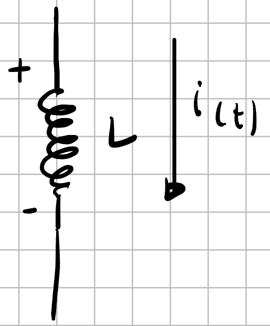
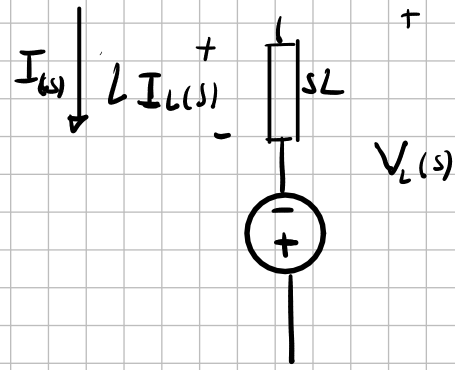
Ejercicio 4.

Transformar las relaciones tensión-corriente del inductor y capacitor del dominio del tiempo al dominio de Laplace, y armar el circuito equivalente serie y paralelo que representan las ecuaciones transformadas.

inductor



$$\xrightarrow{\mathcal{L}}$$



$$V_L(t) = L \frac{di_L}{dt}$$

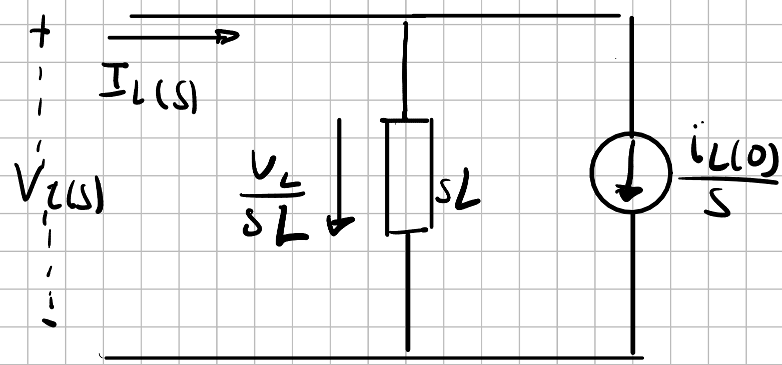
$$\Big\downarrow \mathcal{L}$$

$$V_L(s) = L\ \mathcal{L}\left(i_L(t)\right)$$

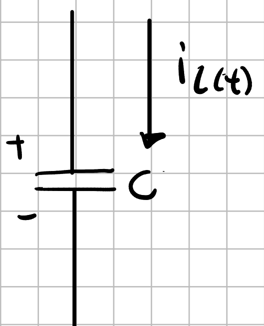
$$V_L(s) = L\left[s\, I_L(s) - i_L(0)\right]$$

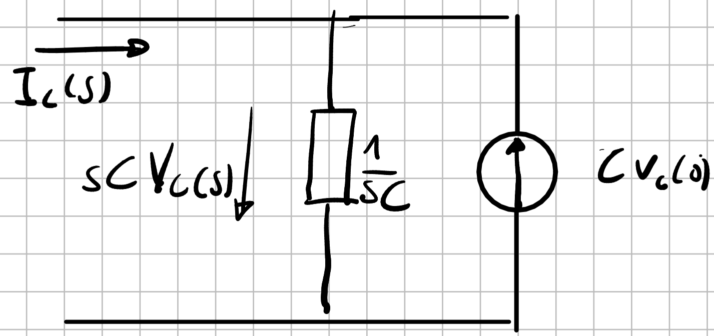
$$V_L(s) = sL\, I_L(s) - L\, i_L(0)$$

$$I_L(s) = \frac{V_L(s)}{sL} + \frac{i_L(0)}{s}$$



capacitor



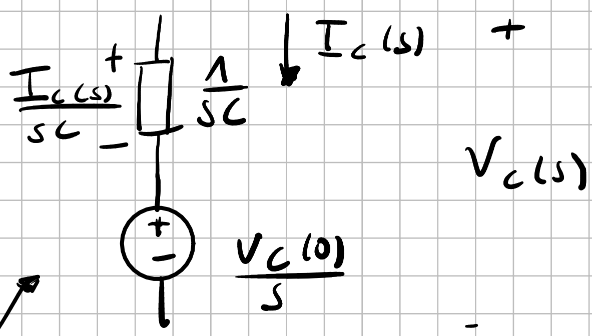


$$i_C(t) = C \frac{dV_C}{dt}$$

$$\Big\downarrow \mathcal{L}$$

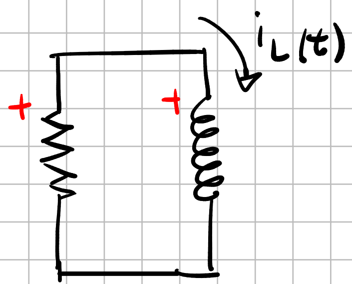
$$I_C(s) = \mathcal{L}\left[i_C(t)\right]$$

$$I_C(s) = C\, s V_C(s) - C\, v_C(0) \quad ; \quad V_C(s) = \frac{I_C(s)}{sC} + \frac{v_C(0)}{s}$$



Con un ejercicio de la guia 3

$i_{L(0)} = 20A$



$$\left.\begin{aligned} V_R(t) &= V_L(t) \\ V_R(t) &= -i_L(t)\,R \\ V_L(t) &= L\,\frac{di_L(t)}{dt} \end{aligned}\right\} \quad i_L(t)R + L\frac{di_L}{dt} = 0$$

$$\boxed{\frac{di_L(t)}{dt} + \frac{R}{L}\,i_{(t)} = 0}$$

$\mathcal{L}$

$$s\,I_{L(s)} - i_{(0)} + \frac{R}{L}\,I_{(s)} = 0$$

$$I_{L(s)}\left(s + \frac{R}{L}\right) - i_{(0)} = 0$$

$$I_{L(s)}\left(s + \frac{R}{L}\right) = i_{(0)}$$

$$I_{L(s)} = \frac{20}{s + \frac{1}{\tau}}\,;\ \tau = \frac{L}{R} \xrightarrow{\mathcal{L}^{-1}} i_{L(t)} = 20\,e^{-\frac{t}{\tau}}$$

➤otra opcion es pasar el S.E a laplace

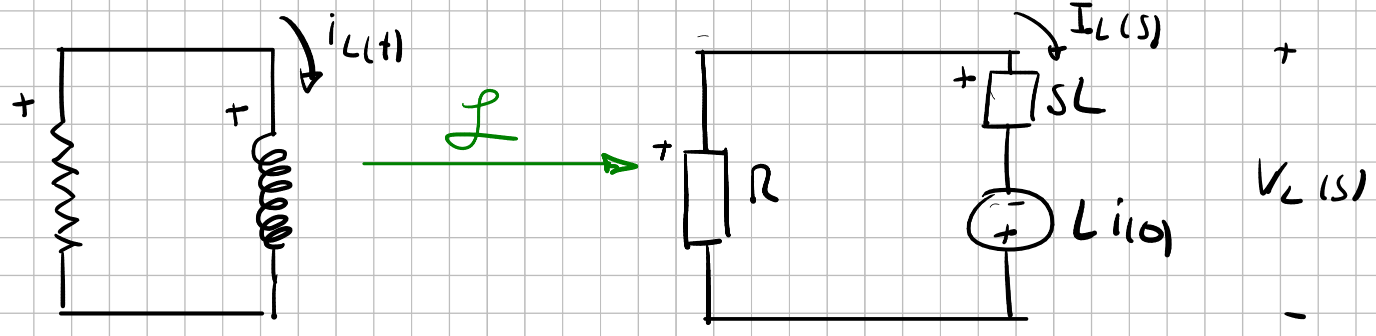
$$\begin{aligned} V_R(t) &= V_L(t) \\ V_R(t) &= -i_L(t)\,R \\ V_L(t) &= L\,\frac{di_L(t)}{dt} \end{aligned} \xrightarrow{\mathcal{L}} \begin{aligned} V_{R(s)} &= V_{L(s)} \\ V_{R(s)} &= -I_{L(s)}\,R \\ V_{L(s)} &= L\,s\,I_{L(s)} - L\,i_{(0)} \end{aligned}$$

$$L s\,I_{L(s)} - L\,i_0 + I_{L(s)}\,R = 0$$

$$I_{L(s)}\left(Ls + R\right) - L\,i_{(0)} = 0$$

$$I_{L(s)}\left(s + \frac{R}{L}\right) = i_{(0)}$$

Otra opcion mas es pasar directamente el circuito



LKV

$$V_{R(s)} = V_{L(s)}$$
$$-R I_{L(s)} = I_{L(s)} \cdot sL - L\, i_{(0)}$$
$$I_{L(s)}\left(sL + R\right) - L\, i_{(0)} = 0$$
$$I_{L(s)}\left(s + \frac{R}{L}\right) = i_{(0)}$$

Ejercicio 5.

En $t = 0$ se aplica al circuito RL serie de la figura 2 una tensión continua de 55V. Encontrar la transformada de la respuesta $i(t)$ para $t > 0$.

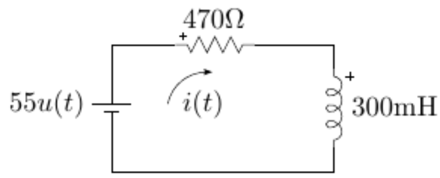


Figura 2: Circuito RL serie con excitación constante.

LKV

$$V_{(t)} = V_{R(t)} + V_{L(t)}$$
$$V_{R(t)} = i_{(t)} R$$
$$V_{L(t)} = L\,\frac{di_L}{dt}$$

$\xrightarrow{\mathcal{L}}$

$$V_{(s)} = 55\,\frac{1}{s}$$
$$V_{(s)} = V_{R(s)} + V_{L(s)}$$
$$V_{R(s)} = I_{(s)} R$$
$$V_{L(s)} = L\left(s I_{L(s)} - i_{L(0)}\right)$$

$$\frac{55}{s} = I_{L(s)} R + L s I_{L(s)} - \underbrace{L\, i_{L(0)}}_{=0}$$

$$\frac{55}{L s} + L\, \cancel{i_{L(0)}} = I_{L(s)}\left(\frac{R}{L} + s\right)$$

$$I_{L(s)} = \frac{\frac{55}{L s}}{\frac{R}{L} + s}$$

$$I_{L(s)} = \frac{\frac{1}{L}\, 55}{s\left(s + \frac{R}{L}\right)}$$

$$I_L(s) = \frac{A}{s} + \frac{B}{s+\frac{R}{L}}$$

A; B?

$$\lim_{s\to 0} s \frac{\frac{55}{L}}{s\left(s+\frac{R}{L}\right)} = \lim_{s\to 0} \cancel{s}\frac{A}{\cancel{s}} + \cancel{\lim_{s\to 0} \frac{B}{s+\frac{R}{L}}}$$

$$A = \frac{\frac{55}{L}}{\frac{R}{L}} = \frac{55}{470} = 0{,}117$$

$$B = \lim_{s\to -\frac{R}{L}} \cancel{\left(s+\frac{R}{L}\right)} \frac{\frac{55}{L}}{s\cancel{\left(s+\frac{R}{L}\right)}} = \frac{\frac{55}{L}}{-\frac{R}{L}} = -0{,}117$$

$$I_L(s) = \frac{0{,}117}{s} - \frac{0{,}117}{s+\frac{1}{\tau}} \quad ; \quad \tau = \frac{L}{R}$$

$$i(t) = 0{,}117\, u(t) - 0{,}117\, e^{-\frac{t}{\tau}} u(t)$$

$$i(t) = \left[0{,}117 - 0{,}117\, e^{-\frac{t}{\tau}}\right] u(t)$$

Ejercicio 6.

El capacitor de la figura 3 tiene una carga inicial de $q_0 = 800 \times 10^{-6}$C con la polaridad indicada. Hallar la respuesta completa de la tensión del capacitor en el dominio de la variable s.

$$V_c(0) = \frac{q_0}{C}$$

$$V_c(0) = 200\,V$$

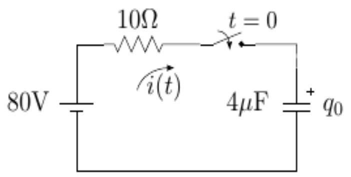


Figura 3: Circuito *RC*.

LKV

$$V(t) = V_R(t) + V_C(t)$$

$$V_R = i(t)\, R$$

$$i(t) = C \frac{dV_c(t)}{dt}$$

$$\xrightarrow{\mathcal{L}}$$

$$V(s) = I(s)\, R + V_C(s)$$

$$I(s) = C\left(s V_C(s) - V_C(0)\right)$$

$$V(s) = RCs V_C - RCV_C(0) + V_C(s)$$

$$V(s) = V_C\left(RCs+1\right) - RCV_C(0)$$

$$V_C(s) = \frac{V(s) + RCV_C(0)}{RCs+1}$$

$$V(s) = \frac{80}{s}$$

$$V_c(s) = \frac{\frac{80}{s}\frac{1}{RC} + V_c(0)}{s + \frac{1}{RC}} \cdot \frac{s}{s}$$

$$V_c(s) = \frac{\frac{80}{RC} + s V_c(0)}{s\left(s + \frac{1}{RC}\right)}$$

$$A = \lim_{s \to 0} \frac{\frac{80}{RC} + s V_c(0)}{s + \frac{1}{RC}} = 80$$

$$B = \lim_{s \to -\frac{1}{RC}} \frac{\frac{80}{RC} + s V_c(0)}{s} = -80 + 200 = 120$$

$$V_c(s) = \frac{80}{s} + \frac{120}{s + \frac{1}{RC}} \quad ; \quad \tau = RC$$

$$V_c(s) = \frac{80}{s} + \frac{120}{s + \frac{1}{\tau}}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$V_c(t) = \left(80 + 120\, e^{-\frac{t}{\tau}}\right) u(t)$$

Ejercicio 7.

La respuesta de corriente de un circuito eléctrico tiene la siguiente transformada

$$I(s) = \frac{\frac{4}{5}}{\left(\frac{s}{5}+1\right)^2+4},$$

se pide:

1. encontrar $i(t)$,
2. encontrar el valor de $i(0)$ aplicando el teorema del valor inicial y comprobar en el tiempo,
3. encontrar el valor de $i(\infty)$ aplicando el teorema del valor final y comprobar en el tiempo.

1. $$I(s) = \frac{4/5}{\frac{s^2}{25} + \frac{2}{5}s + 5}$$

$$I(s) = \frac{20}{s^2 + 10s + 125} \quad (1) \quad \begin{cases} s_1 = -5+10j \\ s_2 = -5-10j \end{cases}$$

$$I(s) = \frac{20}{(s+5-10j)(s+5+10j)}$$

$$A = \lim_{s \to -5+10j} \cancel{(s+5-10j)} \frac{20}{\cancel{(s+5-10j)}(s+5+10j)} = -j$$

$$B = \lim_{s \to -5-10j} \cancel{(s+5+10j)} \frac{20}{(s+5-10j)\cancel{(s+5+10j)}} = j$$

$$I(s) = \frac{-j}{s+5-10j} + \frac{j}{s+5+10j}$$

$$i(t) = -j\, e^{(-5+10j)t} + j\, e^{(-5-10j)t}$$

$$i(t) = -j\, e^{-5t} \cdot e^{j10t} + j\, e^{-5t}\, e^{-j10t} = e^{-5t}\left(-j\, e^{j10t} + j\, e^{-j10t}\right)$$

$$i(t) = e^{-5t}\left(\frac{e^{j10t} - e^{-j10t}}{j}\right)$$

$$\boxed{i_{(t)} = e^{-5t}\left(2\,\text{sen}(10t)\right)\mathcal{N}_{(t)}}$$

* Sabiendo que son raices complejas lo haremos pero sin frac simples, con la idea de que en laplace nos quede directamente la anti transformada del seno y coseno

buscamos la forma

$$i_{(t)} = e^{-\alpha t}\left(C\,\text{sen}(\omega t) + D\cos(\omega t)\right)$$

$$I_{(s)} = C\frac{\omega}{(s+\alpha)^2+\omega^2} + D\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$$

desde 1

$$I_{(s)} = C\frac{10}{(s+5)^2+10^2} + D\frac{s+5}{(s+5)^2+10^2}$$

$s = -5$

$$\frac{20}{(-5)^2+10(-5)+125} = C\frac{10}{(-5+5)^2+10^2}$$

$$C = \frac{20}{25-50+125}\;\frac{10^2}{10} = 2 = C$$

$s = 0$

$$\frac{20}{125} = 2\frac{10}{5^2+10^2} + D\frac{5}{5^2+10^2}$$

$$D = 0$$

$$I_{(s)} = 2\frac{10}{(s+5)^2+10^2} + 0$$

$$\boxed{i_{(t)} = 2\,\text{sen}(10t)\,e^{-5t}\,\mathcal{N}_{(t)}}$$

2. TVI

$$i_{(0)} = \lim_{s\to\infty} s\, I_{(s)}$$

$$= \lim_{s\to\infty} s \frac{20}{s^2+10s+125}$$

$$= \frac{\frac{s}{s^2}\, 20}{\frac{s^2}{s^2} + 10\frac{s}{s^2} + \frac{125}{s^2}}$$

$$i_{(0)} = 0$$

3. TVF

$$i_{(\infty)} = \lim_{s\to 0} s\, I_{(s)}$$

$$= \lim_{s\to 0} s \frac{20}{s^2+10s+125}$$

$$i_{(\infty)} = 0$$

Ejercicio 17.

Dado el circuito de la figura 12 en el dominio de s.

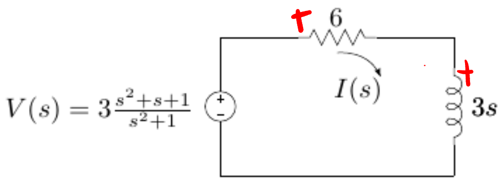


Figura 12: Dominio de s.

- Encontrar $I(s)$ y su correspondiente $i(t) = \mathcal{L}^{-1}[I(s)]$
- Tiene el circuito condición inicial no nula? Verificar utilizando el TVI.
- Encontrar $V_L(s)$.

a) $\left.\begin{array}{l} V_s = V_R + V_2 \\ V_R = R I_s \\ V_2 = sL I_s \end{array}\right\}$

$$V_s = R I_s + sL I_s$$

$$V_s = I_s (R + sL)$$

$$I_s = \frac{V_s}{R+sL}$$

$$I_s = \frac{3s^2+3s+3}{(s^2+1)} \cdot \frac{1}{R+sL} = \frac{\cancel{3}s^2+\cancel{3}s+\cancel{3}}{(s^2+1)} \cdot \frac{1}{\underset{2}{\cancel{6}}+s\cancel{3}}$$

$$I_s = \frac{s^2+s+1}{(s^2+1)(2+s)} = \frac{s^2+s+1}{(s+2)(s-j)(s+j)}$$

No trabajamos con la forma $\frac{A}{s+2} + \frac{B}{s-j} + \frac{C}{s+j}$ pq de la otra ya te queden los cosenos

$$I(s) = \frac{A}{s+2} + B\frac{1}{(s^2+1)} + C\frac{s}{(s^2+1)}$$

$$A = \lim_{s \to -2} \cancel{(s+2)} \frac{s^2+s+1}{(s^2+1)\cancel{(s+2)}} = \frac{3}{5}$$

Para B y para C? lo valuamos

$s = 0$

$$\frac{(0)^2+(0)+1}{(0^2+1)(0+2)} = \frac{3/5}{0+2} + B\frac{1}{(0^2+1)} + C\frac{0}{(0^2+1)}$$

$$\frac{1}{2} = \frac{3}{10} + B + 0 \longrightarrow B = \frac{1}{5}$$

$s = 1$

$$\frac{(1)^2+(1)+1}{(1^2+1)(1+2)} = \frac{3/5}{1+2} + \frac{1}{5}\frac{1}{(1^2+1)} + C\frac{1}{(1^2+1)}$$

$$C = \frac{2}{5}$$

$$I_{(s)} = \frac{3}{5}\,\frac{1}{s+2} + \frac{1}{5}\,\frac{1}{(s^2+1)} + \frac{2}{5}\,\frac{s}{(s^2+1)}$$

$$i_{(t)} = \underbrace{\frac{3}{5}\,e^{-2t}}_{\text{nat}} + \underbrace{\frac{1}{5}\,\operatorname{sen}(t) + \frac{2}{5}\cos(t)}_{\text{forz}}$$

b) parece no tener CI

$$I_{(s)} = \frac{s^2+s+1}{s^3+2s^2+s+2}$$

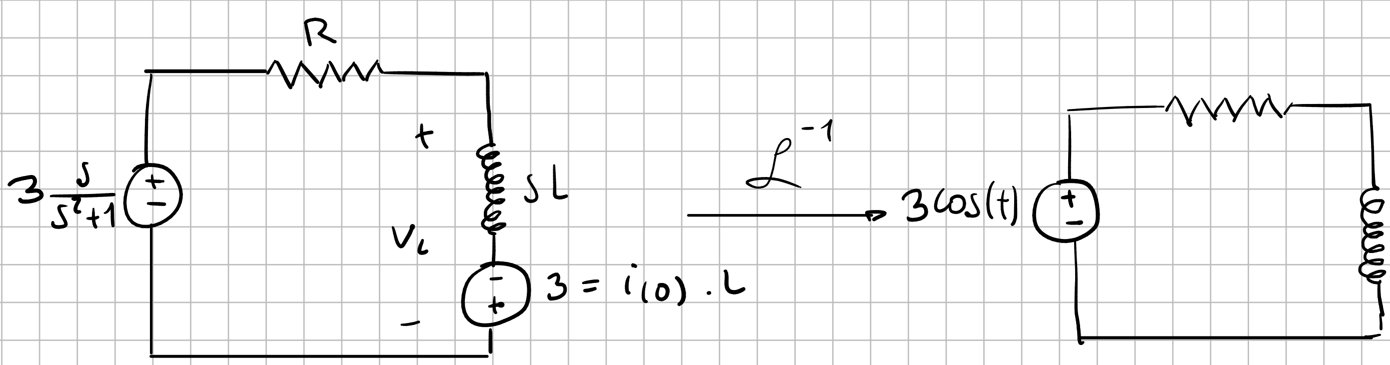
$$i_{(0)} = \lim_{s\to\infty} s\,\frac{s^2+s+1}{s^3+2s^2+s+2} = \lim_{s\to\infty} \frac{s^3+s^2+s}{s^3+2s^2+s+2}$$

$i_{(0)} = 1 \longrightarrow$ entonces?

que forma tiene $V_{(t)}$?

$$V_{(s)} = \frac{3s^2+3s+3}{s^2+1} \longrightarrow \begin{array}{r|l} 3s^2+3s+3 & s^2+1 \\ -\;3s^2+0s+3 & \overline{3} \\ \hline 0\;+3s+0 & \end{array}$$

$$V_{(s)} = 3 + 3\,\frac{s}{s^2+1} \searrow \text{coseno}$$



Ejercicio 18.

Un circuito RL serie tiene como función de transferencia

$$H(s) = \frac{I(s)}{V(s)} = \frac{1}{36 + s18}. \tag{3}$$

Si se lo excita con un escalón $v(t) = 36u(t)[\mathrm{V}]$, encontrar por convolución la respuesta $i(t) = h(t) * v(t)$.

$$h(t) = \frac{1}{18} e^{-2t}$$

$$i(t) = \int_0^t h(\tau)\, v(t-\tau)\, d\tau = \int_0^t \frac{1}{18} e^{-2\tau} u(\tau)\, 36\, u(t-\tau)\, d\tau$$

$$i(t) = \frac{36}{18} \int_0^t e^{-2\tau}\, d\tau = 2 \left. \frac{e^{-2\tau}}{-2} \right|_0^t = 2\, \frac{e^{-2t} - 1}{-2}$$

$$i(t) = -1 \left(e^{-2t} - 1\right) u(t) = 1 - e^{-2t} u(t)$$

Ejercicio 22.

Obtener la respuesta al impulso del circuito de la figura 15 considerando $H(s) = \frac{I_R(s)}{V(s)}$; donde $I_R(s) = \mathcal{L}[i_R(t)]$ y $V(s) = \mathcal{L}[v(t)]$.

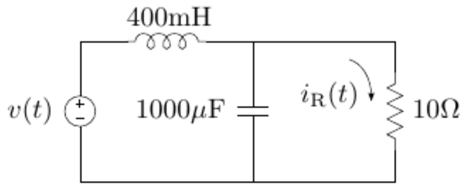
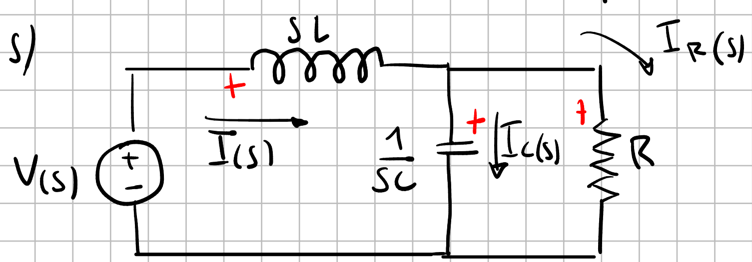


Figura 15: Cálculo de respuesta al impulso.

Como CI = 0 ; circ. eq



LKV

$$V(s) = V_L(s) + V_c(s)$$

$$V_R(s) = V_C(s)$$

$$V_L(s) = sL\, I(s)$$

$$V_C(s) = \frac{I_C(s)}{sC} \rightarrow I_C(s) = V_R(s)\, sC$$

$$V_R(s) = R\, I_R(s)$$

LKI

$$I(s) = I_C(s) + I_R(s)$$

$$V(s) = sL\, I(s) + R\, I_R(s)$$

$$= sL\, I_C(s) + sL\, I_R(s) + R\, I_R(s) = sL\left(V_R(s)\, sC\right) + I_R(s)\left(sL + R\right) = V(s)$$

$$V(s) = I_R(s)\left(s^2 LC\, R + sL + R\right)$$

$$\frac{I_R(s)}{V(s)} = H(s) = \frac{1}{s^2 LCR + sL + R} \quad \frac{\frac{1}{LCR}}{\frac{1}{LCR}} = \frac{\frac{1}{LCR}}{s^2 + \frac{1}{CR}s + \frac{1}{LC}}$$

$$H(s) = \frac{250}{s^2 + 100^{-3}s + 500s} = \frac{250}{(s+50)(s+50)}$$

$$H(s) = \frac{A}{(s+50)^2} + \frac{B}{(s+50)}$$

$$\lim_{s \to -50} \cancel{(s+50)^2} \frac{250}{\cancel{(s+50)^2}} = \lim_{s \to -50} A + \underbrace{\cancel{\lim_{s \to -50} (s+50) B}}_{=0}$$

$$A = 250$$

$$\lim_{s \to -50} \frac{d}{ds} 250 = \lim_{s \to -50} \frac{d}{ds} A + \lim_{s \to -50} (s+50) \frac{d}{ds} B$$

$$B = \frac{d}{ds} 250 = 0$$

Ejercicio 24.

Para el circuito de la figura 17 de condiciones iniciales $i_L(0) = 1[A]$ y $v_C(0) = 1[V]$ se pide:

1. encontrar la respuesta completa de corriente $i(t)$ para $t > 0$ utilizando el modelo de circuito equivalente de Laplace,
2. decir que parte de la respuesta corresponde a la natural y cuál es la forzada.

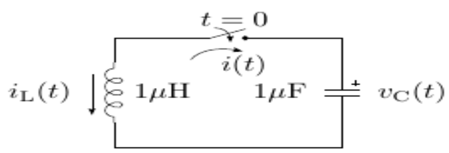
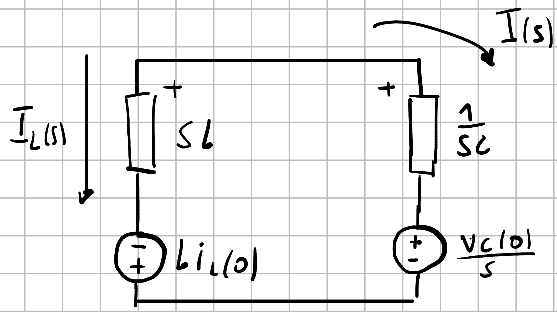


Figura 17: Equivalente de Laplace.



LKI $\quad I_L(s) = -I(s)$

LKV

$$I(s)\frac{1}{sC} + \frac{V_C(0)}{s} + L\,i_L(0) - I_L(s)\,sL = 0$$

$$I(s)\left(\frac{1}{sC} + sL\right) = -\frac{V_C(0)}{s} - L\,i_L(0)$$

$$I(s) = \frac{-\frac{1}{s} - 1.10^{-6}}{\frac{10^6}{s} + s\,1.10^{-6}}$$

$$= \frac{-1 - 10^{-6} s}{10^{6} + 10^{-6} s^2}$$

$$= \frac{-10^{6} - s}{s^2 + 10^{12}} \quad \begin{cases} s_1 = j1 \cdot 10^{6} \longrightarrow \text{la parte imaginaria} = \omega \\ s_2 = -j1 \cdot 10^{6} \end{cases}$$

$$= \frac{A}{s + j1.10^{6}} + \frac{B}{s - j1.10^{6}}$$

$$= A \frac{\omega}{s^2 + \omega^2} + B \frac{s}{s^2 + \omega^2}$$

$s = 0$

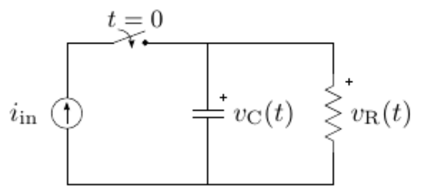
$$-10^{-6} = A \frac{\omega}{\omega^2} = \frac{A}{\omega} \longrightarrow A = -1$$

$s = -10^{6}$

$$0 = A \frac{\omega}{s^2 + \omega^2} + B \frac{s}{s^2 + \omega^2} \longrightarrow B = -1$$

$$I_{(s)} = -10^{12} \frac{\omega}{s^2 + \omega^2} - 10^{12} \frac{s}{s^2 + \omega^2}$$

$$i_{(t)} = -1 \operatorname{sen}(10^{6} t) - 1 \cos(10^{6} t)$$



Datos

$i_{\mathrm{in}} = 10\,\mathrm{sen}(2\pi 50 t)[\mathrm{A}]$

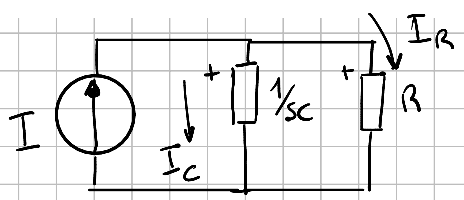
$C = 10000\mu\mathrm{F}$

$R = 20\Omega$

Figura 5: Circuito RC paralelo.

Ejercicio 10.

En el circuito de la figura 5 encontrar la respuesta $v_{\mathrm{C}}(t)$ para $t > 0$ utilizando la transformada de Laplace como herramienta. La tensión inicial sobre el capacitor es cero.



$$\left.\begin{aligned} I &= I_C + I_R \\ V_C &= I_R R \\ I_C &= C s V_C \end{aligned}\right\} \quad I = V_C \left(\frac{1}{R} + sC \right)$$

$$I = 10 \frac{2\pi 50}{s^2 + (2\pi 50)^2}$$

$$V_C = \frac{10 \dfrac{2\pi 50}{s^2 + (2\pi 50)^2}}{\dfrac{1}{R} + sC}$$

$$V_c = \frac{(10\ 2\pi 50)\,100}{(s^2+(2\pi 50)^2)(s+5)}$$

$$V_c = A\,\frac{2\pi 50}{s^2+2\pi 50^2} + B\,\frac{s}{s^2+2\pi 50^2} + C\,\frac{1}{s+5}$$

$$C = \lim_{s\to -5}\ (s+5)\,\frac{(10\ 2\pi 50)\,100}{(s^2+(2\pi 50)^2)(s+5)} = 3,18$$

$s=0$

$$\frac{100000\pi}{(2\pi 50)^2\ 5} = A\,\frac{\cancel{2\pi 50}}{2\pi 50^{\cancel{2}}} + \frac{3,18}{5}$$

$$A = 0,19$$

$s=1$

$$\frac{100000\pi}{(2\pi 50)^2\ 6} = 0,19\,\frac{2\pi 50}{1+2\pi 50^2} + B\,\frac{1}{1+2\pi 50^2} + \frac{3,18}{6}$$

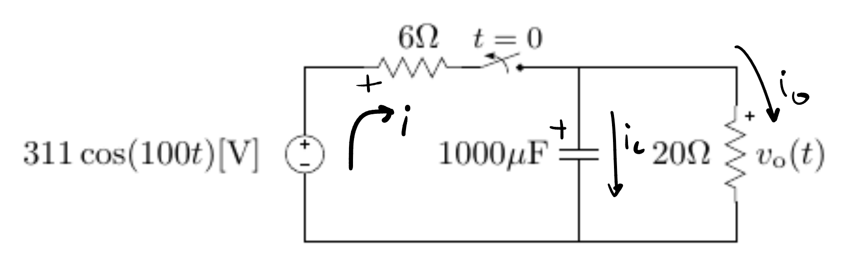
$$-8,83\cdot 10^{-5} = B\ 1,013\cdot 10^{-5}$$

$$B = -9$$

$$V_c = 0,19\,\frac{2\pi 50}{s^2+2\pi 50^2} - 9\,\frac{s}{s^2+2\pi 50^2} + \frac{3,18}{s+5}$$

Ejercicio 23.

Aplicando transformada de Laplace encontrar la tensión $v_o(t)$ indicada en el circuito de la figura 16.



$t<0$:

LKV

$V - V_R - V_c = 0 \longrightarrow$

$V_c = V - V_R$

$V_c = V - i\,R$

$V_c = V - i_c R$

$V_c = V - RC\dfrac{dV_c}{dt}$

$V_c + RC\dfrac{dV_c}{dt} = V$

$V_C = 153{,}65\sqrt{2}\cos(\omega t + 22{,}77°)$

LKI

$i = i_c$

V-I

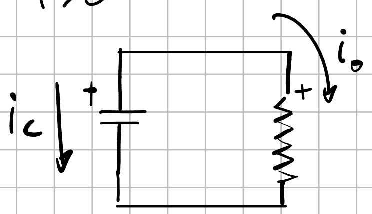
$V_R = i\,R$

$i_c = C\dfrac{dV_c}{dt}$

$t=0$

$V_c(0) = 197{,}3\ V$

$t>0$



$V_c = V_o$

$i_c + i_o = 0$

$i_c = C\dfrac{dV_c}{dt} = CS\,V_c(s) - C\,V_c(0)$

$i_o = \dfrac{V_o}{R_o} \longrightarrow I_{(o)} = \dfrac{V_{o(s)}}{R_o}$

$CS\,V_{o(s)} - C\,V_{c(0)} + \dfrac{V_{o(s)}}{R_o} = 0$

$V_{o(s)}\left(CS + \dfrac{1}{R_o}\right) = C\,V_{c(0)}$

$V_{o(s)} = \dfrac{C\,V_c(0)}{CS + \frac{1}{R_o}} = \dfrac{V_c(0)}{S + \frac{1}{R_o C}} = \dfrac{197{,}3}{S + 50}$

$V_{o(s)} = 197{,}3\ e^{-50t}$

Ejercicio 25.

Aplicando la transformada de Laplace, determinar $i_1(t)$ y $i_2(t)$ del circuito de la figura 18 para $t > 0$, siendo $V = 10V$, $R1 = 3\Omega$, $R_2 = 4\Omega$, $L_1 = 1H$, $L_2 = 4H$ y $k = 0{,}6$.

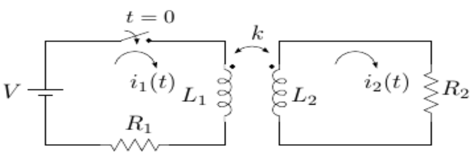


Figura 18: Circuito con acoplamiento inductivo.

$t>0$

LKV

$$V = V_{L1} + V_{R1} \longrightarrow V = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt} + i_1 R_1$$

$$V_{L2} = V_{R2} \qquad\qquad 0 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt} + i_2 R_2$$

V-I

$$V_{R1} = i_1 R_1$$

$$V_{R2} = i_2 R_2$$

$$V_{L1} = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$V_{L2} = -L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

$$\Big\downarrow \mathcal{L} \qquad i_1(0) = i_2(0) = 0$$

$$\frac{V}{s} = L_1 s I_1(s) - Ms\, I_2(s) + R_1 I_1(s)$$

$$0 = L_2 s I_2(s) - Ms I_1(s) + R_2 I_2(s)$$

$$\downarrow$$

$$\frac{V}{s} = I_1(s)(L_1 s + R_1) + I_2(s)(-Ms)$$

$$0 = I_1(s)(-Ms) + I_2(L_2 s + R_2)$$

$$\begin{bmatrix} \frac{V}{s} \\ 0 \end{bmatrix} = \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} \begin{bmatrix} L_1 s + R_1 & -Ms \\ -Ms & L_2 s + R_2 \end{bmatrix} \quad ; \quad I_1 = \frac{\Delta_1}{\Delta_p} \; ; \; I_2 = \frac{\Delta_2}{\Delta_p}$$

$$\Delta_p = (L_1 s + R_1)(L_2 s + R_2) - [(-Ms)(-Ms)] = L_1 L_2 s^2 + L_1 R_2 s + L_2 R_1 s + R_1 R_2 - M^2 s^2$$

$$\Delta_p = (L_1 L_2 - M^2)s^2 + (L_1 R_2 + L_2 R_1)s + R_1 R_2$$

$$L_1 = 1; L_2 = 4; R_1 = 3, R_2 = 4$$

$$M = K\sqrt{L_1 L_2} = 0{,}6\sqrt{1 \cdot 4} = 1{,}2$$

$$\Delta_p = 2{,}56 s^2 + 16 s + 12$$

$$\Delta_1 = \begin{bmatrix} \frac{V}{s} & -Ms \\ 0 & L_2 s + R_2 \end{bmatrix} = \left(\frac{V}{s} \cdot L_2 s + R_2\right) - 0 = \frac{40s + 40}{s}$$

$$\Delta_2 = \begin{bmatrix} L_1 s + R_1 & \frac{V}{s} \\ -Ms & 0 \end{bmatrix} = -\frac{V}{s}(-Ms) = 12$$

$$I_1(s) = \frac{\frac{40s+40}{s}}{2{,}56s^2 + 16s + 12} = \frac{40s + 40}{s(2{,}56s^2 + 16s + 12)} = \frac{40 \quad (s+1)}{2{,}56\, s(s^2 + 6{,}25 s + 4{,}68)} = \frac{15{,}625\,(s+1)}{s(s+0{,}8)(s+5{,}3)}$$

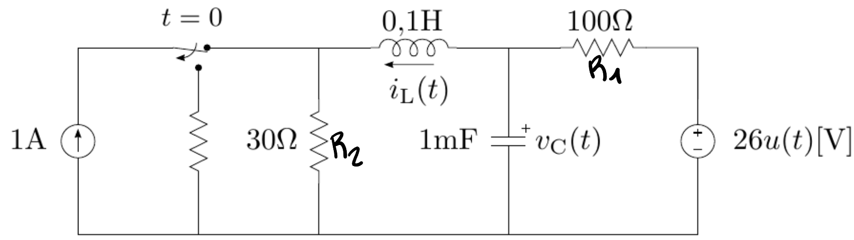
$$= \frac{3{,}3}{s} - \frac{0{,}86}{s+0{,}8} - \frac{2{,}81}{s+5{,}3} \xrightarrow{t} 3{,}3\,u(t) - 0{,}8\, e^{-0{,}8t} - 2{,}81\, e^{-5{,}3t}$$

$$I_2(s) = \frac{12}{2{,}56s^2+16s+12} = \frac{12}{2{,}56\,(s+0{,}8)(s+5{,}3)} = \frac{4{,}69}{(s+0{,}8)(s+5{,}3)} = \frac{1{,}04}{s+0{,}8} - \frac{1{,}04}{s+5{,}3}$$

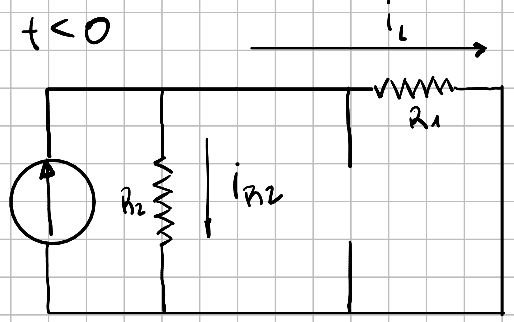
$$i_2(t) = 1{,}04\, e^{-0{,}8t} - 1{,}04\, e^{-5{,}3t}$$

Ejercicio 15.

Para el circuito de la figura 10 se pide encontrar $I_L(s)$ y $i_L(t)$ para $t > 0$.



$t<0$



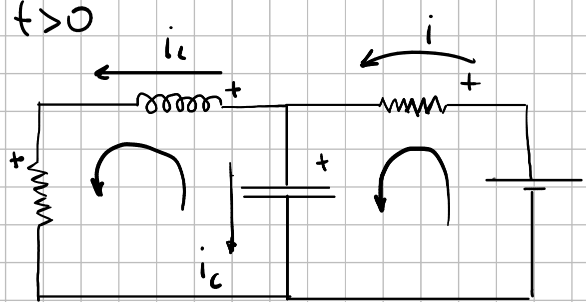
$$1A = i_{R2} + i_L \qquad V_{R2} = V_{R1}$$

$$1A = \frac{V_{R2}}{R2} + \frac{V_{R1}}{R1}$$

$$1A = V_{R2}\left(\frac{1}{R2}+\frac{1}{R1}\right) \longrightarrow V_{R2} = \frac{300}{13}\,V$$

$$1A = \frac{\frac{300}{13}}{30} + i_L \longrightarrow i_L(0) = \frac{3}{13}A$$

$t>0$



LKV:

① $V - V_{R1} - V_C = 0$

② $V_C - V_L - V_{R2} = 0$

LKI:

$i = i_C + i_L$

V-I

$V_{R1} = i\,R_1$

$V_{R2} = i_L R_2$

$V_L = L\frac{di_L}{dt}$

$i_C = C\frac{dV_C}{dt}$

① $V_C = V - V_{R1} = V - (i_C + i_L)R_1$

$$V_C = V - R_1 C\frac{dV_C}{dt} + i_L R_1$$

$$V_C - R_1 C\frac{dV_C}{dt} = V + i_L R_1$$

②

$$V_C = L\frac{di_L}{dt} + i_L R_2 \longrightarrow \frac{dV_C}{dt} = L\frac{d^2 i_L}{dt} + R_2\frac{di_L}{dt}$$

$$\left(L\frac{di_L}{dt} + i_L R_2\right) - R_1 C\left(L\frac{d^2 i_L}{dt} + R_2\frac{di_L}{dt}\right) - i_L R_1 = V \longrightarrow \mathcal{L}$$

$$\frac{di_L}{dt} = s I_L(s) - i_L(0)\,; \quad \frac{d^2 i_L}{dt} = s^2 I(s) - s\,i(0) - i(0)$$

$$L\left(Ls\,I_L(s) - L\,i_L(0)\right) + I_L(s)R_2 - R_1 C\left[L\left(L s^2 I(s) - L s\, i(0) - i(0)\right) + R_2\left(Ls\,I_L(s) - L\,i_L(0)\right)\right] - I_L R_1 = \frac{V}{s}$$

$$L^2 s\,I_L(s) - L^2 i_L(0) + I_L(s)R_2 - R_1 C L^2 s^2 I(s) - R_1 C L^2 s\, i(0) - R_1 C\, i(0) + R_2 L s\, I_L(s) - R_2 L\, i_L(0)$$
$$- I_L R_1 = \frac{V}{s}$$

$$I_L(s)\left[L^2 s + R_2 - R_1 C L^2 s^2 + R_2 L s - R_1\right] - L^2 i(0) - R_1 C L^2 s\, i(0) - R_1 C\, i(0) - R_2 L\, i_L(0) = \frac{V}{s}$$

$L=0{,}1$; $C = 1m$; $R_2 = 30$; $R_1 = 100$; $V = 26$; $i_L(0) = \frac{3}{13}$

$$I_L(s)\left[-1.10^{-3} s^2 + 3{,}01 s - 70\right] - \frac{3}{1300} - \frac{3}{13000}s - \frac{3}{130} - \frac{9}{13} = \frac{26}{s}$$

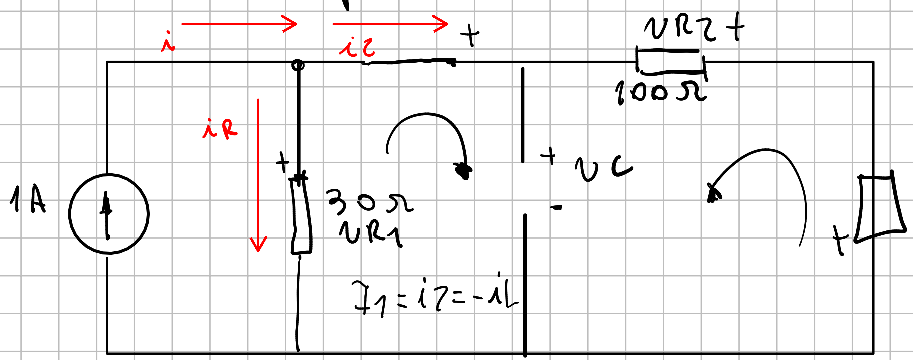
$$s\,I_L(s)\left[-1.10^{-3} s^2 + 3{,}01 s - 70\right] = 26 + \frac{3}{13000}s^2 + \frac{933}{1300}s$$

$$I_L(s) = \frac{\frac{3}{13000}s^2 + \frac{933}{1300}s + 26}{s\left(-1.10^{-3}s^2 + 3{,}01 s - 70\right)} = \frac{\frac{3}{13000}s^2 + \frac{933}{1300}s + 26}{s\,(s-2986)(s-23{,}43)}$$

$$I_L(s) = \frac{A}{s} + \frac{B}{s-2986} + \frac{C}{s-23{,}43} = \frac{-\frac{13}{35}}{s} + \frac{0{,}00047}{s-2986} - \frac{0{,}00061}{s-23{,}43}$$

Resolución rao, error en los reemplazos

Analizamos para $t<0$



$$\boxed{i = i_2 + i_R}$$

$$\boxed{\begin{array}{l} V_{R2} + V_C = 0 \rightarrow V_C = -V_{R2} \\ V_C - V_{R1} = 0 \rightarrow V_C = V_{R1} \end{array}}$$

$V_{R1} = R_1\, i_R$

$V_{R2} = -i_2 \cdot R_2 \rightarrow i_2 = -\frac{V_{R2}}{R_2}$

$i_2 = i - i_R$

$-\frac{V_{R2}}{R_2} + \frac{V_{R1}}{R_1} = i$

$\frac{V_C}{R_2} + \frac{V_C}{R_1} = i$

$\rightarrow V_C\left(\frac{13}{300}\right) = 1$

$$\boxed{V_C = \frac{300}{13}}$$

$i_2 = -\frac{V_{R2}}{R_2}$

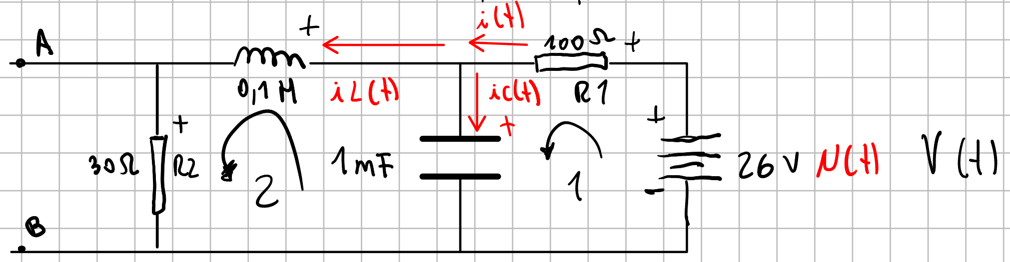
$\cancel{-}i_L = \cancel{-}\frac{V_{R2}}{R_2}$

$i_L = \frac{V_{R2}}{R_2}$

$\rightarrow i_L = -\frac{V_C}{R_2}$

$$\boxed{i_L = -0{,}2307}$$

para $t>0$ el circuito en el tiempo queda



LKV para $t>0$ (dominio t)

$\boxed{\text{malla 2} \quad -v_C(t) + v_L(t) + V_{R2}(t) = 0}$

$\boxed{\text{malla 1} \quad -V(t) + v_{R1} + v_C(t) = 0}$

$\boxed{\text{LKI} \quad i(t) = i_L + i_C}$

Rel tensión corriente

$V_{R2} = R_2 \cdot i_L(t)$

$V_{R1} = R_1 \cdot i(t)$

$i_C = C \dfrac{d v_C(t)}{dt}$

$v_L = L \dfrac{d i_L(t)}{dt}$

pasamos a laplace

Malla 2

$-V_C(S) + V_L(S) + V_{R2}(S) = 0$

Malla 1

$-V(S) + V_{R1}(S) + V_C(S) = 0$

LKI

$I(S) = I_C(S) + I_L(S)$

Rel tensión corriente

$V_{R2}(S) = R_2\, I_L(S)$

$I_C(S) = C\left(S V_C(S) + v_C(0)\right)$

$V_L(S) = L\left(S I_L(S) + i_L(0)\right)$

de la malla 2

$V_L(S) = V_C(S) - V_{R2}(S)$

$LS\, I_L(S) + L \cdot i_L(0) = V_C(S) - V_{R2}(S)$

$\bullet\ I_L(S) = \dfrac{V_C(S)}{LS} - \dfrac{V_{R2}(S)}{LS} - \dfrac{i_L(0)}{S}$

$\bullet\ V_C(S) = LS\left(I_L(S) + i_L(0)\right) + S\, V_{R2}(S)$

de la malla 1

$V_C(S) = V(S) - V_{R1}(S)$

$I_L(S) = \dfrac{V(S)}{LS} - \dfrac{V_{R1}(S)}{LS} - \dfrac{V_{R2}(S)}{LS} - \dfrac{i_L(0)}{S}$

$I_L(S) = \dfrac{26}{S^2 L} - \dfrac{R_1 I(S)}{LS} - \dfrac{R_2 I_L(S)}{LS} - \dfrac{i_L(0)}{S}$

$I_L(S) = \dfrac{26}{S^2 L} - \dfrac{R_1 I_L(S)}{LS} - \dfrac{R_1 I_C(S)}{LS} - \dfrac{R_2 I_L(S)}{LS} - i_L(0)$

$I_L(S) = \dfrac{26}{S^2 L} - \dfrac{R_1 I_L(S)}{LS} - \dfrac{R_1 C S V_C(S)}{LS} - \dfrac{R_1 C V_C(0)}{LS} - \dfrac{R_2 I_L(S)}{LS} - \dfrac{i(0)}{S}$

$I_L(S) = \dfrac{26}{S^2 L} - \dfrac{R_1 I_L(S)}{LS} - \dfrac{R_1 C \cancel{L} \cancel{S} I_L S}{\cancel{L} \cancel{S}} - \dfrac{R_1 C \cancel{L} \cancel{S} i_L(0)}{\cancel{L} \cancel{S}} - \dfrac{R_1 C S R_2 I_L(S)}{L \cancel{S}} - \dfrac{R_1 C V_C(0)}{LS} - \dfrac{R_2 I_L(S)}{LS} - \dfrac{i_L(0)}{S}$

$I_L(S) + \dfrac{R_1 I_L(S)}{LS} + R_1 C\, I_L(S) + \dfrac{R_1 C R_2 I_L(S)}{L} + \dfrac{R_2 I_L(S)}{LS} = \dfrac{26}{S^2 L} - R_1 C L\, i_L(0) - \dfrac{R_1 C V_C(0)}{LS} - \dfrac{i_L(0)}{S}$

$$IL(s)\left(1+\frac{R1}{Ls}+R1\cdot C+\frac{R1cR2}{L}+\frac{R2}{Ls}\right)=\frac{260}{s^2}+0{,}0006921-\frac{6{,}9230}{s}+\frac{0{,}2307}{s}$$

$$IL(s)\left(1+\frac{300}{s}+\frac{3}{100}+30+\frac{1000}{s}\right)=\frac{260}{s^2}+0{,}0006921-\frac{6{,}9230}{s}+\frac{0{,}2307}{s}$$

$$IL(s)\left(31{,}03+\frac{1300}{s}\right)=\frac{260}{s^2}-\frac{6{,}6923}{s}+0{,}0006921$$

$$IL(s)=\left(\frac{260}{s^2}-\frac{6{,}6923}{s}+0{,}0006921\right):\left(31{,}03+\frac{1300}{s}\right)$$

$$IL(s)=-\frac{6{,}6923}{s}\cdot\frac{1}{31{,}03}-$$

$$=\frac{8{,}3789}{s^2}+\frac{0{,}2}{s}$$

¡¡¡muchos errores

Rehacer