PROVA-2 - SSTD

1) $w[n] = x[n] \cos\left[\frac{\pi}{2} n\right] \cos\left[\frac{\pi}{4} n\right]$

$$w[n] = x[n] \cdot \frac{1}{2}\left(e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}\right) \cdot \frac{1}{2}\left(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}\right)$$

$$w[n] = x[n] \cdot \frac{1}{4}\left(e^{j\frac{3\pi}{4}n} + e^{+j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n} + e^{-j\frac{3\pi}{4}n}\right)$$

$\Updownarrow$

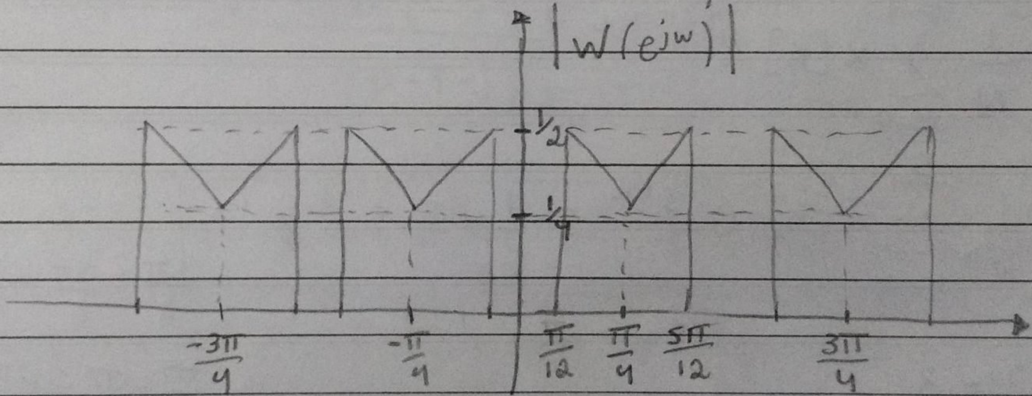
$$W(e^{jw}) = \frac{1}{4}\left(X\left(e^{j(w-\frac{3\pi}{4})}\right) + X\left(e^{j(w-\frac{\pi}{4})}\right) + X\left(e^{j(w+\frac{\pi}{4})}\right) + X\left(e^{j(w+\frac{3\pi}{4})}\right)\right)$$

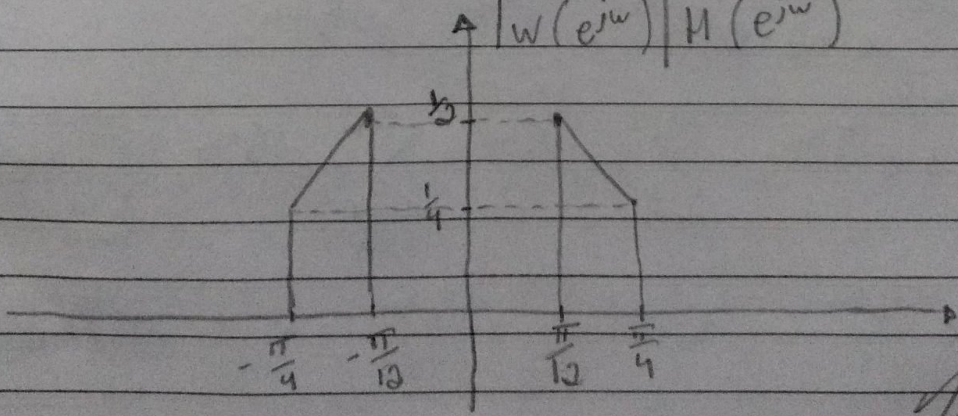
$\frac{\pi}{4} + \frac{\pi}{6} = \frac{3\pi + 2\pi}{12} = \frac{5\pi}{12}$

$\frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi - 2\pi}{12}$

$\frac{\pi}{12}$

4,6	2
2,3	3
2,1	2
1,1	12





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2) a) $N = 6$

$$\tilde{x}[K] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi}{N}Kn} = \sum_{n=0}^{5} x[n] \cdot e^{-j\frac{\pi}{3}Kn}$$

$$\tilde{x}[K] = 4 + 5e^{-j\frac{\pi}{3}\cdot 1K} + 4e^{-j\frac{\pi}{3}\cdot 2K} + 2e^{-j\frac{\pi}{3}\cdot 3K} + e^{-j\frac{\pi}{3}\cdot 4K} + 2e^{-j\frac{\pi}{3}\cdot 5K}$$

$-\frac{\pi}{4} \leq \omega \leq \frac{\pi}{4}$ como $\omega = \frac{\pi}{3} \cdot K$, somente em $K=0$ o filtro deixa passar

$$\tilde{X}[0] = 4 + 5 + 4 + 2 + 1 + 2 = 18$$

$$\tilde{x}[K] = 0, \quad K \neq 0.$$

b) $$x[n] = \frac{1}{N} \sum_{K=0}^{N-1} \tilde{x}[K] e^{j\frac{2\pi}{N}Kn}$$

$$x[n] = \frac{1}{6} \sum_{K=0}^{5} \tilde{x}[K] e^{j\frac{\pi}{3}Kn} = \frac{1}{6} \cdot 18 = 3$$

$$\frac{5\pi}{3} - \frac{5\pi}{6} = \frac{10\pi - 5\pi}{6} = \frac{5\pi}{6}$$

$$x[n] = 3$$

$$\frac{\pi}{3} - \frac{\pi}{6} = \frac{2\pi - \pi}{6} = \frac{\pi}{6}$$

$$x[K] = \sum_{n=0}^{5} x[n] \cdot e^{-j\frac{2\pi}{6}Kn} = \sum_{n=0}^{5} 3 \cdot e^{-j\frac{\pi}{3}Kn}$$

$$x[K] = \sum_{n=0}^{\infty} 3e^{-j\frac{\pi}{3}Kn} - \sum_{n=5}^{\infty} 3e^{-j\frac{\pi}{3}Kn} = \sum_{n=0}^{\infty} 3e^{-j\frac{\pi}{3}Kn} - \sum_{n=0}^{\infty} 3e^{-j\frac{\pi}{3}K(n+5)}$$

$$x[K] = 3\left(\sum_{n=0}^{\infty} e^{-j\frac{\pi}{3}Kn} - e^{-j\frac{5\pi}{3}K} \sum_{n=0}^{\infty} e^{-j\frac{\pi}{3}Kn}\right) = 3\left(1 - e^{-j\frac{5\pi}{3}K}\right) \sum_{n=0}^{\infty} e^{-j\frac{\pi}{3}Kn}$$

$$x[K] = 3 - 3e^{-j\frac{5\pi}{3}K} \cdot \frac{1}{1 - e^{-j\frac{\pi}{3}K}} = 3 \cdot \frac{1 - e^{-j\frac{5\pi}{3}K}}{1 - e^{-j\frac{\pi}{3}K}} \cdot \frac{e^{j\frac{5\pi}{6}K}}{e^{j\frac{5\pi}{6}K}} \cdot \frac{e^{j\frac{\pi}{6}K}}{e^{j\frac{\pi}{6}K}}$$

$$x[K] = 3 \cdot \frac{e^{j\frac{5\pi}{6}K} - e^{-j\frac{5\pi}{6}K}}{e^{j\frac{\pi}{6}K} - e^{-j\frac{\pi}{6}K}} \cdot e^{jK\left(\frac{\pi}{6} - \frac{5\pi}{6}\right)} = \boxed{3 \cdot \frac{\operatorname{sen}\left(K\frac{5\pi}{6}\right)}{\operatorname{sen}\left(K \cdot \frac{\pi}{6}\right)} \cdot e^{-jK\frac{4\pi}{6}}}$$

spiral

3) a) Para $N = 2048$, por meio da DFT temos N^2 operações:

$$(2048)^2 = 4194304 \text{ operações}$$

$$t = 4194304 \cdot 0{,}1 \cdot 10^{-6} = 0{,}4194304 \text{ s}$$

b) Para a FFT a complexidade computacional é de $N \log_2 N$ operações:

$$2048 \log_2 2048 = 2048 \cdot 11 = 22528 \text{ operações}$$

$$t = 22528 \cdot 0{,}1 \cdot 10^{-6} = 0{,}0022528 \text{ s}$$

4) $$H(p) = \frac{(p+1-j)(p-j)(p+j)(p+1+j)}{(p-1-j)(p-1+j)(p-1)(p+1)}$$

$$H(p) = \frac{(p^2-j^2)(p^2+p+\cancel{pj}+p+1+\cancel{j}-\cancel{jp}-\cancel{j}-\overset{-1}{\cancel{j^2}})}{(p^2-1)(p^2-p+\cancel{pj}-p+1-\cancel{j}-\cancel{jp}+\cancel{j}-j^2)}$$

$$H(p) = \frac{(p^2+1)(p^2+2p+2)}{(p^2-1)(p^2-2p+2)}$$

$$H(z) = \frac{(z^2+1)(z^2+2z+2)}{(z^2-1)(z^2-2z+2)} \cdot \frac{z^{-4}}{z^{-4}}$$

$$H(z) = \frac{(1+z^{-2})(1+2z^{-1}+2z^{-2})}{(1-z^{-2})(1-2z^{-1}+2z^{-2})}$$

$$H(z) = \frac{(1+z^{-2})}{(1-z^{-2})} \cdot \frac{(1+2z^{-1}+2z^{-2})}{(1-2z^{-1}+2z^{-2})}$$