Simulation of dynamic systems

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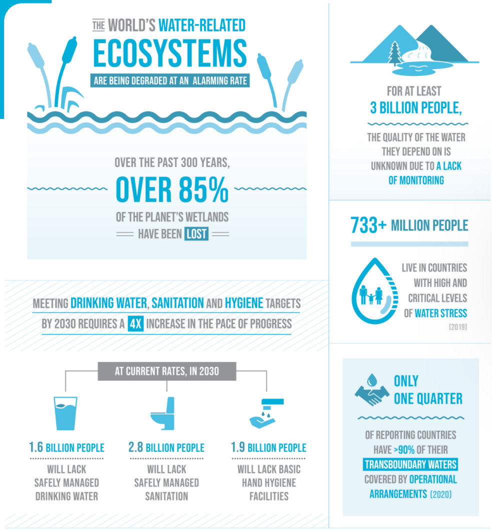


THE SUSTAINABLE DEVELOPMENT GOALS REPORT 2022: UNSTATS.UN.ORG/SDGS/REPORT/2022/

Brief recap

We did...

- Formulate the **system block representation** by establishing a viable definition for a function that algebraically relates a system's output to its input.
- This function will allow separation of the input, system and output into three separate and distinct parts.
- Use Laplace transform as a mapping from the time domain to the s-domain (or Laplace domain) and vice versa.

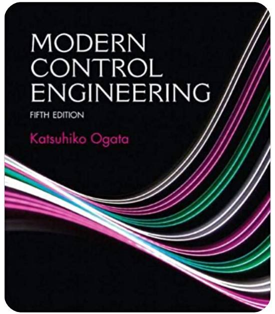
Goals

Today's lecture is about ...

- Find the transfer function for:
  - Linear, time-invariant **electrical** networks.
  - Linear, time-invariant translational **mechanical** systems.
- Linearise and simulate a nonlinear system.

Reading list

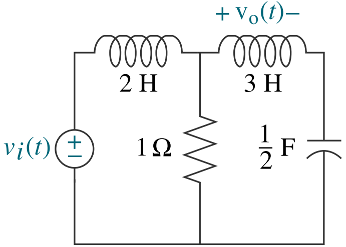
- Nise, *Control Systems Engineering* (6th Edition)[1]
- Ogata and Severo *Engenharia de Controle Moderno* (3rd Edition)



[1] Exercises from **Ch.2**

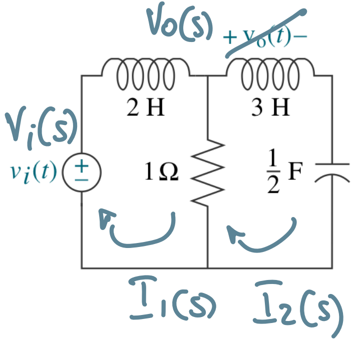
Electrical system

Example: Find the transfer function, $G(s) = V_o(s)/V_i(s)$ for the network:



1. Replace passive element values with their impedances.
2. Replace all sources and time variables with their Laplace transform.
3. Assume a transform current and a current direction in each mesh.
4. Write Kirchhoff's voltage law around each mesh.
5. Solve the simultaneous equations for the output.
6. Form the transfer function.

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$



Write the MESH EQUATIONS

$(2s+1)I_1(s) - I_2(s) = V_i(s)$

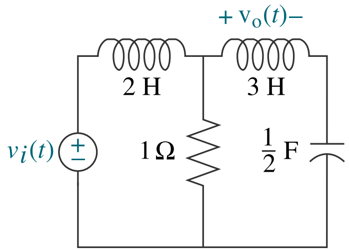
$-I_1(s) + 3\left(s + \frac{2}{s} + 1\right)I_2(s) = 0$

Solving for $I_2(s)$

$$\frac{\begin{vmatrix} 2s+1 & V_i(s) \\ -1 & 0 \end{vmatrix}}{\begin{vmatrix} 2s+1 & -1 \\ -1 & \frac{3s^2+s+2}{s} \end{vmatrix}} = \frac{sV_i(s)}{6s^2+5s+4s+2}$$

Electrical system

Example: Find the transfer function, $G(s) = V_o(s)/V_i(s)$ for the network:



Solving for $I_2(s) / V_i(s)$

$$\frac{I_2(s)}{V_i(s)} = \frac{s}{6s^3 + 5s^2 + 4s + 2}$$

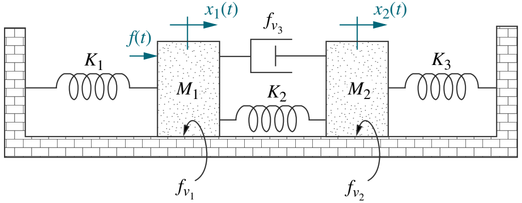
Noticing that $V_o(s) = I_2(s)3s$

We have

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{3s^2}{6s^3 + 5s^2 + 4s + 2}$$

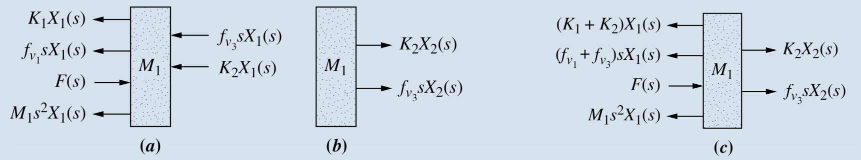
Mechanical system

Example: Find the transfer function, $G(s) = X_2(s)/F_i(s)$ for the system below:

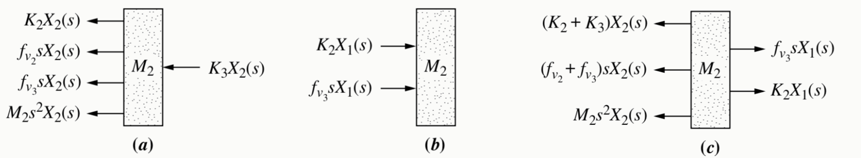


Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper	$f(t) = f_v v(t)$	$f(t) = f_v \frac{dx(t)}{dt}$	$f_v s$
Mass	$f(t) = M\frac{dv(t)}{dt}$	$f(t) = M\frac{d^2x(t)}{dt^2}$	Ms^2

M1



M2



$$[M_1s^2(f_{v_1} + f_{v_3})s + (K_1+K_2)]X_1(s) - (f_{v_3}s + K_2)X_2(s) = F(s)$$

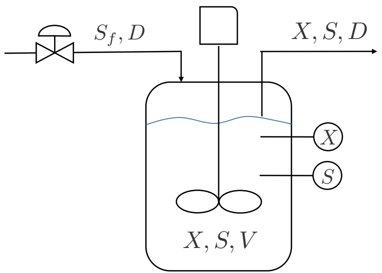
$$-(f_{v_3}s + K_2)X_1(s) + [M_2s^2 + (f_{v_2} + f_{v_3})s + (K_2 + K_3)]X_2(s) = 0$$

$$G(s) = \frac{X_2(s)}{F(s)} = \frac{f_{v_3}s + K_2}{\Delta}$$

$\Delta = \dots$

Biological system

Example: For the system shown below, write the nonlinear state-space model and compare it with a linearised one.



Assumptions :

- continuous CSTR
- constant volume
- substrate consumption ⇝ biomass
- Kinetics given by Monod
- isothermal operation.

S = Substrate [g/l]
X = Biomass [g/l]

$$\frac{dX}{dt} = \mu(S)X - DX \qquad \mu(S) = \frac{\mu_{max} S}{K_s + S}$$

$$\frac{dS}{dt} = D(S_f - S) - \frac{\mu(S)}{Y} X$$

1. Define/Verify the nonlinear model ⇝ SIMULATE
2. Find equilibrium points
3. Linearize → Evaluate the Jacobians
   → SIMULATE

COMPARE !!