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Lista 4 SSTD Pedro Duarte Alvim 180108042

① a) Com $N = 10$

$$X(k) = \sum_{m=0}^{9} x(m) e^{-j\frac{2\pi}{10}km}$$

$$X(k) = \sum_{m=0}^{4} e^{-j\frac{\pi}{5}km} = \sum_{m=0}^{\infty} e^{-j\frac{\pi}{5}km} - \sum_{m=5}^{\infty} e^{-j\frac{\pi}{5}km}$$

$$= \sum_{m=0}^{\infty} e^{-j\frac{\pi}{5}km} - \sum_{m=5}^{\infty} e^{-j\frac{\pi}{5}k(m+5)} = \sum_{m=0}^{\infty} e^{-j\frac{\pi}{5}km} - e^{-j\pi k} \sum_{m=0}^{\infty} e^{-j\frac{\pi}{5}km}$$

$$X(k) = (1 - e^{-j\pi k}) \sum_{m=0}^{\infty} (e^{-j\frac{\pi}{5}k})^m$$

$$X(k) = \frac{1 - e^{-j\pi k}}{1 - e^{-j\frac{\pi}{5}k}}$$

b) $N = 10$

$$X(k) = \sum_{m=0}^{9} x(m) e^{-j\frac{\pi}{5}km}$$

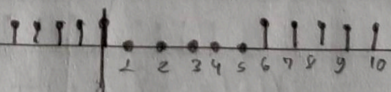
$$X(k) = 1 + 2\cdot e^{-j\frac{\pi}{5}k} + 3\cdot e^{-j\frac{\pi}{5}\cdot 2k} + 2\cdot e^{-j\frac{\pi}{5}\cdot 3k} + e^{-j\frac{\pi}{5}4k} + e^{-j\frac{\pi}{5}5k} + e^{-j\frac{\pi}{5}6k} + e^{-j\frac{\pi}{5}7k} + e^{-j\frac{\pi}{5}8k}$$

c) $N = 10$

$$X(k) = \sum_{m=0}^{9} x_3(m) e^{-j\frac{\pi}{5}km}$$

$$X(k) = 1 - 2e^{-j\frac{\pi}{5}k} - 2e^{-j\frac{\pi}{5}k2} - 2e^{-j\frac{\pi}{5}k3} + e^{-j\frac{\pi}{5}k4} + e^{-j\frac{\pi}{5}k5} - 2e^{-j\frac{\pi}{5}k6} - 2e^{-j\frac{\pi}{5}k7} - 2e^{-j\frac{\pi}{5}k8} + e^{-j\frac{\pi}{5}k9}$$

② $x_3(m)$ Ⓝ $x_1(m)$ $N = 10$



$Y(0) = 1\cdot1 - 2\cdot0 - 2\cdot0 - 2\cdot0 + 1\cdot0 + 1\cdot0 - 2\cdot1 - 2\cdot1 - 2\cdot1 + 1\cdot1 = -4$

$Y(1) = 1\cdot1 - 2\cdot1 - 2\cdot0 -$ " " $-2\cdot0 - 2\cdot1 - 2\cdot1 + 1\cdot1 = -4$

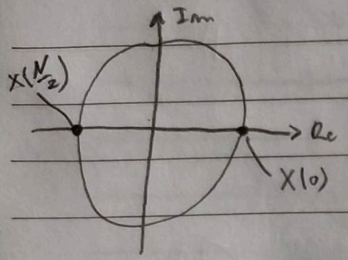
$Y(2) = -4$

$Y(3) = -4$ $Y(5) = -4$ $Y(7) = -4$ $Y(9) = -4$

$Y(4) = -4$ $Y(6) = -4$ $Y(8) = -4$

spiral

③ $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$



Assim $X(k)$ é puramente real com $k=0$ e $k=\frac{N}{2}$

Usando $N=6$

$$X(k) = \sum_{n=0}^{5} x(n) e^{-j\frac{2\pi}{6}kn}$$

$$x(k) = 1 + 2e^{-j\frac{\pi}{3}k} + 3e^{-j\frac{\pi}{3}\cdot 2k} + 2e^{-j\frac{\pi}{3}3k} + e^{-j\frac{\pi}{3}4k}, \quad k = 0 \text{ ou } 3$$

④ $X_1((n))_8$

$$X_1((n-4)_8) = X_2(n)$$

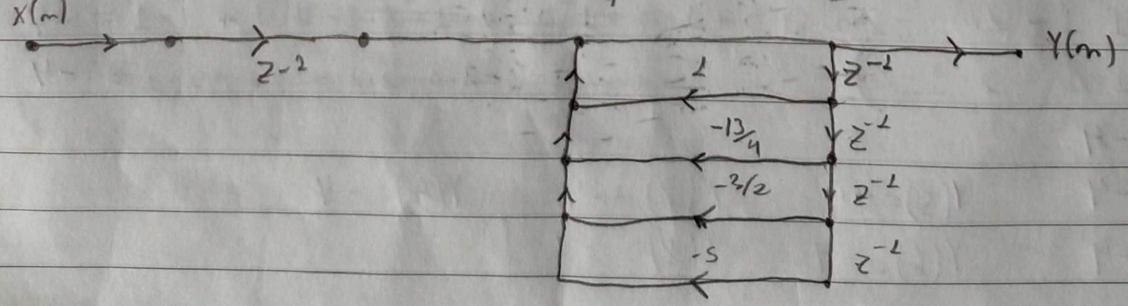
$$e^{-j\frac{2\pi}{8}\cdot 4} X_1(k) = X_2(k)$$

Assim, $X_2(k) = e^{-j\pi} X_1(k)$

⑤ a) $H(z) = \dfrac{z^3}{(z-2)^2(z-j+1/2)(z+j+1/2)} = \dfrac{z^3}{z^4 - z^3 + \frac{13}{4}z^2 + \frac{3}{2}z + 5} \cdot \dfrac{z^{-4}}{z^{-4}}$

$$H(z) = \frac{z^{-1}}{1 - z^{-1} + \frac{13}{4}z^{-2} + \frac{3}{2}z^{-3} + 5z^{-4}} = \frac{Y(z)}{X(z)}$$

$$Y(n) = x(n-1) + Y(n-1) - \frac{13}{4}Y(n-2) - \frac{3}{2}Y(n-3) - 5Y(n-4)$$



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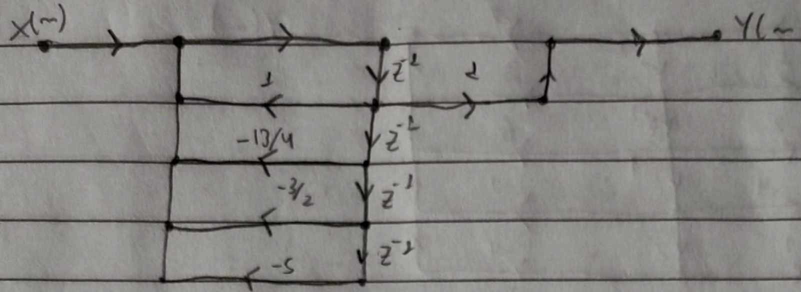
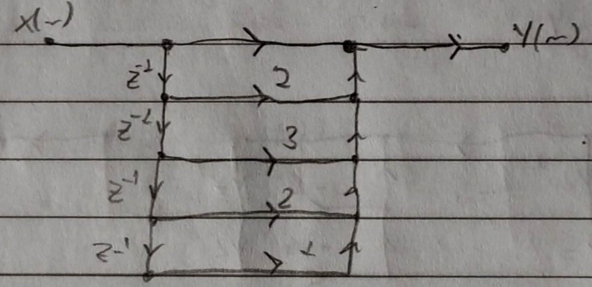
a) forma direta 2:



b) $H(z) = 1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + 1z^{-4} = \dfrac{Y(z)}{X(z)}$

$y(n) = x(n) + 2x(n-1) + 3x(n-2) + 2x(n-3) + x(n-4)$

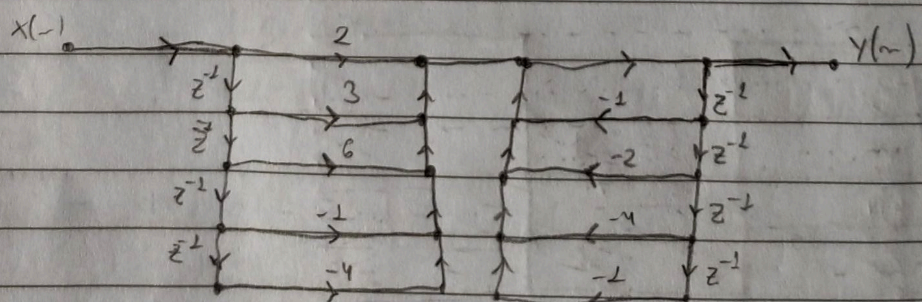
Forma direta 1:   Forma direta 2 = fd 1



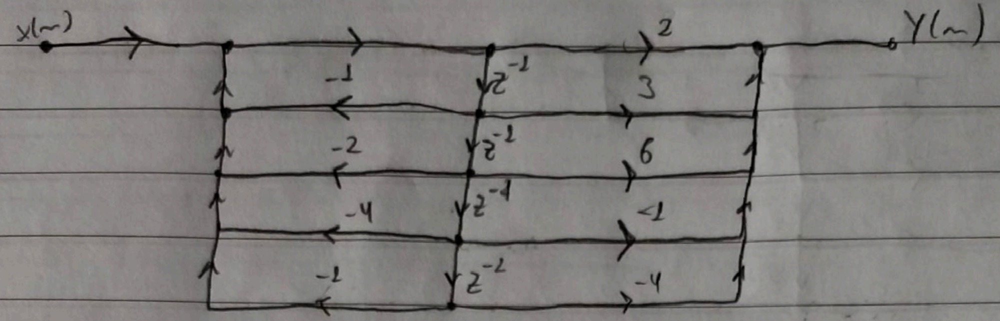
c) $H(z) = \dfrac{2+3z^{-1}+6z^{-2}-z^{-3}-4z^{-4}}{1+z^{-1}+2z^{-2}+4z^{-3}+z^{-4}} = \dfrac{Y(z)}{X(z)}$

$y(n) = 2x(n) + 3x(n-1) + 6x(n-2) - x(n-3) - 4x(n-4) - y(n-1) - 2y(n-2)$
$- 4y(n-3) - y(n-4)$

Forma direta 1:



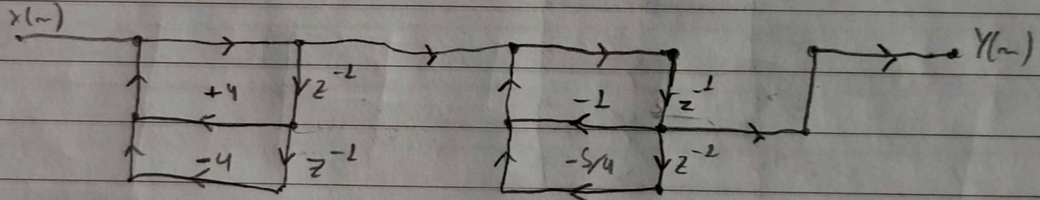
Forma direta 2:



⑥ Forma em cascata:

$$H(z) = \frac{z^2}{z^2-4z+4} \cdot \frac{z}{z^2+z+\frac{5}{4}} \cdot \frac{z^{-4}}{z^{-4}}$$

$$H(z) = \frac{1}{1-4z^{-1}+4z^{2}} \cdot \frac{z^{-1}}{1+z^{-1}+5/4\, z^{-2}}$$

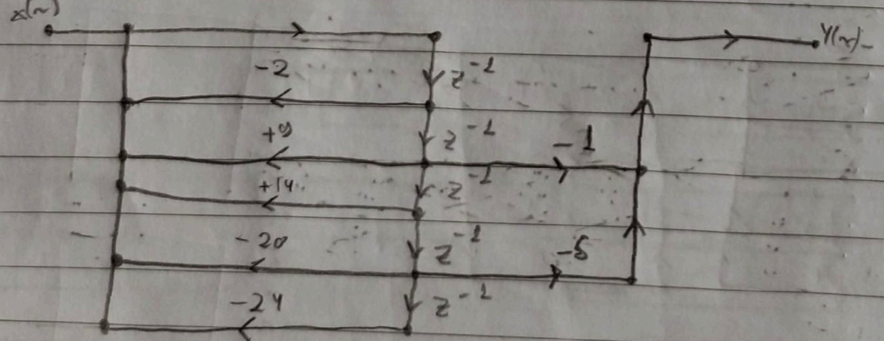


⑦

$$H(z) = \frac{5z-3z^3}{(z-2)^2(z+1)(z+2)(z+3)} \cdot \frac{z^{-5}}{z^{-5}}$$

$$H(z) = \frac{5z^{-4}-z^{-2}}{1+2z^{-1}-9z^{-2}-14z^{-3}+20z^{-4}+24z^{-5}}$$

Direta 2:



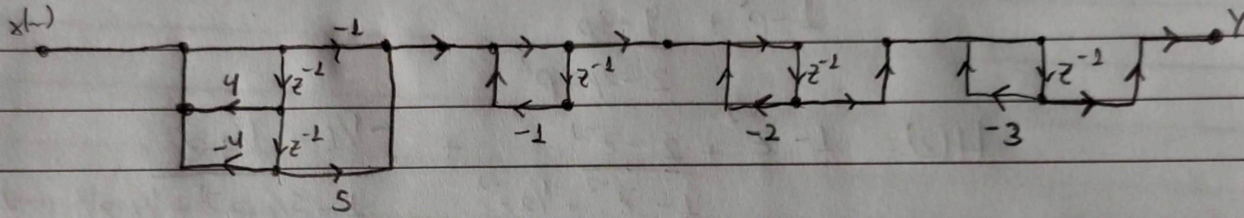
Cascata

$$H(z) = \frac{5z - z^{+3}}{(z^2 - 4z + 4)(z+1)(z+2)(z+3)}$$

$$= \frac{z(5 - z^2)}{(z^2 - 4z + 4)(z+1)(z+2)(z+3)}$$

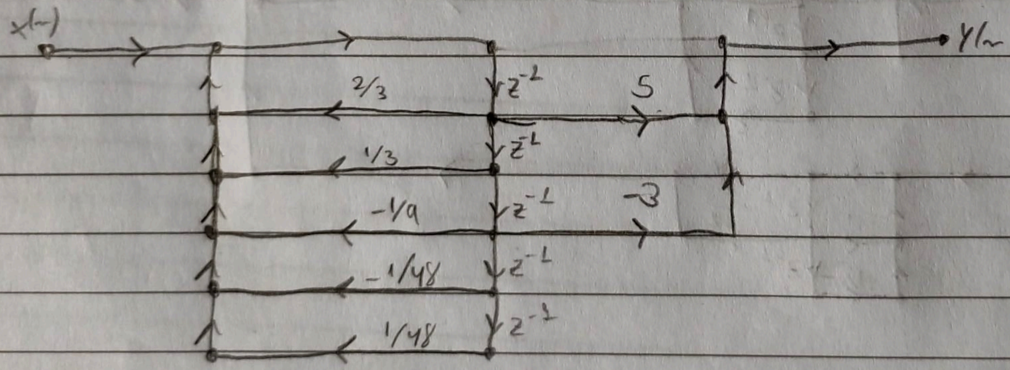
$$= \frac{(5 - z^2)}{(z^2 - 4z + 4)} \cdot \frac{z^{-2}}{z^{-2}} \cdot \frac{z}{(z+1)} \cdot \frac{z^{-1}}{z^{-1}} \cdot \frac{1}{(z+2)} \cdot \frac{z^{-1}}{z^{-1}} \cdot \frac{1}{(z+3)} \cdot \frac{z^{-1}}{z^{-1}}$$

$$= \frac{5z^{-2} - 1}{1 - 4z^{-1} + 4z^{-2}} \cdot \frac{1}{(1 + z^{-1})} \cdot \frac{z^{-1}}{(1 + 2z^{-1})} \cdot \frac{z^{-1}}{(1 + 3z^{-1})}$$

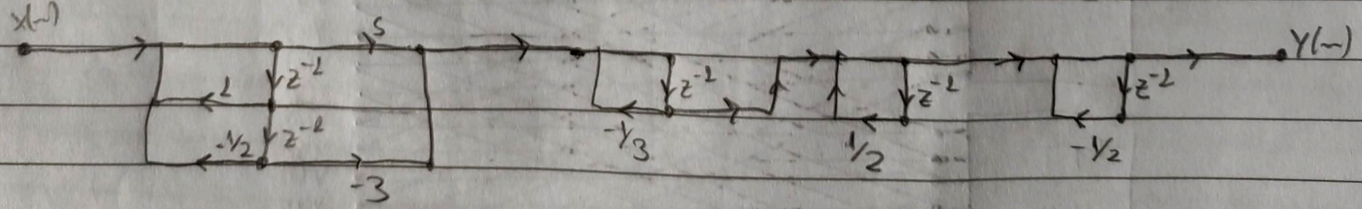


(8)

Direta 2: $H(z) = \dfrac{5z^{-1} - 3z^{-3}}{1 - \frac{2}{3}z^{-2} - \frac{1}{3}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{48}z^{-4} - \frac{1}{48}z^{-5}}$



Cascata: $H(z) = \dfrac{(5 - 3z^{-2})}{(1 - z^{-1} + \frac{1}{2}z^{-2})} \cdot \dfrac{z^{-1}}{(1 + \frac{1}{3}z^{-1})} \cdot \dfrac{1}{(1 - \frac{1}{2}z^{-1})} \cdot \dfrac{1}{(1 + \frac{1}{2}z^{-1})}$



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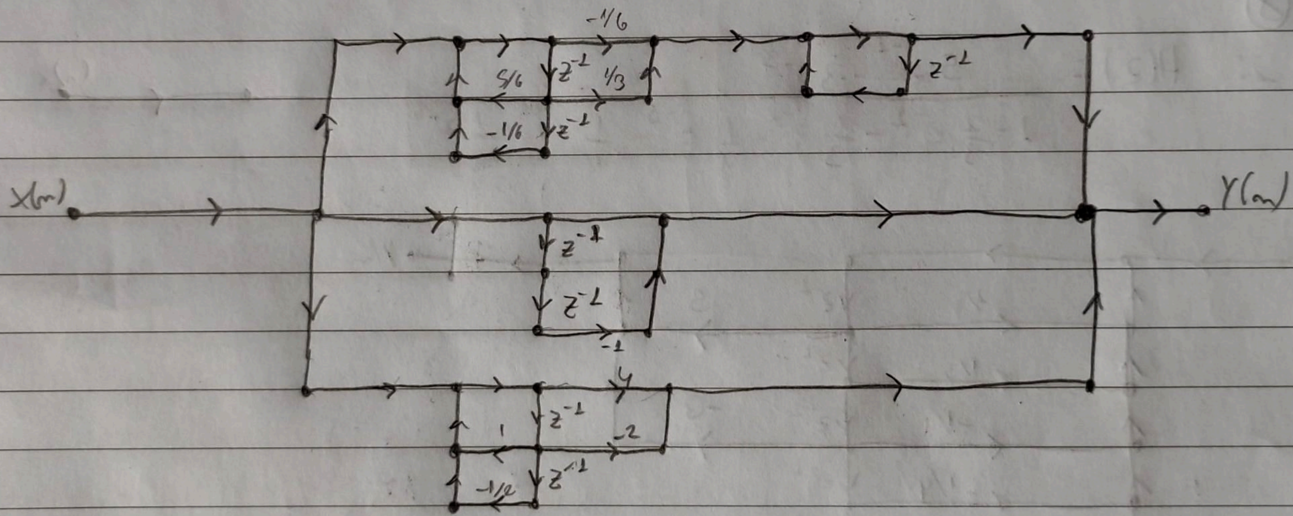
$$H(z) = \frac{6 - 4z^{-1} - z^{-2} + 2z^{-3} - z^{-4}}{2 - z^{-1} + z^{-2}} + \frac{-1/6 + 1/3\, z^{-1}}{(1-z^{-1})(1-1/2\, z^{-1})(1-1/3\, z^{-2})}$$

$$\underbrace{}_{\dfrac{6-4p-p^2+2p^3-p^4}{2-2p+p^2}}$$

$$\begin{array}{l|l} -p^4 + 2p^3 - p^2 - 4p + 6 & p^2 - 2p + 2 \\ \hline \cancel{+p^4} \cancel{- 2p^3} + 2p^2 & -p^2 + 1 \\ \quad p^2 - 4p + 6 & \\ \quad -p^2 + 2p - 2 & \\ \hline \quad -2p + 4 & \end{array}$$

$$1 - p^2 + \frac{4 - 2p}{2 - 2p + p^2}$$

$$H(z) = 1 - z^{-2} + \frac{2 - z^{-1}}{1 - z^{-1} + 1/2\, z^{-2}} + \frac{-1/6 + 1/3\, z^{-1}}{1 - 5/6\, z^{-1} + 1/6\, z^{-2}} \cdot \frac{1}{1 - z^{-2}}$$



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