/Q_{GB}	wallclock (min)	time/step (s)	Δt
16	32	7.4 ± 0.9	467	0.14	0.01
★ 8	16	7.3 ± 0.9	61	0.037	0.02
6	12	7.5 ± 0.7	31	0.023	0.025
4	8	7.4 ± 0.9	9	0.011	0.04
★ 4	6	7.3 ± 0.8	6	0.0092	0.05

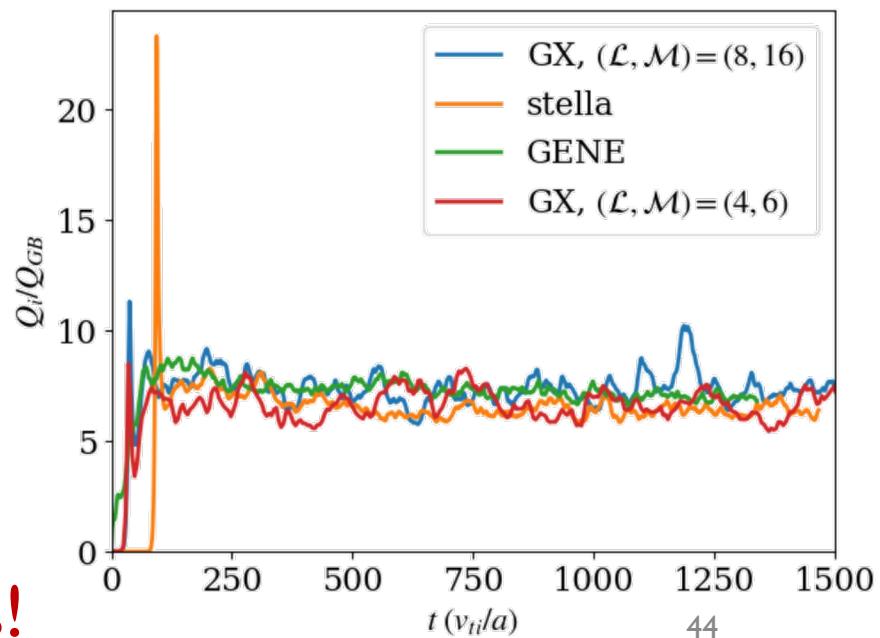
GX code (gyrokinetics in GPUs)

Hermite-Laguerre decomposition

DESC+GX optimization currently

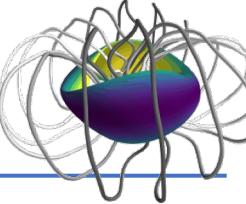
underway

We can obtain an accurate heat flux in just 6 minutes!



Direct approaches covered today

- Direct constructions using the near-axis expansion
- Direct fast particle optimization
- Direct turbulence optimization: quasi-linear and nonlinear approaches
- Direct coil optimization: single-stage approaches

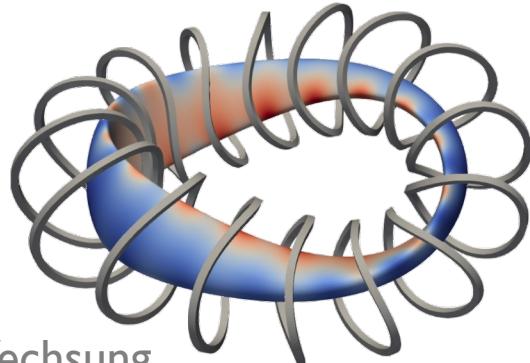


Coil optimization

Coil performance

Aim for $B \cdot n = 0$ to achieve:

- Quasi-symmetry
- Rotational transform
- Particle confinement



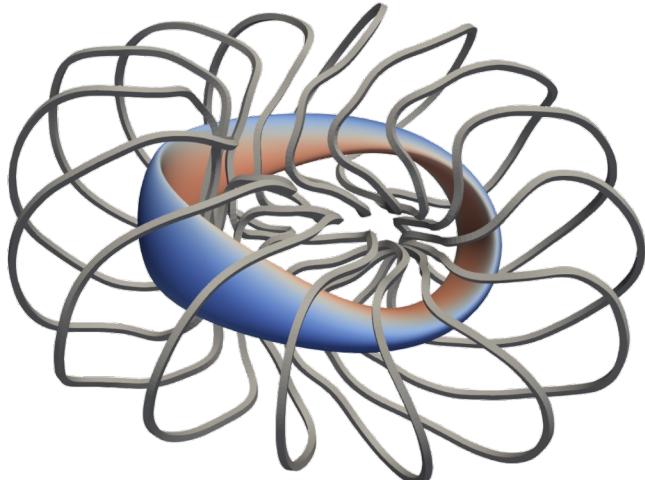
F.Wechsung
Simons 2022

Simple coils far away from the plasma are difficult to find

Coil complexity

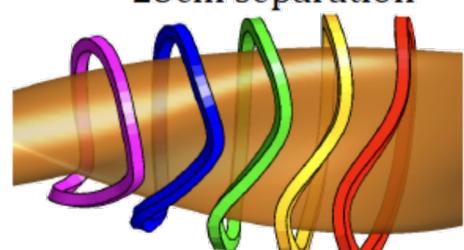
Include regularization terms to control:

- Coil curvature
- Coil-to-coil separation
- Coil length

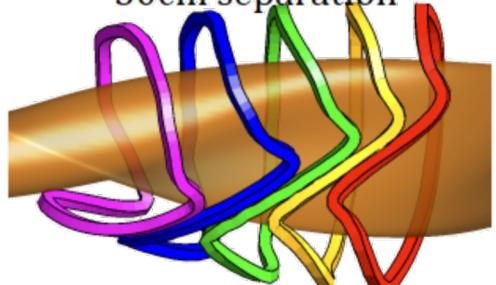


Coils offset a uniform distance from W7-X plasma:

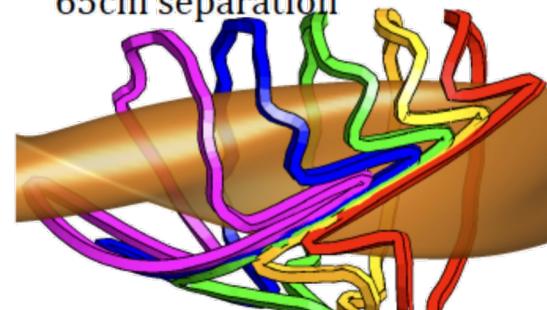
25cm separation



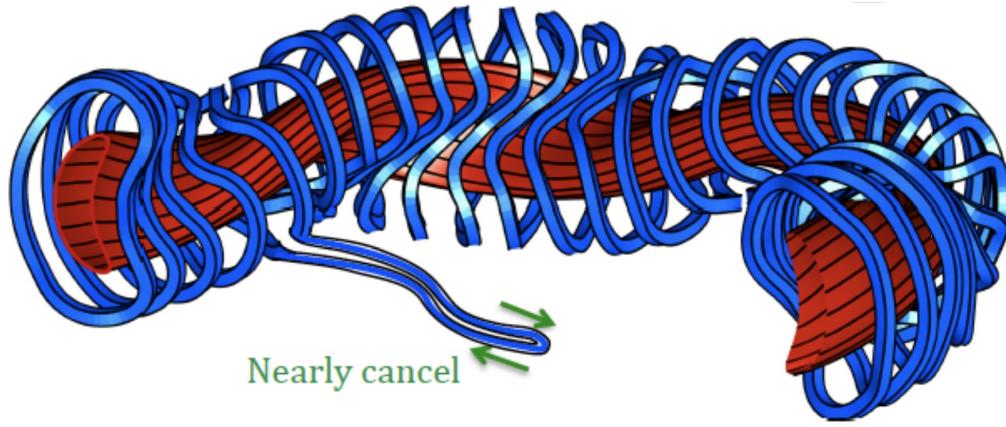
50cm separation



65cm separation



Finding coils that reproduce a given field is an ill-posed problem



M. Landreman
Simons 2019

Direct coil + near-axis optimization – A. Giuliani et al. 2021

arXiv > physics > arXiv:2010.02033

Search...

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Physics > Plasma Physics

[Submitted on 1 Oct 2020 (v1), last revised 15 Mar 2022 (this version, v2)]

Single-stage gradient-based stellarator coil design: Optimization for near-axis quasi-symmetry

Andrew Giuliani, Florian Wechsung, Antoine Cerfon, Georg Stadler, Matt Landreman

From the near-axis expansion, combining

$$\mathbf{r} = \mathbf{r}_0 + \sqrt{s} \left[\frac{\bar{\eta}}{\kappa} \cos \theta \mathbf{n} + \frac{\kappa}{\bar{\eta}} (\sin \theta + \boxed{\sigma} \cos \theta) \mathbf{b} \right]$$

with the definition of Boozer coordinates

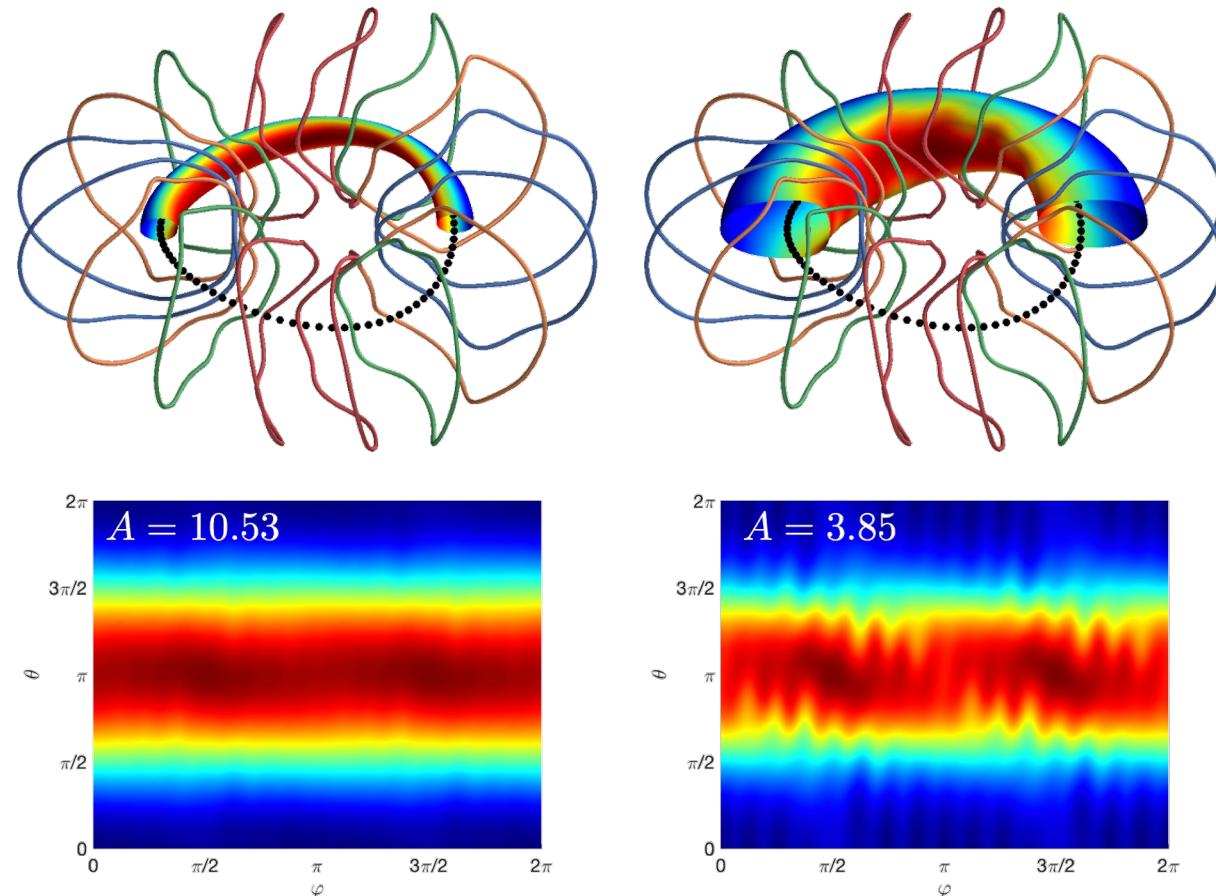
$$\mathbf{B} = \nabla \Psi \times \nabla \theta + \iota \nabla \varphi \times \nabla \Psi$$

$$\mathbf{B} = G \nabla \varphi$$

we can find coils that minimize

$$\hat{J}(\mathbf{c}, \mathbf{a}, \sigma, \iota) = \frac{1}{2} \int_{\Gamma_a} \|\mathbf{B}_{\text{coils}}(\mathbf{c}) - \mathbf{B}_{\text{QS}}(\mathbf{a})\|^2 dl + \frac{1}{2} \int_{\Gamma_a} \|\nabla \mathbf{B}_{\text{coils}}(\mathbf{c}) - \nabla \mathbf{B}_{\text{QS}}(\mathbf{a}, \sigma, \iota)\|^2 dl$$

Resulting solution



Direct surface and coil optimization – A. Giuliani et al. 2023

arXiv > physics > arXiv:2210.03248

Physics > Plasma Physics

[Submitted on 6 Oct 2022 (v1), last revised 13 Mar 2023 (this version, v3)]

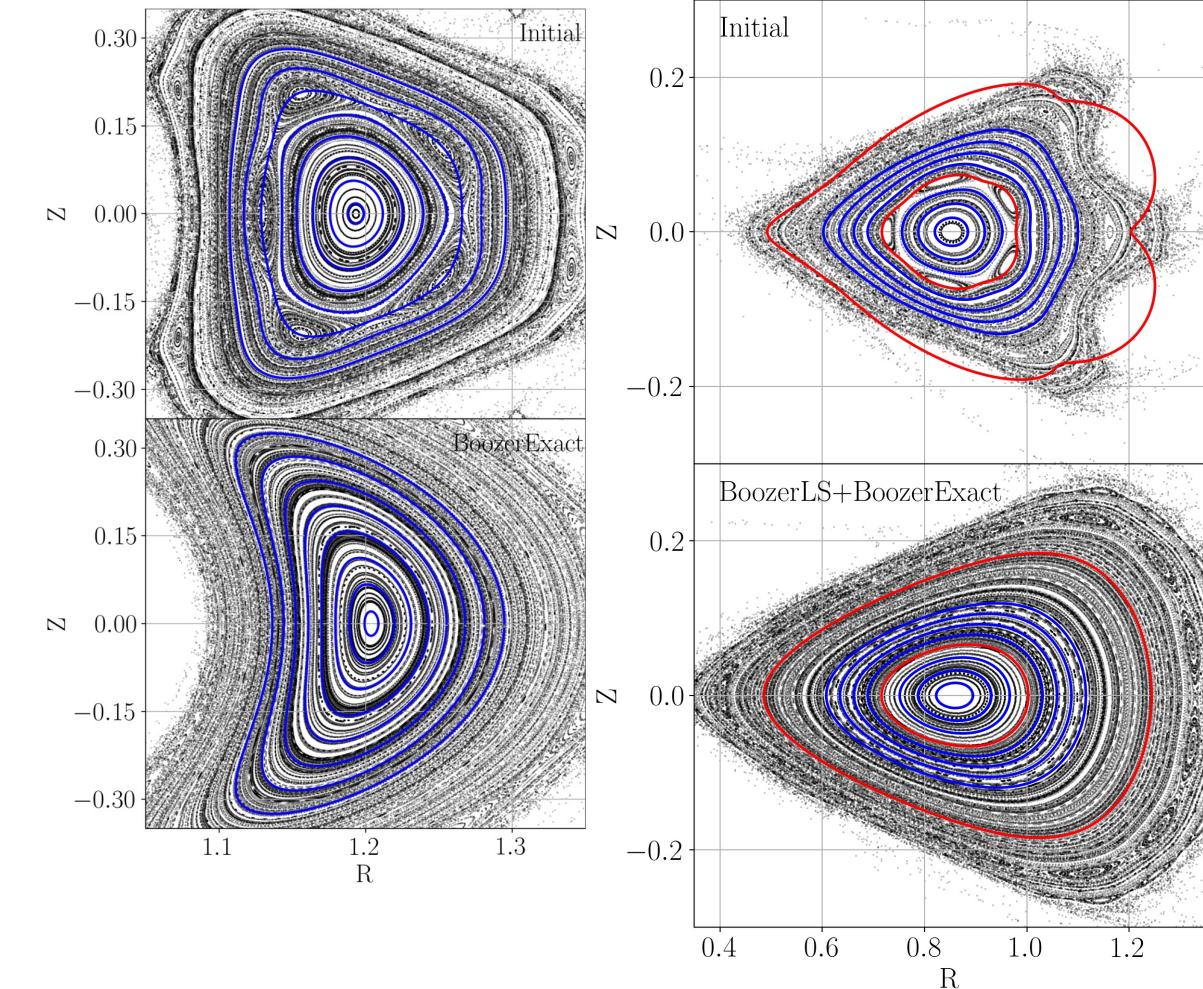
Direct stellarator coil optimization for nested magnetic surfaces with precise quasi-symmetry

Andrew Giuliani, Florian Wechsung, Antoine Cerfon, Matt Landreman, Georg Stadler

We can now heal magnetic islands
finding surfaces in the Biot-Savart
magnetic field directly (“Boozer” surfaces)

$$G\mathbf{B} - \|\mathbf{B}\|^2 \left(\frac{\partial \Sigma}{\partial \varphi} + \iota \frac{\partial \Sigma}{\partial \theta} \right) = 0,$$

$$V(\Sigma) - V_{\text{target}} = 0.$$



No need for MHD solvers!

Direct coil optimization (fixed boundary) – R. Jorge et al. 2023

arXiv > physics > arXiv:2302.10622

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Physics > Plasma Physics

[Submitted on 21 Feb 2023]

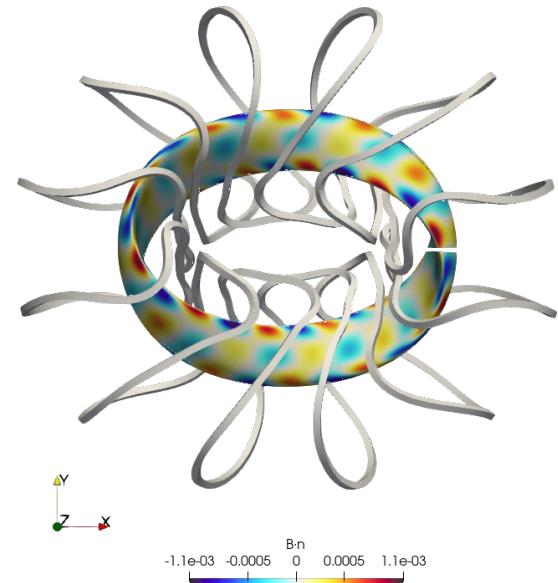
Single-Stage Stellarator Optimization: Combining Coils with Fixed Boundary Equilibria

R. Jorge, A. Goodman, M. Landreman, J. Rodrigues, F. Wechsung

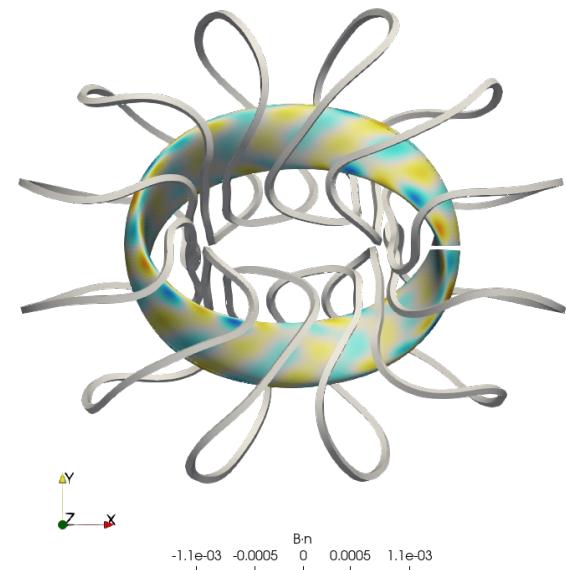
Minimize simultaneously

$$J = J_{QS} + J_{coils}$$
 using an expanded array of degrees of freedom

Stage 1 + Stage 2



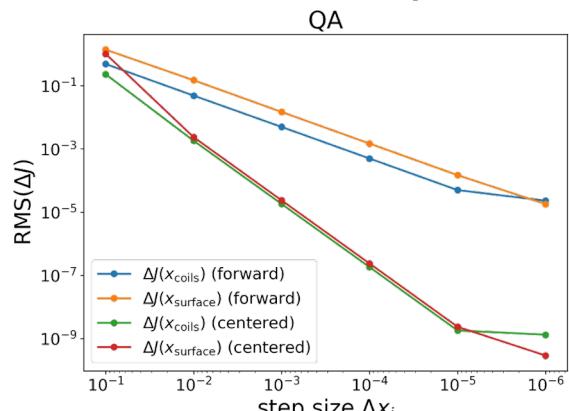
Single Stage



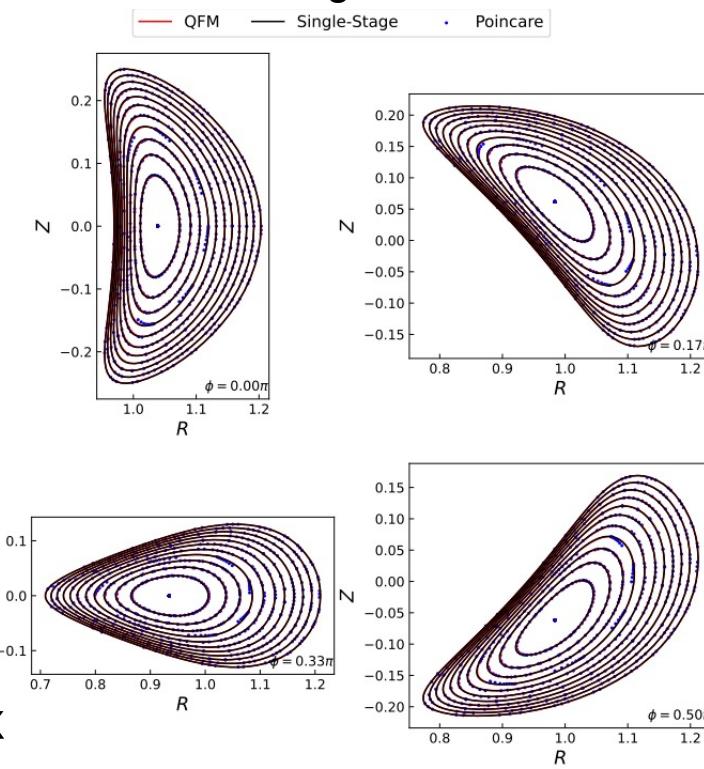
Coil cost function J_{coils} :
the squared flux

$$\int_S \left(\frac{\mathbf{B}_{\text{ext}} \cdot \mathbf{n}}{|\mathbf{B}_{\text{ext}}|} \right)^2 dS$$

Validation of the analytical derivatives of the squared flux



Validation of resulting Biot-Savart field



Single-Stage Stellarator Optimization
Of Fixed Boundary Equilibria
R Jorge et al 2023 Plasma Phys.
Control. Fusion 65 074003

Direct coil optimization (fixed boundary) – R. Jorge et al. 2023

arXiv > physics > arXiv:2302.10622

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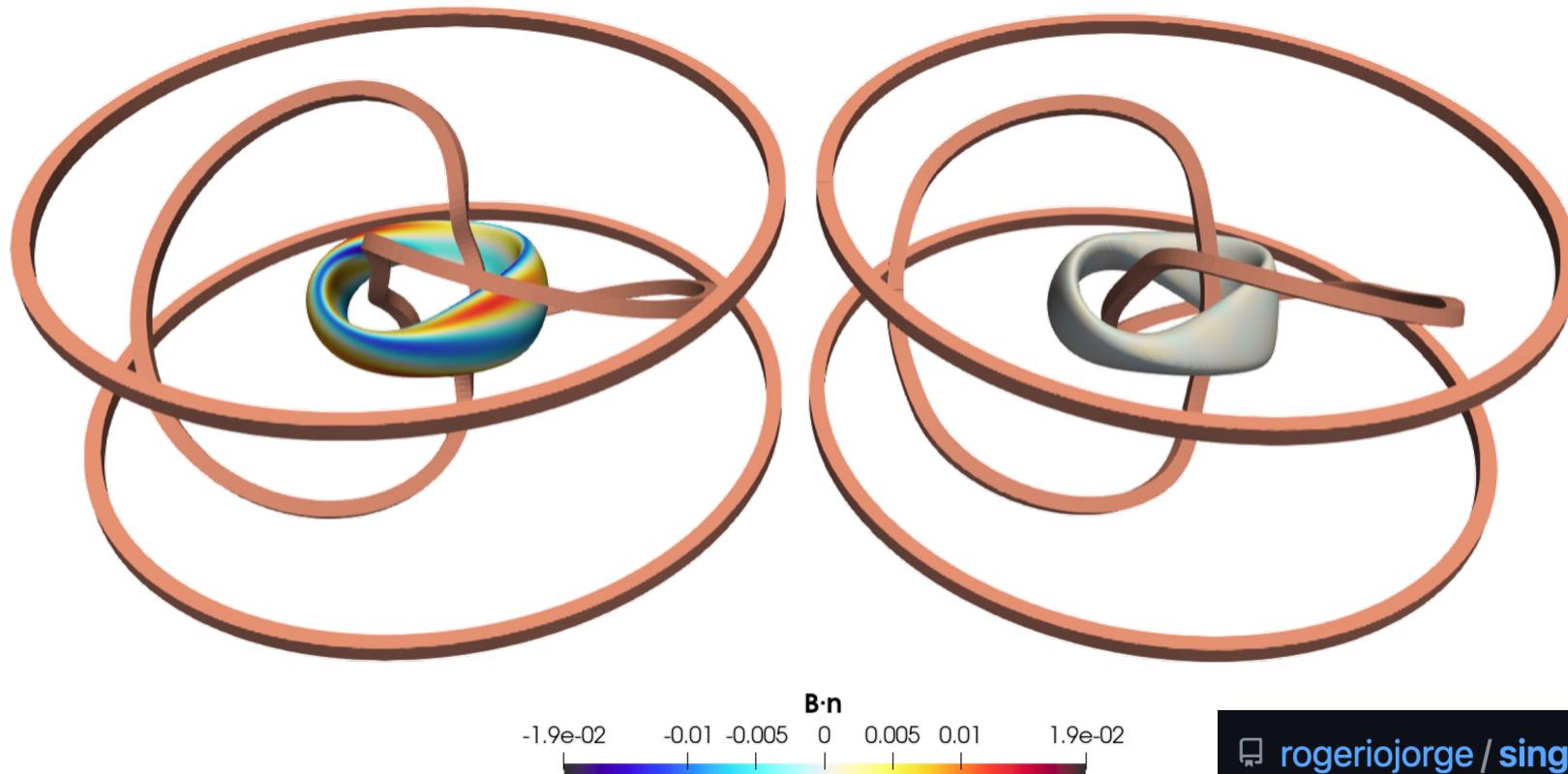
Physics > Plasma Physics

[Submitted on 21 Feb 2023]

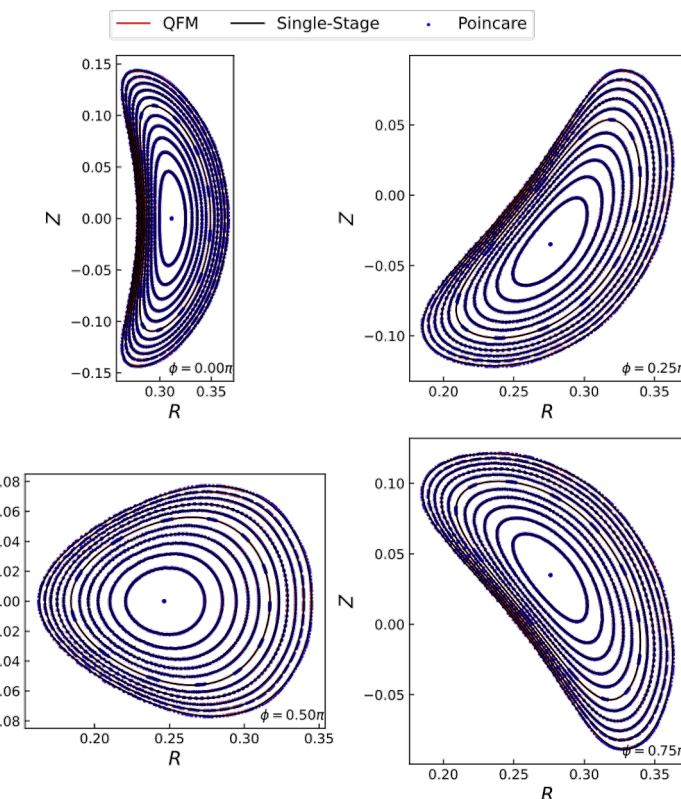
Single-Stage Stellarator Optimization: Combining Coils with Fixed Boundary Equilibria

R. Jorge, A. Goodman, M. Landreman, J. Rodrigues, F. Wechsung

4-coil quasisymmetric stellarator designed using single-stage



Validation of resulting Biot-Savart field



Single-Stage Stellarator
Optimization Of Fixed
Boundary Equilibria, R. Jorge et al,
Submitted to PPCF
<https://arxiv.org/abs/2302.10622>



Direct coil optimization with finite β – R. Jorge et al. 2023

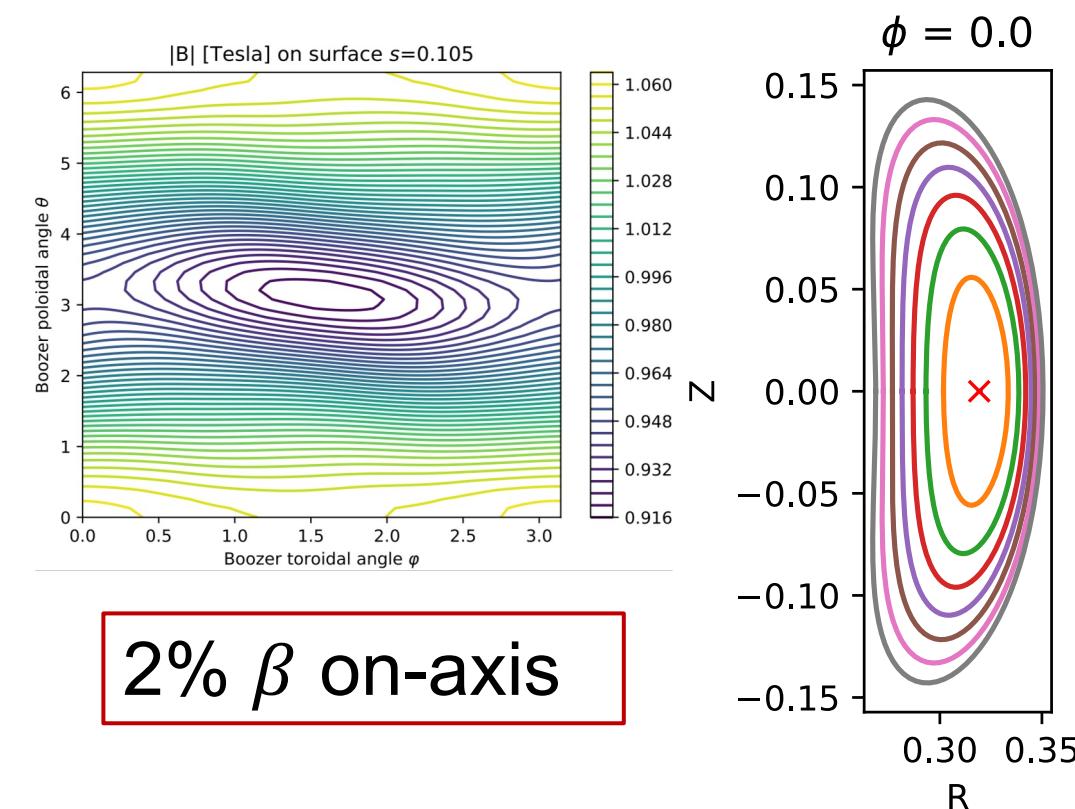
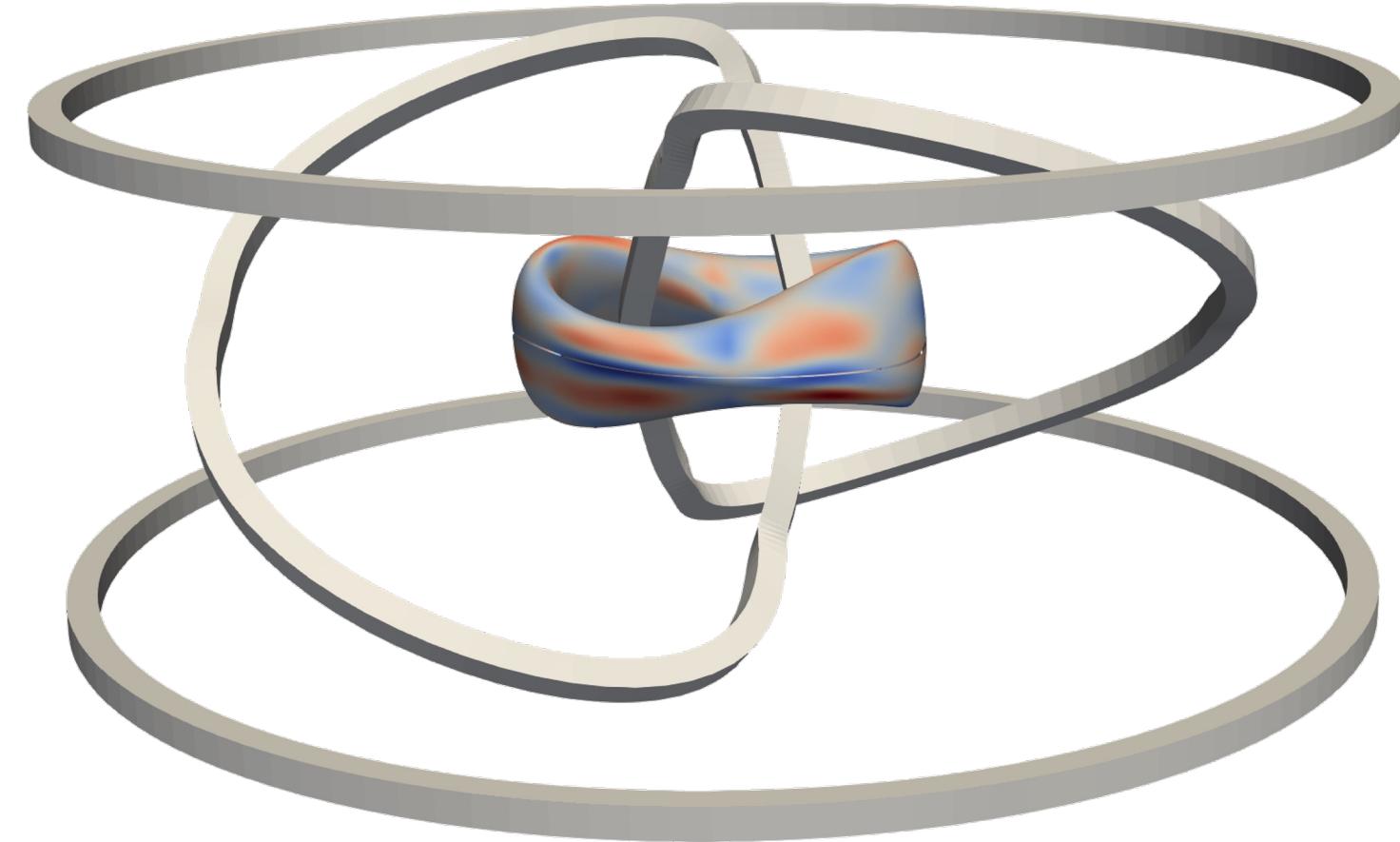
New coil cost function J_{coils} taking into account plasma pressure

$$\int_S \left[\frac{(\mathbf{B}_{\text{ext}} - \mathbf{B}_{\text{target}}) \cdot \mathbf{n}}{|\mathbf{B}_{\text{ext}}|} \right]^2 dS$$

VMFC \mathbf{B}

$$\mathbf{B} = \mathbf{B}_{\text{plasma}} + \mathbf{B}_{\text{target}}$$

Coils targeted to reproduce vacuum field



2% β on-axis

$\mathbf{B}_{\text{plasma}}$ calculated using the Virtual Casing principle (BIEST code)

hiddenSymmetries / virtual-casing Public

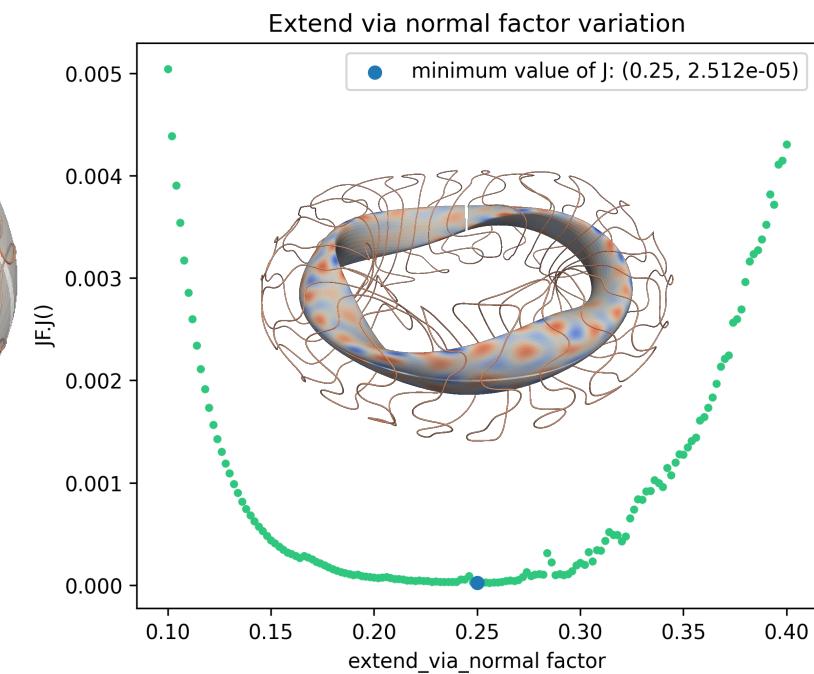
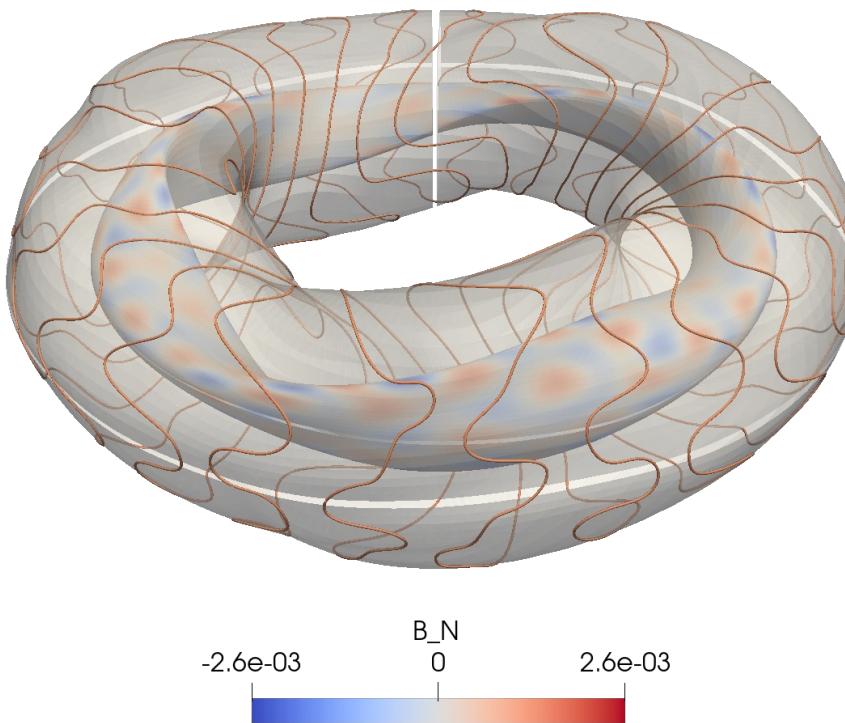
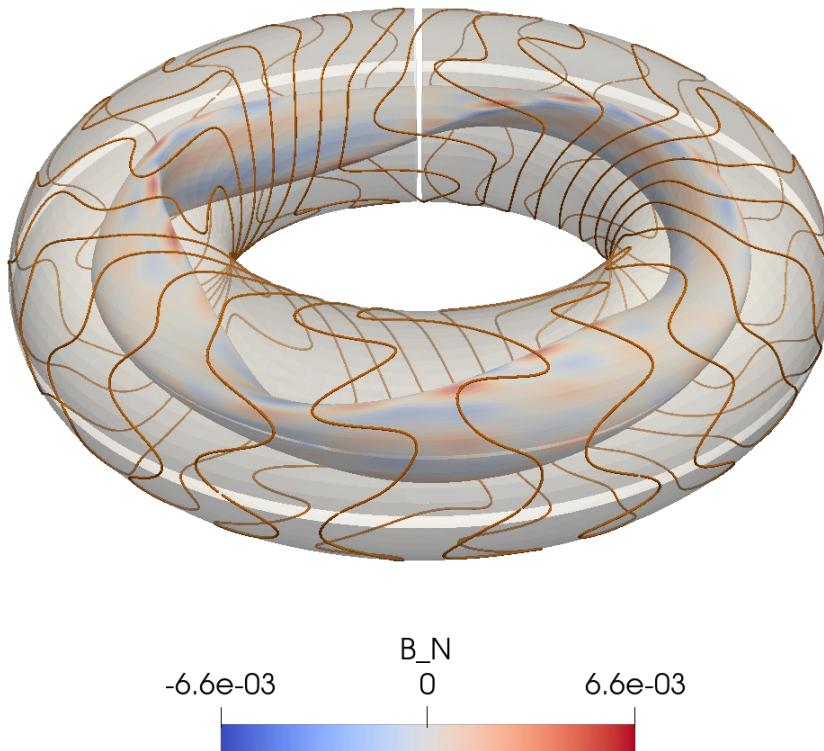
dmalhotra / BIEST Public

Coil Winding Surface optimization – J. Biu (IST Lisbon)

Single-stage optimized plasma
for an axisymmetric coil surface

Coil surface that follows the
plasma boundary improves ripple

**Optimal plasma to surface
distance: 0.25/Major radius**

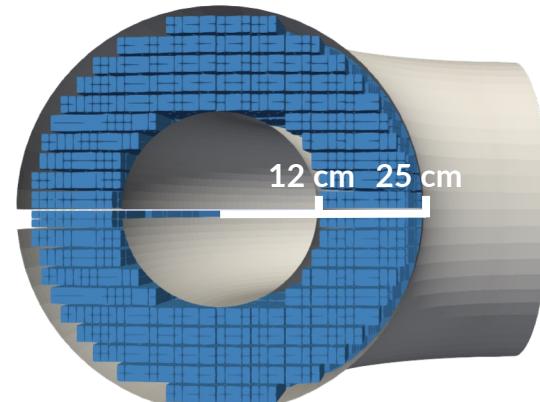


Single-stage & Permanent Magnets – M. Madeira (IST Lisbon)

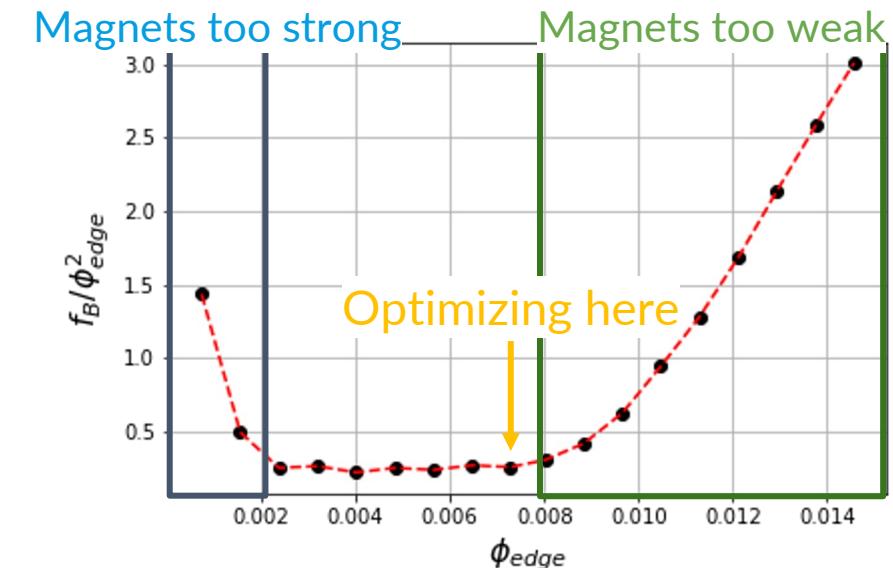
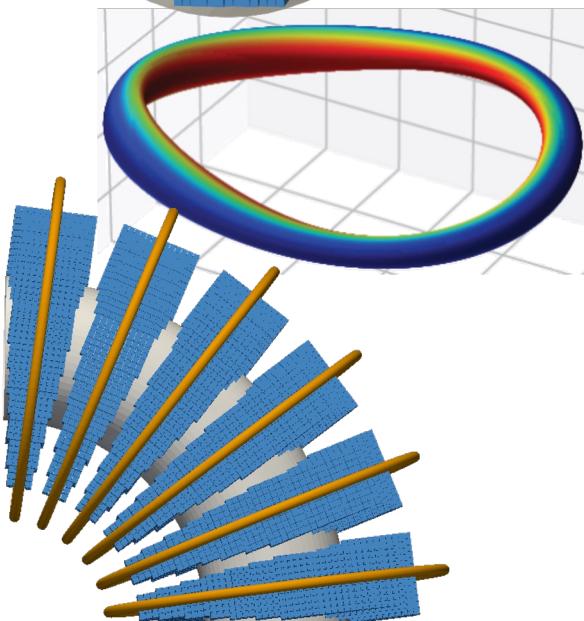
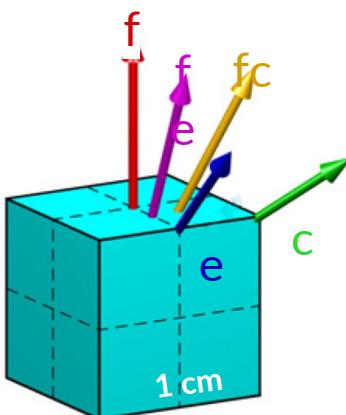
This is ISTTOK + Miguel Madeira



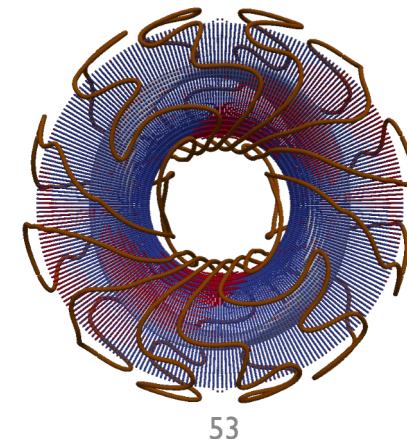
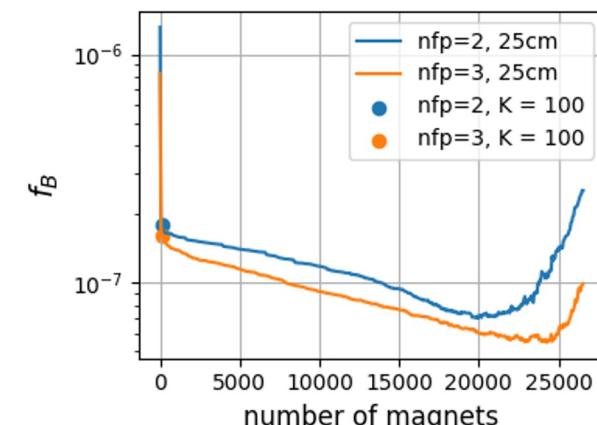
This is ISTTELL – a QA stellarator
to fit into ISTTOK



Use permanent magnets to
convert a tokamak into a
stellarator without new coils



Permanent magnets can also be
used to fix coil ripple in stellarators



Conclusions

- Proxies start to be replaced by accurate figures of merit
- Near-axis expansion is the Swiss Army Knife of stellarator optimization
- Direct fast particle may require global optimization methods
- Direct turbulence optimization enabled by new turbulence codes
- Direct coil optimization performed using single stage methods
- Next steps: *direct plasma stability, particle flux, maximum-J, help from AI*