
LLM4EO: LARGE LANGUAGE MODEL FOR EVOLUTIONARY OPTIMIZATION IN FLEXIBLE JOB SHOP SCHEDULING

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ABSTRACT

Customized static operator design has enabled widespread application of Evolutionary Algorithms (EAs), but their search performance is transient during iterations and prone to degradation. Dynamic operators aim to address this but typically rely on predefined designs and localized parameter control during the search process, lacking adaptive optimization throughout evolution. To overcome these limitations, this work leverages Large Language Models (LLMs) to perceive evolutionary dynamics and enable operator-level meta-evolution. The proposed framework, LLMs for Evolutionary Optimization (LLM4EO), comprises three components: knowledge-transfer-based operator design, evolution perception and analysis, and adaptive operator evolution. Firstly, initialization of operators is performed by transferring the strengths of classical operators via LLMs. Then, search preferences and potential limitations of operators are analyzed by integrating fitness performance and evolutionary features, accompanied by corresponding suggestions for improvement. Upon stagnation of population evolution, gene selection priorities of operators are dynamically optimized via improvement prompting strategies. This approach achieves co-evolution of populations and operators in the search, introducing a novel paradigm for enhancing the efficiency and adaptability of EAs. Finally, a series of validations on multiple benchmarks of the flexible job shop scheduling problem demonstrate that LLM4EO accelerates population evolution and outperforms mainstream evolutionary programming and optimization algorithms.

Keywords Large language model · Operator evolution · Meta-evolution · Evolutionary optimization · Flexible job shop scheduling

1 Introduction

Differentiated design of operators makes evolutionary algorithms (EAs) widely applied in combinatorial optimization scenarios, such as production, logistics, and medicine. The traditional operator design relies on expert knowledge of problem structures to balance exploration and exploitation by adjusting the direction and intensity of the search. This search paradigm primarily adopts fixed operator structures and static parameter sets, making it difficult to adapt to dynamic changes in evolutionary states, such as neighborhood structures and fitness distributions. Consequently, the adaptability and generalization of search operators are constrained in complex and diverse scenarios. An intriguing question arises: how can evolutionary information be leveraged during the search process to achieve the self-adaptive evolution of operators?

Dynamic adjustment of the search paradigm is crucial for improving the adaptability of EAs and overcoming the limitations of static meta-search strategies. Existing research on the dynamic setting of operators focuses on parameter

control methods and evolutionary procedure induction algorithms. Although the parametric control methods influence the search strategy by adjusting initial parameters [1], local tuning is still constrained by the fixed solving paradigm of the operator. In addition, the evolutionary program induction algorithms [2], such as Genetic Programming (GP) and Genetic Evolutionary Programming (GEP), search for operators expressed by symbols in particular instances based on an evolutionary framework. However, the search procedure is inefficient due to the large number of iterations, and the quality of the operators is subject to validation sets. These limitations reveal two key issues: 1) the inability to perceive and utilize evolutionary information to guide operator evolution: static strategies struggle to meet optimization demands at different search stages; and 2) the lack of an operator-level self-evolutionary mechanism: algorithms depend on local parameter tuning or pre-search operator design, hindering deep, dynamic adaptive optimization.

To bridge these research gaps, we propose the Large Language Model for Evolutionary Optimization (LLM4EO) framework, which leverages the semantic capabilities of LLMs to perceive information about population evolution and drive the operator evolution. In contrast to methods that directly generate solutions or design rules, LLM4EO dynamically analyzes and improves operators in the search based on comprehension of population characteristics and evolutionary states. When population evolution is struggling, LLM4EO replaces the gene selection strategy of the operator in time to achieve operator-level meta-evolution, thus speeding up the convergence rate and enhancing the adaptability of EAs.

We design a LLM-driven meta-operator structure and implement the iterative co-evolution of solutions and operators. The core components are as follows: 1) operator design based on knowledge transfer, where the LLM leverages prior knowledge of problem structures and classic operators to generate a high-quality initial operator population; 2) perception and analysis of evolution, in which fitness indicators of solutions and operators are calculated to capture key evolutionary characteristics such as population distribution and convergence trends. Based on this information, the LLM analyzes the search preferences and potential limitations of the operator; and 3) operator self-adaptive evolution, where the gene selection strategy of the meta-operator is dynamically optimized by the LLM based on prompt-driven strategies for operator improvement. These three components are closely integrated through a structured prompt framework, forming a closed-loop optimization process of perception, analysis, and generation for the meta-operator in LLM4EO. Furthermore, the optimization potential of LLM4EO is validated on multiple benchmark datasets of the flexible job shop scheduling problem (FJSP). The results demonstrate that LLM4EO significantly outperforms mainstream methods in convergence speed and optimization performance by creatively generating operators suited to the current search stage when the algorithm falls into a local optimum. This work illustrates how to fully exploit the strengths of LLMs to dynamically improve operators during the search process, offering a new perspective on enhancing the performance of evolutionary algorithms and the adaptability of search strategies to diverse problems.

2 Related Work

2.1 Operator Control and Evolution.

Traditional operator design, constrained by fixed operator structures and static parameter configurations, struggles to cope with the dynamic nature of evolutionary search. Usually, some local parameters can be controlled according to a pre-defined adaptive strategies [3], such as crossover/mutation rates [4], mutation step sizes [5], strategy selection [6] and individual selection [7]. However, as the evolutionary progresses, the effectiveness of these heuristic-based adaptive strategies tends to decline due to changes in the optimization environment, such as population distribution. Although machine learning methods enable parameter self-adaptation by learning from historically successful experiences [8, 9, 10], these methods are inherently limited by the high cost of model training. To reduce human labor required, evolutionary program induction algorithms are proposed to construct dynamic scheduling strategies by automatic rule evolution [11, 12], such as Genetic Programming (GP) and Gene Expression Programming (GEP). Due to the lack of semantic awareness, rule structures are constrained by the limited search space of syntactic frameworks. Meanwhile, convergence speed depends on representation structures and fitness landscapes, while extensive iterations hinder algorithmic efficiency [13]. To overcome these challenges, our approach utilizes the LLM to dynamically assess the evolutionary state during the search process and intelligently generate tailored strategies adapted to different optimization stages.

2.2 Combination Optimization based on LLMs.

In recent years, LLMs have shown great potential in combinatorial optimization, primarily through two approaches: algorithm design and solution generation [14], as shown in Figure 1 (a)-(b). In the algorithm design paradigm, LLMs construct hybrid methods based on algorithmic components [15] or improve algorithms on the iterative optimization framework [16, 17, 18, 19]. However, due to limited global search capabilities and difficulties in handling complex constraints, the resulting heuristic algorithms often lack effectiveness and generalization. In solution generation, given

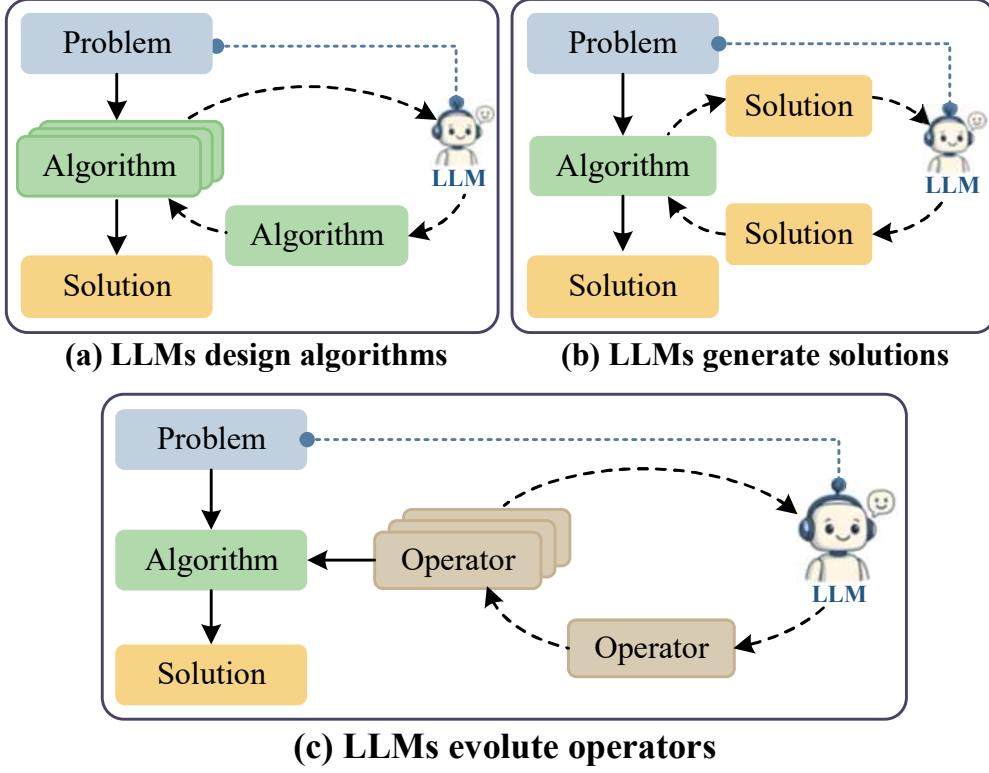


Figure 1: The primary approaches for leveraging LLMs to assist in solving optimization problems.

the optimization problems described in natural language, LLMs are capable of iteratively producing new solutions from historical solutions [20]. Moreover, as search operators, they perform evolutionary operations such as crossover and mutation [21]. However, in large-scale problems, these methods require significant computational resources, and their solution representations are often too long, preventing the generation of feasible solutions [22, 23]. In contrast, LLM4EO uses LLMs to evolve operators, thereby enhancing algorithmic performance and improving solution quality, as shown in Figure 1 (c).

3 Problem

The flexible job shop scheduling problem (FJSP) involves scheduling I jobs, where each job $i \in \{1, \dots, I\}$ consists of J_i operations that must be processed in a specific sequence. Each operation $O_{ij}(j = 1, \dots, J_i)$ can be performed on any machine m in a set of eligible machines M_{ij} , with processing time $T_{ijm} > 0$. The start and finish time of operation O_{ij} are $S_{ij} \geq 0$ and $F_{ij} > 0$, respectively. The FJSP aims to minimize the makespan C_{max} , and the formulations are as follows:

$$\min C_{max} = \min \left(\max_{j \in \{1, \dots, J_i\}} F_{ij} \right), \forall i \quad (1)$$

subject to:

$$\sum_{m \in M_{ij}} x_{ijm} = 1, \forall i, \forall j \quad (2)$$

$$\sum_{i \in \{1, \dots, I\}} \sum_{j \in \{1, \dots, J_i\}} x_{ijm}(t) \leq 1, \forall m, \forall t \in \mathbb{R}^+ \quad (3)$$

$$F_{ij} = S_{ij} + \sum_{m \in M_{ij}} x_{ijm} T_{ijm}, \forall i, \forall j \quad (4)$$

$$S_{ij+1} \geq F_{ij}, \forall i, \forall j \in \{1, \dots, J_i - 1\} \quad (5)$$

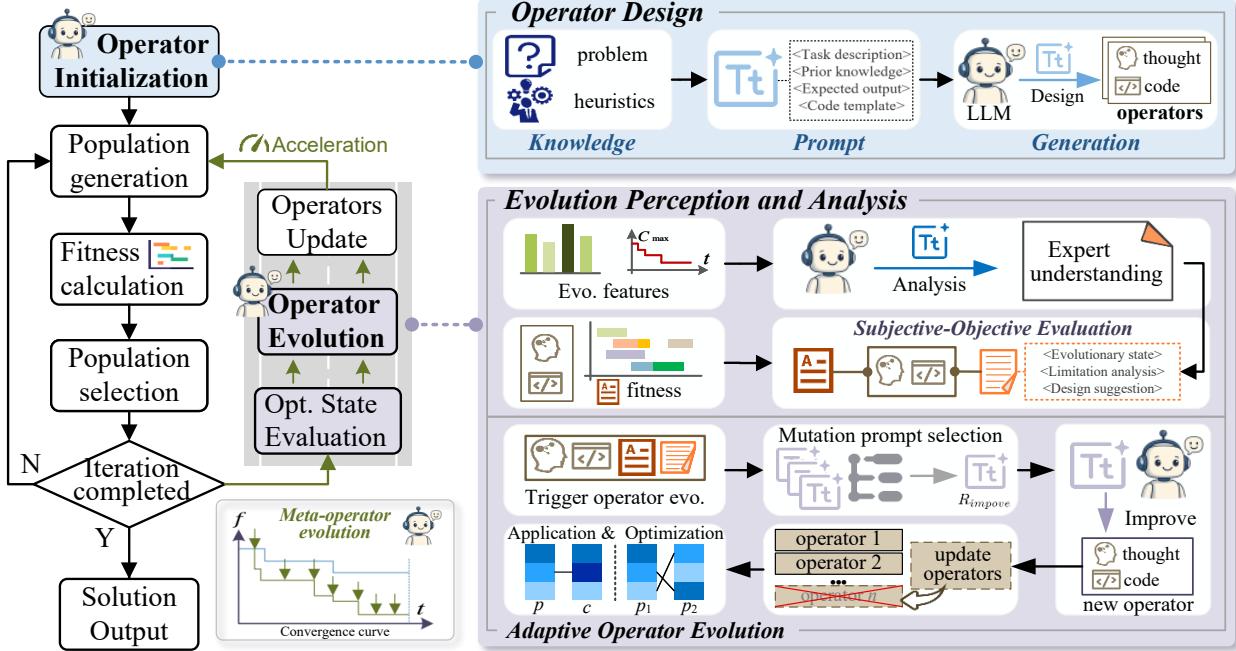


Figure 2: The framework of LLM4EO.

Eq.(1) represents the optimization objective. Eq.(2) ensures that each operation must be strictly assigned to a machine, where the binary variable $x_{ijm} = 1$ if operation O_{ij} is processed to machine m , and 0 otherwise. Eq.(3) denotes that the machine cannot process more than one operation at a time t . Eq.(4) shows the finish time of an operation. Eq.(5) determines that operations should be processed according to the given processing routes, where the j th operation must be finished before the $j + 1$ th operation starts.

4 LLM4EO

The proposed framework, LLMs for evolutionary optimization (LLM4EO), leverages the LLM to enhance the optimization performance of EAs, as shown in Figure 2. There are three core components: 1) knowledge-transfer-based operator design, 2) evolution perception and analysis, and 3) adaptive operator evolution. First, we use the LLM to design gene selection strategies of operators based on prior knowledge of the flexible job shop scheduling domain, then construct a high-quality initial operator population. During the search process, individuals with higher fitness are selected from both the solution population and the operator population to generate new solutions. To prevent premature convergence, we evaluate the optimization state based on the fitness changes of solutions and set a dynamic threshold to determine when to trigger the adaptive operator evolution mechanism. Before operator evolution, the LLM is employed to identify current optimization bottlenecks through evolutionary features, as well as to analyze the search preferences and limitations of operators. The resulting design suggestions guide the LLM to further generate promising new operators, thus accelerating the search process. LLM4EO facilitates the collaborative evolution of solutions and operators, providing an intelligently adaptive optimization approach for EAs.

4.1 Solution Representation and Initialization

4.1.1 Solution Representation.

In LLM4EO, Genetic Algorithm (GA) is adopted to solve the FJSP, where solutions are encoded into two parts: operation sequence vector (OSV) and machine assignment vector (MAV). Both vectors are arrays of integers, the length of which is the total number of operations. OSV denotes the sequence of operations and MAV represents the processing machine for each operation.

4.1.2 Solution Initialization.

We employ the initialization method proposed from [24], which combines two machine assignment rules and three operation dispatching rules to generate the initial population. Assignment rules: 1) Rule 1 allocates the operation corresponding to the global minimum in the processing time matrix to its corresponding machine. 2) Rule 2 involves randomly permuting the jobs and machines before applying the positioning method. Dispatching rules: 1) Rule 1 is the Random rule, where a job is randomly assigned to a machine for processing. 2) Rule 2 is the Most Work Remaining (MWR) rule, which prioritizes processing the jobs with the longest remaining processing time. 3) Rule 3 is the Most Operations Remaining (MOR) rule, which prioritizes processing the jobs with the greatest number of remaining operations.

Fitness reflects the quality of the solution, calculated as $f_{soln} = 1/C_{max}$, ensuring that shorter makespans correspond to higher fitness values. To speed up convergence, the tournament selection method is used to select individuals to generate solutions. After each iteration, the best individual in the current population is retained and directly enters the next generation.

4.2 Meta-operator

To reduce dependence on manual design and enhance evolutionary adaptability, we propose a meta-operator that employs an LLM-driven gene selection strategy to dynamically explore high-fitness regions of the search space while generating better solutions through neighborhood moves.

4.2.1 Neighborhood Moves.

In this work, four typical neighborhood moves are adopted as structural transformation operations on selected genes to generate new solutions.

- 1) OSV crossover: The Precedence Preserving Order-based crossover (POX) [25]. First, a job is selected from the first parent, and all its operations are copied to the first offspring. Then, the remaining operations are filled in according to the operation sequence of the second parent.
- 2) MAV crossover: Select some operations and exchange their machines between the two parents.
- 3) OSV mutation: The Precedence Preserving Shift mutation (PPS) [25]. The sequence of operations is changed by moving one of them to a different position, while satisfying the process path constraints.
- 4) MAV mutation: Select an alternative machine to replace the original machine assigned to an operation.
- 5) Critical operation swapping: The critical path is the longest path in the scheduling scheme. In each iteration, for every individual, two critical operations are selected for swapping. If a better solution is found, the process of swapping operations continues along the new critical path. Otherwise, the local search process is terminated.

4.2.2 Gene Selection.

The gene selection strategy determines both the perturbation range and search direction by identifying which genes on the chromosome should be modified through neighborhood moves to generate new solutions. A gene is selected when a uniformly distributed random number in $[0,1]$ exceeds its selection probability. Considering the characteristics of FJSP, we design a hierarchical gene selection method based on job and operation levels. At the job level, if the selection probability γ_i of a job i is less than the random number u_i , all genes corresponding to its operations are added to the gene set $\delta = \{J_i | \gamma_i > u_i, \forall i\}$. At the operation level, if the selection probability γ_{ij} of an operation O_{ij} is less than the random number u_{ij} , its gene will be selected into the gene set $\delta = \{O_{ij} | \gamma_{ij} > u_{ij}, \forall i, j\}$. During crossover, offspring c_1 and c_2 are generated by exchanging selected genes δ between parent individuals p_1 and p_2 , denoted as $c_1, c_2 = \text{crossover}(p_1, p_2, \delta)$. In mutation, a parent individual p modifies its selected genes δ to produce offspring $c = \text{mutation}(p, \delta)$.

The gene selection controls the size of the search neighborhood to be explored, which directly impacts solution quality and algorithm performance. However, random or single-heuristic approaches often lead to poor exploration and early convergence. To address this issue, we use the LLM to construct gene selection heuristics, enabling the algorithm to adaptively explore potential neighborhoods. Gene selection is formulated as a probability function $g(t, Q, \beta)$, where t denotes the iteration number, Q is the operator population, and β represents the genetic features. This function maps the hidden correlations among genes to their selection probabilities. To fully harness the knowledge embedded in LLM, we design seven genetic features at both the job and operation levels that capture the core properties of the FJSP and serve as effective prompts for guiding model inference, as shown in Table 1. Notably, *process span* denotes the time interval

Table 1: The proposed genetic features.

Type	Feature	Formulation
job	process span	$F_{iJ_i} - S_{i1}, \forall i$
	minimal process span	$\sum_{j \leq J_i} \min_{m \in M_{ij}} T_{ijm}, \forall i$
	operation number	$J_i, \forall i$
operation	start time	$S_{ij}, \forall i, j$
	earliest start time	$F_{ij-1}, j \in \{2, \dots, J_i\}, \forall i$
	processing time	$\sum_{m \in M_{ij}} x_{ijm} T_{ijm}, \forall i, j$
	machine number	$ M_{ij} , \forall i, j$

between starting the first operation and completing the last operation of a job. The *minimal process span* represents the sum of the shortest processing times for all operations of a job, while the *earliest start time* indicates the completion time of the preceding operation.

4.2.3 Fitness Evaluation.

To guide the evolution toward high-quality solutions, the fitness of each operator in the population is evaluated, and superior operators are preferentially selected using the roulette wheel method. The optimization success rate of an operator is defined as its fitness value $f_{op} = n_s/n_v$, where n_s denotes the number of times it successfully generates better solutions, and n_v is the total number of times it has been selected. A higher f_{op} reflects a stronger performance of the operator.

4.3 Collaborative Evolution Architecture

Transcending the limitations of static meta-heuristics in traditional EAs, we propose a collaborative evolutionary architecture that harnesses LLMs for experience transfer, perception, analysis, and generation. It facilitates adaptive operator evolution throughout the evolutionary process of solutions.

4.3.1 Knowledge-transfer-based Operator Design.

In initialization, a structured prompt is designed to activate prior knowledge of the LLM, enabling the generation of the high-quality initial operator population. This initial prompt includes a task description, expected output, code template, and special hints.

- 1) Task description: First, we need to describe the target problem and requirements, such as “For the flexible job shop scheduling problem aiming to minimize makespan (total completion time), design an algorithm to compute priority values for jobs requiring sequence adjustments and operations needing machine reassignment.”
- 2) Prior knowledge: To enhance the quality of initial operators, we inform the LLM that effective classical heuristics can be used as references, such as the Shortest Processing Time (SPT) rule. In addition, the brief descriptions of the genetic features defined in Table 1 help the LLM use its prior knowledge of the problem to generate gene selection heuristics.
- 3) Expected output: To avoid long and noisy responses, the LLM must summarize the thought of the algorithm in one sentence within {} and output a structured Python function using the given template.
- 4) Code template: The name, input and output of the code block are defined, with detailed explanations of the meaning and type of each parameter, to ensure the accuracy of results generated by the LLM. It is worth noting that the input parameters are derived from the genetic features and the two output lists are the priorities of all jobs and the priorities of all operations.

The executable function generated by the LLM takes the proposed genetic features as input and outputs priority values as gene selection probabilities at the job and operation levels. The function is tested for validity, and if the code contains errors, a new heuristic is generated. In addition, the probabilities are normalized using the cumulative distribution function of the normal distribution, ensuring all values fall within the range [0, 1].

Algorithm 1 Operator evolution

Input: Initial prompt R_{init} , solution population P , operator population Q
Output: New operator population Q^*

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1:  $Q^* \leftarrow \{\}$ .
2: Calculate the evolutionary features of  $P$  and produce textual description  $R_{pop}$ . //Perception and Analysis
3: Construct textual description  $R_{op}$ , describing the operators and fitness values  $f_{op}$  of  $Q$ .
4: Use LLM to perceive the optimization bottlenecks based on  $R_{pop}$ , analyze the limitations of operators based on  $R_{op}$ , give design suggestions, and output the result  $R_{as}$ .
5: Combine  $R_{pop}$ ,  $R_{op}$ ,  $R_{as}$  and  $R_{init}$  to form the improved prompt  $R_{improve}$ . // Adaptive Operator Evolution
6: Use LLM to fine-tune the task description of  $R_{improve}$ .
7:  $valid \leftarrow False$ .
8: while  $valid$  is  $False$  do
9:   Generate new operator  $op_{new}$  based on  $R_{improve}$ .
10:  if  $op_{new}$  can run then
11:     $op_{worst} \leftarrow \arg \min_{op \in Q} f_{op}$ .
12:     $Q^* \leftarrow (Q \setminus \{op_{worst}\}) \cup \{op_{new}\}$ .
13:     $valid \leftarrow True$ .
14:  end if
15: end while
16: return  $Q^*$ 

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4.3.2 Evolution Perception and Analysis.

Evolutionary changes in the solution population distribution may degrade the effectiveness of search strategies, necessitating the identification of when, why, and how to improve operators. To address this, we design the following steps.

1) Convergence evaluation: We measure the convergence state by the number Δt of consecutive iterations without improvement in the global best makespan, and then introduce a dynamic threshold $\theta = 1/(\epsilon \times \Delta t)$ to adaptively trigger operator evolution, where ϵ is a coefficient controlling the frequency of evolution evolution. Based on extensive experimental experience, $\epsilon = 0.05$ is selected to balance the frequency of LLM invocation and the overall algorithmic efficiency. In each iteration, if a uniform random number is greater than the threshold, the operator evolution mechanism will be triggered.

2) Perception and analysis: The complete procedure for operator evolution is presented in algorithm 1. To help LLM understand the evolutionary state at the current search stage, we record the solution population changes and the operator population performance. The solution population changes include the minimum and average fitness values, as well as their rates of change since the last operator evolution. The operator population performance contains the fitness value and thought of each operator. Based on these information, LLM perceives the optimization bottlenecks faced by the solution population, analyzes the limitations of the gene selection strategy in each operator, and provides design suggestions for operator improvement.

4.3.3 Adaptive Operator Evolution.

Based on the initial prompt, the solution population changes, the operator population performance, and the results of perception and analysis are integrated to produce a new prompt aimed at improving operators. This prompt explicitly emphasizes “develop a completely new algorithm distinct from the previous ones.” To mitigate response inertia caused by a fixed prompt, the LLM further fine-tunes the task description to increase output diversity. Using mutation-based prompting, the LLM generates a potential operator, replacing the worst-performing one to form a new operator population. Before the next iteration, the fitness values of all operators are reset. With LLM assistance, operators gain self-adaptive evolutionary capabilities that guide the algorithm to continuously search towards potential regions in the solution space.

Table 2: The comparison results of different LLM APIs.

LLM	Run 1	Run 2	Run 3	RPD_{aver}	Cost	Ratio
GPT-4.1-mini	217	230	231	36.97	0.0564\$	0.0015
GPT-4o	230	224	222	36.57	0.1770\$	0.0048
DeepSeek-Chat	230	223	228	37.58	0.2847\$	0.0076
Qwen-Max	224	230	228	37.78	0.3721\$	0.0098
Claude-4-Sonnet	233	233	235	41.62	0.5914\$	0.0142
Gemini 2.5 Pro	225	226	223	36.16	2.7024\$	0.0747

5 Experiments

5.1 Experimental Details

Experimental Design. To evaluate our proposed LLM4EO, extensive computational experiments are conducted in this section. First, the setting of experimental environment and algorithm parameters is described. Then, an ablation study is designed to analyze the impact of each improvement on the algorithm. Subsequently, LLM4EO is compared with mainstream algorithms for automatic program generation to demonstrate the potential of LLMs in designing operators. Furthermore, the superior performance of LLM4EO is verified by comparing it with other optimization algorithms. Finally, we extended the experiment to the distributed flexible job shop scheduling problem (DFJSP), showing the excellent generalization capability of LLM4EO.

Settings. The size of the operator population is 3, the size of the solution population is 100 and the maximum number of iterations is 200. In the solution initialization, for machine assignment, 10% of the population adopts Rule 1 and 90% adopts Rule 2. For operation dispatching, 20% uses the Random rule, 40% uses the MWR rule, and the remaining 40% uses the MOR rule [24]. The probabilities of crossover and mutation are set to 0.9 and 0.9. All programs of the experiments are implemented with Python 3.9 and execute on a computer with an AMD Ryzen 7-4800H @ 2.90 GHz and 40.0 GB of RAM.

To ensure operator performance, we evaluate the quality and cost of the algorithm across different LLM APIs. The tested models include GPT-4.1-mini, GPT-4o, DeepSeek-Chat, Qwen-Max, Claude-4-Sonnet and Gemini 2.5 Pro. LLM4EO is repeated three times on the largest instance MK10 of the Brandimarte benchmark [26]. Model performance is compared based on three metrics: average relative percent deviation (RPD_{aver}), running cost and quality-price ratio. As shown in Table 2, GPT-4.1-mini offers the best cost-effectiveness. Notably, despite its small model size, it achieves relatively satisfactory results, primarily due to the inherent robustness of evolutionary algorithms, which maintain strong performance under limited resources. In contrast, Gemini-2.5-Pro achieves the best quality of results, but incurs significant costs. Taking into account both algorithmic effectiveness and economic viability, GPT-4.1-mini is considered the most suitable option for operator design.

5.2 Performance Comparisons and Analysis

5.2.1 Ablation Study.

To evaluate the effectiveness of each improvement on performance, we compare algorithm variants with different components. First, the LLM autonomously designs an initial operator for the GA, resulting in a variant algorithm named LLM4OD. Then, the operator is further evolved by LLM, yielding LLM4EV. Finally, LLM4EO extends the previous variants by generating multiple initial operators to form an operator population. Unlike LLM4EO, LLM4EO-NPA removes the “evolution perception and analysis” module. Each Brandimarte benchmark instance is solved 10 times by these algorithms to obtain the best makespan (BM) and average makespan (AM). The relative percentage deviation (RPD) is used to quantify the deviation of each solution from the known optimal solution. The lower RPD indicates that the result is closer to the theoretical minimum, reflecting better algorithm performance. We calculate the relative percentage deviation for both the best makespan $RPD_{BM} = \frac{BM - LB}{LB} \times 100\%$ and the average makespan $RPD_{AM} = \frac{AM - LB}{LB} \times 100\%$, with LB denoting the lower bound.

As shown in Table 3, the algorithms are ranked by the mean values of RPD_{BM} and RPD_{AM} , from best to worst: LLM4EO, LLM4EO-NPA, LLM4EV, and LLM4OD. Notably, LLM4EV outperforms LLM4OD, indicating that the operator evolution mechanism can dynamically adjust the search strategy, thereby improving adaptability. However, the LLM4EV can only select the unique operator per iteration, limiting search scope and reducing population diversity. To balance exploration and exploitation, LLM4EO maintains a diverse operator pool and adaptively replaces poorly

Table 3: The results of RPD for variant algorithms on Brandimarte benchmark.

Instance	RPD_{BM}				RPD_{AM}			
	LLM4OD	LLM4EV	LLM4EO-NPA	LLM4EO	LLM4OD	LLM4EV	LLM4EO-NPA	LLM4EO
MK01	11.11	11.11	11.11	11.11	14.17	13.06	13.33	12.78
MK02	16.67	16.67	12.50	12.50	16.67	17.08	15.83	15.00
MK03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MK04	27.08	25.00	27.08	25.00	33.33	32.50	31.88	30.42
MK05	2.98	4.76	2.98	2.98	5.54	5.71	5.30	4.64
MK06	93.94	90.91	90.91	90.91	101.21	99.39	99.39	97.58
MK07	8.27	8.27	8.27	8.27	11.80	9.85	9.17	9.10
MK08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MK09	4.01	4.01	4.01	4.01	5.38	5.45	4.88	4.68
MK10	38.18	38.18	36.36	31.52	44.30	41.21	38.36	36.67
Mean	20.22	19.89	19.32	18.63	23.24	22.43	21.82	21.09

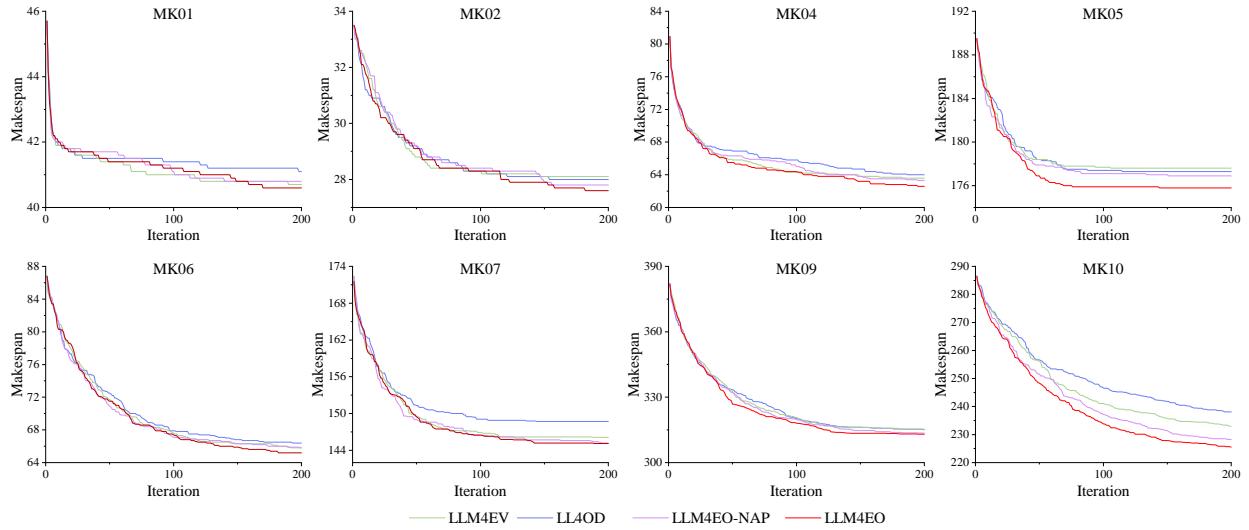


Figure 3: The average convergence curves of MK01, MK02, MK04, MK05, MK06, MK07, MK09 and MK10.

performing operators based on search feedback. Compared to LLM4EO-NPA, LLM4EO leverages the perception and analysis capabilities of the LLM to identify optimization bottlenecks and limitations, enabling the generation of more effective operators tailored to the current search stage. As a result, LLM4EO achieves the best performance among comparative algorithms. Figure 3 presents the average convergence of four algorithms after running 10 times on the MK01, MK02, MK04, MK05, MK06, MK07, MK09 and MK10 instances. It can be observed that LLM4EO converges faster than other algorithms in most cases and finally achieves better solutions. The results demonstrate that the collaborative evolution of solutions and operators effectively enhances optimization performance.

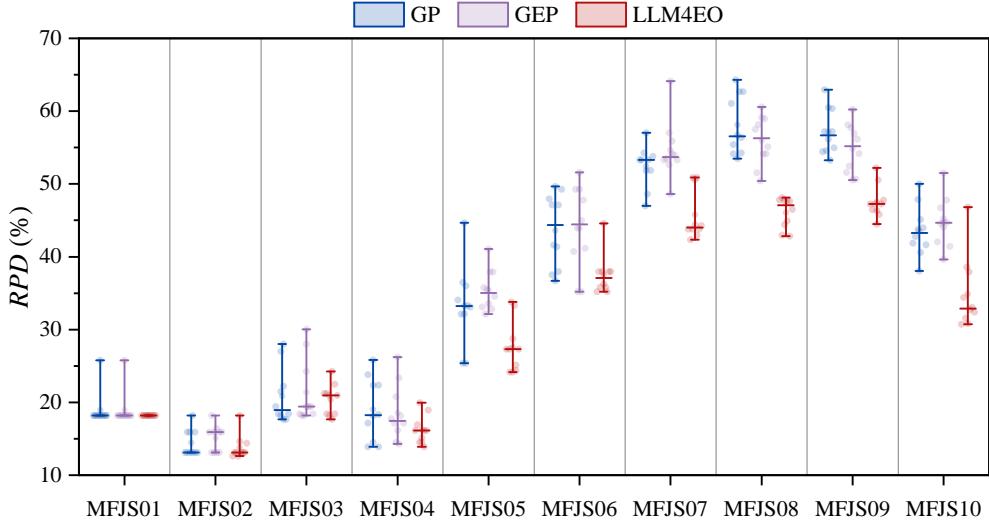
5.2.2 Comparison of Automatic Operator Design Methods.

Genetic Programming (GP) and Gene Expression Programming (GEP), as traditional symbolic evolution-based approaches, are employed to generate operators for GA and compared with LLM4EO to assess the potential of LLM in operator design. Since the LLM generates initial operators by referencing the Shortest Processing Time (SPT) heuristic, this heuristic is also adopted as the initial operator for both GP and GEP. To ensure experimental fairness, each method uses the legitimate result produced by a single iteration as the new operator during every operator evolution.

Table 4 presents the BM and AM obtained by solving MFJS instances of Fattahi benchmark [27] 10 times. Additionally, the box plot of the RPD values for each run is shown in Figure 4. It can be observed that LLM4EO outperforms both GP and GEP in terms of BM and AM values, demonstrating its excellent generation capability. This result is closely related to the inherent characteristics of each method. First, traditional symbolic approaches lack semantic understanding, resulting in lower-quality operators without sufficient iterations. In contrast, LLMs have powerful

Table 4: The result of *BM* and *AM* for comparison algorithms on Fattahi benchmark.

Instance	Size	LB	<i>BM</i>			<i>AM</i>		
			GP	GEP	LLM4EO	GP	GEP	LLM4EO
MFJS01	5x6	396	468	468	468	471.00	471.00	468.00
MFJS02	5x7	396	448	448	446	453.30	455.70	450.70
MFJS03	6x7	396	466	468	466	478.60	480.70	476.70
MFJS04	7x7	496	565	567	565	590.40	588.00	576.50
MFJS05	7x7	414	519	547	514	551.70	560.70	528.10
MFJS06	8x7	469	641	634	634	673.60	674.90	644.70
MFJS07	8x7	619	910	920	881	943.60	957.30	902.30
MFJS08	9x8	619	950	931	884	978.70	965.40	904.10
MFJS09	11x8	764	1171	1150	1104	1200.70	1183.10	1127.20
MFJS10	12x8	944	1303	1318	1234	1355.20	1366.60	1273.50

Figure 4: Partial box plots of *RPD* for GP, GEP and LLM4EO on Fattahi benchmark.

representational capabilities and natural language understanding, enabling them to integrate expert knowledge into search strategies and adaptively optimize operators based on search feedback. By overcoming the limitations of traditional methods in text comprehension and program generation, LLM4EO exhibits strong generalization and adaptability, highlighting the potential of LLMs to drive a more effective evolutionary search.

5.2.3 Comparison with Genetic Algorithms.

To further evaluate the impact of LLM-generated meta-operators on the convergence of evolutionary algorithms, LLM4EO is compared with several genetic algorithms. These algorithms include the traditional genetic algorithm proposed by F.Pezzella et al. (F'GA) [24], the self-learning genetic algorithm based on reinforcement learning (SLGA) [28], and the proposed genetic algorithm (GA). The components of GA and LLM4EO are essentially the same, with the only difference being that GA generates offspring by randomly selecting genes rather than relying on LLM-generated operators. These algorithms run independently 10 times on each Brandimarte benchmark instance.

As shown in Table 5, LLM4EO achieves optimal *BM* and *AM* values for all instances except MK07, which demonstrates its outstanding performance. Furthermore, the average relative percentage deviation (RPD_{aver}) of LLM4EO is lower than that of the comparison algorithms, thus validating its effectiveness in solving the FJSP. Notably, LLM4EO is based on GA but employs meta-operators to select promising genes for generating individuals. With the assistance of LLMs, the RPD_{aver} of *BM* decreased from 13.19 to 12.71, achieving a 3.64% performance improvement; the RPD_{aver} of *AM* dropped from 14.58 to 14.12, yielding a 3.16% performance gain. These improvements demonstrate that the proposed meta-operator effectively enhances the exploration capability and stability of evolutionary algorithms.

Figure 5 presents the average convergence of four algorithms after running 10 times on the MK01, MK02, MK04, MK05, MK06, MK07, MK09, and MK10 instances. For the MK03 and MK08 instances, no significant convergence

Table 5: The results of genetic algorithms on Brandimarte benchmark.

Instance	Size	LB	BM				AM			
			F'GA	SLGA	GA	LLM4EO	F'GA	SLGA	GA	LLM4EO
MK01	10×6	36	42	42	40	40	42.6	42.1	40.8	40.6
MK02	10×6	24	29	28	27	27	30.6	28.8	28.0	27.6
MK03	15×8	204	204	204	204	204	204.0	204.0	204.0	204.0
MK04	15×8	48	69	66	62	60	72.0	67.0	64.1	62.6
MK05	15×4	168	177	176	174	173	178.5	177.7	176.0	175.8
MK06	10×15	33	71	74	63	63	75.1	78.1	65.2	65.2
MK07	20×5	133	150	145	143	144	154.1	149.5	144.9	145.1
MK08	20×10	523	523	523	523	523	527.4	523.0	523.0	523.0
MK09	20×10	299	362	360	311	311	368.1	366.1	313.2	313.0
MK10	20×15	165	251	269	224	217	258.7	273.9	229.0	225.5
<i>RPD_{aver}</i>			18.36	17.97	13.19	12.71	20.19	19.18	14.58	14.12

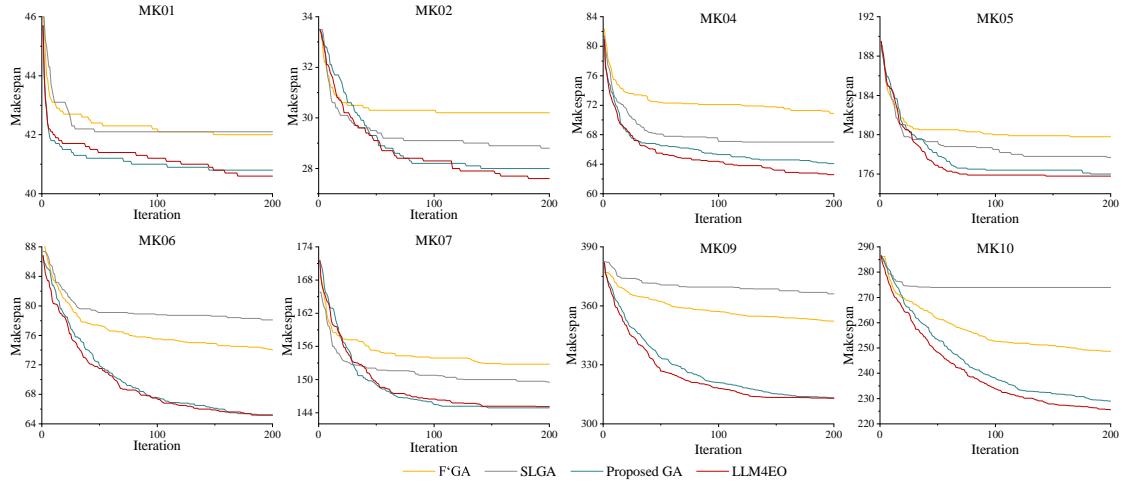


Figure 5: The average convergence curves of MK01, MK02, MK04, MK05, MK06, MK07, MK09 and MK10.

process is observed since the initial solutions already attain the theoretical lower bound. It can be seen that, in most cases, LLM4EO converges faster than other algorithms and ultimately yields higher-quality solutions. This phenomenon validates that meta-operators based on LLMs effectively accelerate the algorithmic evolution process.

5.2.4 Comparison with Other Optimization Algorithms.

In order to validate the performance of LLM4EO in solving the FJSP, it is compared with various traditional optimization algorithms, such as TS [26], PSO [29] and ACO [30], as well as hybrid optimization algorithms, including HGWO [31], HQPSO [32], HLO-PSO [33] and SLABC [34].

As shown in Table 6, LLM4EO outperforms traditional optimization algorithms in 8 out of 10 instances and achieves the *LB* values on MK03 and MK08. Although KBACO achieves the lower *BM* on MK01 and MK09, its performance on the remaining instances is inferior to LLM4EO. Table 7 shows that LLM4EO outperforms the compared hybrid optimization algorithms in both *BM* and *AM* for all instances. Overall, LLM4EO demonstrates superior performance in solving the FJSP with better solution quality and higher stability.

To validate the generalization capability of the proposed algorithm, LLM4EO is compared with several other optimization algorithms on the Fattahi dataset, including HSA/TS [27], HTS/SA [27], MILP [35], AIA [36], IGAR [37] and the proposed GA. Table 8 shows the best makespan values in ten running times. Since these algorithms all find solutions equal to the *LB* values on SFJS instances with the relatively simple nature, only the results for MFJS instances are presented. Due to LLM4EO achieving lower *BM* than comparison algorithms in 7 out of 10 instances, its superior consistency across different datasets is demonstrated, thereby validating its strong generalization capability.

Table 6: The results of traditional optimization algorithms on Brandimarte benchmark.

Instance	Size	LB	BM				AM			
			TS	PSO	ACO	LLM4EO	TS	PSO	ACO	LLM4EO
MK01	10x6	36	42	40	39	40	43.0	41.60	39.8	40.6
MK02	10x6	24	32	29	29	27	38.0	31.10	29.1	27.6
MK03	15x8	204	211	204	204	204	221.0	204.00	204.0	204.0
MK04	15x8	48	81	66	65	60	100.0	68.05	66.1	62.6
MK05	15x4	168	186	175	173	173	188.0	178.05	173.8	175.8
MK06	10x15	33	86	77	67	63	111.0	80.70	69.1	65.2
MK07	20x5	133	157	146	144	144	232.0	148.70	145.4	145.1
MK08	20x10	523	523	523	523	523	523.0	523.00	523.0	523.0
MK09	20x10	299	369	320	311	311	379.0	338.05	312.2	313.0
MK10	20x15	165	296	239	229	217	301.0	248.85	233.7	225.5

Table 7: The results of hybrid optimization algorithms on Brandimarte benchmark.

Instance	LB	BM					AM				
		HGWO	HQPSO	HLO-PSO	SLABC	LLM4EO	HG-WO	HQPSO	HLO-PSO	SLABC	LLM4EO
MK01	36	40	42	40	42	40	41.6	42.0	41.3	43.0	40.6
MK02	24	29	28	28	29	27	30.3	29.2	28.8	31.5	27.6
MK03	204	204	204	204	204	204	204.1	204.0	204.0	219.4	204.0
MK04	48	65	70	63	69	60	67.4	72.0	66.1	75.6	62.6
MK05	168	175	179	175	175	173	178.2	182.6	177.3	191.9	175.8
MK06	33	79	68	71	80	63	79.9	72.8	73.6	89.4	65.2
MK07	133	149	149	144	155	144	156.4	155.5	145.1	159.3	145.1
MK08	523	523	523	523	523	523	523.0	523.5	523.0	525.0	523.0
MK09	299	325	342	326	368	311	342.3	355.7	331.3	379.1	313.0
MK10	165	253	246	238	283	217	262.7	254.9	246.7	294.0	225.5

5.2.5 Comparison with Other Algorithms in Distributed Flexible Job Shop Scheduling.

This subsection expands the scope of the experiment from FJSP to distributed flexible job shop scheduling problem (DFJSP), aiming to explore the generalization and robustness of LLM4EO. The benchmark instances proposed by Giovanni and Pezzella [38] are used to test the performance. This dataset extends traditional FJSP benchmarks by assuming uniform processing capacities across factories and disregarding inter-factory transportation time. Four algorithms, Giovanni'GA [38], Lu'GA [39], Wu'GA [40], CRO [41] and the basic genetic algorithm proposed in this paper (GA), are used to compare with LLM4EO.

In order to adapt to DFJSP, the proposed algorithm incorporates adjustments across three key components. 1) Add factory crossover: Select multiple operations from each parent and swap their assigned factories sequentially to generate new offspring. 2) Add factory mutation: Assign the selected operations to other feasible machines within the current factory. 3) Modified critical operation swapping: Select an operation on the critical path and attempt to swap it with other operations on the critical path. If fitness improves after swapping, repeat this local search process on the new critical path until no further improvement can be achieved by swapping the selected operation with any other operation.

Ten repeated runs are performed on each of the 20 instances with two factories, and the results are shown in Table 9. It can be found that all algorithms achieved optimal solutions in LA01-LA05 and LA16-LA20. Meanwhile, LLM4EO obtained the optimal *BM* values in LA06, LA09, LA10, LA12, and LA14, as well as the optimal *AM* values in LA06 and LA11-LA15. LLM4EO achieved the second-best *RPD_{aver}* value for *BM*, trailing only Wu'GA, while delivering the optimal *RPD_{aver}* value for *AM*. The results demonstrate the outstanding generalization of LLM4EO.

Figure 6 presents the box plot of the makespan values obtained by LLM4EO and GA in ten runs. It is evident that the result distribution of LLM4EO is more concentrated than that of GA, with a shorter box positioned closer to the lower end. This indicates that most results of LLM4EO exhibit superior performance and fewer fluctuations. The above experimental analysis demonstrates that LLM4EO outperforms GA in both solution quality and stability. Therefore, the proposed LLM-driven meta-operator is still effective in enhancing the performance of evolutionary algorithms when solving the DFSP.

Table 8: The result of BM for comparison algorithms on Fattahi benchmark.

Instance	Size	LB	HSA/TS	HTS/SA	MILP	AIA	IGAR	GA	LLM4EO
MFJS01	5x6	396	491	469	468	468	468	468	468
MFJS02	5x7	396	482	468	466	468	468	446	446
MFJS03	6x7	396	538	538	466	468	468	468	466
MFJS04	7x7	496	650	618	564	554	554	565	565
MFJS05	7x7	414	662	625	514	527	514	519	514
MFJS06	8x7	469	785	730	635	635	634	634	634
MFJS07	8x7	619	1081	947	935	879	881	879	881
MFJS08	9x8	619	1122	922	905	884	884	884	884
MFJS09	11x8	764	1243	1105	1192	1088	1097	1109	1104
MFJS10	12x8	944	1615	1384	1276	1267	1275	1258	1234

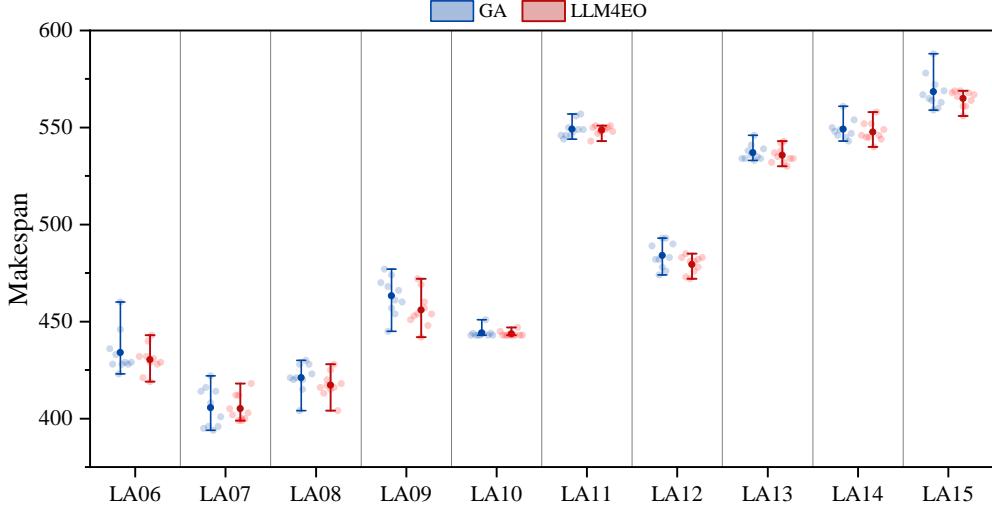


Figure 6: Partial box plots of LLM4EO and GA on LA06-LA15 instances.

6 Conclusion

We propose LLM4EO that utilizes LLMs to enhance the optimization performance of EAs. By leveraging prior knowledge of problem structures and heuristics, it constructs high-quality gene selection strategies of operators. When population evolution stagnates, the LLM4EO perceives evolutionary states, analyzes operator limitations, and provides improvement suggestions. The resulting new operators are tailored to the current search stage and guide the EAs to explore potential neighborhoods. On the Brandimarte and Fattahi benchmarks for FJSP, LLM4EO outperforms other algorithms in most instances, demonstrating its superior performance. With the addition of LLM-driven meta-operators, algorithmic performance improves by at least 3%, consistently accelerating population evolution and improving optimization quality, even under constrained iteration conditions. Furthermore, when the problem is extended from FJSP to DFJSP, LLM4EO still outperforms mainstream algorithms, showing its excellent generalization capability.

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Table 9: Computational results for the DFJSP with two factories.

Instance	Size	LB	LLM4EO		GA		Giovanni'GA		Lu'GA		Wu'GA		CRO
			BM	AM	BM	AM	BM	AM	BM	AM	BM	AM	
LA01	10×5	413	413	413.0	413	413.0	413	413.0	413	413.0	413	413.0	413
LA02	10×5	394	394	394.0	394	394.0	394	394.0	394	394.0	394	394.0	394
LA03	10×5	349	349	349.0	349	349.0	349	349.0	349	349.0	349	349.0	349
LA04	10×5	369	369	369.0	369	369.0	369	369.0	369	369.0	369	369.0	369
LA05	10×5	380	380	380.0	380	380.0	380	380.0	380	380.0	380	380.0	380
LA06	15×5	413	419	430.4	423	434.0	445	449.6	424*	435.8	424	432.7	424
LA07	15×5	376	399	405.0	394	405.6	412	419.2	398	408.5	390	403.6	398
LA08	15×5	369	404	417.2	404	421.0	420	427.8	406	417.4	397	411.7	406
LA09	15×5	382	442	456.0	445	463.2	469	474.6	447	459.0	444	455.7	463
LA10	15×5	443	443	443.6	443	444.1	445	448.6	443	444.1	443	443.2	445
LA11	20×5	413	543	548.7	544	549.1	570	571.6	548	557.1	541	549.9	553
LA12	20×5	408	472	479.4	474	484.0	504	508.0	480	492.5	474	482.3	500
LA13	20×5	382	530	535.7	533	537.0	542	552.2	533	538.4	529	538.1	551
LA14	20×5	443	540	547.7	543	549.2	570	576.0	542*	557.3	544	553.7	581
LA15	20×5	378	556	564.9	559	568.5	584	588.8	562	568.7	554	566.6	597
LA16	10×10	717	717	717.0	717	717.0	717	717.0	717	717.0	717	717.0	717
LA17	10×10	646	646	646.0	646	646.0	646	646.0	646	646.0	646	646.0	646
LA18	10×10	663	663	663.0	663	663.0	663	663.0	663	663.0	663	663.0	663
LA19	10×10	617	617	617.0	617	617.0	617	617.2	617	622.1	617	617.0	617
LA20	10×10	756	756	756.0	756	756.0	756	756.0	756	756.0	756	756.0	756
<i>RPD_{aver}</i>			9.38	10.41	9.55	10.75	12.05	12.75	9.82	11.07	9.26	10.51	11.50

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