Ejercicio 24.

Para el circuito de la figura 17 de condiciones iniciales $i_L(0) = 1[A]$ y $v_C(0) = 1[V]$ se pide:

1. encontrar la respuesta completa de corriente $i(t)$ para $t > 0$ utilizando el modelo de circuito equivalente de Laplace,
2. decir que parte de la respuesta corresponde a la natural y cuál es la forzada.

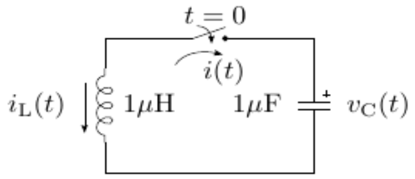
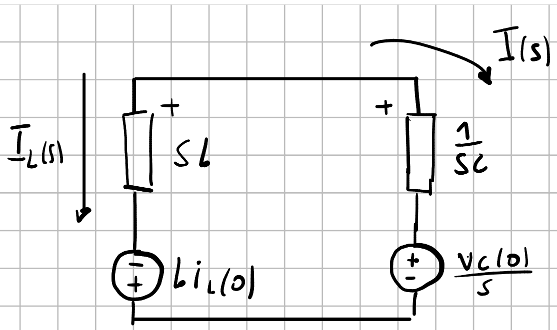


Figura 17: Equivalente de Laplace.



LKI

$$I_{L(s)} = -I_{(s)}$$

LKV

$$I_{(s)} \frac{1}{sC} + \frac{V_C(0)}{s} + L\, i_L(0) - I_L(s)\, sL = 0$$

$$I_{(s)}\left(\frac{1}{sC} + sL\right) = -\frac{V_C(0)}{s} - L\, i_L(0)$$

$$I_{(s)} = \frac{-\frac{1}{s} - 1.10^{-6}}{\frac{10^6}{s} + s\,1.10^{-6}}$$

$$= \frac{-1 - 10^{-6}s}{10^6 + 10^{-6}s^2}$$

$$= \frac{-10^6 - s}{s^2 + 10^{12}}$$

$S_1 = j1.10^6$ → la parte imaginaria $= \omega$

$S_2 = -j1.10^6$

$$= \frac{A}{s + j1.10^6} + \frac{B}{s - j1.10^6}$$

$$= A\frac{\omega}{s^2+\omega^2} + B\frac{s}{s^2+\omega^2}$$

$S = 0$

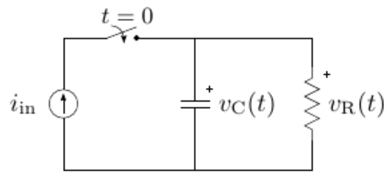
$$-10^{-6} = A\frac{\omega}{\omega^2} = \frac{A}{\omega} \longrightarrow A = -1$$

$S = -10^6$

$$0 = A\frac{\omega}{s^2+\omega^2} + B\frac{s}{s^2+\omega^2} \longrightarrow B = -1$$

$$I_{(s)} = -10^{12}\frac{\omega}{s^2+\omega^2} - 10^{12}\frac{s}{s^2+\omega^2}$$

$$i_{(t)} = -1\ \text{sen}(10^6 t) - 1\cos(10^6 t)$$



Datos

$i_{\text{in}} = 10\,\text{sen}(2\pi 50 t)[\text{A}]$

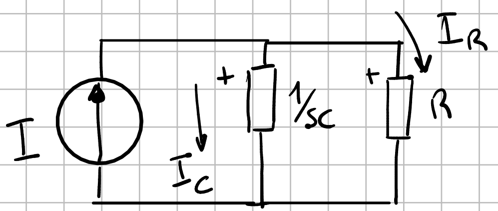
$C = 10000\mu\text{F}$

$R = 20\Omega$

Figura 5: Circuito RC paralelo.

Ejercicio 10.

En el circuito de la figura 5 encontrar la respuesta $v_C(t)$ para $t > 0$ utilizando la transformada de Laplace como herramienta. La tensión inicial sobre el capacitor es cero.



$$\left.\begin{aligned} I &= I_C + I_R \\ V_C &= I_R R \\ I_C &= C s V_C \end{aligned}\right\} \quad I = V_C\left(\frac{1}{R} + sC\right)$$

$$I = 10\,\frac{2\pi 50}{s^2 + (2\pi 50)^2}$$

$$V_C = \frac{10\,\dfrac{2\pi 50}{s^2+(2\pi 50)^2}}{\dfrac{1}{R} + sC}$$

$$V_C = \frac{(10\ 2\pi 50)\,100}{(s^2 + (2\pi 50)^2)(s+5)}$$

$$V_C = A\,\frac{2\pi 50}{s^2 + 2\pi 50^2} + B\,\frac{s}{s^2 + 2\pi 50^2} + C\,\frac{1}{s+5}$$

$$C = \lim_{s\to -5} (s+5)\,\frac{(10\ 2\pi 50)\,100}{(s^2+(2\pi 50)^2)(s+5)} = 3{,}18$$

$s = 0$

$$\frac{100000\pi}{(2\pi 50)^2\ 5} = A\,\frac{\cancel{2\pi 50}}{2\pi 50^2} + \frac{3{,}18}{5}$$

$A = 0{,}19$

$s = 1$

$$\frac{100000\pi}{(2\pi 50)^2 \; 6} = 0{,}19 \frac{2\pi 50}{1+2\pi 50^2} + B \frac{1}{1+2\pi 50^2} + \frac{3{,}18}{6}$$

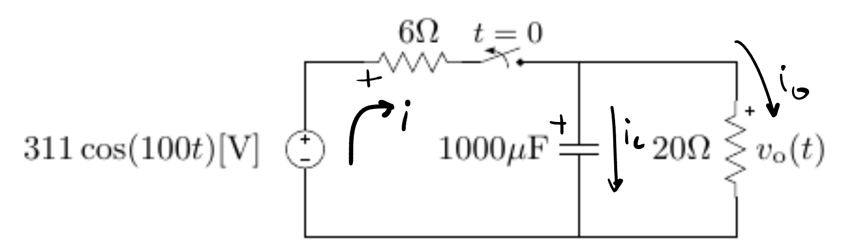
$$-8{,}83.10^{-5} = B \; 1{,}013.10^{-5}$$

$$B = -9$$

$$V_C = 0{,}19 \frac{2\pi 50}{S^2 + 2\pi 50^2} - 9 \frac{S}{S^2 + 2\pi 50^2} + \frac{3{,}18}{S+5}$$

Ejercicio 23.

Aplicando transformada de Laplace encontrar la tensión $v_o(t)$ indicada en el circuito de la figura 16.



$t<0$:

LKV

$$V - V_R - V_c = 0 \longrightarrow$$

LKI

$$i = i_c$$

V-I

$$V_R = i\,R$$

$$i_c = C\,\frac{dV_c}{dt}$$

$$V_c = V - V_R$$

$$V_c = V - i\,R$$

$$V_c = V - i_c R$$

$$V_c = V - RC\,\frac{dV_c}{dt}$$

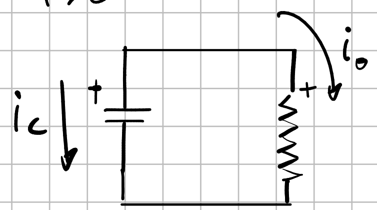
$$V_c + RC\,\frac{dV_c}{dt} = V$$

$$V_c = 153{,}65\,\sqrt{2}\,\cos(\omega t + 22{,}77^\circ)$$

$t=0$

$$V_c(0) = 197{,}3\ \text{V}$$

$t>0$



$$V_c = V_o$$

$$i_c + i_o = 0$$

$$i_c = C\,\frac{dV_c}{dt} = CS\,V_c(s) - C\,V_c(0)$$

$$i_o = \frac{V_o}{R_o} \longrightarrow I_{(o)} = \frac{V_{o(s)}}{R_o}$$

$$CS\,V_{o(s)} - C\,V_{c(0)} + \frac{V_{o(s)}}{R_o} = 0$$

$$V_{o(s)}\left(CS + \frac{1}{R_o}\right) = C\,V_{c(0)}$$

$$V_{o(s)} = \frac{C\,V_c(0)}{CS + \frac{1}{R_o}} = \frac{V_c(0)}{S + \frac{1}{R_o C}} = \frac{197{,}3}{S + 50}$$

$$V_{o(s)} = 197{,}3\ e^{-50t}$$

Ejercicio 25.

Aplicando la transformada de Laplace, determinar $i_1(t)$ y $i_2(t)$ del circuito de la figura 18 para $t > 0$, siendo $V = 10\text{V}$, $R1 = 3\Omega$, $R_2 = 4\Omega$, $L_1 = 1\text{H}$, $L_2 = 4\text{H}$ y $k = 0{,}6$.

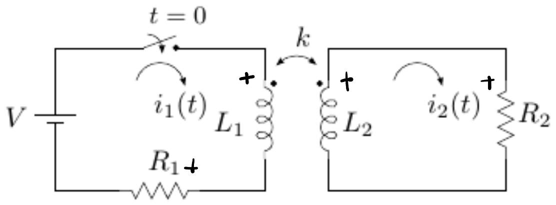


Figura 18: Circuito con acoplamiento inductivo.

$t > 0$

LKV

$$V = V_{L1} + V_{R1} \longrightarrow V = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt} + i_1 R_1$$

$$V_{L2} = V_{R2} \qquad 0 = L_2 \frac{di_2}{dt} - M\frac{di_1}{dt} + i_2 R_2$$

V-I

$$V_{R1} = i_1 R_1$$

$$V_{R2} = i_2 R_2$$

$$V_{L1} = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$V_{L2} = -L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}$$

$\downarrow \mathcal{L}$ $\qquad i_1(0) = i_2(0) = 0$

$$V = L_1 s I_1(s) - Ms\, I_2(s) + R_1 I_1(s)$$

$$0 = L_2 s I_2(s) - Ms\, I_1(s) + R_2 I_2(s)$$

$\downarrow$

$$V = I_1(s)(L_1 s + R_1) + I_2(s)(-Ms)$$

$$0 = I_1(s)(-Ms) + I_2(L_2 s + R_2)$$

$$\begin{bmatrix} V \\ 0 \end{bmatrix} = \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} \begin{bmatrix} L_1 s + R_1 & -Ms \\ -Ms & L_2 s + R_2 \end{bmatrix} \quad ; \quad I_1 = \frac{\Delta_1}{\Delta_p} \quad ; \quad I_2 = \frac{\Delta_2}{\Delta_p}$$

$$\Delta_p = (L_1 s + R_1)(L_2 s + R_2) - [(-Ms)(-Ms)] = L_1 L_2 s^2 + L_1 R_2 s + L_2 R_1 s + R_1 R_2 - M^2 s^2$$

$$\Delta_p = (L_1 L_2 - M^2)s^2 + (L_1 R_2 + L_2 R_1)s + R_1 R_2$$

$$L_1 = 1;\ L_2 = 4;\ R_1 = 3;\ R_2 = 4$$

$$M = k\sqrt{L_1 L_2} = 0{,}6\sqrt{1 \cdot 4} = 1{,}2$$

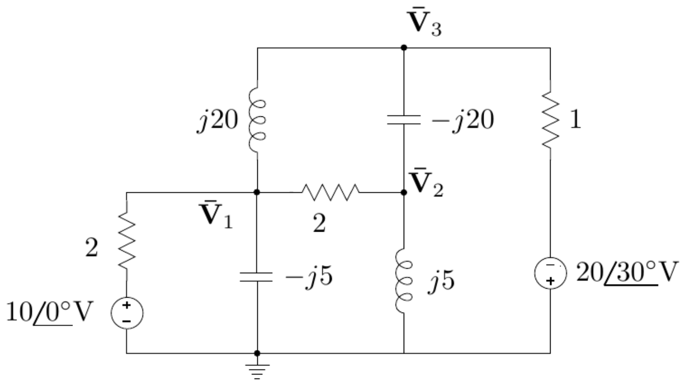
$\Delta_p = 2{,}56\,s^2 + 16s + 12$

$\Delta_1 =$

Guia 6:

Ejercicio 10.

En el circuito de la figura 8 se pide, aplicando el método de tensiones en los nudos, obtener la matriz de admitancia $[\mathbf{Y}]$ y el vector de corrientes $[\mathbf{I}]$, tal que $[\mathbf{Y}]\,[\mathbf{V}] = [\mathbf{I}]$, con $[\mathbf{V}] = [\bar{\mathbf{V}}_1, \bar{\mathbf{V}}_2, \bar{\mathbf{V}}_3]^T$ según las referencias.



$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$[Y] = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} + \frac{1}{j20} - \frac{1}{j5} & -\frac{1}{2} & -\frac{1}{j20} \\ -\frac{1}{2} & \frac{1}{2} + \frac{1}{j5} - \frac{1}{j20} & -\frac{1}{j20} \\ -\frac{1}{j20} & -\frac{1}{j20} & \frac{1}{j20} - \frac{1}{j20} + 1 \end{bmatrix}$$

$$[I] = \left[\begin{matrix} \frac{10}{2} \\ \\ \\ \end{matrix} \right.$$

```
octave:18> Y
Y =

   1.0000 + 0.1500i  -0.5000 +      0i        0 + 0.0500i
  -0.5000 +      0i   0.5000 - 0.1500i        0 - 0.0500i
        0 + 0.0500i        0 - 0.0500i   1.0000 - 0.1000i

octave:19> I
I =

    5.0000 +       0i
         0 +       0i
  -17.3205 - 10.0000i

octave:20> V=inv(Y)*I
V =

    9.5662 +  0.1996i
   10.2315 +  1.6457i
  -16.2338 - 11.5901i
```