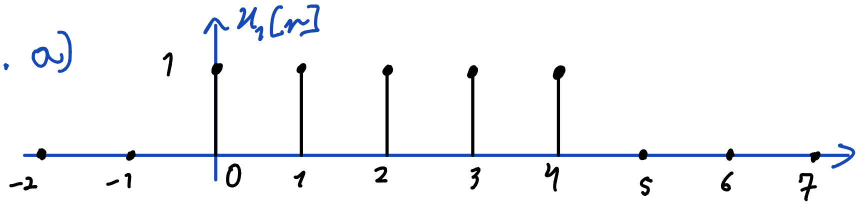
1. a)



Tomando $N=10$

(Escolher $N=5$ faz $X[k]=0$)

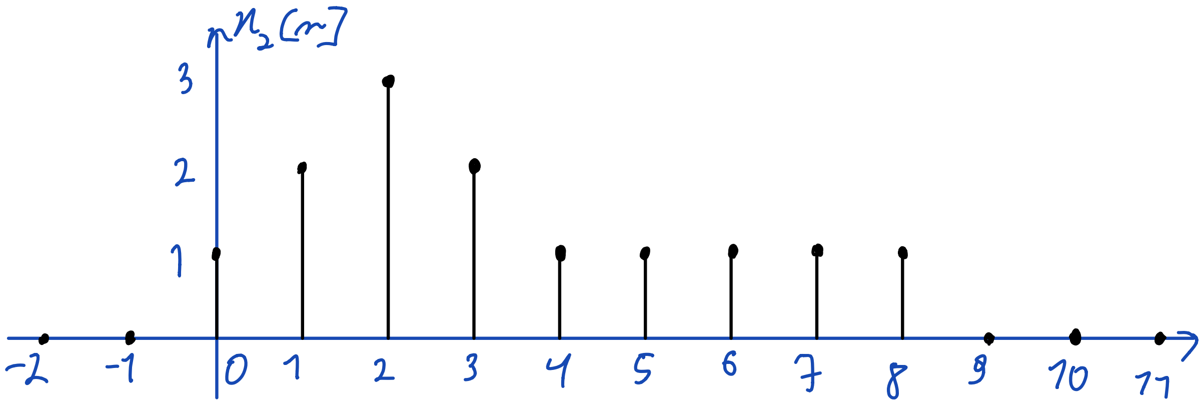
$$X_1[k] = \sum_{n=0}^{N-1} x_1[n] e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{4} e^{-j\frac{2\pi}{10}kn}$$

$$X_1[k] = \sum_{n=0}^{\infty} e^{-j\frac{\pi}{5}k\cdot n} - \sum_{n=5}^{\infty} e^{-j\frac{\pi}{5}k\cdot n} = \sum_{n=0}^{\infty}\left(e^{-j\frac{\pi}{5}k\cdot n} - e^{-j\frac{\pi}{5}k\cdot(n+5)}\right)$$

$$X_1[k] = (1-e^{-jk\pi}) \sum_{n=0}^{\infty} e^{-j\frac{\pi}{5}k\cdot n} = \frac{1-e^{-jk\pi}}{1-e^{-j\frac{\pi}{5}k}} \cdot \frac{e^{j\frac{k\pi}{2}}}{e^{jk\frac{\pi}{2}}} \cdot \frac{e^{j\frac{\pi}{10}k}}{e^{j\frac{\pi}{10}k}}$$

$$X_1[k] = \frac{e^{j\frac{k\pi}{2}} - e^{-j\frac{k\pi}{2}}}{e^{j\frac{k\pi}{10}} - e^{-j\frac{k\pi}{10}}} \cdot e^{jk(\frac{\pi}{10}-\frac{\pi}{2})} = \frac{sen[\frac{\pi}{2}k]}{sen[\frac{\pi}{10}k]} e^{-j\frac{2\pi}{5}k}$$

b)



Tomando $N=10$: $X_2[k] = \sum_{n=0}^{9} x_2[n] e^{-j\frac{2\pi}{10}kn} = \sum_{n=0}^{8} x_2[n] e^{-j\frac{\pi}{5}kn}$

$$X_2[k] = 1 + 2e^{-j\frac{\pi}{5}\cdot k} + 3e^{-j\frac{\pi}{5}\cdot 2k} + 2e^{-j\frac{\pi}{5}\cdot 3k} + \sum_{n=4}^{8} e^{-j\frac{\pi}{5}k\cdot n}$$

$$X_2[k] = 1 + 2e^{-j\frac{\pi}{5}k} + 3e^{-j\frac{2\pi}{5}k} + 2e^{-j\frac{3\pi}{5}k} + e^{-j\frac{4\pi}{5}k} + e^{-jk\pi} + e^{-j\frac{6\pi}{5}k} + e^{-j\frac{7\pi}{5}k} + e^{-j\frac{8\pi}{5}k}$$

$X_2[0] = 13$

$X_2[1] = 0,309 - 4,0287j$

$X_2[2] = -2,4271 - 2,4899j$

$X_2[3] = -0{,}809 - 0{,}1388j$

$X_2[4] = 0{,}9271 - 0{,}2245j$

$X_2[5] = 1$

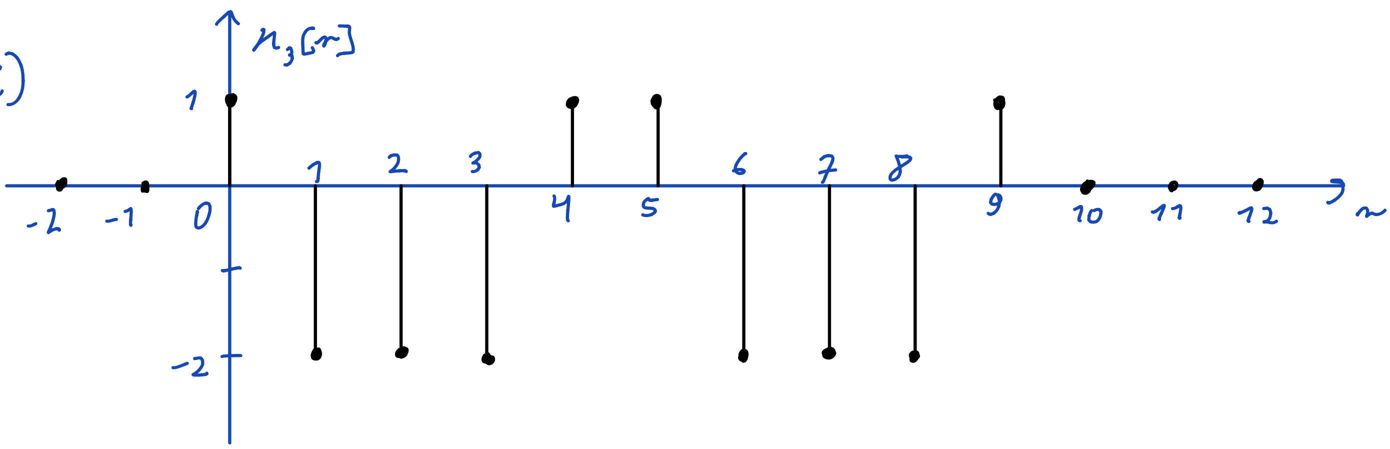
$X_2[6] = 0{,}9271 + 0{,}2245j$

$X_2[7] = -0{,}809 + 0{,}1388j$

$X_2[8] = -3{,}4271 + 2{,}4899j$

$X_2[9] = 0{,}309 + 4{,}0287j$

c)



Tomando $N=10$: $X_3[k] = \sum_{n=0}^{9} x_3[n] e^{-j\frac{\pi}{5}k\cdot n}$

$$X_3[k] = 1 - 2\left(e^{-j\frac{\pi}{5}k} + e^{-j\frac{2\pi}{5}k} + e^{-j\frac{3\pi}{5}k}\right) + e^{-j\frac{4\pi}{5}k} + e^{-jk\pi} - 2\left(e^{-j\frac{6\pi}{5}k} + e^{-j\frac{7\pi}{5}k} + e^{-j\frac{8\pi}{5}k}\right) + e^{-j\frac{9\pi}{5}k}$$

$X_3[0] = 1 - 2\cdot 3 + 2 - 2\cdot 3 + 1 = -8$

$X_3[1] = 0$

$X_3[2] = 7{,}8541 - 5{,}7063j$

$X_3[3] = 0$

$X_3[4] = 1{,}1459 - 3{,}5267j$

$X_3[5] = 0$

$X_3[6] = 1{,}1459 + 3{,}5267j$

$X_3[7] = 0$

$X_3[8] = 7{,}8541 + 5{,}7063j$

$X_3[9] = 0$

2. $x_1[n] \circledast x_3[n] = ?$

Sabemos que $x_1[n] \circledast x_3[n] \longleftrightarrow X_1[k]X_3[k]$

→ Escolhendo $N=10$, podemos utilizar os DFTs obtidos anteriormente.

$$X_1 = \begin{bmatrix} 5 \\ 1+3{,}077j \\ 0 \\ 1+0{,}7265j \\ 0 \\ 1 \\ 0 \\ 1-0{,}7265j \\ 0 \\ 1-3{,}077j \end{bmatrix} \qquad X_3 = \begin{bmatrix} -8 \\ 0 \\ 7{,}8541-5{,}7063j \\ 0 \\ 1{,}7459-3{,}5267j \\ 0 \\ 1{,}7459+3{,}5267j \\ 0 \\ 7{,}8541+5{,}7063j \\ 0 \end{bmatrix}$$

Segue: $Y[k] = X_1[k]X_3[k]$.

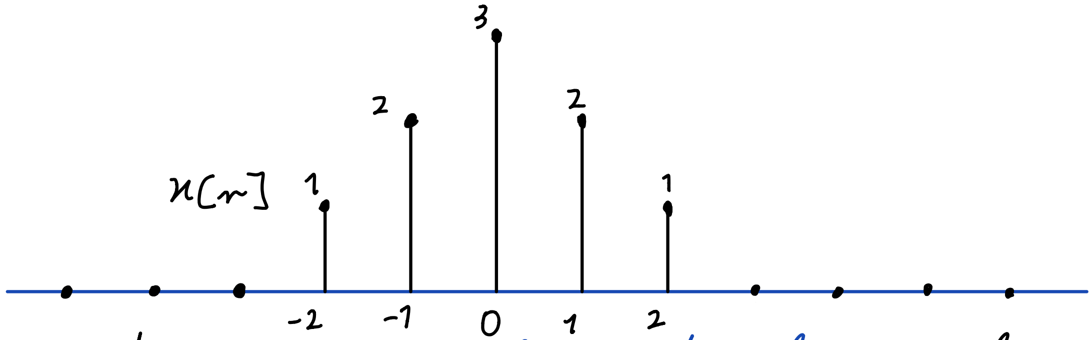
$$Y = \begin{bmatrix} -40 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Logo, $Y[0] = -40$ e $Y[k]=0$, $1 \leq k \leq 9$.

Como $y[n] = \frac{1}{N}\sum_{k=0}^{N-1} Y[k]e^{j\frac{2\pi}{N}kn}$,

segue que $y[n] = \frac{-40}{10} = -4$

3.



- puramente par → projeção sobre a parte real → DFT real
- puramente ímpar → projeção sobre a parte imaginário → DFT imaginário

Fazendo $N=5$:

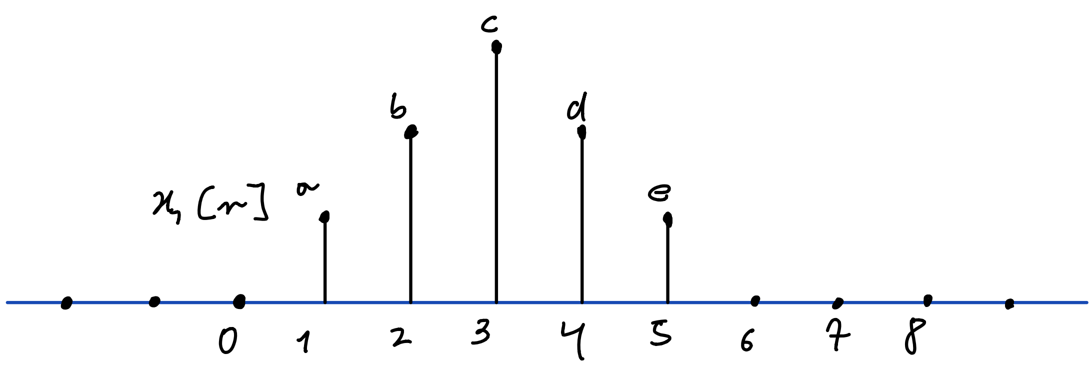
$$X[k]=\sum_{n=-2}^{2} x[n]\, e^{-j\frac{2\pi}{5}k\cdot n} = e^{j\frac{4\pi}{5}k} + 2e^{j\frac{2\pi}{5}k} + 3 + 2e^{-j\frac{2\pi}{5}k} + e^{-j\frac{4\pi}{5}k}$$

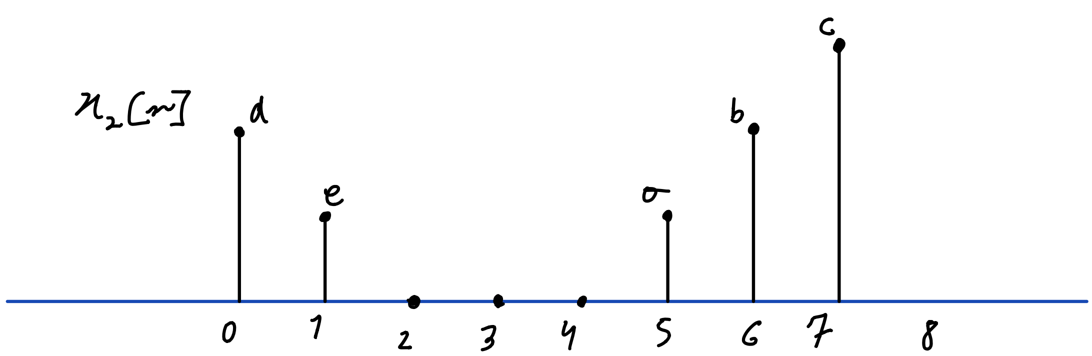
$$X[k] = \left(e^{j\frac{4\pi}{5}k}+e^{-j\frac{4\pi}{5}k}\right) + 2\left(e^{j\frac{2\pi}{5}k}+e^{-j\frac{2\pi}{5}k}\right) + 3$$

$$X[k] = 2\cos\left[\frac{4\pi}{5}k\right] + 4\cos\left[\frac{2\pi}{5}k\right] + 3$$

Portanto, fazendo $N=5$ e tomando $x[0]=3$, $X[k]$ é puramente real.

4.





$\rightarrow x_1[(n)_8] \rightarrow x_2[(n)_8] = x_1[(n-4)_8]$

Com isso, $X_2[k] = e^{-j\frac{2\pi}{8}\cdot 4k} X_1[k]$

$$X_2[k] = e^{-jk\pi} X_1[k]$$

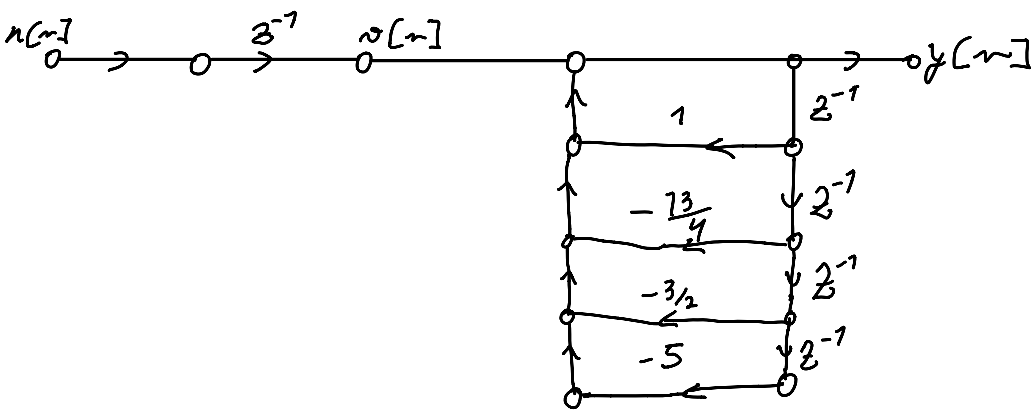
5. a) $H(z) = \dfrac{z^3}{(z-2)^2(z-j+1/2)(z+j+1/2)}$

Forma Direta I: $H(z) = \dfrac{z^3}{(z^2-2z+4)(z^2+z+\frac{5}{4})} = \dfrac{z^3}{z^4-z^3+\frac{13}{4}z^2+\frac{3}{2}z+5}\cdot\dfrac{z^{-4}}{z^{-4}}$

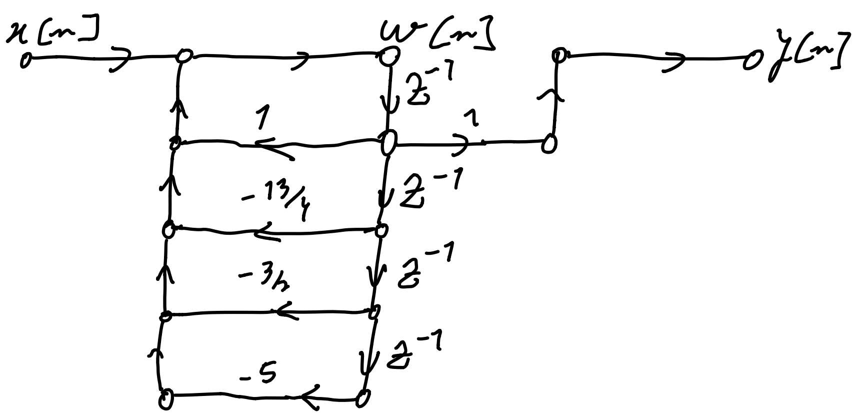
$$H(z) = \frac{\overset{\sum_{n=0}^{M} b_n z^{-n}}{z^{-1}}}{\underbrace{1-z^{-1}+\frac{13}{4}z^{-2}+\frac{3}{2}z^{-3}+5z^{-4}}_{1+\sum_{k=1}^{N} a_k z^{-k}}} = \frac{Y(z)}{X(z)}$$

$$\longrightarrow y[n] = \sum_{n=0}^{M} b_n x[n-n] - \sum_{k=1}^{N} a_k y[n-k]$$

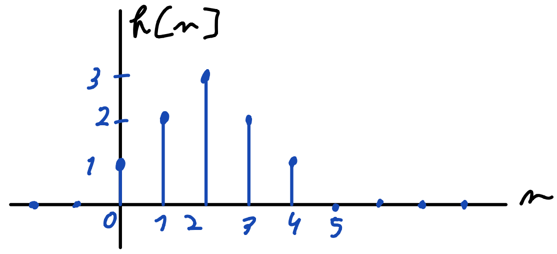
$$y[n] = x[n-1] + y[n-1] - \frac{13}{4}y[n-2] - \frac{3}{2}y[n-3] - 5y[n-4]$$



Forma direta II:

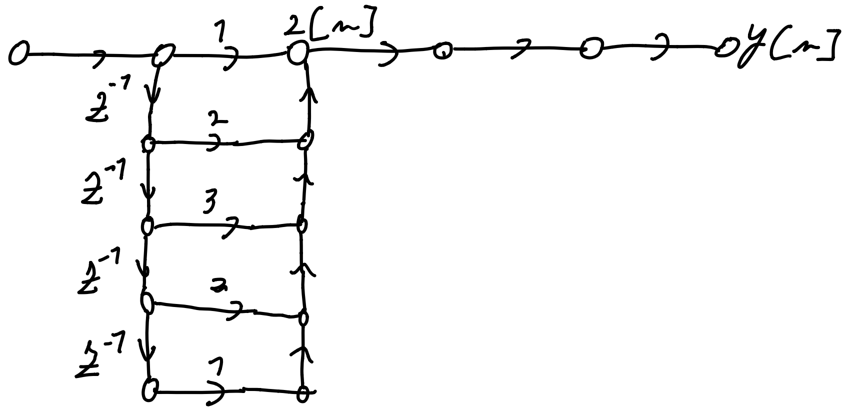


b)



$\rightarrow H(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$

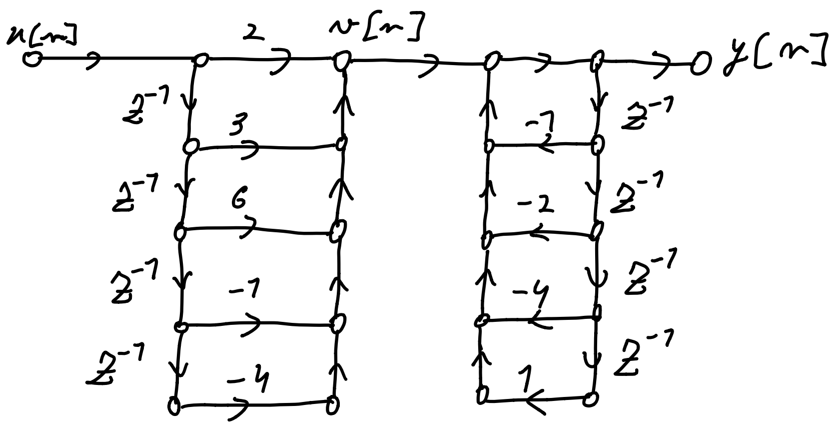
Forma direta I:



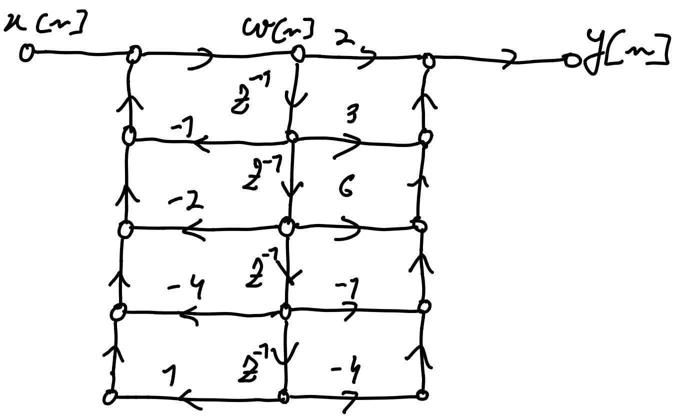
Forma direta II: fica igual à forma direta I.

c) $H(z) = \dfrac{2 + 3z^{-1} + 6z^{-2} - z^{-3} - 4z^{-4}}{1 + z^{-1} + 2z^{-2} + 4z^{-3} - z^{-4}}$

Forma Direta I:



Forma direta II:



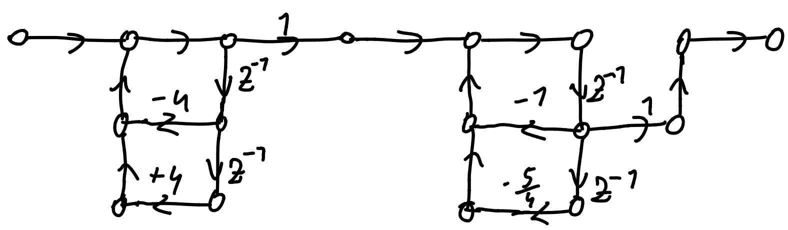
Obs.: $y[n] = 2x[n] + 3x[n-1] + 6x[n-2] - x[n-3] - 4x[n-4] - y[n-1] - 2y[n-2]$
$-4y[n-3] + y[n-4]$

6. (5a) $H(z)=\dfrac{z^3}{(z-2)^2(z-j+1/2)(z+j+1/2)}$

Forma em cascata:

$$H(z)=\frac{z^2}{z^2-4z+4}\cdot\frac{z}{z^2+z+\frac{5}{4}}\cdot\frac{z^{-4}}{z^{-4}}$$

$$H(z)=\frac{1}{1-4z^{-1}+4z^{-2}}\cdot\frac{z^{-1}}{1+z^{-1}+\frac{5}{4}z^{-2}}$$



Realização em paralelo: $H(p)=\dfrac{p}{1-p+\frac{13}{4}p^2+\frac{3}{2}p^3+5p^4}$

$$H(p)=\frac{p}{(p+4-0{,}8j)(p+4+0{,}8j)(p-0{,}25-0{,}433j)(p-0{,}25+0{,}433j)}$$

$$H(p)=\frac{K_1}{p+4-0{,}8j}+\frac{K_1^*}{p+4+0{,}8j}+\frac{K_2}{p-0{,}25-0{,}433j}+\frac{K_2^*}{p-0{,}25+0{,}433j}$$

$$\rightsquigarrow K_1=(p+4-0{,}8j)\cdot\left.\frac{p}{(p+4-0{,}8j)(p+4+0{,}8j)(p-0{,}25-0{,}433j)(p-0{,}25+0{,}433j)}\right|_{p=-4+0{,}8j}$$

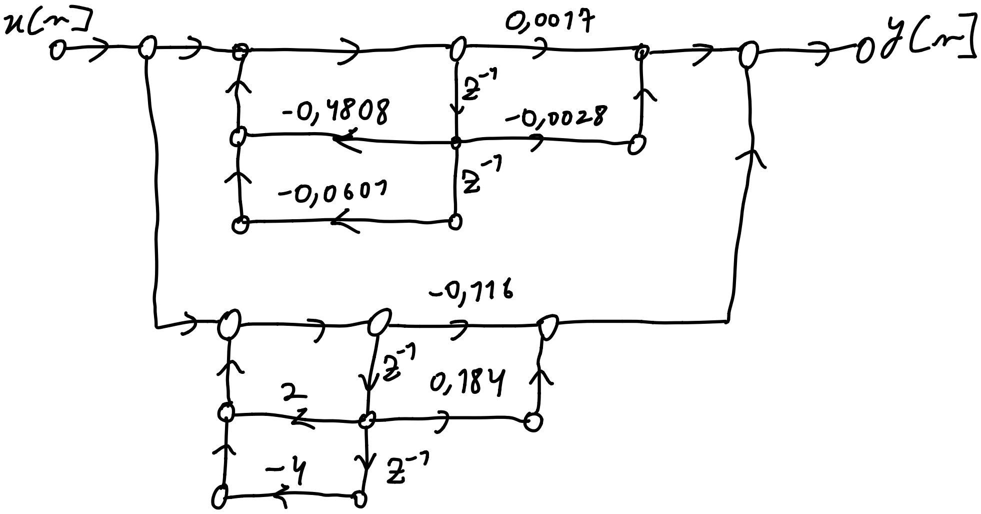
$K_1=-0{,}023+0{,}1331j$; $K_1^*=-0{,}023-0{,}1331j$;

$K_2=0{,}023-0{,}0202j$; $K_2^*=0{,}023+0{,}0202j$;

$$\rightarrow H(p)=\frac{0{,}029-0{,}046p}{16{,}64+8p+p^2}+\frac{-0{,}029+0{,}046p}{0{,}25-\frac{1}{2}p+p^2}$$

$$H(z) = \frac{0{,}029 - 0{,}046 z^{-1}}{16{,}64 + 8z^{-1} + z^{-2}} + \frac{-0{,}029 + 0{,}046 z^{-1}}{0{,}25 - \frac{1}{2} z^{-1} + z^{-2}}$$

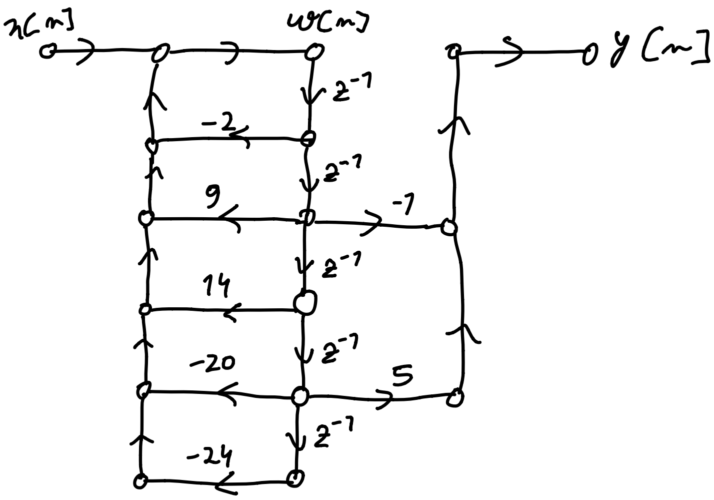
$$H(z) = \frac{0{,}0017 - 0{,}0028 z^{-1}}{1 + 0{,}4808 z^{-1} + 0{,}0601 z^{-2}} + \frac{-0{,}116 + 0{,}184 z^{-1}}{1 - 2z^{-1} + 4z^{-2}}$$



7. $$H(z) = \frac{5z - z^3}{(z-2)^2(z+1)(z+2)(z+3)}$$

(1) Forma direta II

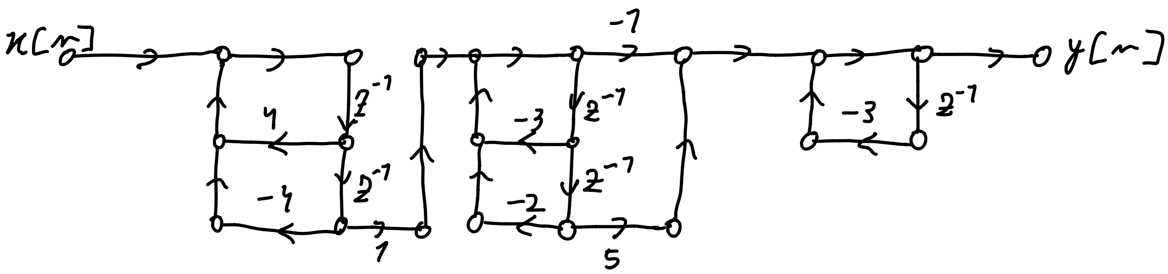
$$H(z) = \frac{5z - z^3}{z^5 + 2z^4 - 9z^3 - 14z^2 + 20z + 24} = \frac{5z^{-4} - z^{-2}}{1 + 2z^{-1} - 9z^{-2} - 14z^{-3} + 20z^{-4} + 24z^{-5}}$$



(2) Forma em cascata

$$H(z) = \frac{5z - z^3}{(z-2)^2(z+1)(z+2)(z+3)} = \frac{z(5-z^2)}{(z^2-4z+4)(z^2+3z+2)(z+3)} \cdot \frac{z^{-5}}{z^{-5}}$$

$$H(z) = \frac{z^{-2}(5z^{-2}-1)}{(1-4z^{-1}+4z^{-2})(1+3z^{-1}+2z^{-2})(1+3z^{-1})} = \frac{z^{-2}}{1-4z^{-1}+4z^{-2}} \cdot \frac{5z^{-2}-1}{1+3z^{-1}+2z^{-2}} \cdot \frac{1}{1+3z^{-1}}$$

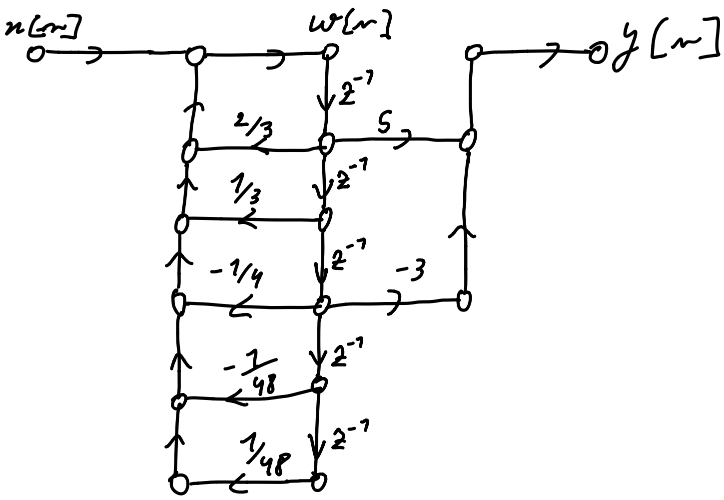


8. $$H(z) = \frac{5z^{-1} - 3z^{-3}}{(1-\frac{1}{2}z^{-1})^2(1+\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})}$$

Forma direta II: $$H(z) = \frac{5z^{-1} - 3z^{-3}}{1 - \frac{2}{3}z^{-1} - \frac{1}{3}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{48}z^{-4} - \frac{1}{48}z^{-5}}$$

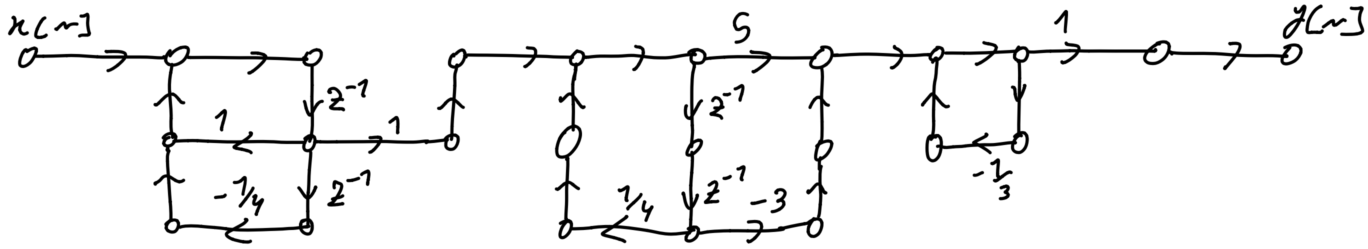
expandindo

D(z)



Cascata: $$H(z) = \frac{z^{-1}(5-3z^{-2})}{(1-z^{-1}+\frac{1}{4}z^{-2})(1+\frac{1}{3}z^{-1})(1-\frac{1}{4}z^{-2})}$$

$$H(z) = \frac{z^{-1}}{1 - z^{-1} + \frac{1}{4}z^{-2}} \cdot \frac{5 - 3z^{-2}}{1 - \frac{1}{4}z^{-2}} \cdot \frac{1}{1 + \frac{1}{3}z^{-1}}$$



9. Transformada direta: pegar $x[n]$ e fazer $X[-k]$

Temos $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}k \cdot n}$

Queremos $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}k \cdot n}$

$X[k] \Rightarrow X[-k] = X^*[k]$

$X[k] \to X[-n] \to$ insere $X[k]$ na DFT direta como $X[-k]$

$$\sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} X[-n] e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} X^*[n] e^{-j\frac{2\pi}{N}kn} =$$

$= \sum_{n=0}^{N-1} \left(X[n] e^{j\frac{2\pi}{N}kn} \right)^* \to$ sinal temporal é real $\Rightarrow$ conjugado é igual a ele mesmo.

↳ dividir por N.

10. $H(z) = \underbrace{\dfrac{6-4z^{-1}-z^{-2}+2z^{-3}-z^{-4}}{2-2z^{-1}+z^{-2}}}_{H_1(z)} + \underbrace{\dfrac{-\frac{1}{6}+\frac{1}{3}z^{-1}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})(1-\frac{1}{3}z^{-1})}}_{H_2(z)}$

$$H_1(p) = \frac{6-4p-p^2+2p^3-p^4}{2-2p+p^2}$$

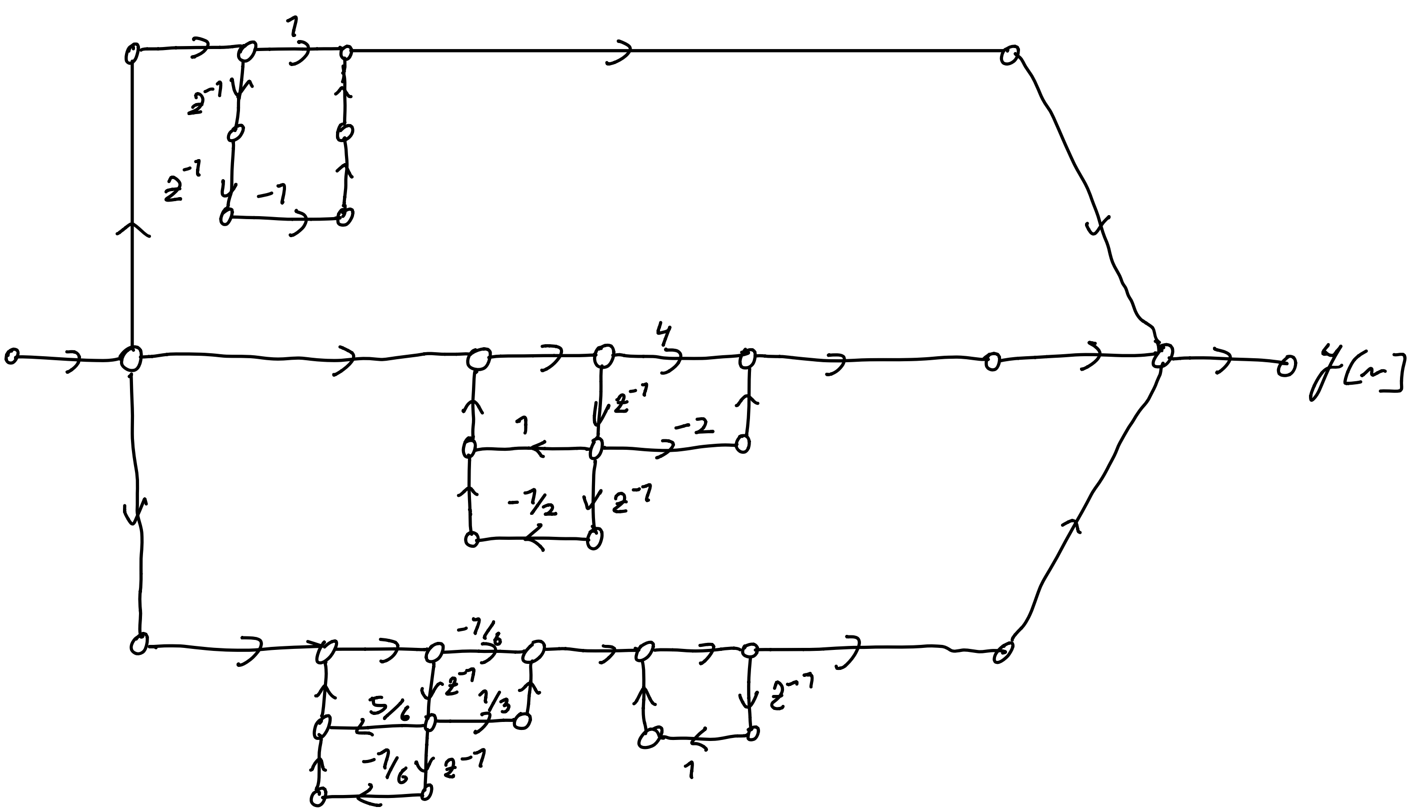
Divisão:

$$\begin{array}{l|l} -p^4+2p^3-p^2-4p+6 & p^2-2p+2 \\ \underline{-(-p^4+2p^3-2p^2)} & -p^2+1 \\ 0+0+p^2-4p+6 & \\ \underline{-(p^2-2p+2)} & \\ 0-2p+4 & \end{array}$$

$$\rightarrow H_1(p) = 1-p^2+\frac{4-2p}{2-2p+p^2} \rightarrow H_1(z) = 1-z^{-2}+\frac{4-2z^{-1}}{2-2z^{-1}+z^{-2}} = 1-z^{-2}+\frac{2-z^{-1}}{1-z^{-1}+\frac{1}{2}z^{-2}}$$

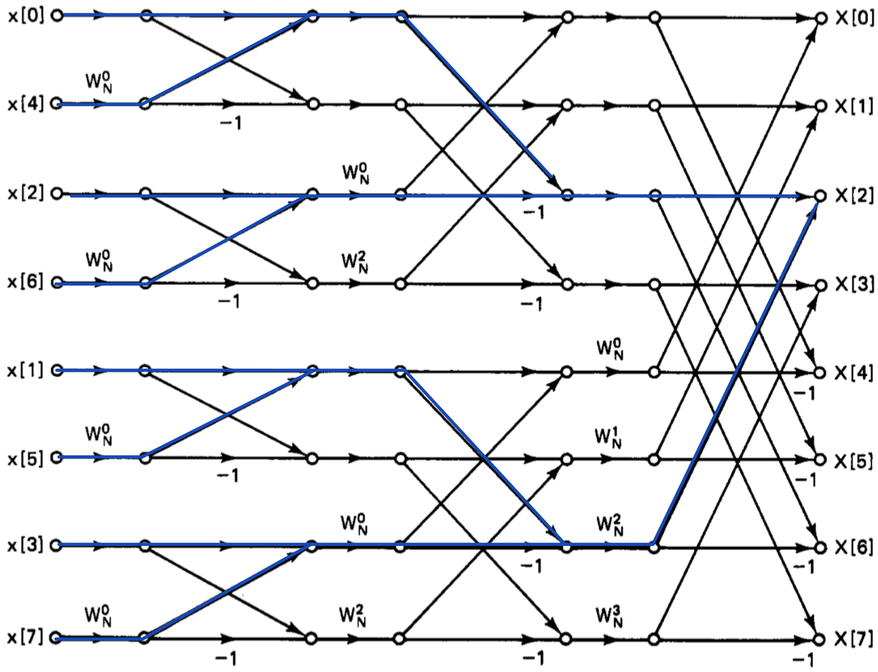
$$H_2(z) = \frac{-1/6+1/3\,z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} \cdot \frac{1}{1-z^{-1}}$$

$$H_1(z) = 1-z^{-2}+\frac{2-z^{-1}}{1-z^{-1}+\frac{1}{2}z^{-2}}+\frac{-1/6+1/3\,z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} \cdot \frac{1}{1-z^{-1}}$$



11.

Diagrama de fluxo da FFT – N = 8.



a) O ganho é $(W_N^0)\cdot(W_N^0)\cdot(-1)\cdot(W_N^2) = -(W_N^0)^2\cdot W_N^2$

$$\rightsquigarrow e^{-j\frac{2\pi}{N}k} = W_N^k \Rightarrow -(W_N^0)^2\cdot W_N^2 = -e^{-j\frac{4\pi}{N}\cdot 0}\cdot e^{-j\frac{4\pi}{N}} = -e^{-j\frac{\pi}{2}} = j$$

b) Como enfatizado na figura, pode-se ver que todos os coeficientes entrada contribuem. Mais especificamente, segue o ganho de cada entrada:

$x[0]: 1$; $x[4]: W_N^0$; $x[2]: -W_N^0$; $x[6]: -(W_N^0)^2$; $x[1]: W_N^2$; $x[5]: W_N^0 W_N^2$;

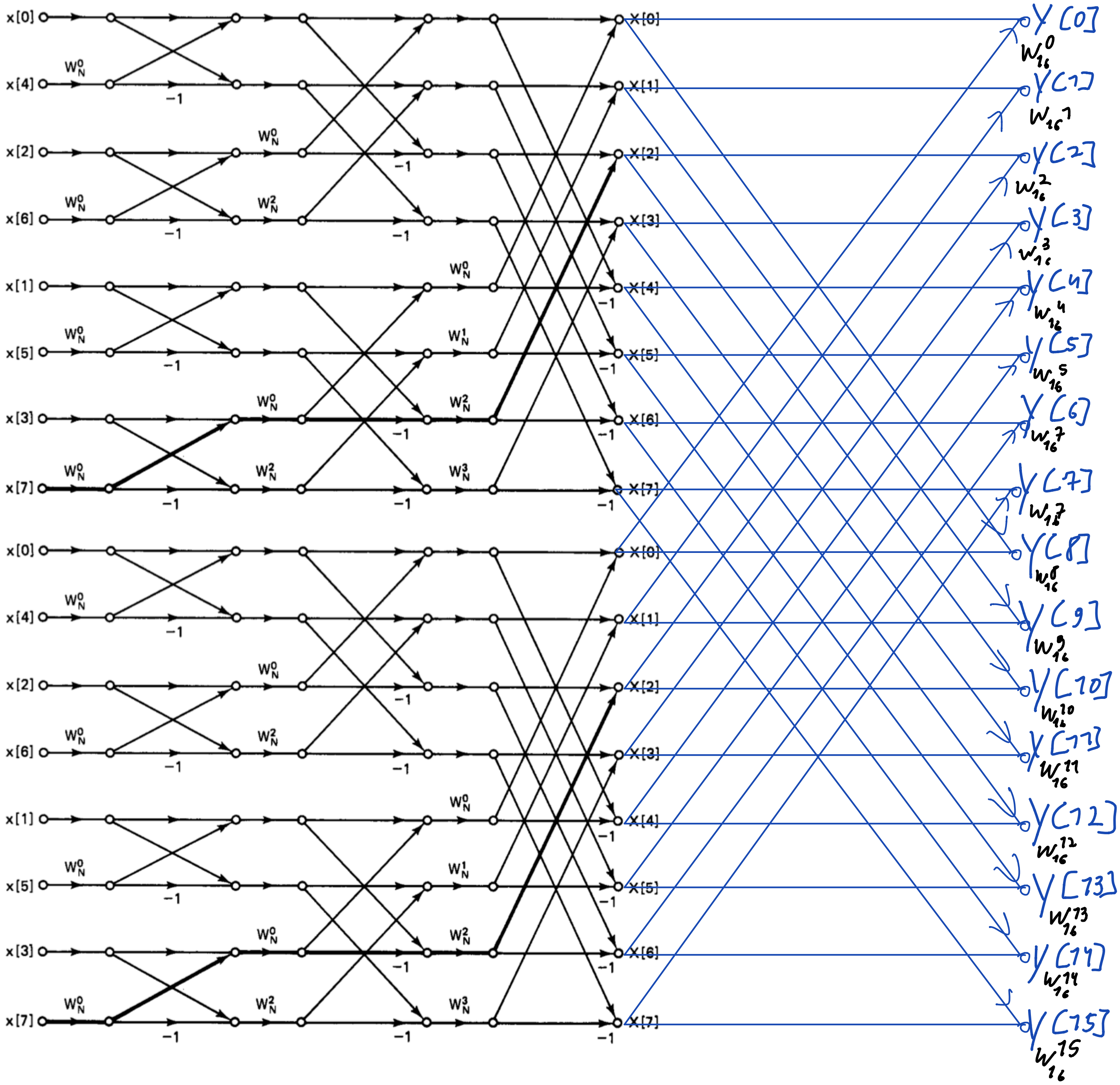
$x[3]: -W_N^0 W_N^2$; $x[7]: -(W_N^0)^2\cdot W_N^2$

$$X[2] = x[0] + W_N^0 x[4] - W_N^0 x[2] - (W_N^0)^2 x[6] + W_N^2 x[1] + W_N^0 W_N^2 x[5] - W_N^0 W_N^2 x[3] - (W_N^0)^2 W_N^2 x[7]$$

$$X[2] = x[0] + x[4] - x[2] - x[6] + e^{-j\frac{\pi}{2}} x[1] + e^{-j\frac{\pi}{2}} x[5] - e^{-j\frac{\pi}{2}} x[3] - e^{-j\frac{\pi}{2}} x[7]$$

$$X[2] = (x[0] + x[4] - x[2] - x[6]) + j\,(x[1] + x[5] - x[3] - x[7])$$

12. Dois sistemas que computam a FFT para comprimento $N=8$. Configure uma topologia para calcular uma FFT de comprimento $N=16$.



x[0]
x[4]
x[2]
x[6]
x[1]
x[5]
x[3]
x[7]
W_N^0
W_N^1
W_N^2
W_N^3
−1
X[0]
X[1]
X[2]
X[3]
X[4]
X[5]
X[6]
X[7]
Y[0]
Y[1]
Y[2]
Y[3]
Y[4]
Y[5]
Y[6]
Y[7]
Y[8]
Y[9]
Y[10]
Y[11]
Y[12]
Y[13]
Y[14]
Y[15]
W_{16}^0
W_{16}^1
W_{16}^2
W_{16}^3
W_{16}^4
W_{16}^5
W_{16}^7
W_{16}^7
W_{16}^8
W_{16}^9
W_{16}^{10}
W_{16}^{11}
W_{16}^{12}
W_{16}^{13}
W_{16}^{14}
W_{16}^{15}