Modelling dynamic systems in the Laplace domain

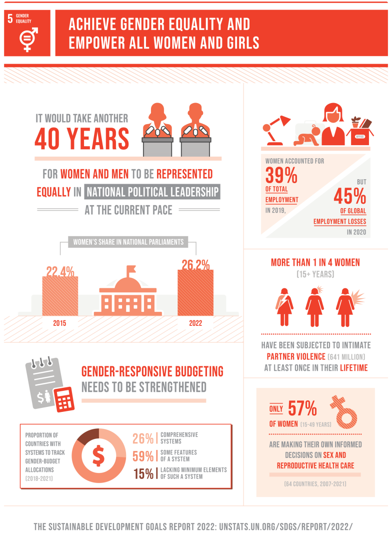
Transfer function examples

Michela Mulas

5 GENDER EQUALITY

EXTRA CLASS TOMORROW!!!

Topic: Simulation of dynamical systems



SALA 11 @ 14:00

Brief recap

We did...

- Formulate the **system block representation** by establishing a viable definition for a function that algebraically relates a system's output to its input.
- This function will allow separation of the input, system and output into three separate and distinct parts.
- Use Laplace transform as a mapping from the time domain to the s-domain (or Laplace domain) and vice versa.

Exercise L5E0a: Write the differential equation for the system shown below

$$R(s) \longrightarrow \boxed{\frac{s^5 + 2s^4 + 4s^3 + s^2 + 4}{s^6 + 7s^5 + 3s^4 + 2s^3 + s^2 + 5}} \longrightarrow C(s)$$

$$s = \sigma + j\omega$$

FOCUS on LINEAR and TIME INVARIANT SYSTEMS - LTI

$$\xrightarrow[R(s)]{\text{INPUT}} \boxed{G(s)} \xrightarrow[C(s)]{\text{OUTPUT}}$$

Nise's NOTATION on INPUT/OUTPUT variables

$$G(s) = \frac{C(s)}{R(s)} = \frac{s^5+2s^4+4s^3+s^2+4}{s^6+7s^5+3s^4+2s^3+s^2+5}$$

$$(s^6+7s^5+3s^4+2s^3+s^2+5)\,C(s) = (s^5+2s^4+4s^3+s^2+4)\,R(s)$$

$$\frac{d^6c(t)}{dt^6} + 7\frac{d^5c(t)}{dt^5} + 3\frac{d^4c(t)}{dt^4} + 2\frac{d^3c(t)}{dt^3} + \frac{d^2c(t)}{dt^2} + 5c(t) =$$

$$= \frac{d^5r(t)}{dt^5} + 2\frac{d^4r(t)}{dt^4} + 4\frac{d^3r(t)}{dt^3} + \frac{d^2r(t)}{dt} + 4r(t)$$

Brief recap

We did...

- Use the partial-fraction expansion and applied the concepts to the solution of differential equations.

  Exercise L5E0b: Find the inverse Laplace transform of the function $F(s)$ below

$$F(s) = \frac{2s^2 - 8}{s(s^2+4)(s+1)}$$

$$F(s) = \frac{2s^2-8}{s(s^2+4)(s+1)} = \frac{R_1}{s} + \frac{R}{s+j2} + \frac{R'}{s-j2} + \frac{R_2}{s+1}$$

① $\mathcal{L}^{-1}\left[\frac{R}{s-p} + \frac{R'}{s-p'}\right] = M e^{\alpha t}\cos(\omega t + \phi)$

Where $p, p' = \alpha + j\omega$ $\quad R, R' = u + jv$

$M = 2|R| = 2\sqrt{u^2+v^2}$

$\phi = \arg(R) = \arctan\left(\frac{v}{u}\right)$

② $\mathcal{L}^{-1}\left[\frac{R}{s-p} + \frac{R'}{s-p'}\right] = B e^{\alpha t}\cos\omega t + C e^{\alpha t}\sin\omega t$

Where $B = 2u \quad C = -2v$

Brief recap

We did...

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Exercise L5E0b: Find the inverse Laplace transform of the function $F(s)$ below

$$F(s) = \frac{2s^2-8}{s(s^2+4)(s+1)}$$

$$R_1 = s \cdot F(s)\Big|_{s\to 0} = \frac{2s^2-8}{(s^2+4)(s+1)}\Big|_{s\to 0} = \frac{-8}{4} = -2$$

$$R_2 = (s+1)F(s)\Big|_{s\to -1} = \frac{2s^2-8}{s(s^2+4)}\Big|_{s\to -1} = \frac{2-8}{-1(1+4)} = \frac{6}{5}$$

$$R = (s+j2)F(s)\Big|_{s\to -j2} = \frac{2s^2-8}{s(s-j2)(s+1)}\Big|_{s\to -j2} =$$

$$= \frac{2\times(-4)-8}{-j2(-j4)(-j2+1)} = \frac{-16}{-8+j16} = \frac{-2}{-1+j2} = \frac{2}{5} + j\frac{4}{5}$$

$$R' = (s-j2)F(s)\Big|_{s\to j2} = \frac{2}{5} - j\frac{4}{5}$$

Brief recap

We did...

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  Exercise L5E0b: Find the inverse Laplace transform of the function $F(s)$ below

$$F(s) = \frac{2s^2 - 8}{s(s^2+4)(s+1)}$$

The Inverse Laplace Transform by Partial Fraction Expansion

Intro | Inverse Laplace by PFE | Direct Calculation | MATLAB | Printable

Contents

Inverse Laplace Transform by Partial Fraction Expansion

This technique uses Partial Fraction Expansion to split up a complicated fraction into forms that are in the Laplace Transform table. As you read through this section, you may find it helpful to refer to the review section on partial fraction expansion techniques. The text below assumes you are familiar with that material.

https://lpsa.swarthmore.edu/LaplaceXform/InvLaplace/InvLaplaceXformPFE.html#Comments_on_the_two_methods

1. $M = 2|R| = 2\sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \frac{4}{5}\sqrt{5}$

$\phi = \arg(R) = \operatorname{atan}\left(\frac{4/5}{2/5}\right) = \operatorname{atan}(2) = 1.1$

$f(t) = -2 + \frac{4}{5}\sqrt{5}\left(e^{0t}\cos(2t + 1.1)\right) + \frac{6}{5}e^{-t} =$

$= -2 + \frac{4}{5}\sqrt{5}\cos(2t + 1.1) + \frac{6}{5}e^{-t}$

2. $B = 2u = 2 \cdot \frac{2}{5} = 4/5$

$C = -2v = -2 \cdot 4/5 = -8/5$

$f(t) = -2 + \frac{4}{5}\cos 2t - \frac{8}{5}\sin 2t + \frac{6}{5}e^{-t}$

Goals

Today's lecture is about ...

- Find the transfer function for:
  - Linear, time-invariant electrical networks.
  - Linear, time-invariant mechanical systems.

Reading list

- Nise, *Control Systems Engineering* (6th Edition)[1]
- Ogata and Severo *Engenharia de Controle Moderno* (3rd Edition)

[1] Today's lecture is mainly from **Ch.2** of Nise.
Same concepts can be found in Ogata, Ch.3

Examples of electrical systems

Electrical network transfer functions

Equivalent circuits for the electric networks that we work with first consist of three passive linear components: **resistors**, **capacitors**, and **inductors**.

The relationships between voltage and current and voltage and charge under zero initial conditions are summarised below.

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

The units are:

$v(t)$	$\rightsquigarrow$	V (volts)	$i(t)$	$\rightsquigarrow$	A (ampers)
$q(t)$	$\rightsquigarrow$	Q (coulombs)	C	$\rightsquigarrow$	F (farads)
R	$\rightsquigarrow$	Ω (ohms)	L	$\rightsquigarrow$	H (henries)

Electrical network transfer functions

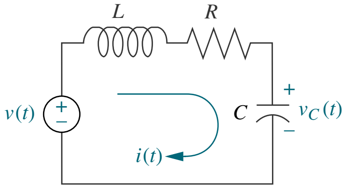
We now combine electrical components into circuits, decide on the input and output, and find the transfer function.

- Our guiding principles are the **Kirchhoff's laws**.
  - ⇝ Sum voltages around loops or currents at nodes
    Choose the technique that involves the least effort in algebraic manipulation
  - ⇝ Them, equate the result to zero.
- From these relationships we write the differential equations for the circuit.
- Then we can take the Laplace transforms of the differential equations.
  - ⇝ Finally solve for the transfer function.

Electrical network transfer functions

Simple circuits via mesh analysis

Transfer functions can be obtained using **Kirchhoff's voltage law** and summing voltages around loops or meshes. We call this method **loop or mesh analysis**.



Example: Find the transfer function relating

- The capacitor voltage, $V_C(s) \rightsquigarrow$ **output**
- The voltage, $V(s) \rightsquigarrow$ **input**

- We sum the voltages around the loop and assume zero initial conditions
- This yields the integro-differential equation for this network as:

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v(t)$$

- Changing variables from current to charge using $i(t) = dq(t)/dt$ yields

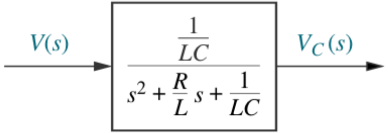
$$L\frac{d^2q(t)}{dt^2} + R\frac{dq(t)}{dt} + \frac{1}{C}q(t) = v(t)$$

- From the voltage-charge relationship for a capacitor in the table $q(t) = Cv_c(t)$:

$$LC\frac{d^2v_C(t)}{dt^2} + RC\frac{dv_C(t)}{dt} + v_c(t) = v(t)$$

- Taking the Laplace transform assuming zero initial conditions, rearranging terms, and simplifying yields:

$$\left(LCs^2 + RCs + 1\right)V_c(s) = V(s)$$



Solving for the transfer function, $V_C(s)/V(s)$:

$$\frac{V_c(s)}{V(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Electrical network transfer functions

Simple circuits via mesh analysis

We generalize and develop a technique for simplifying the solution for future problems:

1. Take the Laplace transform of the equations in the voltage-current assuming zero initial conditions.
   - For the **capacitor**: $V(s) = \frac{1}{Cs} I(s)$;
   - For the **resistor**: $V(s) = RI(s)$;
   - For the **inductor**: $V(s) = LsI(s)$;
2. Then, define the following transfer function: $\frac{V(s)}{I(s)} = Z(s)$
   - This function is similar to the definition of resistance.
   - This function is applicable to capacitors and inductors and carries information on the dynamic behaviour of the component.
   - We call this particular transfer function **impedance**.

Electrical network transfer functions

Simple circuits via mesh analysis

The concept of impedance simplifies the solution for the transfer function:

- The Laplace transform of the original differential equation, assuming zero initial conditions, is:

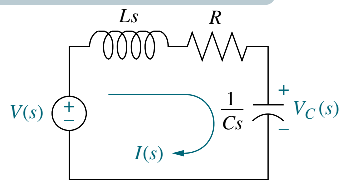
$$\left(Ls + R + \frac{1}{Cs}\right) I(s) = V(s)$$

- This is in the form:

$$\Big[\text{Sum of impedances}\Big] I(s) = \text{Sum of applied voltages}$$

The circuit can be modified to take into account the impedances.

We call this altered circuit the **transformed circuit**.



Rather than writing the differential equation first and then taking the Laplace transform, **we can draw the transformed circuit and obtain the Laplace transform** of the differential equation simply by applying Kirchhoff's voltage law to the transformed circuit.

Summarising:

1. Redraw the original network showing all time variables, such as $v(t), i(t)$, and $v_C(t)$, as Laplace transforms $V(s), I(s)$ and $V_C(s)$, respectively.
2. Replace the component values with their impedance values.
   - This replacement is similar to the case of dc circuits, where we represent resistors with their resistance values.

Electrical network transfer functions

Complex circuits via mesh analysis

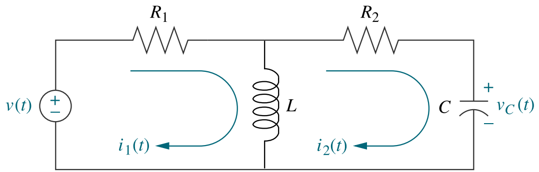
To solve complex electrical networks - those with multiple loops and nodes - using mesh analysis, we can perform the following steps:

1. Replace passive element values with their impedances.
2. Replace all sources and time variables with their Laplace transform.
3. Assume a transform current and a current direction in each mesh.
4. Write Kirchhoff's voltage law around each mesh.
5. Solve the simultaneous equations for the output.
6. Form the transfer function.

Electrical network transfer functions

Complex circuits via mesh analysis

Example: Given the following network, find the transfer function $I_2(s)/V(s)$.

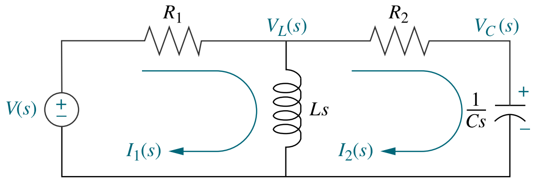


Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
Capacitor	$v(t) = \frac{1}{C}\int_0^1 i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^1 v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Electrical network transfer functions

Complex circuits via mesh analysis

The circuit requires **two simultaneous equations** to solve for the transfer function that can be found by summing voltages around each mesh through which the assumed currents, $I_1(s)$ and $I_2(s)$, flow.



- **Mesh 1**: $R_1 I_1(s) + LsI_1(s) - LsI_2(s) = V(s)$
- **Mesh 2**: $LsI_2(s) + R_2 I_2(s) + \frac{1}{Cs} I_2(s) - LsI_1(s) = 0$

Combining the two equations, we have:

$$(R_1 + Ls)I_1(s) - LsI_2(s) = V(s)$$

$$-LsI_1(s) + \left(Ls + R_2 + \frac{1}{Cs}\right) I_2(s) = 0$$

CRAMER RULE for I_2

$$\frac{\begin{vmatrix} R_1 + Ls & V \\ -Ls & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + Ls & -Ls \\ -Ls & Ls + R_2 + \frac{1}{Cs} \end{vmatrix}} = \frac{LsV}{(R_1 + Ls)\left(Ls + R_2 + \frac{1}{Cs}\right) + L^2 s^2}$$

$$= \frac{LCs^2 V}{(R_1 + R_2)LCs^2 + (R_1 R_2 C + L)s + R} = I_2$$

Electrical network transfer functions
Complex circuits via mesh analysis

Forming the transfer function, $G(s)$, yields

$$G(s) = \frac{I_2(s)}{V(s)} = \frac{LCs^2}{(R_1 + R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

$$V(s) \longrightarrow \boxed{\frac{LCs^2}{(R_1 + R_2)LCs^2 + (R_1R_2C + L)s + R_1}} \longrightarrow I_2(s)$$

In general, we have:

$+$	Sum of impedances around Mesh 1	$I_1(s)$	$-$	Sum of impedances common to the two meshes	$I_2(s) =$	Sum of applied voltages around Mesh 1
$-$	Sum of impedances common to the two meshes	$I_1(s)$	$+$	Sum of impedances around Mesh 2	$I_2(s) =$	Sum of applied voltages around Mesh 2

Electrical network transfer functions

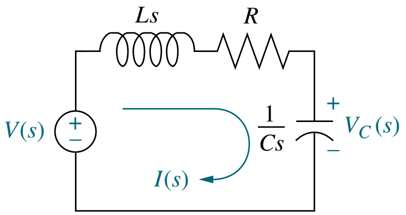
Simple circuits via nodal analysis

Transfer functions also can be obtained using Kirchhoff's current law and summing currents flowing from nodes. We call this method **nodal analysis**.

- The transfer function can be obtained by summing currents flowing out of the node whose voltage is $V_C(s)$.

- We assume that currents leaving the node are positive and currents entering the node are negative.
- The currents consist of the current through the capacitor and the current flowing through the series resistor and inductor

$$I(s) = \frac{V(s)}{Z(s)}$$



- $V_C(s)/(1/Cs)$ is the current flowing out of the node through the capacitor.
- $[V_C(s) - V(s)]/(R + Ls)$ is the current flowing out of the node through the series resistor and inductor.

Electrical network transfer functions

Complex circuits via nodal analysis

Often, the easiest way to find the transfer function is to **use nodal analysis rather than mesh analysis**.

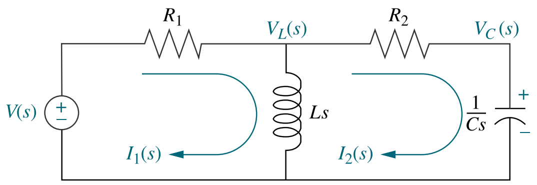
- The number of simultaneous differential equations that must be written is equal to the number of nodes whose voltage is unknown.
- Previously, we wrote simultaneous mesh equations using Kirchhoff's voltage law. For multiple nodes we **use Kirchhoff's current law** and sum currents flowing from each node.
- As a convention
  - Currents flowing from the node are assumed to be positive
  - Currents flowing into the node are assumed to be negative.
- We define the **admittance** as the reciprocal of impedance:

$$Y(s) = \frac{1}{Z(s)} = \frac{I(s)}{V(s)}$$

Electrical network transfer functions

Complex circuits via nodal analysis

Example: We sum currents at the nodes around the mesh.



The sum of currents flowing from the nodes marked $V_L(s)$ and $V_C(s)$ are:

$$\frac{V_L(s)-V(s)}{R_1}+\frac{V_L(s)}{Ls}+\frac{V_L(s)-V_C(s)}{R_2}=0$$

$$CsV_C(s)+\frac{V_C(s)-V_L(s)}{R_2}=0$$

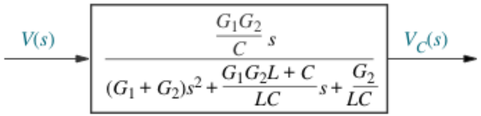
Rearranging and expressing the resistances as conductances, $G_1=1/R_1$ and $G_2=1/R_2$, we obtain:

$$\left(G_1+G_2+\frac{1}{Ls}\right)V_L(s)-G_2V_C(s)=V(s)G_1$$

$$-G_2V_L(s)+(G_2+Cs)V_C(s)=0$$

Solving for the transfer function, $V_C(s)/V(s)$, yields

$$\frac{V_C(s)}{V(s)}=\frac{\frac{G_1G_2}{C}s}{(G_1+G_2)s^2+\frac{G_1G_2L+C}{LC}s+\frac{G_2}{LC}}$$



Electrical network transfer functions

Complex circuits via nodal analysis

Alternative way: **replace voltage sources by current sources**.

- A voltage source presents a constant voltage to any load; conversely, a current source delivers a constant current to any load.
- Practically, a current source can be constructed from a voltage source by placing a large resistance in series with the voltage source.
- Thus, variations in the load do not appreciably change the current, because the current is determined approximately by the large series resistor and the voltage source.

Theoretically, we rely on **Norton's theorem**:

- A voltage source, $V(s)$, in series with an impedance, $Z_s(s)$, can be replaced by a current source, $I(s) = V(s)/Z(s)$, in parallel with $Z_s(s)$.

Electrical network transfer functions

Complex circuits via nodal analysis

To handle multiple-node electrical networks, we can perform the following steps:

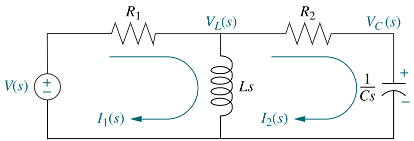
1. Replace passive element values with their admittances.
2. Replace all sources and time variables with their Laplace transform.
3. Replace transformed voltage sources with transformed current sources.
4. Write Kirchhoff's current law at each node.
5. Solve the simultaneous equations for the output.
6. Form the transfer function.

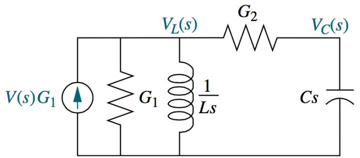
Electrical network transfer functions

Complex circuits via nodal analysis

Example: Find the transfer function $V_C(s)/V(s)$, using nodal analysis and transformed circuit with current.

- Convert all impedances to admittances and all voltage sources in series with an impedance to current sources in parallel with an admittance using Norton's theorem.





Summing the currents at the node $V_L(s)$ and using $I(s) = Y(s)V(s)$:

$$G_1 V_L(s) + \frac{1}{Ls} V_L(s) + G_2 \Big[V_L(s) - V_C(s) \Big] = V(s) G_1$$

Summing the currents at the node $V_C(s)$, we get:

$$C(s) V_C(s) + G_2 \Big[V_C(s) - V_L(s) \Big] = 0$$

Combining terms, these equations become simultaneous equations in $V_C(s)$ and $V_L(s)$, which lead to the previous solution.

Electrical network transfer functions
Complex circuits via nodal analysis

In general, we have:

$$
\begin{array}{cccccc}
+ & \begin{bmatrix}\text{Sum of admittances}\\ \text{connected to Node 1}\end{bmatrix} & V_L(s) & - & \begin{bmatrix}\text{Sum of admittances}\\ \text{common to the two nodes}\end{bmatrix} & V_C(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{currents at Node 1}\end{bmatrix} \\
- & \begin{bmatrix}\text{Sum of admittances}\\ \text{common to the two nodes}\end{bmatrix} & V_L(s) & + & \begin{bmatrix}\text{Sum of admittances}\\ \text{connected to Node 2}\end{bmatrix} & I_2(s) = \begin{bmatrix}\text{Sum of applied}\\ \text{currents at Node 2}\end{bmatrix}
\end{array}
$$

Electrical network transfer functions

A problem solving technique

In all of the previous examples, we have seen a repeating pattern in the equations that we can use to our advantage.

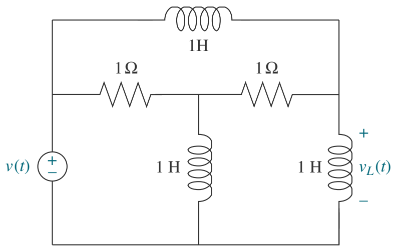
- If we recognize this pattern, we do not need to write the equations component by component.
- We can sum impedances around a mesh in the case of mesh equations or sum admittances at a node in the case of node equations.

Electrical network transfer functions

Exercise L5E2: For the given circuit, find the transfer function:

$$G(s) = V_L(s)/V(s)$$

1. Solve the problem two ways: mesh analysis and nodal analysis.
2. Show that the two methods yield the same result.



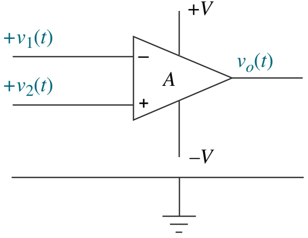
ToDo!!

Electrical network transfer functions
Operational amplifiers

An operational amplifier is an electronic amplifier used as a basic building block to implement transfer functions.

It has the following characteristics:

1. Differential input, $v_2(t) - v_1(t)$
2. High input impedance, $Z_1 = \infty$ (ideal)
3. Low output impedance, $Z_o = 0$ (ideal)
4. High constant gain amplification $A = \infty$ (ideal)



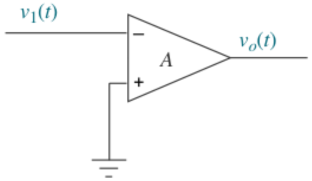
The output, $v_o(t)$, is given by:

$$v_o(t) = A\Big(v_2(t) - v_1(t)\Big)$$

If $v_2(t)$ is grounded, the amplifier is called **inverting operational amplifier**.

We have:

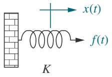
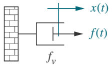
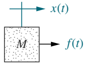
$$v_o(t) = -Av_1(t)$$



Examples of mechanical systems

Translational mechanical system transfer functions

Mechanical systems, as electrical networks, can be modelled by a transfer function, $G(s)$, that algebraically relates the Laplace transform of the output to the Laplace transform of the input.

Component	Force-velocity	Force-displacement	Impedence $Z_M(s) = F(s)/X(s)$
Spring 	$f(t) = K\int_0^t v(\tau)d\tau$	$f(t) = Kx(t)$	K
Viscous damper 	$f(t) = f_v v(t)$	$f(t) = f_v \dfrac{dx(t)}{dt}$	$f_v s$
Mass 	$f(t) = M\dfrac{dv(t)}{dt}$	$f(t) = M\dfrac{d^2x(t)}{dt^2}$	Ms^2

K,f_v and M are called **spring constant**, **coefficient of viscous friction**, and **mass**, respectively.

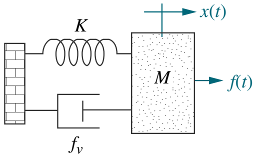
Translational mechanical system transfer functions

Mechanical systems, like electrical networks, have three passive, linear components.

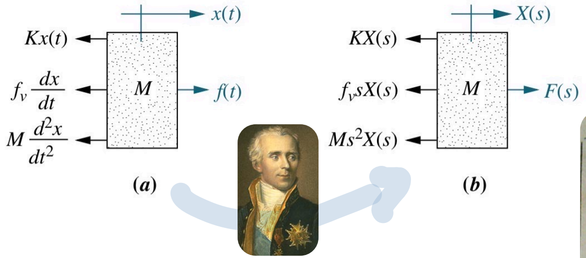
- Two of them, the **spring** and the **mass**, are energy-storage elements; one of them, the **viscous damper**, dissipates energy.
- The two energy-storage elements are analogous to the two electrical energy-storage elements, the inductor and capacitor.
- The energy dissipator is analogous to electrical resistance.

Example: Find the transfer function, $X(s)/F(s)$, for the system below:

1. Draw the FREE-BODY DIAGRAM
2. Write the DIFFERENTIAL EQUATIONS
3. Convert to LAPLACE domain



$$G(s) = \frac{X(s)}{F(s)}$$

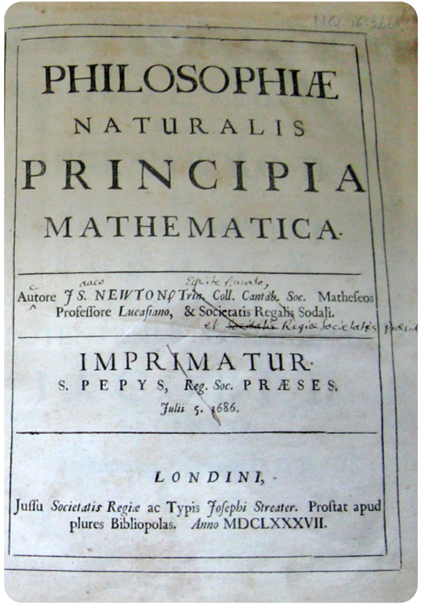


$$M\frac{d^2x(t)}{dt^2} + f_v\frac{dx(t)}{dt} + Kx(t) = f(t)$$

$$Ms^2X(s) + f_v sX(s) + KX(s) = F(s)$$

$$(Ms^2 + f_v s + K)X(s) = F(s)$$

$$G(s) = \frac{1}{Ms^2 + f_v s + K}$$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S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

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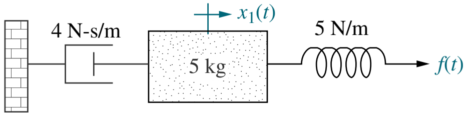
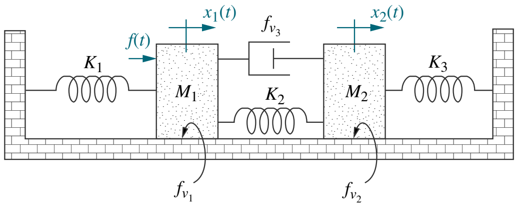
Translational mechanical system transfer functions

Exercise L5E3: Find the transfer function, $G(s) = X_1(s)/F(s)$, for the system below:



Exercise L5E4: Find the transfer function, $X_2(s)/F(s)$, for the system below:



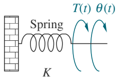
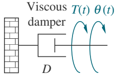
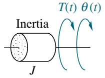
ToDo!!

(Check Nise)

Rotational mechanical system transfer functions

Rotational mechanical systems can be handled the same way as translational mechanical systems, except that torque replaces force and angular displacement replaces translational displacement.

The mechanical components for rotational systems are the same as those for translational systems, except that the components undergo rotation instead of translation.

Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\dfrac{d\theta(t)}{dt}$	Ds
	$T(t) = J\dfrac{d\omega(t)}{dt}$	$T(t) = J\dfrac{d^2\theta(t)}{dt^2}$	Js^2

Example: Write, but do not solve, the Laplace transform of the equations of motion for the system shown below: