Lista de Exercícios - Transformada Z

1)

a) $y(K) = \left(\frac{1}{2}\right)^K U(K) + \left(\frac{1}{4}\right)^K U(K)$

i) $\downarrow Z$

$$\boxed{Y(Z) = \underset{\text{I}}{\frac{Z}{Z-\frac{1}{2}}} + \underset{\text{II}}{\frac{Z}{Z-\frac{1}{4}}}}$$

ii) I : ROC: $|Z| > |1/2|$
II : ROC: $|Z| > |1/4|$ } $\boxed{|Z| > 1/2}$

b) $y(K) = \left[(1)^{K-1} + (-1)^{K-1}\right] U(K-1)$

i) $\downarrow Z$

$$\boxed{Y(Z) = \underset{\text{I}}{\frac{1}{Z-1}} + \underset{\text{II}}{\frac{1}{Z+1}}}$$

ii) I : ROC: $|Z| > |1|$
II : ROC: $|Z| > |-1|$ } $\boxed{|Z| > 1}$

c) $y(k) = a^k u(k) + b^k u(k-1)$

i) $= \left[a^k u(k)\right] - \left[-b^k u(k-1)\right]$

$\downarrow Z$

$$\boxed{Y(z) = \frac{z}{z-a} + \frac{z}{z-b}}$$

ii) I: ROC : $|z| > |a|$

II: ROC : $|z| < |b|$

$$\boxed{|a| < |z| < |b|}$$

2

a) $x(k) = 2 \cdot (0{,}8)^k u(k)$

$\downarrow Z$

$$\boxed{X(z) = 2 \cdot \frac{z}{z-0{,}8}}$$

b) $x(k) = 0{,}5^k u(k)$

$\downarrow Z$

$$\boxed{X(z) = \frac{z}{z-0{,}5}}$$

c) $X(k) = [0, 1, 2, 3, 4]$

$\downarrow Z$

$$\boxed{X(z) = 1 \cdot z^{-1} + 2 \cdot z^{-2} + 3 \cdot z^{-3} + 4 z^{-4}}$$

d) $X(k) = u(k) - u(k-4)$

$\downarrow Z$

$$X(z) = \frac{z}{z-1} - \frac{z \cdot z^{-4}}{z-1}$$

$$\boxed{X(z) = \frac{z - z^{-3}}{z-1}}$$

③

a) $$X(z) = \frac{(z+0.8)(z-1)}{(z+0.2)(z-0.5)} = \frac{z^2 - z\cdot 0.2 - 0.8}{z^2 - z\cdot 0.3 - 0.1}$$

i)

$$= \frac{z^2 - 0{,}3\cdot z - 0{,}1}{z^2 - 0{,}3\cdot z - 0{,}1} + \frac{-0{,}2\cdot z - 0{,}8 + 0{,}3\cdot z + 0{,}1}{z^2 - 0{,}3\cdot z - 0{,}1}$$

ii)

$$= 1 + \underbrace{\frac{0{,}1\cdot z - 0{,}7}{(z+0{,}2)(z-0{,}5)}}$$

$$\frac{A}{z+0{,}2} + \frac{B}{z-0{,}5} = \frac{A(z-0{,}5) + B(z+0{,}2)}{\ldots} = \frac{0{,}1\cdot z - 0{,}7}{\ldots}$$

iii)

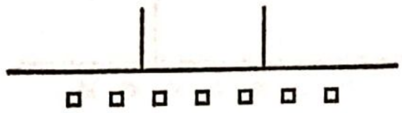
$$\left.\begin{array}{l} z(A+B) + \;=\; 0{,}1\cdot z + \\ -(0{,}5 A - 0{,}2 B) \;=\; -0{,}7 \end{array}\right\} \begin{array}{l} A = 36/35 \\ B = -13/14 \end{array}$$

iv)

$$X(z) = 1 + \frac{36}{35}\cdot\frac{1}{z+0{,}2} - \frac{13}{14}\cdot\frac{1}{z-0{,}5}$$

$\downarrow Z^{-1}$

$$\boxed{x(k) = \delta(k) + \left[\frac{36}{35}\cdot\left(\frac{-1}{5}\right)^{k-1} - \frac{13}{14}\left(\frac{1}{2}\right)^{k-1}\right] u(k-1)}$$



b) $X(z) = \dfrac{z^3+z+1}{(z-1)(z^2-0,5z+0,25)} = \dfrac{z^3+z+1}{z^3-1,5z^2-0,25z-0,25}$

i)

$$= 1 + \frac{(z+1)+1,5z^2+0,25\cdot z+0,25}{z^3-1,5z^2-0,25z-0,25}$$

$$= 1 + \frac{1,5\cdot z^2+1,25\cdot z+1,25}{(z-1)\underbrace{(z^2-0,5z+0,25)}}$$

$$\Delta = 0,25-4\cdot 1\cdot 0,25$$

$$\Delta = -0,75 = -3/4$$

$$r_{+,-} = \frac{\frac{1}{2} \pm j\frac{\sqrt{3}}{2}}{2} = \underbrace{\frac{1}{2}\cdot e^{j\pi/3}}_{\frac{1}{4}+j\frac{\sqrt{3}}{4}} \text{ e } \underbrace{\frac{1}{2}e^{-j\pi/3}}_{\frac{1}{4}-j\frac{\sqrt{3}}{4}}$$

$$\frac{A}{z-1} + \frac{B}{z-r_+} + \frac{C}{z-r_-} =$$

$$= A(z-r_+)(z-r_-) + B(z-1)(z-r_-) + C(z-1)(z-r_+)$$

$$= A(z^2 - 0,5z+0,25) + B(z^2-(1+r_-)z\, r_-) + C(z^2-(1+r_+)z + r_+)$$

$$1,5\cdot z^2 + 1,25\cdot z + 1,25 = z^2(A+B+C) + z\left[0,5A - B(1+r_-) - C(1+r_+)\right] + 0,25\cdot A + B\cdot r_- + C\cdot r_+$$

obs:

$$1,25 + j\cdot 0 = \left[\frac{A}{2} - B - C - B\cdot\left(\frac{5}{4} - j\frac{\sqrt{3}}{4}\right) - C\left(\frac{5}{4} + j\frac{\sqrt{3}}{4}\right)\right]$$

$$\frac{5}{4} = \left[\frac{A}{2} - \frac{9B}{4} - \frac{9C}{4} + j\underbrace{\left(\frac{B\sqrt{3}}{4} - \frac{C\sqrt{3}}{4}\right)}_{B=C}\right]$$

$$\frac{5}{4} = \frac{2A}{4} - \frac{18B}{4}$$

$$5 = 2A - 18B$$

$$1,5 = A + 2B$$

$A = 37/22$

$B = C = -1/11$

$$X(z) = 1 + \frac{37}{22} \cdot \frac{1}{z-1} - \frac{1}{11} \cdot \left(\frac{1}{z - r_+} + \frac{1}{z - r_-} \right)$$

z^{-1}

$$\boxed{x(k) = \delta(k) + \left\{ \frac{37}{22} \cdot (1)^{k-1} - \frac{1}{11} \left[\left(\frac{1}{2} e^{j\pi/3} \right)^{k-1} + \left(\frac{1}{2} e^{-j\pi/3} \right)^{k-1} \right] \right\} u(k-1)}$$

c) $$X(z) = \frac{(z+0{,}8)(z^2-1)}{(z+0{,}2)(z-0{,}5)^2} = \frac{z^3 + 0{,}8 \cdot z^2 - z - 0{,}8}{z^3 - 0{,}8 \cdot z^2 + 0{,}05 \cdot z + 0{,}05}$$

$$= 1 + \frac{(0{,}8 \cdot z^2 - z - 0{,}8) + 0{,}8 \cdot z^2 - 0{,}05 \cdot z - 0{,}05}{\ldots}$$

$$X(z) = 1 + \frac{1{,}6 z^2 - 1{,}05 \cdot z - 0{,}85}{(z+0{,}2)(z-0{,}5)^2}$$

$$\frac{A}{z+0,2}+\frac{B}{z-0,5}+\frac{C}{(z-0,5)^2}=$$

$$A(z-0,5)^2+B(z+0,2)(z-0,5)+C(z+0,2)=$$
$$=1,6z^2-1,05z-0,85$$

$$\begin{aligned} A(z^2-z+0,25)+ & \quad 1,6z^2+\\ B(z^2-0,3\cdot z-0,1)+ & = -1,05z+\\ C(\quad z+0,2) & \quad -0,85z \end{aligned}$$

$$\left.\begin{aligned} z^2(A+B)+ & \quad 1,6\cdot z^2+\\ -z(A+0,3\cdot B-1)+ & = -1,05\cdot z+\\ -(-A0,25+0,1\cdot B-0,2\cdot C) & \quad -0,85\cdot z \end{aligned}\right\} \begin{aligned} A&=-288/245\\ B&=136/49\\ C&=-39/28 \end{aligned}$$

$$X(z)=1-\frac{288}{245}\cdot\frac{1}{z+0,2}+\frac{136}{49}\cdot\frac{1}{z-0,5}-\frac{39}{28}\cdot\frac{1}{(z-0,5)^2}$$

z^{-1}

$$\boxed{x(k)=\delta(k)-\frac{288}{245}\cdot\left(\frac{-1}{5}\right)^{k-1}u(k-1)+\frac{136}{49}\cdot\left(\frac{1}{2}\right)^{k-1}u(k-1)-\frac{39}{28}\cdot\left(\frac{1}{2}\right)^{k-2}\cdot(k-1)\,u(k-1)}$$

4 T.V.F: $X(\infty) = \lim_{z \to 1} \left[\frac{z-1}{z} \cdot X(z) \right]$

a) $\circ\, x(k \to \infty) = 0$

$\circ \lim_{z \to 1} \left[\frac{z-1}{z} \cdot X(z) \right] = \lim_{z \to 1} \left[\frac{1}{z} \cdot \frac{(z+0{,}8)(z-1)^2}{(z+0{,}2)(z-0{,}5)} \right] = 0$

b) $\circ\, X(k \to \infty) = 37/22$

$\circ \lim_{z \to 1} \left[\frac{z-1}{z} X(z) \right] = \lim_{z \to 1} \left[\frac{z^2 + 1 + 1/z}{z^2 - 0{,}5 \cdot z + 0{,}25} \right] = 4$

$\hookrightarrow$ Os valores diferentes ocorrem devido à presença de um polo em $z=1$ que contribui p/ diferença.

c) $\circ\, X(k \to \infty) = 0$

$\circ \lim_{z \to 1} \left[\frac{z-1}{z} X(z) \right] = \lim_{z \to 1} \left[\frac{(z^2 + 0{,}8z - 1 - 0{,}8/z)(z-1)}{z^3 - 0{,}8z + 0{,}05 \cdot z + 0{,}05} \right] = 0$

⑤ $y(k) + 3\,y(k-1) + 2\,y(k-2) = 2u(k) - u(k-1)$

$y(-1) = 0$ e $y(-2) = 1$ $\quad\xrightarrow{Z}$

i) $$Y(z) + 3\left[\cancel{y(-1)}^{\,0} + z^{-1}Y(z)\right] + 2\left[y(-2) + \cancel{y(-1)}^{\,0} z^{-1} + z^{-2}Y(z)\right] =$$

$$= 2\cdot\frac{z}{z-1} - \frac{\cancel{z\cdot z^{-1}}\ 1}{z-1}$$

ii) $$Y(z)\left[1 + 3\cdot z^{-1} + 2\cdot z^{-2}\right] = \frac{2\cancel{z}-1}{z-1} - \frac{2(\cancel{z}-1)}{z-1} = \frac{1}{z-1}$$

iii) $$Y(z) = \frac{1}{z-1}\cdot\frac{1}{1+\frac{3}{z}+\frac{2}{z^2}} \Rightarrow Y(z) = \frac{z^2}{(z-1)(z+1)(z+2)}$$

iv) $$\frac{A}{z-1} + \frac{B}{z+1} + \frac{C}{z+2} = \frac{z^2}{(z-1)(z+1)(z+2)}$$

$$\begin{aligned} &A(z^2+3z+2) + \\ &B(z^2+1z-2) + \\ &C(z^2 \quad -1) \end{aligned} = z^2 \quad \therefore \quad \begin{aligned} &z^2(A+B+C) + \\ &z(3A+B) + \\ &(2A-2B-C) \end{aligned} = \begin{aligned} &z^2\cdot 1 + \\ &0 + \\ &0 \end{aligned}$$

$$\begin{cases} A + B + C = 1 \\ 3A + B = 0 \\ 2A - 2B - C = 0 \end{cases} \Longrightarrow \boxed{\begin{aligned} A &= 1/6 \\ B &= -1/2 \\ C &= 4/3 \end{aligned}}$$

v) $$Y(z) = \frac{1/6}{z-1} + \frac{(-1/2)}{z+1} + \frac{(4/3)}{z+2}$$

$\xrightarrow{Z^{-1}}$

$$\boxed{y(k) = \frac{1}{6}(1)^{k-1}u(k-1) - \frac{1}{2}(-1)^{k-1}u(k-1) + \frac{4}{3}(-2)^{k-1}u(k-1)}$$

6

a) $y(k) + 0{,}5\, y(k-1) = 2x(k)$

$$\Big\downarrow Z$$

i)

$$Y(z) + 0{,}5\left[\overset{0}{y(-1)} + z^{-1}Y(z)\right] = 2X(z)$$

$$Y(z)\left[1 + \frac{z^{-1}}{2}\right] = 2X(z)$$

ii)

$$Y(z)/X(z) = \frac{2}{1 + \frac{1}{2z}} \quad \therefore \quad \boxed{G(z) = \frac{2z}{z+0{,}5}}$$

b) $y(k) + y(k-1) + 0{,}25\, y(k-2) = x(k) - x(k-1)$

$$Z\Big\downarrow$$

i)

$$Y(z) + \left[\overset{0}{y(-1)} + z^{-1}Y(z)\right] + \frac{1}{4}\left[\overset{0}{y(-2)} + \overset{0}{y(-1)}z^{-1} + z^{-2}Y(z)\right] =$$

$$= X(z) + \left[\overset{0}{x(-1)} + z^{-1}X(z)\right]$$

$$Y(z)\left[1 + z^{-1} + \frac{z^{-2}}{4}\right] = X(z)\left[1 + z^{-1}\right]$$

ii)

$$Y(z)/X(z) = \frac{\frac{z+1}{\cancel{z}}}{\frac{4z^2+4z+1}{z^2}} \quad \therefore \quad \boxed{G(z) = \frac{z^2+z}{(2z+1)^2}}$$

c) $y(k) + y(k-2) = 2x(k) - x(k-1)$

$\downarrow z$

i)

$$Y(z) + z^{-2} \cdot Y(z) = 2X(z) - z^{-1}X(z)$$

$$Y(z)\left[1 + z^{-2}\right] = X(z)\left[2 - z^{-1}\right]$$

ii)

$$Y(z)/X(z) = \frac{\frac{2z-1}{z}}{\frac{z^2+1}{z^2}} \quad \therefore \quad \boxed{G(z) = \frac{2z^2 - z}{z^2+1}}$$

d) $y(k) = x(k) - 2x(k-1) + x(k-2)$

$\downarrow z$

$$Y(z) = X(z) - 2z^{-1}X(z) + z^{-2}X(z)$$

$$= X(z)\left[1 - 2z^{-1} + z^{-2}\right]$$

ii)

$$Y(z)/X(z) = 1 - \frac{2}{z} + \frac{1}{z^2} \quad \therefore \quad \boxed{G(z) = \left(\frac{z-1}{z}\right)^2}$$

e) $y(k) + 2y(k-1) - y(k-2) = 2x(k) - x(k-1) + 2x(k-2)$

$\downarrow z$

i)

$$Y(z)\left[1 + 2z^{-1} - z^{-2}\right] = X(z)\left[2 - z^{-1} + 2z^{-2}\right]$$

ii) $$Y(z)/X(z) = \frac{2z^2 - z + 2}{z^2 + 2z - 1} \quad \therefore \quad \boxed{G(z) = \frac{2z^2 - z + 2}{z^2 + 2z - 1}}$$

⑦ Supondo CAUSAL

a) $G(z) = \dfrac{z-0{,}5}{z+0{,}75}$ ⟶ Zeros: 0,5 ; ⟶ Polos: −0,75

Polos: $\frac{3}{4}\angle 180°$ ⇒ ROC: $|z| > \frac{3}{4}$ inclui o círculo unitário.

$\boxed{G(z) \text{ é estável}}$ ✓

b) $G(z) = \dfrac{z^2-0{,}25}{\underbrace{z^2+z+4{,}25}}$ ⟶ Zeros: −0,5; 0,5 ; ⟶ Polos: $\frac{-1}{2}+\frac{4j}{2}$; $\frac{-1}{2}-\frac{4j}{2}$

$\Delta = 1 - 4\cdot 1\cdot 4{,}25$

$\Delta = -16$

$z_{+,-} = \dfrac{-1\pm 4j}{2}$ ∴ $|z_{+,-}| = \dfrac{\sqrt{17}}{2}$ ⇒ ROC: $|z| > \dfrac{\sqrt{17}}{2} \cong 2{,}06$

n̄ inclui o círculo unitário

$\boxed{G(z) \text{ NÃO é estável}}$ ✓

c) $G(z) = \dfrac{z+2}{z^2+1}$ ⟶ zeros: −2 ; ⟶ Polos: $-j, +j$

ROC: $|z| > |j| = 1$ n̄ inclui o círculo unitário

$\boxed{G(z) \text{ NÃO é estável}}$ ✓

d) $G(z) = \dfrac{z-0{,}5}{z-0{,}75}$ → Zeros: 0,5 → Polos: 0,75

ROC: $|z| > 0{,}75$ inclui o círculo unitário

$\boxed{G(z) \text{ é estável}}$

e) $G(z) = \dfrac{z}{z^2 - 0{,}5z - 0{,}5}$ → Zeros: 0 → Polos: $\frac{1}{4} + j\frac{\sqrt{3}}{4}$; $\frac{1}{4} - j\frac{\sqrt{3}}{4}$

$\Delta = \frac{1}{4} - 4 \cdot 1 \cdot \frac{1}{4}$

$\Delta = -3/4$

$z_{+,-} = \dfrac{\frac{1}{2} \pm j\frac{\sqrt{3}}{2}}{2}$ ∴ $|z_{+,-}| = \frac{1}{2}$ ⇒ ROC: $|z| > \frac{1}{2}$

inclui o círculo unitário

$\boxed{G(z) \text{ é estável}}$

(B) Resposta ao degrau unitário:

P/ 7a: $Y(z) = G(z) \cdot X(z)$

i)

$$Y(z) = \frac{z-0,5}{z+0,75} \cdot \frac{z}{z-1} = \frac{z(z-0,5)}{(z+0,75)(z-1)}$$

ii)

$$\frac{z-0,5}{(z+0,75)(z-1)} = \frac{A}{z+0,75} + \frac{B}{z-1} = \frac{A(z-1)+B(z+0,75)}{(z+0,75)(z-1)}$$

$$z - 0,5 = z(A+B) - \left(A - \frac{3B}{4}\right)$$

iii)

$$\begin{cases} A+B=1 \\ A - \frac{3B}{4} = \frac{1}{2} \end{cases} \quad \boxed{\begin{array}{l} A = 5/7 \\ B = 2/7 \end{array}}$$

iv) $Y(z) = \frac{5}{7} \cdot \frac{z}{z+0,75} + \frac{2}{7} \cdot \frac{z}{z-1}$

z^{-1}

$$\boxed{Y(K) = \left[\frac{5}{7} \cdot \left(\frac{-3}{4}\right)^K + \frac{2}{7} \cdot (1)^K\right] U(K)}$$

P/7b: $Y(z) = G(z) \cdot X(z)$

i)

$$Y(z) = \underbrace{\frac{z^2 - 0{,}25}{z^2 + z + 4{,}25}}\cdot\frac{z}{z-1}$$

$p_+ = -0.5 + 2j$ $\quad p_+ = \frac{\sqrt{17}}{2} e^{j\alpha}$ $\quad , \quad \alpha = tg^{-1}(-4)$

$p_- = -0{,}5 - 2j$ $\quad p_- = \frac{\sqrt{17}}{2} e^{-j\alpha}$

ii)

$$\frac{z^2 - 0{,}25}{(z - p_+)(z - p_-)(z-1)} = \frac{A}{z - p_+} + \frac{B}{z - p_-} + \frac{C}{z-1}$$

$$= A(z - p_-)(z-1) + B(z - p_+)(z-1) + C(z^2 + z + 4{,}25)$$

$z^2 - 0{,}25 = A(z^2 - (1 + p_-)z + p_-) +$
$B(z^2 - (1 + p_+)z + p_+) +$
$C(z^2 + z + 4{,}25)$

iii)

$z^2 \cdot 1 \quad = z^2(A + B + C)$ ⊛

$z \cdot 0 \quad z[-A(1 + p_-) - B(1 + p_+) + C]$

$0{,}25 \quad p_- \cdot A + p_+ \cdot B + 4{,}25C$

dos: $0 = [-A(0{,}5 - 2j) - B(0{,}5 + 2j) + C]$

$$0 + 0j = \left[\left(-\frac{A}{2} - \frac{B}{2} + C\right) + j(2A - 2B)\right]$$

$A = B = C \qquad A = B$

⊛ $\Rightarrow \boxed{A = B = C = 1/3}$

iv)

$$Y(z) = \frac{1}{3}\left[\frac{z}{z - p_+} + \frac{z}{z - p_-} + \frac{z}{z-1}\right]$$

$z^{-1} \downarrow$

$$\boxed{y(k) = \frac{1}{3}\left[\left(\frac{\sqrt{17}}{2} e^{j\alpha}\right)^k + \left(\frac{\sqrt{17}}{2} e^{-j\alpha}\right)^k + (1)^k\right] U(k)}$$

[redacted]

p/ 7e: $Y(z) = G(z) \cdot X(z)$

i)

$$Y(z) = \underbrace{\frac{z}{z^2 - 0.5z - 0.5}} \cdot \frac{z}{z-1}$$

ii) $r_+ = 0.5\, e^{j\pi/3} = 1/4 + j\sqrt{3}/4$

$r_- = 0.5\, e^{-j\pi/3} = 1/4 - j\sqrt{3}/4$

iii)

$$\frac{z^2}{(z-r_+)(z-r_-)(z-1)} = \frac{A}{z-r_+} + \frac{B}{z-r_-} + \frac{C}{z-1}$$

$$z^2 = A(z-r_-)(z-1) + B(z-r_+)(z-1) + C(z^2 - 0.5z - 0.5)$$

$$\begin{array}{lcl} z^2 & & A(z^2 - (1+r_-)z + r_-) + \\ 0 \cdot z & \equiv & B(z^2 - (1+r_+)z + r_+) + \\ 0 & & C(z^2 - 0.5z - 0.5) \end{array}$$

$$\begin{array}{lcl} z^2 \cdot 1 & & z^2(A+B+C) \circledast \\ 0 \cdot z & \equiv & z\left[-A(1+r_-) - B(1+r_+) - 0.5C\right] \\ 0 & & A \cdot r_- + B r_+ - 0.5C \end{array}$$

iii)

obs: $0 = \left(-A - B - \frac{C}{2}\right) - A\left(\frac{1}{4} + \frac{j\sqrt{3}}{4}\right) - B\left(\frac{1}{4} - \frac{j\sqrt{3}}{4}\right)$

$$0 = \left(-\frac{5A}{4} - \frac{5B}{4} - \frac{2C}{4}\right) \underbrace{- \frac{Aj\sqrt{3}}{4} + \frac{Bj\sqrt{3}}{4}}$$

$-10A = 2C$ $\quad A = B$

$\boxed{C = -5A}$

$\circledast$ $A + A - 5A = 1 \therefore$ $\boxed{\begin{array}{l} A = B = -1/3 \\ C = 5/3 \end{array}}$

iv)

$$Y(z) = -\frac{1}{3} \cdot \left[\frac{1}{z-r_+} + \frac{1}{z-r_-}\right] + \frac{5}{3} \cdot \frac{1}{z-1}$$

z^{-1}

$$y(k) = \left[-\frac{1}{3}\left((0.5 e^{j\pi/3})^{k-1} + (0.5 e^{-j\pi/3})^{k-1}\right) + \frac{5}{3} \cdot (1)^{k-1}\right] U(k-1)$$

(9)

1º: $y(k) = x(k) - x(k-1) + x(k-2)$

a) $\downarrow Z$

$$Y(z) = X(z)\left[1 - z^{-1} + z^{-2}\right] \Rightarrow \boxed{G(z) = 1 - z^{-1} + z^{-2}}$$

$$\cancel{G(z) = \frac{z^2 - z + 1}{z^2}}$$

b) $Y(z) = G(z) \cdot X(z)$

$$Y(z) = \cancel{}\, 1 - z^{-1} + z^{-2} \xrightarrow{Z^{-1}} \boxed{y(k) = \delta(k) - \delta(k-1) + \delta(k-2)}$$

c) $G(z) = \dfrac{z^2 - z + 1}{z^2} \longrightarrow$ Polos: 0,0 $\therefore$ ROC: $|z| > 0$ inclui círculo unitário $\Rightarrow$

$\Rightarrow$ Estável

d) $Y(z) = \left(1 - z^{-1} + z^{-2}\right) \dfrac{z}{z-3}$

$$Y(z) = \frac{z}{z-3} - \frac{1}{z-3} + \frac{z^{-1}}{z-3}$$

$$\boxed{y(k) = (3)^k \left[u(k) - u(k-1) + u(k-2)\right]}$$

2º

a) $y(K) - 3y(K-1) + 2y(K-2) = X(K)$

$\downarrow Z$

$$Y(z)\left[1 - 3z^{-1} + 2z^{-2}\right] = X(z)$$

$$\boxed{G(z) = \frac{z^2}{z^2 - 3z + 2}}$$

→ Zeros: 0, 0

→ Polos: 1, 2

b) $Y(z) = G(z) \cdot X(z)$

$$Y(z) = \frac{z^2}{(z-1)(z-2)} = z \cdot \frac{z}{(z-1)(z-2)} = z \cdot \left[\frac{A}{z-1} + \frac{B}{z-2}\right]$$

$$= z \cdot \left[\frac{A(z-2) + B(z-1)}{(z-1)(z-2)}\right]$$

$$\begin{matrix} z(A+B) \\ -(2A+B) \end{matrix} = \begin{matrix} 1 \cdot z + \\ 0 \end{matrix} \qquad \boxed{\begin{matrix} A = -1 \\ B = 2 \end{matrix}}$$

$$Y(z) = -1 \cdot \frac{z}{z-1} + 2 \cdot \frac{z}{z-2}$$

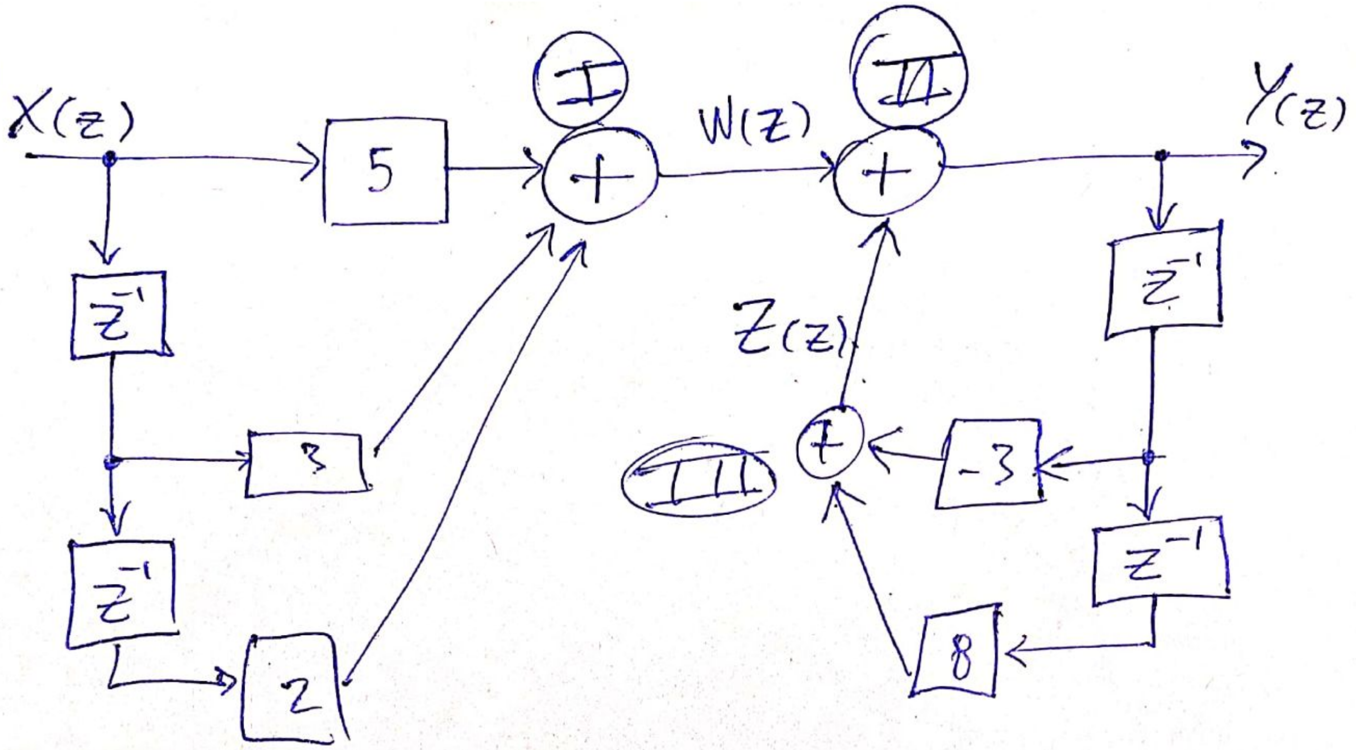
$\downarrow z^{-1}$

$$\boxed{y(K) = \left[-(1)^K + 2 \cdot (2)^K\right] U(K)}$$

c) $|z| > 2$ NÃO inclui o círculo unitário

Instável

10



i) $\boxed{W(z) = 5X(z) + 3z^{-1}X(z) + 2z^{-2}X(z)}$ I

ii) $\boxed{Z(z) = \left[-3z^{-1} + 8z^{-2}\right]Y(z)}$ III

iii) $\boxed{W(z) + Z(z) = Y(z)}$ II

De iii): $Y(z) - Z(z) = W(z)$ ∴

$$Y(z)\left[1 + 3z^{-1} - 8z^{-2}\right] = X(z)\left[5 + 3z^{-1} + 2z^{-2}\right]$$

$$G(z) = \frac{5 + \frac{3}{z} + \frac{2}{z^2}}{1 + \frac{3}{z} - \frac{8}{z^2}} \; z^2 \Rightarrow \boxed{G(z) = \frac{5z^2 + 3z + 2}{z^2 + 3z - 8}}$$