2D Ising model

We studied the two-dimensional Ising model using the Monte Carlo method, with the Metropolis algorithm implemented on a discretized lattice in which each site contains a spin variable s_x taking values ± 1. The energy of a given spin configuration is defined as:

$$H = -\sum_{\langle x,y \rangle} s_x s_y \;, \tag{1}$$

where the sum runs over all pairs of nearest neighbours. The corresponding action is given by:

$$S = \beta H = -\beta \sum_{\langle x,y \rangle} s_x s_y \;, \tag{2}$$

with $\beta = 1/T$, where T is the temperature of the system.

Simulations were performed on square lattices with periodic boundary conditions and volumes $V = 32 \times 32$, 24×24, 16×16, and 12×12. The Metropolis algorithm generates new configurations by scanning each site of the lattice and proposing a spin flip. If the proposed flip lowers the energy of the system, it is accepted unconditionally. Otherwise, it is accepted with probability

$$p = e^{-\beta \Delta H} \;, \tag{3}$$

where $\Delta H = H[s'] - H[s]$, with $[s]$ representing the current configuration and $[s']$ the proposed configuration. In this way, a random number $r \in [0, 1]$ is drawn, and the spin is flipped if $r < p$. A full update of all lattice sites constitutes a single *sweep*.

After thermal equilibrium is reached (typically after 1000 sweeps), physical observables were measured, including:

Average energy: $\langle H \rangle$

Specific heat: $c_v = \beta^2 \frac{\langle H^2 \rangle - \langle H \rangle^2}{V}$

Total magnetization: $M = \sum_i s_i$

Magnetization density: $m = \frac{\langle |M| \rangle}{V}$

Magnetic susceptibility: $\chi_m = \frac{\langle M^2 \rangle - \langle |M| \rangle^2}{V}$

All the above quantities were computed over a temperature range $T \in [1, 5]$. The results are displayed in the plots of Figure 2. From these, it is observed that the critical temperature T_c lies within the interval $2 < T_c < 3$, in good agreement with the theoretical value $T_c = 2.27$. Near this critical point, both the specific heat and the magnetic susceptibility show a pronounced peak, indicating a second-order phase transition. This is further corroborated by the behavior of the magnetization shown in Figure 2c, where magnetization tends to zero for $T > T_c$. Additionally, the plots illustrate that the values of the thermodynamic observables grow more sharply with increasing lattice volume, as expected due to finite-size effects.

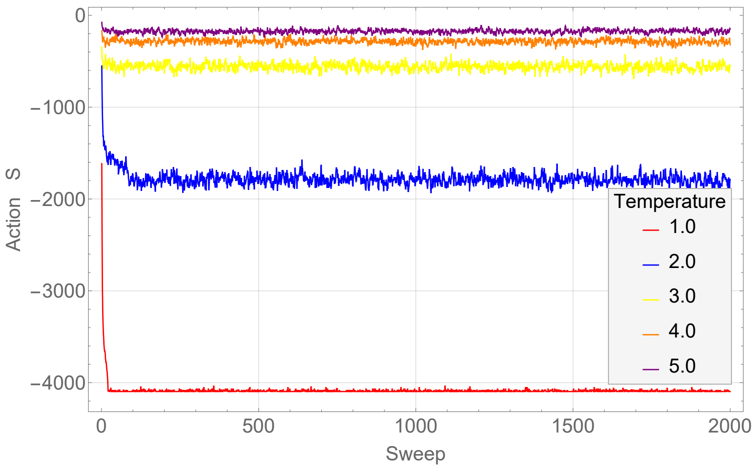
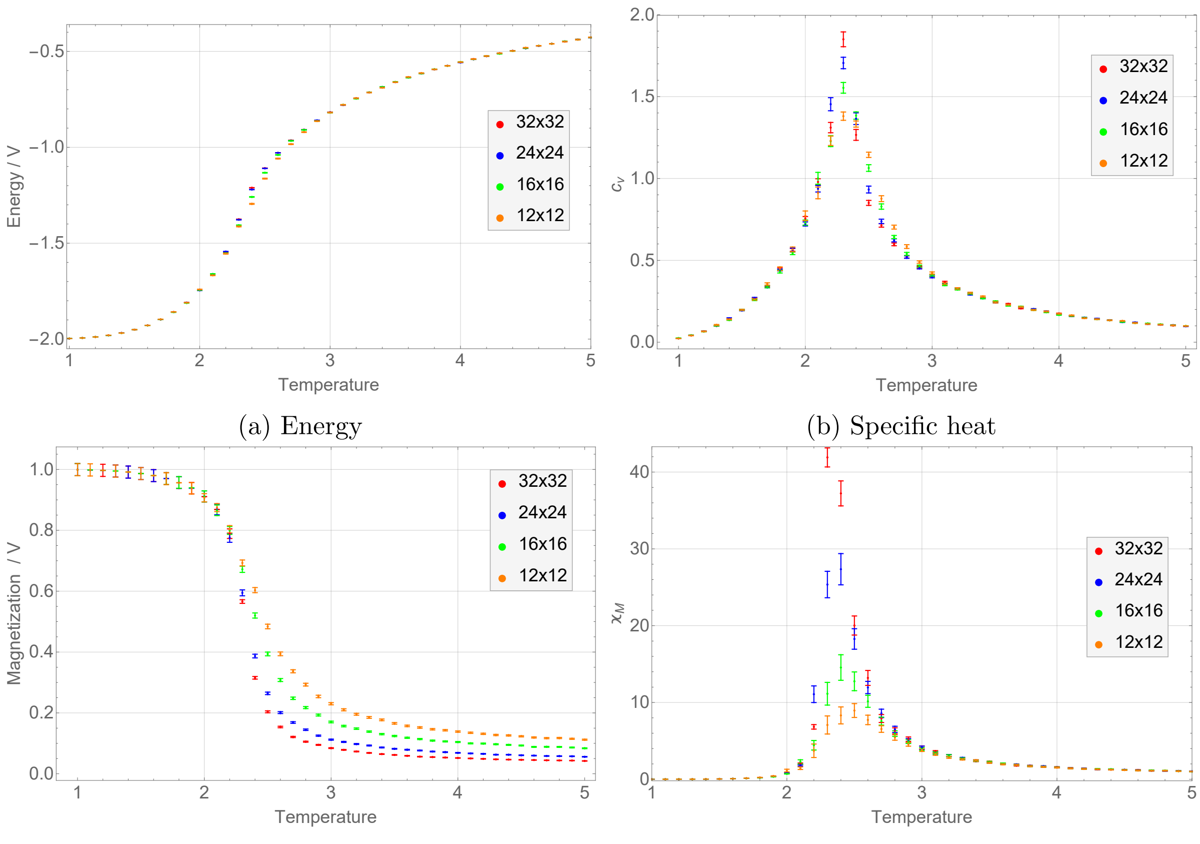


Figure 1: Thermalization of the Ising model, showing the action S as a function of Monte Carlo sweeps. After an initial transient period, the system reaches a stable plateau. Results shown correspond to a 32×32 lattice.



(a) Energy

(b) Specific heat

(c) Magnetization

(d) Magnetic Susceptibility

Figure 2: Physical observables of the Ising model in two dimensions as a function of temperature for different volumes.