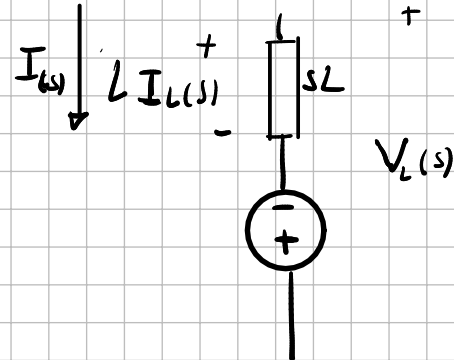
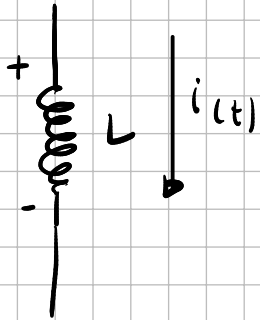


Ejercicio 4.

Transformar las relaciones tensión-corriente del inductor y capacitor del dominio del tiempo al dominio de Laplace, y armar el circuito equivalente serie y paralelo que representan las ecuaciones transformadas.

inductor



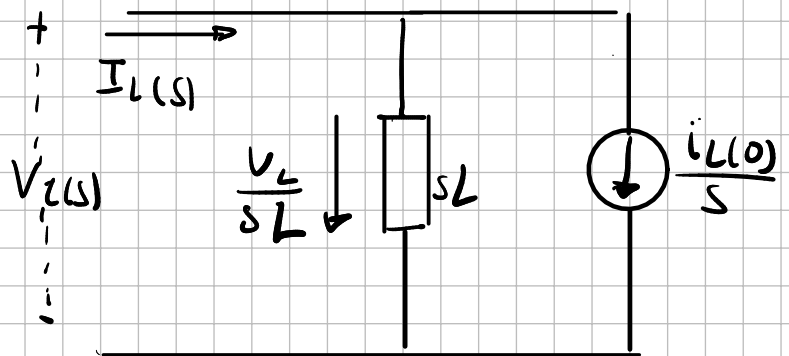
$$V_L(t) = L \frac{di_L}{dt}$$

$$V_L(s) = L \mathcal{L}[i_L(t)]$$

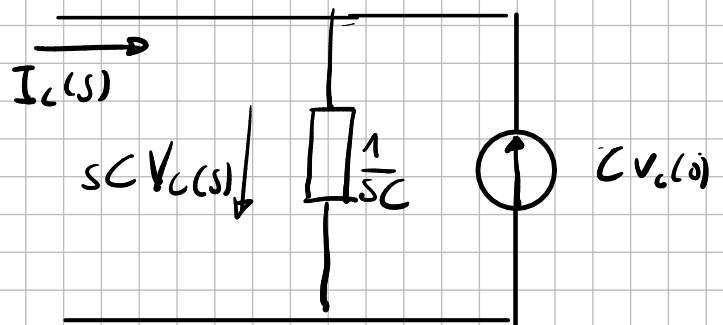
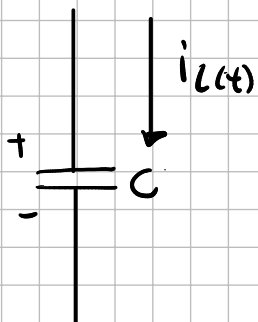
$$V_L(s) = L [s I_L(s) - i_L(0)]$$

$$V_L(s) = sL I_L(s) - L i_L(0)$$

$$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0)}{s}$$



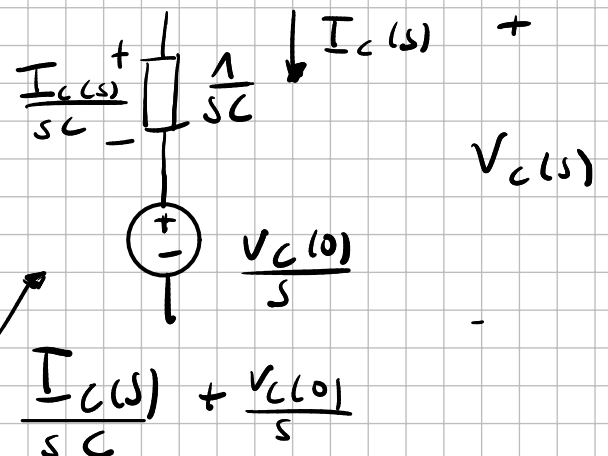
capacitor



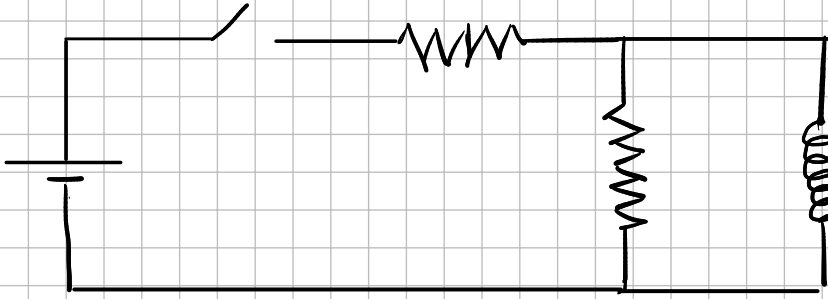
$$i_C(t) = C \frac{dV_C}{dt}$$

$$I_C(s) = \mathcal{L}[i_C(t)]$$

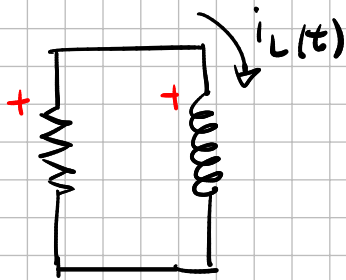
$$I_C(s) = C s V_C(s) - C V_C(0) \quad ; \quad V_C(s) = \frac{I_C(s)}{sC} + \frac{V_C(0)}{s}$$



Con un ejercicio de la guía 3



$$i_L(0) = 20A$$



$$\left. \begin{aligned} V_R(t) &= V_L(t) \\ V_R(t) &= -i_L(t) R \\ V_L(t) &= L \frac{di_L(t)}{dt} \end{aligned} \right\}$$

$$i_L(t) R + L \frac{di_L}{dt} = 0$$

$$\frac{di_L(t)}{dt} + \frac{R}{L} i_L(t) = 0$$

$$s I_L(s) - i(0) + \frac{R}{L} I_L(s) = 0$$

$$I_L(s) \left(s + \frac{R}{L} \right) - i(0) = 0$$

$$I_L(s) \left(s + \frac{R}{L} \right) = i(0)$$

$$I_L(s) = \frac{20}{s + \frac{1}{\tau}} ; \tau = \frac{L}{R} \xrightarrow{\mathcal{L}^{-1}} i_L(t) = 20 e^{-\frac{t}{\tau}}$$

> otra opción es pasar el S.E a laplace

$$\begin{aligned} V_R(t) &= V_L(t) \\ V_R(t) &= -i_L(t) R \\ V_L(t) &= L \frac{di_L(t)}{dt} \end{aligned}$$

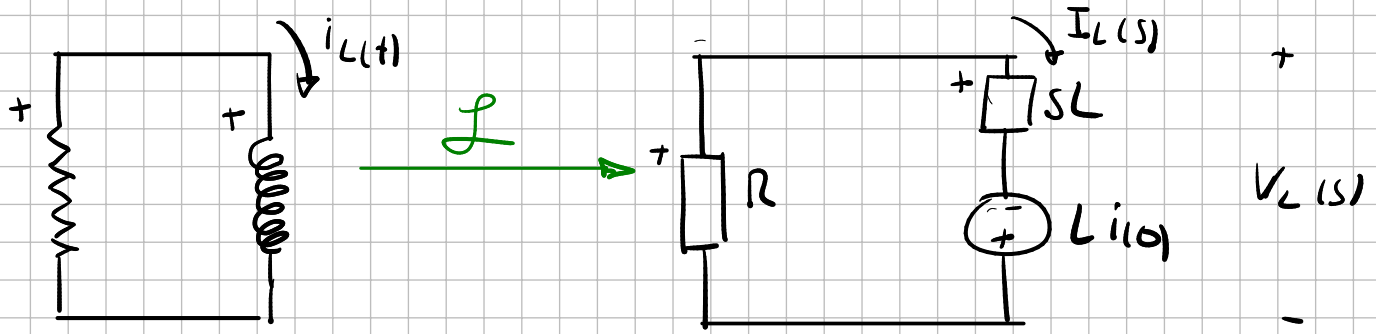
$$\xrightarrow{\mathcal{L}} \begin{aligned} V_R(s) &= V_L(s) \\ V_R(s) &= -I_L(s) R \\ V_L(s) &= L s I_L(s) - L i(0) \end{aligned}$$

$$L s I_L(s) - L i(0) + I_L(s) R = 0$$

$$I_L(s) (L s + R) - L i(0) = 0$$

$$I_L(s) \left(s + \frac{R}{L} \right) = i(0)$$

➤ Otra opción más es pasar directamente el circuito



LKV

$$V_R(s) = V_L(s)$$

$$-RI_L(s) = I_L(s) \cdot sL - Li(0)$$

$$I_L(s)(sL + R) - Li(0) = 0$$

$$I_L(s)\left(s + \frac{R}{L}\right) = i(0)$$

Ejercicio 5.

En $t = 0$ se aplica al circuito RL serie de la figura 2 una tensión continua de 55V. Encontrar la transformada de la respuesta $i(t)$ para $t > 0$.

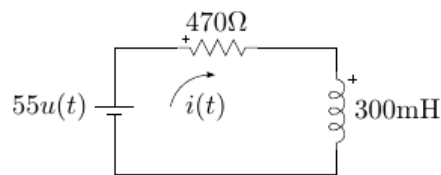


Figura 2: Circuito RL serie con excitación constante.

LKV

$$V(t) = V_R(t) + V_L(t)$$

$$V_R(t) = i(t) R$$

$$V_L(t) = L \frac{di_L}{dt}$$

\xrightarrow{L}

$$V(s) = 55 \frac{1}{s}$$

$$V(s) = V_R(s) + V_L(s)$$

$$V_R(s) = I(s) R$$

$$V_L(s) = L(sI_L(s) - i_L(0))$$

$$\frac{55}{s} = I_L(s) R + LsI_L(s) - Li_L(0)$$

$$\frac{55}{s} + L \frac{i_L(0)}{s} = I_L(s) \left(\frac{R}{L} + s \right)$$

$$I_L(s) = \frac{\frac{55}{Ls}}{\frac{R}{L} + s}$$

$$I_L(s) = \frac{\frac{1}{L} 55}{s \left(s + \frac{R}{L} \right)}$$

$$I_L(s) = \frac{A}{s} + \frac{B}{s + \frac{R}{L}}$$

A; B?

$$\lim_{s \rightarrow 0} s \frac{\frac{55}{L}}{s(s + \frac{R}{L})} = \lim_{s \rightarrow 0} \cancel{s} \frac{A}{\cancel{s}} + \lim_{s \rightarrow 0} \frac{B}{s + \frac{R}{L}}$$

$$A = \frac{\frac{55}{L}}{\frac{R}{L}} = \frac{55}{470} = 0,117$$

$$B = \lim_{s \rightarrow -\frac{R}{L}} \cancel{\left(s + \frac{R}{L}\right)} \frac{\frac{55}{L}}{s \cancel{\left(s + \frac{R}{L}\right)}} = \frac{\frac{55}{L}}{-\frac{R}{L}} = -0,117$$

$$I_L(s) = \frac{0,117}{s} - \frac{0,117}{s + \frac{1}{\tau}} ; \tau = \frac{L}{R}$$

$$i(t) = 0,117 \mu(t) - 0,117 e^{-\frac{t}{\tau}} \mu(t)$$

$$i(t) = \left[0,117 - 0,117 e^{-\frac{t}{\tau}} \right] \mu(t)$$

Ejercicio 6.

El capacitor de la figura 3 tiene una carga inicial de $q_0 = 800 \times 10^{-6} \text{C}$ con la polaridad indicada. Hallar la respuesta completa de la tensión del capacitor en el dominio de la variable s .

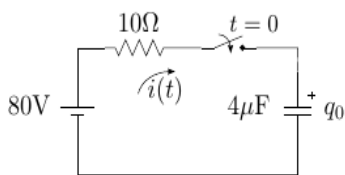


Figura 3: Circuito RC.

$$V_C(s) = \frac{q_0}{C}$$

$$V_C(s) = 200V$$

LKV

$$V(t) = V_R(t) + V_C(t)$$

$$V_R = i(t) R$$

$$i(t) = C \frac{dV_C(t)}{dt}$$

\xrightarrow{L}

$$V(s) = I(s) R + V_C(s)$$

$$I(s) = C (s V_C(s) - V_C(0))$$

$$V(s) = R C s V_C - R C V_C(0) + V_C(s)$$

$$V(s) = V_C (R C s + 1) - R C V_C(0)$$

$$V_C(s) = \frac{V(s) + R C V_C(0)}{R C s + 1}$$

$$V(s) = \frac{80}{s}$$

$$V_C(s) = \frac{\frac{80}{s R_C} + V_C(0)}{s + \frac{1}{R_C}} \cdot \frac{s}{s}$$

$$V_C(s) = \frac{\frac{80}{R_C} + s V_C(0)}{s \left(s + \frac{1}{R_C} \right)}$$

$$A = \lim_{s \rightarrow 0} \frac{\frac{80}{R_C} + s V_C(0)}{s + \frac{1}{R_C}} = 80$$

$$B = \lim_{s \rightarrow -\frac{1}{R_C}} \frac{\frac{80}{R_C} + s V_C(0)}{s} = -80 + 200 = 120$$

$$V_C(s) = \frac{80}{s} + \frac{120}{s + \frac{1}{R_C}} ; \tau = R_C$$

$$V_C(s) = \frac{80}{s} + \frac{120}{s + \frac{1}{\tau}}$$

$\downarrow \mathcal{L}^{-1}$

$$V_C(t) = \left(80 + 120 e^{-\frac{t}{\tau}} \right) u(t)$$

Ejercicio 7.

La respuesta de corriente de un circuito eléctrico tiene la siguiente transformada

$$I(s) = \frac{\frac{4}{5}}{\left(\frac{s}{5} + 1\right)^2 + 4},$$

se pide:

1. encontrar $i(t)$,
2. encontrar el valor de $i(0)$ aplicando el teorema del valor inicial y comprobar en el tiempo,
3. encontrar el valor de $i(\infty)$ aplicando el teorema del valor final y comprobar en el tiempo.

$$1. I(s) = \frac{4/5}{\frac{s^2}{25} + \frac{2}{5}s + 5}$$

$$I(s) = \frac{20}{s^2 + 10s + 125} \quad \textcircled{1}$$
$$\begin{cases} s_1 = -5 + 10j \\ s_2 = -5 - 10j \end{cases}$$

$$I(s) = \frac{20}{(s + 5 - 10j)(s + 5 + 10j)}$$

$$A = \lim_{s \rightarrow -5 + 10j} \cancel{(s + 5 - 10j)} \frac{20}{\cancel{(s + 5 - 10j)}(s + 5 + 10j)} = -j$$

$$B = \lim_{s \rightarrow -5 - 10j} \cancel{(s + 5 + 10j)} \frac{20}{(s + 5 - 10j)\cancel{(s + 5 + 10j)}} = j$$

$$I(s) = \frac{-j}{s + 5 - 10j} + \frac{j}{s + 5 + 10j}$$

$$i(t) = -j e^{(-5 + 10j)t} + j e^{(-5 - 10j)t}$$

$$i(t) = -j e^{-5t} \cdot e^{j10t} + j e^{-5t} e^{-j10t} = e^{-5t} (-j e^{j10t} + j e^{-j10t})$$

$$i(t) = e^{-5t} \left(\frac{e^{j10t} - e^{-j10t}}{j} \right)$$

$$i(t) = e^{-5t} (2 \sin(10t)) N(t)$$

* Sabiendo que son raíces complejas lo haremos pero sin fracciones simples, con la idea de que en Laplace nos quede directamente la anti transformada del seno y coseno

buscamos la forma

$$i(t) = e^{-\alpha t} (C \sin(\omega t) + D \cos(\omega t))$$

$$I(s) = C \frac{\omega}{(s+\alpha)^2 + \omega^2} + D \frac{s+\alpha}{(s+\alpha)^2 + \omega^2}$$

desde ①

$$I(s) = C \frac{10}{(s+5)^2 + 10^2} + D \frac{s+5}{(s+5)^2 + 10^2}$$

$$s = -5$$

$$\frac{20}{(-5)^2 + 10(-5) + 125} = C \frac{10}{(-5+5)^2 + 10^2}$$

$$C = \frac{20}{25 - 50 + 125} \cdot \frac{10^2}{10} = 2 = C$$

$$s = 0$$

$$\frac{20}{125} = 2 \frac{10}{5^2 + 10^2} + D \frac{5}{5^2 + 10^2}$$

$$D = 0$$

$$I(s) = 2 \frac{10}{(s+5)^2 + 10^2} + 0$$

$$i(t) = 2 \sin(10t) e^{-5t} N(t)$$

2. TVI

$$\begin{aligned} i_{(0)} &= \lim_{s \rightarrow \infty} s I(s) \\ &= \lim_{s \rightarrow \infty} s \frac{20}{s^2 + 10s + 12s} \\ &= \frac{\frac{s}{s^2} 20}{\frac{s^2}{s^2} + 10 \frac{s}{s^2} + \frac{12s}{s^2}} \end{aligned}$$

$$i_{(0)} = 0$$

3. TVF

$$\begin{aligned} i_{(\infty)} &= \lim_{s \rightarrow 0} s I(s) \\ &= \lim_{s \rightarrow 0} s \frac{20}{s^2 + 10s + 12s} \end{aligned}$$

$$i_{(\infty)} = 0$$

Ejercicio 17.

Dado el circuito de la figura 12 en el dominio de s .

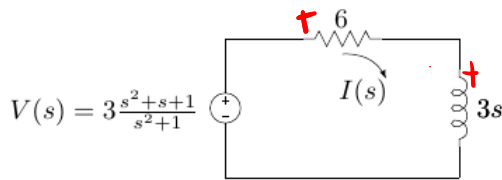


Figura 12: Dominio de s .

$$\left. \begin{aligned} a) \quad V_s &= V_R + V_L \\ V_R &= R I_s \\ V_L &= sL I_s \end{aligned} \right\}$$

$$V_s = R I_s + sL I_s$$

$$V_s = I_s (R + sL)$$

$$I_s = \frac{V_s}{R + sL}$$

- Encontrar $I(s)$ y su correspondiente $i(t) = \mathcal{L}^{-1}[I(s)]$
- Tiene el circuito condición inicial no nula? Verificar utilizando el TVI.
- Encontrar $V_L(s)$.

$$I_s = \frac{3 \frac{s^2 + s + 1}{s^2 + 1}}{6 + s} = \frac{3(s^2 + s + 1)}{(s^2 + 1)(s + 6)}$$

$$I_s = \frac{s^2 + s + 1}{(s^2 + 1)(s + 2)} = \frac{s^2 + s + 1}{(s + 2)(s - j)(s + j)}$$

No trabajamos con la forma $\frac{A}{s+2} + \frac{B}{s-j} + \frac{C}{s+j}$ pq de la otra ya te quedan los cosenos

$$I(s) = \frac{A}{s+2} + B \frac{1}{(s^2+1)} + C \frac{s}{(s^2+1)}$$

$$A = \lim_{s \rightarrow -2} (s+2) \frac{s^2 + s + 1}{(s^2 + 1)(s+2)} = \frac{3}{5}$$

Para B y para C lo evaluamos

$$\frac{(0)^2 + (0) + 1}{(0^2 + 1)(0 + 2)} = \frac{3/5}{0 + 2} + B \frac{1}{(0^2 + 1)} + C \frac{0}{(0^2 + 1)}$$

$$\frac{1}{2} = \frac{3}{10} + B + 0 \rightarrow B = \frac{1}{5}$$

$$s = 1$$

$$\frac{(1)^2 + (1) + 1}{(1^2 + 1)(1 + 2)} = \frac{3/5}{1 + 2} + \frac{1}{5} \frac{1}{(1^2 + 1)} + C \frac{1}{(1^2 + 1)}$$

$$C = \frac{2}{5}$$

$$I(s) = \frac{3}{5} \frac{1}{s+2} + \frac{1}{5} \frac{1}{(s^2+1)} + \frac{2}{5} \frac{s}{(s^2+1)}$$

$$i(t) = \underbrace{\frac{3}{5} e^{-2t}}_{nat} + \underbrace{\frac{1}{5} \sin(t) + \frac{2}{5} \cos(t)}_{forz}$$

b) parece no tener CI

$$I(s) = \frac{s^2 + s + 1}{s^3 + 2s^2 + s + 2}$$

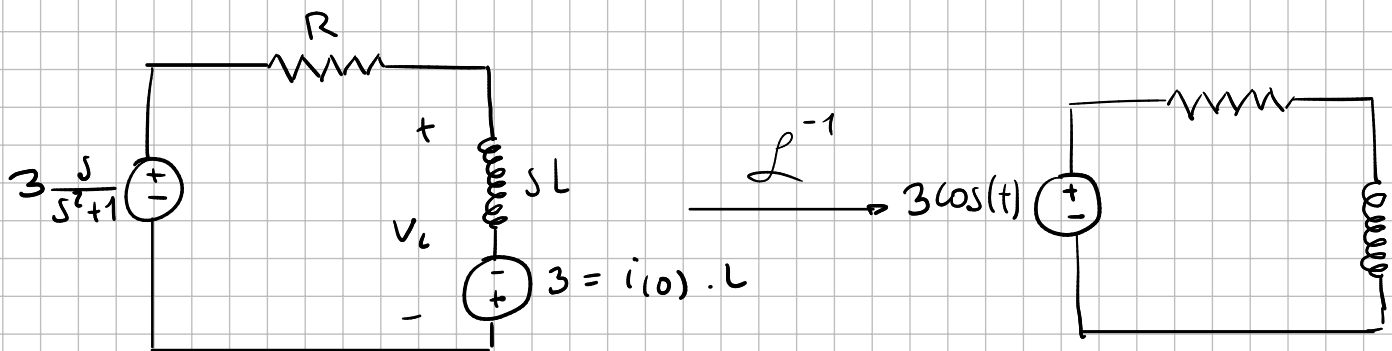
$$i(0) = \lim_{s \rightarrow \infty} s \frac{s^2 + s + 1}{s^3 + 2s^2 + s + 2} = \lim_{s \rightarrow \infty} \frac{s^3 + s^2 + s}{s^3 + 2s^2 + s + 2}$$

$$i(0) = 1 \rightarrow \text{entonces?}$$

que forma tiene $V(t)$?

$$V(s) = \frac{3s^2 + 3s + 3}{s^2 + 1} \rightarrow \frac{3s^2 + 3s + 3}{s^2 + 1} = \frac{3s^2 + 0s + 3}{0 + 3s + 0} \frac{s^2 + 1}{3}$$

$$V(s) = 3 + 3 \frac{s}{s^2 + 1} \rightarrow \cos$$



Ejercicio 18.

Un circuito RL serie tiene como función de transferencia

$$H(s) = \frac{I(s)}{V(s)} = \frac{1}{36 + s18}. \quad (3)$$

Si se lo excita con un escalón $v(t) = 36u(t)[V]$, encontrar por convolución la respuesta $i(t) = h(t) * v(t)$.

$$h(t) = \frac{1}{18} e^{-2t}$$

$$i(t) = \int_0^t h(\tau) v(t-\tau) d\tau = \int_0^t \frac{1}{18} e^{-2\tau} 36 u(t-\tau) d\tau$$

$$i(t) = \frac{36}{18} \int_0^t e^{-2\tau} d\tau = 2 \left. \frac{e^{-2\tau}}{-2} \right|_0^t = 2 \frac{e^{-2t} - 1}{-2}$$

$$i(t) = -1 (e^{-2t} - 1) u(t) = 1 - e^{-2t} u(t)$$

Ejercicio 22.

Obtener la respuesta al impulso del circuito de la figura 15 considerando $H(s) = \frac{I_R(s)}{V(s)}$; donde $I_R(s) = \mathcal{L}[i_R(t)]$ y $V(s) = \mathcal{L}[v(t)]$.

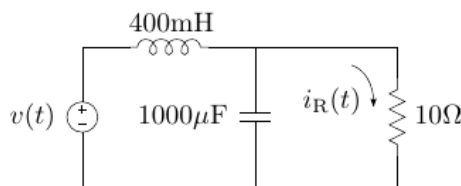
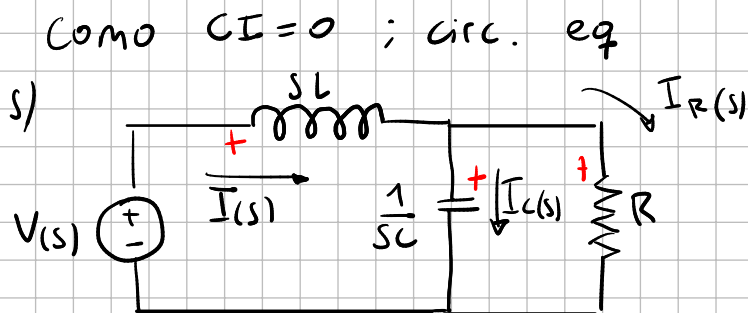


Figura 15: Cálculo de respuesta al impulso.



L K V

$$V(s) = V_L(s) + V_C(s) \quad I(s) = I_C(s) + I_R(s)$$

$$V_R(s) = V_C(s)$$

$$V_L(s) = sL I(s)$$

$$V_C(s) = \frac{I_C(s)}{sC} \rightarrow I_C(s) = V_R(s) sC$$

$$V_R(s) = R I_R(s)$$

$$V(s) = sL I(s) + R I_R(s)$$

$$= sL I_C(s) + sL I_R(s) + R I_R(s) = sL (V_R(s) sC) + I_R(s) (sL + R) = V(s)$$

$$V(s) = I_R(s) (s^2 LC R + sL + R)$$

$$\frac{I_R(s)}{V(s)} = H(s) = \frac{1}{s^2 LCR + sL + R} \cdot \frac{\frac{1}{LCR}}{\frac{1}{LCR}} = \frac{\frac{1}{LCR}}{s^2 + \frac{1}{CR}s + \frac{1}{LC}}$$

$$H(s) = \frac{250}{s^2 + 100^{-3}s + 500s} = \frac{250}{(s+50)(s+50)}$$

$$H(s) = \frac{A}{(s+50)^2} + \frac{B}{(s+50)}$$

$$\lim_{s \rightarrow -50} \frac{(s+50)^2}{(s+50)^2} \frac{250}{(s+50)^2} = \lim_{s \rightarrow -50} A + \lim_{s \rightarrow -50} \frac{(s+50)}{(s+50)^2} B = 0$$

$$A = 250$$

$$\lim_{s \rightarrow -50} \frac{d}{ds} 250 = \lim_{s \rightarrow -50} \frac{d}{ds} A + \lim_{s \rightarrow -50} (s+50) \frac{d}{ds} B$$

$$B = \frac{d}{ds} 250 = 0$$

Ejercicio 24.

Para el circuito de la figura 17 de condiciones iniciales $i_L(0) = 1[A]$ y $v_C(0) = 1[V]$ se pide:

1. encontrar la respuesta completa de corriente $i(t)$ para $t > 0$ utilizando el modelo de circuito equivalente de Laplace,
2. decir que parte de la respuesta corresponde a la natural y cuál es la forzada.

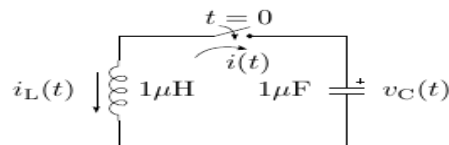
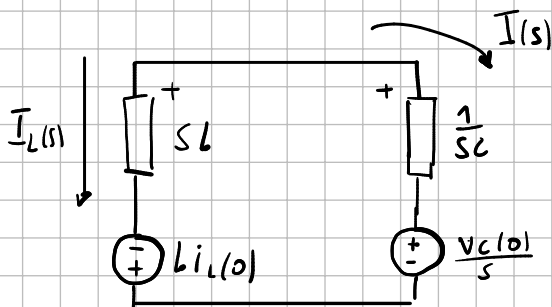


Figura 17: Equivalente de Laplace.



$$LKI \quad I_L(s) = -I(s)$$

$$LKV$$

$$I(s) \frac{1}{sC} + \frac{v_C(0)}{s} + L i_L(0) - I_L(s) sL = 0$$

$$I(s) \left(\frac{1}{sC} + sL \right) = - \frac{v_C(0)}{s} - L i_L(0)$$

$$I(s) = \frac{-\frac{1}{s} - 1 \cdot 10^{-6}}{\frac{10^{-6}}{s} + s \cdot 1 \cdot 10^{-6}}$$

$$= \frac{-1 - 10^{-6}s}{10^6 + 10^{-6}s^2}$$

$$= \frac{-10^6 - s}{s^2 + 10^{12}}$$

$$\begin{aligned} & \begin{aligned} & \swarrow s_1 = j1 \cdot 10^6 \rightarrow \text{la parte imaginaria} = \omega \\ & \searrow s_2 = -j1 \cdot 10^6 \end{aligned} \end{aligned}$$

$$= \frac{A}{s + j1 \cdot 10^6} + \frac{B}{s - j1 \cdot 10^6}$$

$$= A \frac{\omega}{s^2 + \omega^2} + B \frac{s}{s^2 + \omega^2}$$

$$s = 0$$

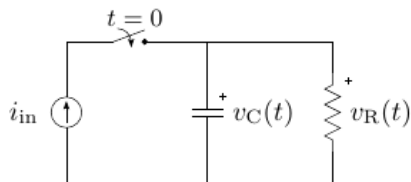
$$-10^6 = A \frac{\omega}{\omega^2} = \frac{A}{\omega} \longrightarrow A = -1$$

$$s = -10^6$$

$$0 = A \frac{\omega}{s^2 + \omega^2} + B \frac{s}{s^2 + \omega^2} \longrightarrow B = -1$$

$$I(s) = -10^{12} \frac{\omega}{s^2 + \omega^2} - 10^{12} \frac{s}{s^2 + \omega^2}$$

$$i(t) = -1 \sin(10^6 t) - 1 \cos(10^6 t)$$



Datos

$$i_{in} = 10 \sin(2\pi 50t) [A]$$

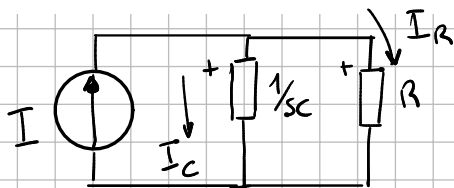
$$C = 10000 \mu F$$

$$R = 20 \Omega$$

Figura 5: Circuito RC paralelo.

Ejercicio 10.

En el circuito de la figura 5 encontrar la respuesta $v_C(t)$ para $t > 0$ utilizando la transformada de Laplace como herramienta. La tensión inicial sobre el capacitor es cero.



$$I = 10 \frac{2\pi 50}{s^2 + (2\pi 50)^2}$$

$$\left. \begin{aligned} I &= I_C + I_R \\ V_C &= I_R R \\ I_C &= C s V_C \end{aligned} \right\} I = V_C \left(\frac{1}{R} + sC \right)$$

$$V_C = \frac{10 \frac{2\pi 50}{s^2 + (2\pi 50)^2}}{\frac{1}{R} + sC}$$

$$V_C = \frac{(10 \cdot 2\pi 50) 100}{(s^2 + (2\pi 50)^2)(s+5)}$$

$$V_C = A \frac{2\pi 50}{s^2 + 2\pi 50^2} + B \frac{s}{s^2 + 2\pi 50^2} + C \frac{1}{s+5}$$

$$C = \lim_{s \rightarrow -5} (s+5) \frac{(10 \cdot 2\pi 50) 100}{(s^2 + (2\pi 50)^2)(s+5)} = 3,18$$

$$s=0$$

$$\frac{100000\pi}{(2\pi 50)^2 \cdot 5} = A \frac{\cancel{2\pi 50}}{2\pi 50^2} + \frac{3,18}{5}$$

$$A = 0,19$$

$$s=1$$

$$\frac{100000\pi}{(2\pi 50)^2 \cdot 6} = 0,19 \frac{2\pi 50}{1 + 2\pi 50^2} + B \frac{1}{1 + 2\pi 50^2} + \frac{3,18}{6}$$

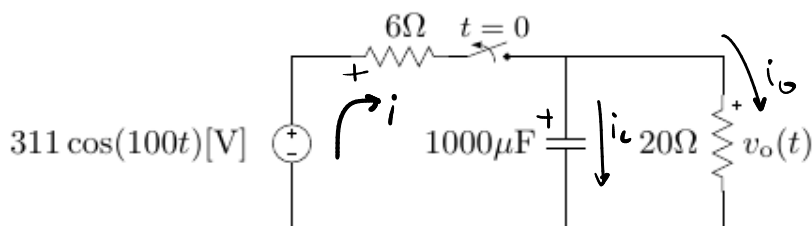
$$-8,83 \cdot 10^{-5} = B \cdot 1,013 \cdot 10^{-5}$$

$$B = -9$$

$$V_C = 0,19 \frac{2\pi 50}{s^2 + 2\pi 50^2} - 9 \frac{s}{s^2 + 2\pi 50^2} + \frac{3,18}{s+5}$$

Ejercicio 23.

Aplicando transformada de Laplace encontrar la tensión $v_o(t)$ indicada en el circuito de la figura 16.



$t < 0$:

LKV

$$V - V_R - V_C = 0 \longrightarrow$$

$$V_C = V - V_R$$

LKI

$$i = i_C$$

$$V_C = V - i R$$

$$V_C = V - i_C R$$

V-I

$$V_R = i R$$

$$V_C = V - R C \frac{dV_C}{dt}$$

$$i_C = C \frac{dV_C}{dt}$$

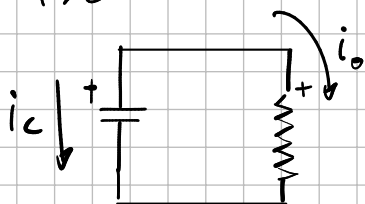
$$V_C + R C \frac{dV_C}{dt} = V$$

$$V_C = 153,65 \sqrt{2} \cos(\omega t + 22,7^\circ)$$

$t = 0$

$$V_C(0) = 197,3 \text{ V}$$

$t > 0$



$$V_C = V_0$$

$$i_C + i_0 = 0$$

$$i_C = C \frac{dV_C}{dt} = C S V_C(s) - C V_C(0)$$

$$i_0 = \frac{V_0}{R_0} \longrightarrow I_0(s) = \frac{V_0(s)}{R_0}$$

$$C S V_0(s) - C V_C(0) + \frac{V_0(s)}{R_0} = 0$$

$$V_0(s) \left(C S + \frac{1}{R_0} \right) = C V_C(0)$$

$$V_0(s) = \frac{C V_C(0)}{C S + \frac{1}{R_0}} = \frac{V_C(0)}{S + \frac{1}{R_0 C}} = \frac{197,3}{S + 50}$$

$$V_0(s) = 197,3 e^{-50t}$$

Ejercicio 25.

Aplicando la transformada de Laplace, determinar $i_1(t)$ y $i_2(t)$ del circuito de la figura 18 para $t > 0$, siendo $V = 10\text{V}$, $R_1 = 3\Omega$, $R_2 = 4\Omega$, $L_1 = 1\text{H}$, $L_2 = 4\text{H}$ y $k = 0,6$.

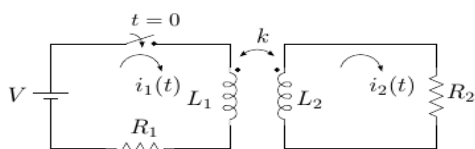


Figura 18: Circuito con acoplamiento inductivo.

$t > 0$

LKV

$$V = V_{L1} + V_{R1}$$

$$V_{L2} = V_{R2}$$

V-I

$$V_{R1} = i_1 R_1$$

$$V_{R2} = i_2 R_2$$

$$V_{L1} = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$V_{L2} = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$V = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} + i_1 R_1$$

$$0 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt} + i_2 R_2$$

$\downarrow \mathcal{L}$

$$i_1(0) = i_2(0) = 0$$

$$\frac{V}{s} = L_1 s I_1(s) - M s I_2(s) + R_1 I_1(s)$$

$$0 = L_2 s I_2(s) - M s I_1(s) + R_2 I_2(s)$$

\downarrow

$$\frac{V}{s} = I_1(s) (L_1 s + R_1) + I_2(s) (-M s)$$

$$0 = I_1(s) (-M s) + I_2(s) (L_2 s + R_2)$$

$$\begin{bmatrix} \frac{V}{s} \\ 0 \end{bmatrix} = \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} \begin{bmatrix} L_1 s + R_1 & -M s \\ -M s & L_2 s + R_2 \end{bmatrix} ; \quad I_1 = \frac{\Delta_1}{\Delta_p} ; \quad I_2 = \frac{\Delta_2}{\Delta_p}$$

$$\Delta_p = (L_1 s + R_1)(L_2 s + R_2) - [(-M s)(-M s)] = L_1 L_2 s^2 + L_1 R_2 s + L_2 R_1 s + R_1 R_2 - M^2 s^2$$

$$\Delta_p = (L_1 L_2 - M^2) s^2 + (L_1 R_2 + L_2 R_1) s + R_1 R_2$$

$$L_1 = 1; L_2 = 4; R_1 = 3; R_2 = 4$$

$$M = K \sqrt{L_1 L_2} = 0,6 \sqrt{1 \cdot 4} = 1,2$$

$$\Delta_p = 2,56 s^2 + 16 s + 12$$

$$\Delta_1 = \begin{bmatrix} \frac{V}{s} & -M s \\ 0 & L_2 s + R_2 \end{bmatrix} = \left(\frac{V}{s} \cdot L_2 s + R_2 \right) - 0 = \frac{40 s + 40}{s}$$

$$\Delta_2 = \begin{bmatrix} L_1 s + R_1 & \frac{V}{s} \\ -M s & 0 \end{bmatrix} = -\frac{V}{s} (-M s) = 12$$

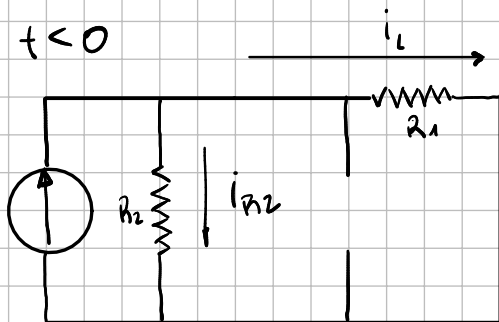
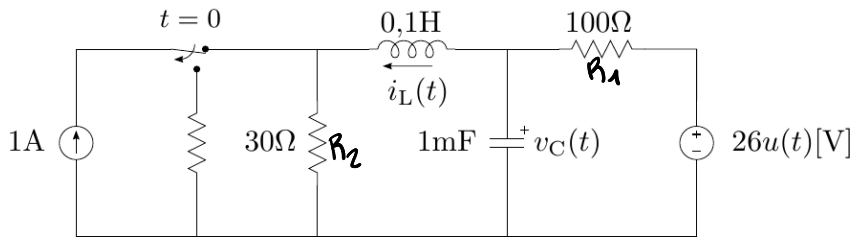
$$\begin{aligned} I_1(s) &= \frac{40 s + 40}{2,56 s^2 + 16 s + 12} = \frac{40 s + 40}{s(2,56 s^2 + 16 s + 12)} = \frac{40 (s+1)}{2,56 s (s^2 + 6,25 s + 4,68)} = \frac{15,625 (s+1)}{s (s+0,8) (s+5,3)} \\ &= \frac{3,3}{s} - \frac{0,86}{s+0,8} - \frac{2,81}{s+5,3} \xrightarrow{t} 3,3 N(t) - 0,8 e^{-0,8 t} - 2,81 e^{-5,3 t} \end{aligned}$$

$$I_2(s) = \frac{12}{2,56s^2 + 16s + 12} = \frac{12}{2,56(s+0,8)(s+5,3)} = \frac{4,68}{(s+0,8)(s+5,3)} = \frac{1,04}{s+0,8} - \frac{1,04}{s+5,3}$$

$$i_2(t) = 1,04 e^{-0,8t} - 1,04 e^{-5,3t}$$

Ejercicio 15.

Para el circuito de la figura 10 se pide encontrar $I_L(s)$ y $i_L(t)$ para $t > 0$.



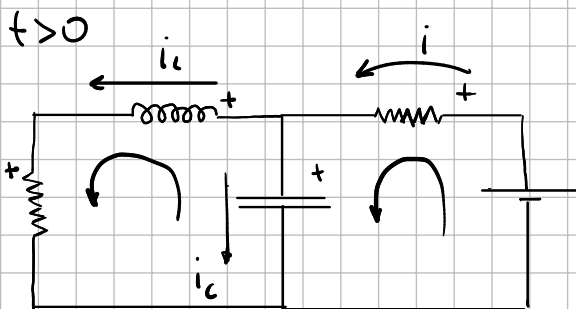
$$1A = i_{R2} + i_L$$

$$V_{R2} = V_{R1}$$

$$1A = \frac{V_{R2}}{R2} + \frac{V_{R1}}{R1}$$

$$1A = V_{R2} \left(\frac{1}{R2} + \frac{1}{R1} \right) \rightarrow V_{R2} = \frac{300}{13} V$$

$$1A = \frac{300}{13} + i_L \rightarrow i_L(0) = \frac{3}{13} A$$



LKV:

$$\textcircled{1} V - V_{R1} - V_C = 0$$

$$\textcircled{2} V_C - V_L - V_{R2} = 0$$

LKI:

$$i = i_C + i_L \quad \textcircled{3}$$

V-I

$$V_{R1} = i R1$$

$$V_{R2} = i_L R2$$

$$V_L = L \frac{di_L}{dt}$$

$$i_C = C \frac{dV_C}{dt}$$

L

$$\textcircled{3} I(s) = I_C(s) + I_L(s)$$

$$I(s) = sC V_C(s) - C V_C(0) + I_L(s)$$

$$\frac{V_{R1}(s)}{R1} = sC (V_L(s) + V_{R2}) - C V_C(0) + I_L(s) \rightarrow \frac{V}{sR1} - \frac{V_C}{R1} = sC V_L(s) + sC V_{R2}(s) - C V_C(0) + I_L(s)$$

$$\frac{V}{sR1} - \frac{V_C}{R1} = sC \left(L s I_L(s) - L i_L(0) \right) + sC V_{R2}(s) - C V_C(0) + I_L(s)$$

$$\frac{V}{sR1} = s^2 CL I_L(s) - sCL i_L(0) + sC V_{R2}(s) - C V_C(0) + I_L(s) + \frac{(V_L + V_{R2})}{R1}$$

$$\frac{V}{sR_1} = s^2 CL I_L(s) - sCL i_L(0) + sC V_{R2}(s) - C V_C(0) + I_L(s) + s \frac{L}{R_1} I_L(s) - \frac{L}{R_1} i_L(0) + \frac{V_{R2}}{R_1}$$

$$\frac{V}{sR_1} = s^2 CL I_L(s) - sCL i_L(0) + sC R_2 I_L(s) - C V_C(0) + I_L(s) + s \frac{L}{R_1} I_L(s) - \frac{L}{R_1} i_L(0) + I_L(s) \frac{R_2}{R_1}$$

$$\frac{V}{sR_1} + C V_C(0) + \frac{L}{R_1} i_L(0) + sCL i_L(0) = I_L(s) \left[s^2 CL + sC R_2 + 1 + s \frac{L}{R_1} + \frac{R_2}{R_1} \right]$$

$$R_1 = 100; R_2 = 30; C = 1 \cdot 10^{-3}; L = 0,1; i_L(0) = \frac{3}{13}; V_C(0) = \frac{300}{13}$$

$$0,26 + s \left(\frac{3}{130} + \frac{3}{13000} \right) + s^2 \frac{3}{130000} = I_L(s) \left[s^2 \frac{1}{10000} + s^2 \frac{31}{1000} + s \frac{13}{10} \right]$$

```
octave:6> factor(I)
ans = (sym)
```

$$\frac{834 \cdot s^2 + 268125 \cdot \pi \cdot s + 9396400}{3614 \cdot s \cdot (s + 50) \cdot (s + 260)}$$

```
octave:7> partfrac(factor(I))
ans = (sym)
```

$$-\frac{3892 + 4125 \cdot \pi}{11676 \cdot (s + 260)} + \frac{-229628 + 268125 \cdot \pi}{758940 \cdot (s + 50)} + \frac{1}{5 \cdot s}$$

```
octave:8> ilaplace(partfrac(factor(I)))
ans = (sym)
```

$$\frac{1}{5} + \left(-\frac{59}{195} + \frac{1375 \cdot \pi}{3892} \right) \cdot e^{-50 \cdot t} + \left(\frac{1}{3} - \frac{1375 \cdot \pi}{3892} \right) \cdot e^{-260 \cdot t}$$

```
octave:13> t=0
```

```
t = 0
```

```
octave:14> eval(i)
```

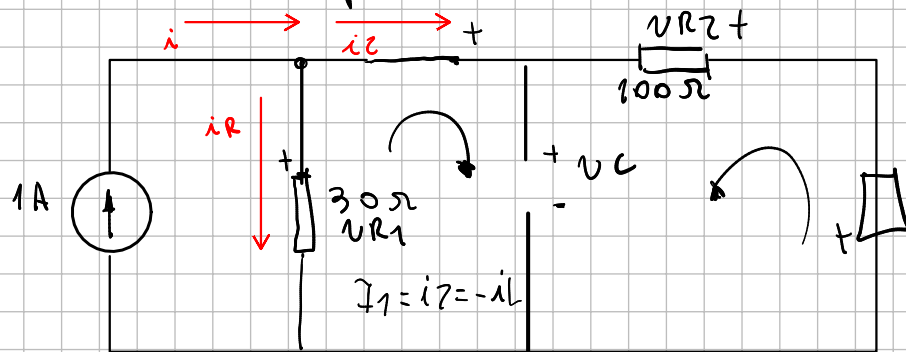
```
ans = 0.2308
```

```
octave:15> 3/13
```

```
ans = 0.2308
```

Resolución rao, error en los reemplazos

Analizamos para $t < 0$



$$i = i_2 + i_R$$

$$V_{R2} + V_C = 0 \rightarrow V_C = -V_{R2}$$

$$V_C - V_{R1} = 0 \rightarrow V_C = V_{R1}$$

$$V_{R1} = R_1 i_R$$

$$V_{R2} = -i_2 \cdot R_2 \rightarrow i_2 = -\frac{V_{R2}}{R_2}$$

$$i_2 = i - i_R$$

$$-\frac{V_{R2}}{R_2} + \frac{V_{R1}}{R_1} = i$$

$$\frac{V_C}{R_2} + \frac{V_C}{R_1} = i$$

$$V_C \left(\frac{13}{300} \right) = 1$$

$$V_C = \frac{300}{13}$$

$$i_2 = -\frac{V_{R2}}{R_2}$$

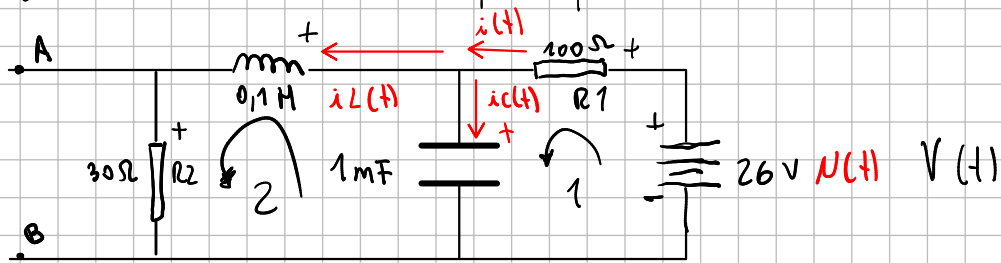
$$-i_2 = \frac{V_{R2}}{R_2}$$

$$i_L = \frac{V_{R2}}{R_2}$$

$$i_L = -\frac{V_C}{R_2}$$

$$i_L = -0,2307$$

para $t > 0$ el circuito en el tiempo que da



L K V para $t > 0$ (dominio t)

malla 2

$$-V_C(t) + V_L(t) + V_{R2}(t) = 0$$

L K I

$$i(t) = i_L + i_C$$

malla 1

$$-V(t) + V_{R1} + V_C(t) = 0$$

Rel tensión
corriente

$$V_{R2} = R_2 \cdot i_L(t)$$

$$V_{R1} = R_1 \cdot i(t)$$

$$i_C = C \frac{dV_C(t)}{dt}$$

$$V_L = L \frac{di_L(t)}{dt}$$

paseamos a laplace

Malla 2

$$-V_C(s) + V_L(s) + V_{R2}(s) = 0$$

$$I_L(s) = \frac{26}{s^2 L} - \frac{R_1 I_L(s)}{L s} - \frac{R_1 C V_C(s)}{L s} - \frac{R_1 C V_C(0)}{L s} - \frac{R_2 I_L(s)}{L s} - \frac{i_L(0)}{s}$$

Malla 1

$$-V(s) + V_{R1}(s) + V_C(s) = 0$$

$$-V(s) + V_{R1}(s) + V_C(s) = 0$$

$$L K I I_L(s) = \frac{26}{s^2 L} - \frac{R_1 I_L(s)}{L s} - \frac{R_1 C V_C(s)}{L s} - \frac{R_1 C V_C(0)}{L s} - \frac{R_2 I_L(s)}{L s} - \frac{i_L(0)}{s}$$

$$I(s) = I_C(s) + I_L(s)$$

$$I(s) = \frac{26}{s^2 L} - \frac{R_1 I_L(s)}{L s} - \frac{R_1 C V_C(s)}{L s} - \frac{R_1 C V_C(0)}{L s} - \frac{R_2 I_L(s)}{L s} - \frac{i_L(0)}{s}$$

$$V_{R2}(s) = R_2 I_L(s)$$

$$I_C(s) = C (s V_C(s) + V_C(0))$$

$$V_L(s) = L (s I_L(s) + i_L(0))$$

de la malla 2

$$V_L(s) = V_C(s) - V_{R2}(s)$$

$$L s I_L(s) + L \cdot i_L(0) = V_C(s) - V_{R2}(s)$$

$$I_L(s) = \frac{V_C(s)}{L s} - \frac{V_{R2}(s)}{L s} - \frac{i_L(0)}{s}$$

$$V_C(s) = L s (I_L(s) + i_L(0)) + s V_{R2}(s)$$

de la malla 1

$$V_C(s) = V(s) - V_{R1}(s)$$

$$I_L(s) = \frac{V(s)}{L s} - \frac{V_{R1}(s)}{L s} - \frac{V_{R2}(s)}{L s} - \frac{i_L(0)}{s}$$

$$I_L(s) = \frac{26}{s^2 L} - \frac{R_1 I(s)}{L s} - \frac{R_2 I_L(s)}{L s} - \frac{i_L(0)}{s}$$

$$I_L(s) = \frac{26}{s^2 L} - \frac{R_1 I_L(s)}{L s} - \frac{R_1 I_C(s)}{L s} - \frac{R_2 I_L(s)}{L s} - \frac{i_L(0)}{s}$$

$$I_L(s) = \frac{26}{s^2 L} - \frac{R_1 I_L(s)}{L s} - \frac{R_1 C V_C(s)}{L s} - \frac{R_1 C V_C(0)}{L s} - \frac{R_2 I_L(s)}{L s} - \frac{i_L(0)}{s}$$

$$I_L(s) = \frac{26}{s^2 L} - \frac{R_1 I_L(s)}{L s} - \frac{R_1 C V_C(s)}{L s} - \frac{R_1 C V_C(0)}{L s} - \frac{R_2 I_L(s)}{L s} - \frac{i_L(0)}{s}$$

$$I_L(s) + R_1 I_L(s) + R_1 C I_L(s) + R_2 I_L(s) = \frac{26}{s^2 L} - \frac{R_1 C V_C(0)}{L s} - \frac{i_L(0)}{s}$$

$$L(s) \left(1 + \frac{R1}{Ls} + R1 \cdot C + \frac{R1 \cdot R2}{L} + \frac{R2}{Ls} \right) = \frac{260}{s^2} + 0,0006921 - \frac{6,9230}{s} + \frac{0,2307}{s}$$

$$IL(s) \left(1 + \frac{200}{s} + \frac{3}{100} + 30 + \frac{1000}{s} \right) = \frac{260}{s^2} + 0,0006921 - \frac{6,9230}{s} + \frac{0,2307}{s}$$

$$L(s) \left(31,03 + \frac{1300}{s} \right) = \frac{260}{s^2} - \frac{6,6923}{s} + 0,0006921$$

$$IL(s) = \left(\frac{260}{s^2} - \frac{6,6923}{s} + 0,0006921 \right) : \left(31,03 + \frac{1300}{s} \right)$$

$$IL(s) = -\frac{6,6923}{s} \cdot \frac{1}{31,03} -$$

$$= \frac{8,3789}{s^2} + \frac{0,2}{s}$$

!!!
... muchos
errores
Rehacer