Robust Differential Evolution via Nonlinear Population Size Reduction and Adaptive Restart: The ARRDE Algorithm

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Abstract

This study is motivated by a robustness issue in numerical optimization of bound-constrained problems: many algorithms that perform well on a particular benchmark suite—such as the IEEE CEC2017 problems—struggle to maintain the same level of performance when applied to other suites that differ in dimensionality, landscape complexity, or the maximum number of function evaluations (N_{max}). To address this, we propose the *Adaptive Restart–Refine Differential Evolution* (ARRDE) algorithm, a new variant of Differential Evolution (DE). ARRDE builds upon the LSHADE algorithm, incorporates key mechanisms from jSO, and introduces a nonlinear population-size reduction strategy combined with an adaptive restart–refine mechanism.

We evaluate ARRDE on five benchmark suites—CEC2011, CEC2017, CEC2019, CEC2020, and CEC2022—which, to the best of our knowledge, constitutes the most extensive experimental study to date in the context of algorithmic comparison, as most prior works consider only one or two suites. This broad evaluation enables a rigorous assessment of generalization across markedly different problem characteristics. To further support fair cross-suite comparisons, we also introduce a bounded accuracy-based scoring metric derived from relative error. Using both rank-based and accuracy-based metrics, and comparing against algorithms that perform strongly on CEC2017 (e.g., jSO and LSHADE-cnEpSin) as well as those that excel on CEC2020 (e.g., j2020 and NLSHADE-RSP), ARRDE consistently demonstrates top-tier performance, ranking first across all benchmark suites considered. These results highlight ARRDE's robustness and its superior generalization capability.

Keywords: Evolutionary Algorithms, Differential Evolution

1. Introduction

Optimization algorithms have emerged as important tools for addressing complex, real-world problems across diverse fields such as engineering, economics, and machine learning. Among these, evolutionary algorithms (EAs), particularly Differential Evolution (DE), have gained widespread recognition for their simplicity and effectiveness in solving continuous optimization problems. Differential Evolution, introduced by Storn and Price [1], is specifically designed to handle challenging optimization tasks. Unlike traditional optimization methods, DE does not require gradient information, making it well-suited for non-differentiable, non-convex, and noisy objective functions. Variants of DE consistently achieve high rankings in the IEEE annual Congress on Evolutionary Computation (CEC) competitions[2], where standardized test suites are used to benchmark newly proposed optimization algorithms.

DE operates by evolving a population of candidate solutions through mutation, crossover, and selection operators. In each generation, new candidates are created by combining individuals from the current population using a mutation strategy, followed by recombination (crossover) with existing solutions. The selection process ensures that only solutions that improve the objective function progress to the next generation. DE's simplicity and its capacity for effective global search have made it a popular choice for optimization tasks. However, the standard DE algorithm can sometimes struggles to balance exploration and exploitation, leading to challenges such as slow convergence and susceptibility to local optima. To address these limitations, numerous DE variants have been proposed, incorporating mechanisms to mitigate these issues.

Over the past two decades, extensive research on DE has resulted in the development of numerous advanced variants [2]. The basic DE algorithm relies on three key hyperparameters: population size, mutation factor (F), and crossover rate (CR). Many DE variants incorporate adaptive mechanisms to dynamically adjust these parameters during the optimization process, enhancing both convergence speed and robustness. One notable example is JADE (Adaptive Differential Evolution with Optional External Archive)[3], which employs adaptive strategies for controlling the mutation and crossover rates and utilizes an external archive to maintain population diversity and avoid premature convergence. Another prominent variant is LSHADE (Success-History Adaptive Differential Evolution)[4], which integrates success-history-based parameter adaptation and features a linear reduction in population size. LSHADE, the win-

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ner of the CEC 2014 competition, has laid the groundwork for state-of-the-art variants such as LSHADE-cnEpSin[5], jSO[6], and NL-SHADE-RSP[7].

Despite their notable success, many DE variants exhibit substantial performance degradation when evaluated on benchmark suites different from the CEC sets for which they were originally fine-tuned, as highlighted in [8]. Moreover, as we demonstrate later, algorithmic performance is highly sensitive to the maximum number of function evaluations ($N_{\max}$). As a result, the ranking of algorithms across benchmark problems depends not only on the specific CEC suite but also on the chosen value of $N_{\max}$. For example, algorithms that perform exceptionally well on CEC2017—whose problems span 10, 30, 50, and 100 dimensions with $N_{\max} = 10\,000 \times D$—often perform poorly on the CEC2020 suite, which consists of 5-, 10-, 15-, and 20-dimensional problems with much larger budgets of 5×10^4, 10^6, 3×10^6, and 10^7 evaluations, respectively. Conversely, algorithms that excel in CEC2020 typically rank among the weakest on CEC2017.

A similar robustness issue has been reported in [9], which compares 30 algorithms on CEC2014, CEC2017, and CEC2022 under evaluation budgets of $N_{\max}$ = 5,000, 50,000, 500,000, and 5,000,000. That study observed a consistent pattern: algorithms that excel under small evaluation budgets often deteriorate sharply when the budget is large, whereas those designed for large-budget search struggle when the budget is restricted. Together, these observations underscore the importance of algorithmic robustness across both benchmark suites and evaluation budgets.

The goal of this study is therefore clear: to develop a robust optimization algorithm that performs consistently across both small and large $N_{\max}$ values, as well as across problems of varying dimensionality and complexity. In real-world scenarios, practitioners often lack the resources or expertise to fine-tune an algorithm for a specific problem setting or a particular evaluation budget. Consequently, a general-purpose optimizer that delivers strong performance without extensive parameter adjustment is highly desirable.

To address these challenges, we propose the *Adaptive Restart–Refine Differential Evolution* (ARRDE) algorithm, designed with robustness as a primary objective. ARRDE is evaluated on five comprehensive CEC benchmark suites: 22 real-world problems from CEC2011 [10], 29 functions from CEC2017 [11] tested at four dimensions (10D, 30D, 50D, 100D), 10 functions from CEC2019 [12], 10 functions from CEC2020 [13] tested at four dimensions (5D, 10D, 15D, 20D), and 12 functions from CEC2022 [14] tested at two dimensions (10D and 20D). In total, this corresponds to 212 problem instances. In addition, ARRDE is tested on CEC2017, CEC2020, and CEC2022 under a wide range of normalized evaluation budgets $N_{\max}/D$, where D is the dimensionality of the problem. To the best of our knowledge, no existing algorithm has been assessed on such an extensive and diverse collection of benchmark problems and evaluation-budget settings. Through this broad evaluation, we aim to encourage more rigorous benchmarking practices and the development of optimizers that remain reliable across varying dimensionalities, landscape complexities, and resource constraints.

All implementations of ARRDE, comparison algorithms, and CEC benchmark suites are conducted using the open-source *Minion* framework[15], a C++ and Python library for designing and evaluating optimization algorithms.

2. Related Works

2.1. Basic Differential Evolution

Differential Evolution (DE) follows the general framework of evolutionary algorithms (EAs) but introduces a unique mutation and recombination strategy that leverages differences between randomly chosen vectors in the population to generate candidate solutions. The algorithm begins by initializing a population of solutions randomly within the boundaries of the search space. In each generation (or iteration), a new candidate population is produced through mutation and crossover operations. The iteration concludes with a greedy selection process: if a candidate solution outperforms its parent, it replaces the parent in the next generation; otherwise, the parent survives. The algorithm terminates when a predefined stopping criterion is met, typically when the number of function evaluations exceeds a maximum limit (*maxevals*).

The generation of new candidate solutions in DE relies on two core operations: mutation and crossover. These operations are summarized as follows:

- **Mutation**: A mutant vector is generated by adding the weighted difference of two or more population vectors to a base vector. Specifically, given a target vector i, a mutant v_i can be obtained as

$$v_i = x_{r_1} + F(x_{r_2} - x_{r_3}) \tag{1}$$

  where $x_j, j = 1, 2, ..., N_P$ is the position vector of an individual from the current population with size N_P, r_k is an index to an individual which is different to i, and F is the mutation factor, also known as the mutation rate. The above mutation scheme is called "rand/1", is one of the simplest and most widely used.

  Other mutation schemes :

$$v_i = x_{best} + F(x_{r_1} - x_{r_2}) \qquad \text{(best/1)} \tag{2}$$

$$v_i = x_i + F(x_{pbest} - x_i) + F(x_{r_1} - x_{r_2}) \qquad \text{(current-to-pbest/2)} \tag{3}$$

- **Crossover**: The mutant vector is combined with a target/parent vector to produce a trial vector. There are two widely used crossover schemes : binary and exponential crossovers. Between these two, binary crossover is more commonly used. It is also employed in the original DE and also used in this work. In binary crossover, a child vector u_i is obtained from a parent vector x_i and a mutant v_i by following the rule :

$$u_{i,j} = \begin{cases} v_{i,j} & \text{if rand[0, 1)} \leq CR \text{ or } j = j_{rand} \\ x_{i,j} & \text{Otherwise} \end{cases} \tag{4}$$

where j denotes the component index, j_{rand} is a random component index chosen for each child vector, and CR is the crossover rate.

The combined mutation-crossover strategy is typically denoted as DE/x/y/z, 'x' denotes for 'rand' indicates random vector selection, 'y' refers to the number of difference vector used, and 'z' represents the crossover scheme.

The standard DE algorithm relies on three hyperparameters: initial population size, mutation factor (F), and crossover probability (CR). Many variants of DE have introduced adaptive mechanisms for these parameters, allowing them to change dynamically during the optimization process. This adaptability improves both the convergence speed and robustness of the algorithm. Additionally, different mutation and crossover strategies have been developed to enhance performance further, depending on the nature of the optimization problem.

2.2. LSHADE

The Linear Population Size Reduction Success-History-based Adaptive Differential Evolution (L-SHADE) algorithm is an advanced variant of DE that won the CEC2014 competition [4]. It builds on the foundation of the Success-History-based Adaptive Differential Evolution (SHADE) algorithm[16], which ranked fourth in the CEC2013 competition. LSHADE and SHADE share several key features, with the primary difference being L-SHADE's linear population size reduction mechanism, which gradually decreases the population size as iterations progress.

The parameter adaptation process in L-SHADE is built around the historical memory of successful parameter settings. This memory stores past successful values for F and CR, and new values are generated by sampling from the memory at each generation. Specifically, let M_{CR} and M_F denote the historical memories for CR and F, respectively. The memories contain H entries, each storing the CR and F values that led to improvements in previous generations. For each individual in the population, a random index r_i is selected from the range $[1, H]$. Then, CR and F are generated by sampling around the stored values in M_{CR} and M_F as follows:

$$CR_i = \begin{cases} 0 & \text{if } M_{CR,r_i} = \perp \\ \text{randn}(M_{CR,r_i}, 0.1) & \text{otherwise} \end{cases} \tag{5}$$

$$F_i = \text{randc}(M_{F,r_i}, 0.1) \tag{6}$$

where randn(m, σ) represents a normal distribution centered around m with a standard deviation σ, and randc(m, σ) is a Cauchy distribution used for generating F. After mutation and crossover, trial vectors are generated. Once the trial vectors are evaluated, the parameter values that contributed to successful offspring (i.e., vectors that performed better than their parents) are stored in S_{CR} and S_F, and these values are used to update the historical memory for the next generation. The historical memory M_{CR} and M_F are updated at the end of each generation using the weighted Lehmer mean, where the weight w_i are the weights associated with each value of F and CR. The weights are derived from the fitness improvements Δf_i of the trial vectors, calculated as:

$$w_i = \frac{\Delta f_i}{\sum_{j=1}^{k} \Delta f_j}$$

where $\Delta f_i = |f(u_i) - f(x_i)|$ is the fitness improvement between the trial vector u_i and the parent vector x_i. This way, larger fitness improvements contribute more to the update of CR and F.

Linear population size reduction (LPSR) controls the population size N_G as the search progresses. Starting with an initial population size N_{init}, the population size is gradually reduced to a minimum size N_{min}. The reduction follows a linear schedule, which is calculated after each generation as:

$$N_{G+1} = \left\lceil \left(\frac{N_{\text{min}} - N_{\text{init}}}{\text{MAX\_NFE}} \right) \times \text{NFE} + N_{\text{init}} \right\rceil$$

Where NFE is the current number of function evaluations and MAX_NFE is the maximum allowed number of evaluations. When N_{G+1} is smaller than N_G, the individuals with the worst fitness values are removed.

This gradual reduction in population size allows for extensive exploration in the early phases of the search and increased exploitation as the population size decreases, which helps fine-tune the solution in the later stages.

2.3. jSO Algorithm

The jSO algorithm [6] is a DE variant derived from LSHADE, designed to improve its performance, particularly in high-dimensional problems. It ranked second in CEC2017 competition. It employs a weighted mutation strategy:

$$v_i = x_i + F_w\, F(x_{pbest} - x_i) + F(x_{r_1} - x_{r_2}), \tag{7}$$

where F_w is a time-dependent weight based on the ratio of function evaluations $\tau = N_{\text{eval}}/N_{\text{max}}$. The value of F_w increases from 0.7 to 1.2 as τ grows.

In addition, the crossover rate (CR), scaling factor (F), and weighting factor (F_w) are each clamped according to τ to slow down the early-stage search and promote more refined steps later in the run. The p-best parameter, which controls the proportion of elite individuals used in the mutation strategy, is also reduced linearly from 0.25 to 0.125 as the search progresses.

Finally, the last entries of the historical memories for F and CR are fixed at 0.9, ensuring that a subset of individuals always explores more aggressively. These adjustments collectively provide smoother parameter transitions and more stable convergence across different problem scales.

2.4. LSHADE-cnEpSin Algorithm

The LSHADE-cnEpSin[5] algorithm extends the LSHADE framework with two main mechanisms: (1) an ensemble of sinusoidal schemes for adapting the scaling factor (F), and (2) a covariance–matrix–based crossover operator constructed from an Euclidean neighborhood.

Algorithm 1 Adaptive Restart–Refine Differential Evolution (ARRDE)

```
1:  Input: objective $f$, dimension $D$, bounds $[L, U]$, max. evaluations $N_{\max}$
2:  Compute initial population size $N_0$ using (8)
3:  Initialize LSHADE components: memories $M_F, M_{CR}$, external archive $A$, population $P$ of size $N_0$
4:  Generate $P$ by Latin hypercube sampling in $[L, U]$; evaluate $f(P)$; set $N_{evals} \leftarrow N_0$, record best $x^\star$
5:  Initialize exclusion intervals $\mathcal{E} \leftarrow \emptyset$ and refinement flag refine ← false
6:  while $N_{evals} < N_{\max}$ do
7:      $t \leftarrow N_{evals}/N_{\max}$                                   ▷ normalized progress
8:      Compute target population size $N_p(t)$ using the reduction schedule in (3)
9:      if $|P| > N_p(t)$ then                                           ▷ population reduction
10:         Remove the worst $|P| - N_p(t)$ individuals from $P$
11:         Resize $A$ to maintain a fixed archive–to–population ratio
12:     $S_F, S_{CR}, \Delta f \leftarrow \emptyset$                    ▷ success histories
13:     for each individual $x_i \in P$ do
14:         Select memory index $k \sim \mathcal{U}\{1, \ldots, H\}$
15:         Sample $F_i$ and $CR_i$ from $M_F[k]$ and $M_{CR}[k]$ (as in LSHADE)
16:         Apply jSO progress-dependent clamping to $F_i$ and $CR_i$ based on $t$
17:         Generate trial vector $u_i$ using LSHADE current-to-pbest mutation and binomial crossover
18:         Apply bound handling and evaluate $f(u_i)$; $N_{evals} \leftarrow N_{evals} + 1$
19:         Perform LSHADE-style selection and update of $P$, $A$, and $S_F, S_{CR}, \Delta f$
20:         Update global best $x^\star$ if improved
21:         if $N_{evals} \geq N_{\max}$ then break
22:     if $S_F \neq \emptyset$ then                                      ▷ memory update
23:         Update $M_F$ and $M_{CR}$ by weighted Lehmer means of $S_F$ and $S_{CR}$; keep $M_F[H], M_{CR}[H]$ locked (jSO)
24:     Compute convergence indicator $s \leftarrow \mathrm{std}(f(P))/\,\mathrm{mean}(f(P))$
25:     if $t \geq 0.9$ and refine = false then
26:         refine ← true
27:     if $s \leq s_{\mathrm{tol}}$ or refine = true then
28:         Store current population and memories in restart archives
29:         Update local exclusion intervals $\mathcal{E}$ from archived populations
30:         Decide between restart and refinement as discussed in Sec. 3
31:         if restart then                                               ▷ restart
32:             Regenerate $P$ by uniform sampling in $[L, U]$ subject to $\mathcal{E}$; evaluate $f(P)$
33:         else                                                          ▷ refinement
34:             Regenerate $P$ from individuals sampled from the archives
35:             if refine = true then Insert best-so-far $x^\star$ into $P$
36:             Evaluate $f(P)$ if needed
37: return $x^\star$ as the final solution
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The ensemble sinusoidal adaptation employs two sinusoidal formulas, a non-adaptive decreasing scheme and an adaptive increasing scheme. Their selection probabilities are updated dynamically according to their recent success rates, enabling F to oscillate while favoring the more effective scheme. This provides a balance between exploration and exploitation throughout the evolutionary process.

The covariance-matrix learning mechanism constructs a local coordinate system around the current best individual. A subset of population members closest to the best in Euclidean distance is selected to estimate a covariance matrix, whose eigenvectors define a rotated basis for the crossover operator. This rotation allows the algorithm to capture variable dependencies and improve the search efficiency in non-separable landscapes.

2.5. *NLSHADE-RSP Algorithm*

TThe NLSHADE-RSP algorithm [7] is an extension of L-SHADE-RSP [17], developed specifically for the CEC2021 benchmark suite, in which it achieved first place. The main idea of NLSHADE-RSP is to incorporates Rank-based Selective Pressure (RSP) in the mutation process. Here, individuals are ranked based on their fitness, and the selective pressure is applied accordingly. This procedure has been show to be beneficial in high dimensional case. Another important feature is the use of an automatic tuning of archive usage probability. As in LSHADE, solutions that do not immediately contribute to population improvement are stored in an archive. These archived solutions are periodically utilized in the mutation process to help explore less-exploited regions of the search space. The archive is dynamically managed based on its impact on performance. The algorithm employs both binomial and exponential crossover techniques, and the crossover rate is controlled in an adaptive manner.

2.6. *j2020 Algorithm*

The j2020 algorithm[18] is a DE variant designed for solving single-objective bound-constrained optimization problems. It builds upon the principles of previous algorithm jDE100[19] which won CEC2019. The algorithm utilizes two populations—a larger population (P_b) and a smaller one (P_s)—which interact to improve both exploration and exploitation. The smaller population focuses on fine-tuning solutions, while the larger one drives global exploration. After crossover, a crowding mechanism is applied to ensure diversity by selecting the individual closest to the trial vector in terms of Euclidean distance for the selection step.The best individual from the larger population P_b is migrated to the smaller population P_s periodically, ensuring that the smaller population benefits from the best solutions found by the larger population. Both populations are restarted separately when diversity is lost. This reinitialization ensures that the algorithm does not stagnate in local optima, improving its global search capabilities.

3. The Proposed ARRDE Algorithm

The Adaptive Restart–Refine Differential Evolution (ARRDE) algorithm is constructed upon the LSHADE and incorporates several mechanisms introduced in jSO. As in LSHADE, ARRDE employs success-history based adaptation of the control parameters F and CR, utilizes an external archive, and adopts the *current*-to-*p*best mutation strategy. From jSO, ARRDE also inherits progress-dependent clamping of CR and F and the locking of the last memory entry. Nevertheless, ARRDE departs from LSHADE in its population initialization strategy, its population reduction schedule, and its mechanism for addressing premature convergence via adaptive restart and refinement.

The initial population is generated by Latin hypercube sampling within the bound constraints. The population size is defined as

$$N_0 = D \times \max\Big[2,\ 2 + 5.756(\eta - 2)^{1.609}\Big], \tag{8}$$

$$\eta = \log_{10}(N_{\max}/D). \tag{9}$$

This formulation yields $N_0 \approx 2D, 8D, 20D, 33D$, and $48D$ when $\eta = 2, 3, 4, 5, 6$, respectively, thereby allowing ARRDE to automatically adjust its initial population size according to the available computational budget. The selected values were determined empirically through experimentation on the CEC2017 and CEC2020 benchmark suites.

During optimization, the population size is reduced exponentially according to

$$N_p(t) = \begin{cases} N_0 - \left(N_0 - \frac{D}{2}\right)\left[1 - \left(\frac{0.9-t}{0.9}\right)^r\right], & t \le 0.9, \\ \frac{N_0}{4} - \left(\frac{N_0}{4} - \frac{D}{2}\right)\left[1 - \left(\frac{1-t}{0.1}\right)^2\right], & 0.9 < t \le 1. \end{cases}$$

Here, $t = N_{evals}/N_{max}$, where N_{evals} is the number of function evaluations. The exponent r governing the strength of the reduction is dimension dependent:

$$r = 1.17 + 2.075\, e^{-0.0567D}. \tag{10}$$

For $D = 5, 10, 30, 50, 100, 200$, the resulting values are $r = 2.74, 2.35, 1.56, 1.29, 1.18$, respectively. Since $r = 1$ corresponds to linear reduction, larger values accelerate decay. Higher decay rates are used for lower-dimensional problems because exploration in these cases typically requires fewer individuals and more opportunities for restart–refine cycles. In contrast, in high-dimensional settings, restarts tend to be less effective, and ARRDE therefore places greater emphasis on the underlying LSHADE/jSO search dynamics. The final population size is set to $D/2$, in contrast to the value of 4 used in LSHADE and jSO, as a larger terminal population promotes more effective refinement and exploration.

To supplement the LSHADE/jSO evolutionary process, ARRDE introduces a lightweight restart–refine mechanism based on the convergence indicator $s = \text{std}(f)/\ \text{mean}(f)$, where $\text{std}(f)$ and $\text{mean}(f)$ denote the standard deviation and mean of the objective values within the current population. A restart or refinement is triggered whenever s falls below a specified threshold s_{tol}, or when refinement is mandated by the schedule (e.g., at $t = 0.9$). At such points, the current population, fitness values, and parameter memories are stored within dedicated archives.

ARRDE determines whether the next stage constitutes a restart or a refinement based on the state of the current run. A restart is selected if the algorithm is in its first cycle, if the previous cycle involved refinement, if the current population has not produced an improvement in the global best solution, and if the number of consecutive restarts has not exceeded $N_{\text{rest,max}}$. Otherwise, a refinement is performed. A refinement step is always invoked at $t = 0.9$. Because the population is aggressively reduced over time, $N_{\text{rest,max}}$ is increased linearly with progress:

$$N_{\text{rest,max}} = 2 + 3t.$$

During refinement, the population is regenerated using individuals sampled from the archive, which may omit the global best solution. However, during the final refinement at $t = 0.9$, the best-so-far individual is explicitly inserted.

A key distinguishing feature of ARRDE is its use of *local exclusion constraints* during restart. When the population is regenerated uniformly within the bounds, the algorithm avoids regions that were previously explored. For each dimension d, ARRDE computes the mean μ_d and standard deviation σ_d across all archived populations and defines the interval

$$[\ell_d, u_d] = \Big[\max(\mu_d - \sigma_d, L_d),\ \min(\mu_d + \sigma_d, U_d)\Big],$$

where $[L_d, U_d]$ denotes the original bounds. Across multiple restarts, these intervals are merged into a collection of disjoint exclusion intervals. When a new candidate solution is generated, any coordinate falling within an exclusion interval is resampled outside the union of all such intervals. This procedure systematically directs restart populations away from previously converged regions while preserving feasibility within the original domain.

The complete ARRDE procedure is summarized in Algorithm 1.

4. Numerical Experiments and Discussions

To evaluate the performance of the proposed ARRDE algorithm, we conducted extensive numerical experiments on benchmark suites from CEC2017 [11], CEC2020 [13], CEC2022 [14], the 100-Digit Challenge from CEC2019[12], and real-world optimization problems from CEC2011 [10]. For all benchmark suites, the maximum number of function evaluations ($N_{\max}$) was set according to the respective competition rules. Each experiment was repeated 51 times to ensure statistical reliability. The run index was used as the seed for the global random number generator to guarantee reproducibility. All algorithms and test functions were implemented in C++ within the Minion framework [15], compiled using MSVC, and executed on a Windows 11 workstation.

The CEC2017, CEC2020, and CEC2022 benchmark suites consist of four categories of problems: unimodal, basic, hybrid, and composition functions. The problems are typically shifted, rotated, and include bias terms in their final objective values. Unimodal functions, such as the Bent Cigar and Zakharov functions, are single-peak problems that evaluate search efficiency. Basic functions include well-known multimodal landscapes such as Rastrigin, Schaffer, Lévy, and Schwefel functions. Hybrid functions combine the values of N basic functions computed over randomly assigned subcomponents of the search space. Composition functions are the most challenging category, formed by weighted mixtures of N basic functions where the weight distribution depends on the decision vector. These weight functions ensure that the global optimum of a composition function coincides with the optimum of one of its constituent basic functions. In the CEC2017 suite, there are 3 unimodal, 7 basic, 10 hybrid, and 10 composition functions. CEC2020 contains 1 unimodal, 3 basic, 3 hybrid, and 3 composition functions, while CEC2022 includes 1 unimodal, 4 basic, 3 hybrid, and 4 composition functions.

For CEC2017, the algorithm was tested on 29 problems at dimensions 10, 30, 50, and 100, with the maximum number of function evaluations set to $N_{\max} = 10^4 \times D$. Following the CEC2017 guidelines, problem F2 (the shifted and rotated Rastrigin function) was excluded due to numerical instability in higher dimensions. For CEC2020, the algorithm was evaluated on 10 problems at dimensions 5, 10, 15, and 20, with the corresponding evaluation budgets set to $N_{\max} = 5 \times 10^4$, 10^6, 3×10^6, and 10^7. For CEC2022, the algorithm was tested on 12 problems at dimensions 10 and 20, using $N_{\max} = 2 \times 10^5$ and 10^6, respectively. It is worth noting that although the CEC2017 suite contains higher-dimensional problems, it uses substantially lower evaluation budgets compared with the CEC2020 and CEC2022 suites. Conversely, CEC2020 represents the opposite extreme: relatively low-dimensional problems paired with exceptionally large evaluation budgets.

For the CEC2019 100-Digit Challenge, algorithms are evaluated under an effectively unlimited time budget. In this study, we impose a practical limit of $N_{\max} = 10^8$. The dimensionality of the problems ranges from 9 to 18, with most being 10-dimensional. For each problem, the number of correctly retrieved digits (up to the 10th decimal place) is recorded, and the final ranking is determined based on the average number of correct digits achieved in the best 25 out of 51 runs.

The CEC2011 real-world optimization suite comprises 22 problems with dimensionalities ranging from 6 to 212. These problems are derived from simplified formulations of practical engineering tasks, including FM sound wave parameter estimation, Lennard–Jones and Tersoff potential minimization, spread-spectrum radar polly phase code design, transmission network expansion planning (TNEP), transmission pricing, circular antenna array design, static and dynamic economic load dispatch (ELD/DED), hydrothermal scheduling, and spacecraft trajectory optimization for the *Messenger* and *Cassini 2* missions. Several of the original problems include inequality constraints in addition to bound constraints. Since our focus in this study is on bound-constrained optimization, these inequality constraints are omitted/ignored. Despite their simplifications, many CEC2011 problems remain very challenging due to their high dimensionality and multimodal landscapes. Following the CEC2011 benchmarking protocol, we evaluate all algorithms under three function-evaluation budgets: $N_{\max} = 5 \times 10^4$, 10^5, and 1.5×10^5.

These benchmark suites collectively assess an algorithm's ability to address a broad spectrum of optimization challenges, including separability, multimodality, rotation, non-linearity, and real-world modeling complexity. The experimental results and comparisons with state-of-the-art algorithms are presented and analyzed in the subsequent sections.

4.1. Scoring

When comparing the performance of different algorithms on a set of problems, it is crucial to define clear scoring metrics on which the final ranking is based. Aggregating multiple performance values into a single scalar score inevitably leads to a loss of information; therefore, the choice of scoring metric can significantly influence the resulting ranking. To obtain a more reliable picture of an algorithm's behavior, it is common to use more than one metric.

In general, two types of scores are frequently employed. The first is a *rank-based* score, which compares algorithms according to their relative performance on each problem (and, if applicable, each run). A key advantage of this scheme is that it is insensitive to the absolute scale of the performance metric and can therefore be applied uniformly across heterogeneous test problems. However, because any amount of improvement—no matter how small—is treated equivalently, an algorithm receives a higher rank whenever it marginally outperforms another. Consequently, in a benchmark suite with many functions, an algorithm that narrowly surpasses its competitors on most problems may achieve the best overall rank even if its mean or median error is substantially worse.

To mitigate this issue, and in line with the objective of this study to design a robust algorithm, a second type of metric—an *accuracy-based* score—is employed. This score evaluates how close the solution produced by an algorithm is to the known global optimum of each problem, typically through the final objective value or a relative error measure. In contrast to rank-based scoring, accuracy-based metrics differentiate between algorithms that achieve only negligible improvements and those that consistently yield solutions of substantially higher quality.

CEC competitions typically employ both rank-based and accuracy-based scores to determine the final ranking of submitted algorithms, thereby providing a more balanced and informative assessment of their performance.

In this work, for the *rank-based* score, we adopt the Friedman ranking. The Friedman test[20] is a widely used non-parametric statistical method for comparing multiple algorithms across multiple benchmark functions. For each problem, the algorithms are ordered according to their performance, and ranks are assigned accordingly, with tied results receiving the average of the corresponding ranks. Let N_A denote the number of algorithms, N_p the number of benchmark problems, and N_{runs} the number of independent trials. Denote by $r_{i,j,k}$ the rank of algorithm k on problem j in trial i. The per-problem Friedman average rank of algorithm k on problem j is defined as

$$R_{k,j} = \frac{1}{N_{\text{runs}}} \sum_{i=1}^{N_{\text{runs}}} r_{i,j,k}. \tag{11}$$

The overall Friedman score for algorithm k is then obtained by averaging its average ranks across all problems:

$$R_k = \frac{1}{N_p} \sum_{j=1}^{N_p} R_{k,j}. \tag{12}$$

Algorithms with smaller R_k values are considered superior. This score forms the *rank-based* metric used to assess overall algorithmic performance. The Friedman ranking reflects how consistently an algorithm outperforms its competitors across the benchmark suite. Importantly, each problem contributes equally to the final score, regardless of differences in scale, dimensionality, or numerical properties, making the metric robust and suitable for heterogeneous collections such as the CEC benchmark suites.

When comparing the performance of two algorithms to determine whether one is significantly better, worse, or equivalent, we employ the Mann–Whitney U test (also known as the Wilcoxon rank-sum test) [21]. This non-parametric test evaluates whether two independent samples originate from the same distribution, making it suitable for optimization results that may be non-normal, skewed, or contain outliers. A performance difference is considered statistically significant when the resulting P-value is smaller than the predefined significance level $\alpha = 0.05$. If significance is detected, the direction of superiority is determined by comparing the sum of ranks across all runs, with the algorithm obtaining the smaller rank sum deemed superior. We note that, although determining the winner using the mean or median of the results may appear intuitive, such criteria are not statistically consistent with the Mann–Whitney test, whose hypothesis and test statistic are defined entirely in terms of ranks rather than raw values.

For the *accuracy-based* scoring, we quantify performance using the relative error. For algorithm k, problem j, and run i, the average relative error across N_{runs} trials is defined as

$$\epsilon_{k,j} = \frac{1}{N_{\text{runs}}} \sum_{i=1}^{N_{\text{runs}}} \frac{f_j(x_i) - f_j(x^*)}{f_j(x^*)}, \tag{13}$$

where x_i denotes the solution obtained on run i and x^* denotes the known global optimum of problem j. For benchmark suites in which the global optimum is explicitly provided (e.g., CEC2017, CEC2019, CEC2020, CEC2022), this value is used directly. However, in the CEC2011 real-world optimization suite, the true global optimum is not available. In this case, we approximate x^* by taking the best solution found across all algorithms and all runs.

Because relative errors may vary by orders of magnitude across different problems, we apply a normalization to ensure that the contribution of each problem to the final score is uniformly bounded. Specifically, we define

$$\mathcal{E}_{k,j} = \frac{\epsilon_{k,j}}{1 + \epsilon_{k,j}}. \tag{14}$$

This transformation has two desirable properties: (i) for small relative errors, $\mathcal{E}_{k,j}$ closely approximates $\epsilon_{k,j}$, since

$$\mathcal{E}_{k,j} = \epsilon_{k,j} + O(\epsilon_{k,j}^2) \quad \text{as } \epsilon_{k,j} \to 0,$$

thereby preserving fine distinctions in high-accuracy results; (ii) for large relative errors, $\mathcal{E}_{k,j}$ asymptotically approaches 1, ensuring that no single poorly solved problem can dominate or distort the overall accuracy score.

The final accuracy-based score for algorithm k is computed by averaging the normalized errors across all N_p benchmark problems:

$$\mathcal{E}_k = \frac{1}{N_p} \sum_{j=1}^{N_p} \mathcal{E}_{k,j}. \tag{15}$$

This formulation highlights the importance of bounding $\mathcal{E}_{k,j}$. If the raw relative error $\epsilon_{k,j}$ were used directly, a single large error—arising, for example, from a difficult or unstable problem—could dominate the aggregated score, violating the principle that each problem should contribute equally to the evaluation. By employing the nonlinear mapping $\epsilon_{k,j} \mapsto \mathcal{E}_{k,j}$, we preserve meaningful differences when the algorithm performs well while preventing excessive influence from outlier cases, resulting in a balanced, problem-independent accuracy measure.

Following the cross-dimension combined scoring methodology adopted in CEC2017 and CEC2020, we construct a unified metric that integrates both the *rank-based* and *accuracy-based* components across all evaluated dimensions. This produces a single overall performance score for each algorithm. For algorithm k, the combined score $S_{tot,k}$ is defined as

$$S_{E,k} = \sum_D w_D \mathcal{E}_l^{(D)}, \tag{16}$$

$$S_{R,k} = \sum_D w_D R_l^{(D)}, \tag{17}$$

$$S_{tot,k} = 50 \left[\frac{\min_l S_{E,l}}{S_{E,k}} + \frac{\min_l S_{R,l}}{S_{R,k}} \right], \tag{18}$$

where $\mathcal{E}_k^{(D)}$ and $R_k^{(D)}$ denote the accuracy-based and rank-based scores of algorithm k at dimension D, respectively, and w_D is a dimension-dependent weight reflecting the relative importance of each dimensional setting. For the CEC benchmark suites, the weights w_D follow the official cross-dimension aggregation rules:

- **CEC2017:** $w_D = 0.1, 0.2, 0.3, 0.4$ for $D = 10, 30, 50, 100$.
- **CEC2020:** $w_D = 0.1, 0.2, 0.3, 0.4$ for $D = 5, 10, 15, 20$.
- **CEC2022:** $w_D = 0.1, 0.2$ for $D = 10, 20$.
- **CEC2011:** $w_D = 1$ for all dimensions and all three prescribed limits on the maximum number of function evaluations $N_{\max}$, since no cross-dimension weighting is required.

For comparing algorithms within each individual dimension, we also define a per-dimension combined score that integrates the accuracy-based and rank-based components at dimension D:

$$S_k^{(D)} = 50 \left[\frac{\min_l \mathcal{E}_l^{(D)}}{\mathcal{E}_k^{(D)}} + \frac{\min_l R_l^{(D)}}{R_k^{(D)}} \right], \tag{19}$$

This yields a unified performance indicator for each dimension, consistent with the overall scoring formulation.

For completeness, we also include the original scoring procedures used in CEC2017 and CEC2020. In CEC2017, the overall scoring formula is structurally similar to (18), but the rank-based and accuracy-based components differ. In the original CEC2017 and CEC2020 rules, the rank-based component R_k is computed from the ranks of the *mean* performance values across runs, rather than from the run-wise rank sums used in (12) and (11). For the accuracy-based component, CEC2017 uses the absolute error

$$\mathcal{E}_k^{\text{CEC2017}} = \sum_j^{N_p} \sum_{i=1}^{N_{\text{runs}}} \Big(f_j(x_i) - f_j(x^*) \Big), \tag{20}$$

where i, j, and k index the run, problem, and algorithm, respectively. In CEC2020, the accuracy score is defined using a normalized error:

$$\mathcal{E}_k^{\text{CEC2020}} = \sum_j^{N_p} \frac{f_j(x_k^{\text{best}}) - f_j(x^*)}{f_j(x^{\text{best,max}}) - f_j(x^*)}, \tag{21}$$

where $f_j(x_k^{\text{best}})$ is the best value obtained by algorithm k, and $f_j(x^{\text{best,max}})$ is the worst among the best values obtained across all algorithms.

For the CEC2019 100-Digit Challenge, in addition to reporting the $\mathcal{E}$ and S scores described earlier, we also include its official scoring procedure. In this scheme, algorithms are ranked according to the average number of correctly recovered digits, computed from their best 25 runs.

As a remark, we note that while these formulations are effective within their respective competitions, they exhibit several limitations compared to the accuracy metric proposed in this work. In particular, the use of absolute errors in the CEC2017 scoring can distort cross-problem aggregation, since functions with large objective-value scales may dominate the final score and violate the principle that each problem should contribute equally. The CEC2020 formulation, although normalized to the interval $[0, 1]$, relies solely on the single best result per algorithm and therefore discards information from all remaining trials. Moreover, its normalization by the difference between the best and worst algorithm can be unstable when this denominator is small, allowing even minor performance differences to be exaggerated.

In contrast, our bounded relative-error formulation preserves fine-grained accuracy information across all runs, ensures consistent weighting among problems, and avoids numerical instabilities arising from small denominators. Given that our primary goal is robustness and fairness across heterogeneous benchmarks, ARRDE is fine-tuned and evaluated using the S_{tot} scoring in (18), rather than the original scoring rules prescribed in the CEC benchmark suites.

4.2. Results

In this subsection, we present the numerical results for all tested algorithms—LSHADE, jSO, LSHADE-cnEpSin (known to perform well on CEC2017), j2020 (strong on CEC2020), and NLSHADE-RSP (strong on CEC2021)—and compare them against ARRDE on the CEC2011, CEC2017, CEC2019, CEC2020, and CEC2022 benchmark suites under their recommended $N_{\max}$ values. Complete error statistics (best, mean, and standard deviation over 51 runs) for every function and algorithm are provided in Appendix A. Here, we focus on aggregated performance across all functions, evaluated using the accuracy-based score $\mathcal{E}$, the rank-based score R, the combined score S, and their corresponding per-dimension variants.

On CEC2017, Tables 1–2 reveal a clear three-tier structure. ARRDE, jSO, and LSHADE-cnEpSin form a tightly grouped top tier, with ARRDE achieving the best overall combined score, followed by jSO and LSHADE-cnEpSin. This ordering mirrors the official CEC2017 competition results, where jSO and LSHADE-cnEpSin ranked second and third. Dimension-wise, ARRDE dominates at 10D and remains extremely close to jSO at 30D and 50D, where jSO attains slightly higher S values. At 100D, LSHADE-cnEpSin takes the lead, with ARRDE a close second. Because the numerical differences among these three algorithms are small, their performance on CEC2017 should be viewed as broadly comparable rather than sharply separated. Indeed, we observed that minor adjustments to ARRDE's parameters can shift the dimension-wise ordering relative to jSO and LSHADE-cnEpSin, which is unsurprising given that ARRDE incorporates several of jSO's design components.

The second tier is represented by LSHADE, whose performance gap relative to the top group is substantial and consistent across all dimensions.

The third tier consists of j2020 and NLSHADE-RSP, which perform markedly worse on CEC2017. While NLSHADE-RSP remains competitive at $D = 10$—ranking third behind ARRDE and jSO—its performance degrades steadily at higher dimensions, placing fifth at $D = 30$, 50, and 100. This behaviour is expected: both NLSHADE-RSP and j2020 were developed and tuned primarily for the CEC2020 and CEC2021 suites, which consist of low-dimensional problems ($D \leq 20$) with very generous evaluation budgets. Their advantages in those settings do not generalize well to the higher-dimensional, stricter-budget environment of CEC2017.

Finally, we note a small discrepancy between the aggregated combined score S_{tot} and the official CEC2017 score S_{2017}. As discussed earlier, this difference stems from the underlying error-normalization schemes: the official score uses absolute final errors, whereas our accuracy-based measure $\mathcal{E}$ employs a relative-error formulation. Nevertheless, under both scoring methods, ARRDE ranks first overall.

The CEC2020 results in Tables 3–4 reveal a markedly different, yet complementary, performance landscape. Although

Table 1: Performance of six algorithms on the CEC2017 benchmark suite for 10D and 30D. Reported are the accuracy-based score $\mathcal{E}$, rank-based score R, combined score S, and win/tie/loss (W/T/L) across all 29 functions, where W/T/L counts ARRDE's wins, ties, and losses against each algorithm. Parentheses denote ranks, and boldface marks the best result.

Algorithm	10D				30D			
	$\mathcal{E}$	R	S	W/T/L	$\mathcal{E}$	R	S	W/T/L
ARRDE	**0.031 (1)**	**3.048 (1)**	**100.000 (1)**	0/29/0	0.094 (2)	2.701 (2)	96.282 (2)	0/29/0
LSHADE	0.037 (4)	3.443 (3)	86.046 (4)	11/12/6	0.105 (4)	2.950 (3)	86.904 (4)	14/6/9
jSO	0.036 (3)	3.246 (2)	90.064 (2)	8/13/8	**0.092 (1)**	**2.551 (1)**	**100.000 (1)**	7/15/7
LSHADE-cnEpSin	0.069 (6)	3.509 (4)	65.870 (6)	13/11/5	0.096 (3)	2.981 (4)	90.730 (3)	13/13/3
j2020	0.037 (5)	4.178 (6)	77.991 (5)	19/8/2	0.200 (6)	5.299 (6)	47.010 (6)	25/4/0
NLSHADE-RSP	0.035 (2)	3.576 (5)	87.396 (3)	12/9/8	0.168 (5)	4.517 (5)	55.599 (5)	22/4/3

Table 2: Performance of six algorithms on the CEC2017 benchmark suite for 50D and 100D, with combined results across dimensions. The column S_{2017} gives the original CEC2017 score. Reported are the accuracy-based score $\mathcal{E}$, rank-based score R, combined score S, and win/tie/loss (W/T/L) across all 29 functions, where W/T/L counts ARRDE's wins, ties, and losses against each algorithm. Parentheses indicate ranks and boldface marks the best result.

Algorithm	50D				100D				Combined	
	$\mathcal{E}$	R	S	W/T/L	$\mathcal{E}$	R	S	W/T/L	S_{tot}	S_{2017}
ARRDE	0.160 (2)	**2.378 (1)**	99.163 (2)	0/29/0	0.252 (2)	**2.273 (1)**	98.599 (2)	0/29/0	**100.000 (1)**	**98.984 (1)**
LSHADE	0.169 (4)	2.995 (4)	86.250 (4)	14/9/6	0.294 (4)	3.311 (4)	76.022 (4)	23/6/0	83.296 (4)	83.284 (4)
jSO	**0.157 (1)**	2.400 (2)	**99.535 (1)**	9/11/9	0.260 (3)	2.550 (3)	91.647 (3)	11/9/9	97.164 (2)	97.637 (2)
LSHADE-cnEpSin	0.164 (3)	2.747 (3)	91.138 (3)	15/9/5	**0.245 (1)**	2.286 (2)	**99.734 (1)**	9/7/13	95.162 (3)	90.048 (3)
j2020	0.403 (6)	5.354 (6)	41.683 (6)	27/2/0	0.596 (6)	5.423 (6)	41.514 (6)	27/0/2	44.644 (6)	22.471 (6)
NLSHADE-RSP	0.362 (5)	5.125 (5)	44.875 (5)	27/2/0	0.546 (5)	5.157 (5)	44.497 (5)	28/1/0	48.816 (5)	38.012 (5)

Table 3: Performance of six algorithms on the CEC2020 benchmark suite for 5D and 10D. Reported are the accuracy-based score $\mathcal{E}$, rank-based score R, combined score S, and win/tie/loss (W/T/L) across all 10 functions, where W/T/L counts ARRDE's wins, ties, and losses against each algorithm. Parentheses denote ranks, and boldface marks the best result.

Algorithm	5D				10D			
	$\mathcal{E}$	R	S	W/T/L	$\mathcal{E}$	R	S	W/T/L
ARRDE	**0.008 (1)**	3.225 (2)	**91.578 (1)**	0/10/0	**0.009 (1)**	2.970 (3)	**93.628 (1)**	0/10/0
LSHADE	0.017 (4)	3.717 (4)	58.982 (4)	2/7/1	0.030 (5)	4.157 (5)	46.375 (5)	6/3/1
jSO	0.018 (6)	3.927 (6)	55.709 (6)	4/5/1	0.029 (4)	3.760 (4)	50.545 (4)	6/1/3
LSHADE-cnEpSin	0.018 (5)	3.789 (5)	57.185 (5)	4/5/1	0.032 (6)	4.615 (6)	42.332 (6)	7/2/1
j2020	0.010 (2)	3.661 (3)	77.504 (3)	4/6/0	0.011 (2)	2.908 (2)	87.475 (2)	4/2/4
NLSHADE-RSP	0.010 (3)	**2.681 (1)**	89.218 (2)	1/5/4	0.018 (3)	**2.591 (1)**	75.426 (3)	2/3/5

Table 4: Performance of six algorithms on the CEC2020 benchmark suite for 15D and 20D, with combined results across dimensions. The column S_{2020} shows the original CEC2020 score. Reported are the accuracy-based score $\mathcal{E}$, rank-based score R, combined score S, and win/tie/loss (W/T/L) across all 10 functions, where W/T/L counts ARRDE's wins, ties, and losses. Parentheses denote ranks, and boldface marks the best result.

Algorithm	15D				20D				Combined	
	$\mathcal{E}$	R	S	W/T/L	$\mathcal{E}$	R	S	W/T/L	S_{tot}	S_{2020}
ARRDE	**0.018 (1)**	2.940 (2)	**93.781 (1)**	0/10/0	**0.023 (1)**	2.904 (2)	**96.691 (1)**	0/10/0	**94.653 (1)**	72.937 (3)
LSHADE	0.034 (4)	3.720 (4)	61.447 (4)	6/2/2	0.037 (4)	3.868 (4)	66.578 (5)	7/1/2	60.805 (4)	46.847 (4)
jSO	0.039 (5)	4.198 (5)	54.405 (5)	7/2/1	0.036 (3)	3.949 (5)	66.892 (4)	7/1/2	59.284 (5)	44.585 (5)
LSHADE-cnEpSin	0.040 (6)	4.285 (6)	53.035 (6)	6/2/2	0.044 (6)	4.380 (6)	57.751 (6)	7/1/2	53.666 (6)	39.606 (6)
j2020	0.027 (3)	3.282 (3)	73.380 (3)	5/2/3	0.039 (5)	3.187 (3)	72.483 (3)	4/1/5	73.846 (3)	86.748 (2)
NLSHADE-RSP	0.022 (2)	**2.575 (1)**	91.491 (2)	2/2/6	0.032 (2)	**2.712 (1)**	86.696 (2)	4/2/4	86.431 (2)	**100.000 (1)**

the CEC2020 problems are mostly low-dimensional, their extremely large evaluation budgets promote extensive exploration. Under our combined metric, ARRDE clearly wins overall: it attains the highest S_{tot} and ranks first in the S–score for all four dimensions (5D, 10D, 15D, and 20D). Interestingly, however, the rank-based score R tells a different story—ARRDE does not rank first in any single dimension. NLSHADE-RSP most frequently achieves the best R–score, with j2020 takes second place at 10D. Yet ARRDE's consistently superior $\mathcal{E}$ values compensate for these rank differences, yielding the highest combined performance.

This behavior highlights a defining characteristic of ARRDE: when it wins, it tends to win decisively. This is especially evident on the composition function F10 (see Tables A.14–A.17), where ARRDE achieves substantially lower errors than all competitors. Conversely, when ARRDE loses to NLSHADE-RSP or j2020 on the simpler basic and hybrid functions, the margins are extremely small—typically within $\lesssim 0.001\%$ relative to the true optimum—producing negligible effects on the accuracy-based score.

The performance of the algorithms that excel on CEC2017—jSO, LSHADE-cnEpSin, and LSHADE—is also noteworthy. Their ranking is almost completely reversed on CEC2020: LSHADE now outperforms jSO and LSHADE-cnEpSin, and jSO surpasses LSHADE-cnEpSin. Apart from ARRDE, which consistently retains first place in S_{tot}, the ranking structure in CEC2020 is largely anti-correlated with that observed in CEC2017. This further illustrates the strong suite-specific behavior of many DE variants and underscores the robustness advantage exhibited by ARRDE.

We also observe that the official CEC2020 scoring produces rankings that differ substantially from those obtained using our $\mathcal{E}$- and R-based metrics. The discrepancy stems from the normalization scheme used in the official score S_{2020}, where each function error is normalized using the worst of the best results among all compared algorithms. Under this scheme, extremely small differences on easy functions—for example, objective values such as 2100.001 vs. 2100.12—are mapped to normalized errors of 0 and 1, respectively. As a result, ARRDE is frequently ranked below NLSHADE-RSP or j2020 on functions where the true numerical difference is negligible, leading to artificially inflated S_{2020} values for those algorithms.

A second contributing factor is that S_{2020} evaluates performance using only the *best* run out of 51, rather than an average across runs. This design places disproportionate weight on rare lucky outcomes and under-represents typical algorithmic behaviour. Consequently, algorithms such as j2020 and NLSHADE-RSP can rank above ARRDE under S_{2020} even when their mean performance is worse and their advantage appears only in isolated runs.

On CEC2022, which includes 10D and 20D problems under relatively large evaluation budgets (though still smaller than those in CEC2020), ARRDE again emerges as the clear overall winner, as shown in Table 5. It achieves the highest combined score S_{tot}, and in 20D it attains the best results in all three metrics ($\mathcal{E}$, R, and S). In 10D, j2020 slightly surpasses ARRDE in the combined score S, but ARRDE still ranks first in R and remains very close in accuracy $\mathcal{E}$. Across both dimensions, ARRDE exhibits the strongest W/T/L profile, consistently winning more pairwise comparisons than it loses on all 12 functions. The detailed error statistics in Table A.19 further show that ARRDE solves the more challenging functions (notably F4 and F11) substantially better than the other algorithms.

What happens when the dimensionality becomes very high while the available function evaluations $N_{\max}$ remain relatively small? The CEC2011 real-world problem suite provides exactly this scenario. Many of its 22 problems are high-dimensional—several have dimension 96, and some reach 120, 140, or even 216—yet the evaluation budgets are limited to only 50,000, 100,000, and 150,000. For comparison, the CEC2017 benchmark prescribes $10{,}000 \times D$ evaluations, which is substantially larger for these dimensions. Under such constrained budgets, one would expect algorithms that excel on high-dimensional artificial benchmarks such as CEC2017 to also perform competitively here. This expectation is largely confirmed by the results: LSHADE-cnEpSin performs strongly across all budgets, and ARRDE ranks first overall according to the combined score S_{tot} (Tables 6-7).

At the smallest evaluation budget $N_{\max} = 50{,}000$ (Table 6), ARRDE clearly dominates. It achieves the best in both $\mathcal{E}$ and R scores. LSHADE-cnEpSin and LSHADE form the next tier, but both show noticeably higher errors and weaker combined scores. When the budget is increased to $N_{\max} = 100{,}000$ and 150,000, the picture becomes more nuanced. At both budgets, ARRDE maintains the best accuracy-based score $\mathcal{E}$ and the best combined score $S^{(D)}$. However, LSHADE-cnEpSin achieves the best rank-based score R, and thus appears more competitive from a purely rank-based perspective. This indicates that LSHADE-cnEpSin narrowly wins on a slightly larger number of individual problems, but with larger error margins on the instances where it loses. In contrast, ARRDE attains systematically lower average errors across the suite, which is captured by $\mathcal{E}$ and reflected in the higher combined score S.

The aggregated results over all three budgets reinforce this picture. ARRDE attains the highest overall combined score $S_{\text{tot}} = 100$, with LSHADE-cnEpSin in second place and LSHADE and jSO forming a clearly weaker middle tier. j2020 and NLSHADE-RSP consistently occupy the last two positions, indicating that their strengths on CEC2020-style settings do not transfer well to these predominantly high-dimensional, low-budget real-world problems.

An interesting observation is the relative behavior of jSO and LSHADE. On CEC2017, jSO typically ranks ahead of LSHADE, whereas on CEC2011 the situation is reversed in terms of the accuracy-based score $\mathcal{E}$: although jSO often obtains slightly better rank-based scores R, LSHADE achieves lower average errors. This suggests that the high dimensionality and real-world structure of many CEC2011 problems amplify

Table 5: Performance of six algorithms on the CEC2022 benchmark suite for 10D and 20D, with combined results across dimensions. Reported are the accuracy-based score $\mathcal{E}$, rank-based score R, combined score S, and win/tie/loss (W/T/L) across all 12 functions, where W/T/L counts ARRDE's wins, ties, and losses. Parentheses denote ranks, and boldface marks the best result.

Algorithm	10D				20D				Combined
	$\mathcal{E}$	R	S	W/T/L	$\mathcal{E}$	R	S	W/T/L	S_{tot}
ARRDE	0.014 (3)	**3.029 (1)**	94.196 (2)	0/12/0	**0.017 (1)**	**2.445 (1)**	**100.000 (1)**	0/12/0	**100.000 (1)**
LSHADE	0.017 (5)	3.958 (6)	76.196 (5)	6/6/0	0.035 (4)	3.605 (4)	58.123 (5)	6/6/0	63.223 (5)
jSO	0.016 (4)	3.530 (4)	82.140 (4)	5/6/1	0.035 (3)	3.354 (2)	60.803 (3)	6/6/0	66.751 (3)
LSHADE-cnEpSin	0.017 (6)	3.771 (5)	76.145 (6)	5/7/0	0.035 (5)	3.580 (3)	58.231 (4)	5/7/0	63.590 (4)
j2020	**0.013 (1)**	3.277 (2)	**96.210 (1)**	2/9/1	0.026 (2)	3.934 (5)	64.072 (2)	7/4/1	73.128 (2)
NLSHADE-RSP	0.013 (2)	3.435 (3)	90.695 (3)	3/8/1	0.037 (6)	4.082 (6)	52.910 (6)	7/4/1	61.664 (6)

Table 6: Performance of six algorithms on the CEC2011 real-world benchmark suite under evaluation budgets of $N_{\max}$ = 50,000 and 100,000. Reported are the accuracy-based score $\mathcal{E}$, rank-based score R, combined score S, and win/tie/loss (W/T/L) across all 22 functions, where W/T/L counts ARRDE's wins, ties, and losses. Parentheses denote ranks, and boldface marks the best result.

Algorithm	N_{max} = 50000				N_{max} = 100000			
	$\mathcal{E}$	R	S	W/T/L	$\mathcal{E}$	R	S	W/T/L
ARRDE	**0.100 (1)**	**2.166 (1)**	**100.000 (1)**	0/22/0	**0.066 (1)**	2.592 (2)	**99.751 (1)**	0/22/0
LSHADE	0.152 (2)	3.344 (4)	65.320 (3)	14/5/3	0.094 (2)	3.128 (4)	76.359 (3)	11/7/4
jSO	0.164 (3)	3.288 (3)	63.307 (4)	13/6/3	0.112 (4)	3.120 (3)	71.007 (4)	9/11/2
LSHADE-cnEpSin	0.168 (4)	2.802 (2)	68.297 (2)	13/4/5	0.099 (3)	**2.579 (1)**	83.537 (2)	6/8/8
j2020	0.415 (6)	5.201 (6)	32.844 (6)	18/4/0	0.349 (6)	5.265 (6)	33.998 (6)	18/3/1
NLSHADE-RSP	0.261 (5)	4.198 (5)	44.951 (5)	17/4/1	0.186 (5)	4.315 (5)	47.736 (5)	16/4/2

Table 7: Performance of six algorithms on the CEC2011 real-world benchmark suite at $N_{\max}$ = 150,000, together with combined results across all three evaluation budgets. Reported are the accuracy-based score $\mathcal{E}$, rank-based score R, combined score S, and win/tie/loss (W/T/L) across all 22 functions, where W/T/L counts ARRDE's wins, ties, and losses. Parentheses denote ranks, and boldface marks the best result.

Algorithm	N_{max} = 150000				Combined
	$\mathcal{E}$	R	S	W/T/L	S_{tot}
ARRDE	**0.072 (1)**	2.706 (2)	**98.049 (1)**	0/22/0	**100.000 (1)**
LSHADE	0.085 (3)	3.067 (3)	84.572 (4)	10/9/3	75.062 (3)
jSO	0.083 (2)	3.094 (4)	85.499 (2)	10/8/4	72.445 (4)
LSHADE-cnEpSin	0.102 (4)	**2.601 (1)**	85.089 (3)	7/5/10	78.952 (2)
j2020	0.317 (6)	5.274 (6)	35.991 (6)	18/3/1	34.719 (6)
NLSHADE-RSP	0.151 (5)	4.258 (5)	54.318 (5)	16/4/2	49.140 (5)

Table 8: Performance of six algorithms on the CEC2019 benchmark suite under a budget of $N_{\max} = 10^8$, together with rankings from the original CEC2019 scoring rule. Reported are the accuracy-based score $\mathcal{E}$, rank-based score R, combined score S, and win/tie/loss (W/T/L) across all 10 functions (seven at 10D and the remainder at 9D, 16D, and 18D). W/T/L counts ARRDE's wins, ties, and losses against each algorithm. Parentheses denote ranks, and boldface marks the best result.

Algorithm	$N_{max} = 1e + 8$				
	$\mathcal{E}$	R	S	W/T/L	S_{2019}
ARRDE	**0.009 (1)**	**2.627 (1)**	**100.000 (1)**	0/10/0	83.000 (2)
LSHADE	0.205 (6)	4.173 (6)	33.666 (6)	5/4/1	63.280 (6)
jSO	0.110 (3)	3.065 (2)	46.943 (3)	4/6/0	75.440 (3)
LSHADE-cnEpSin	0.179 (5)	4.006 (5)	35.295 (5)	5/5/0	64.560 (5)
j2020	0.009 (2)	3.368 (3)	87.251 (2)	3/6/1	**87.251 (1)**
NLSHADE-RSP	0.127 (4)	3.762 (4)	38.448 (4)	4/6/0	66.720 (4))

the advantages of LSHADE's more conservative adaptation dynamics and its ability to maintain larger effective population diversity, leading to more stable and reliable progress under tight evaluation budgets.

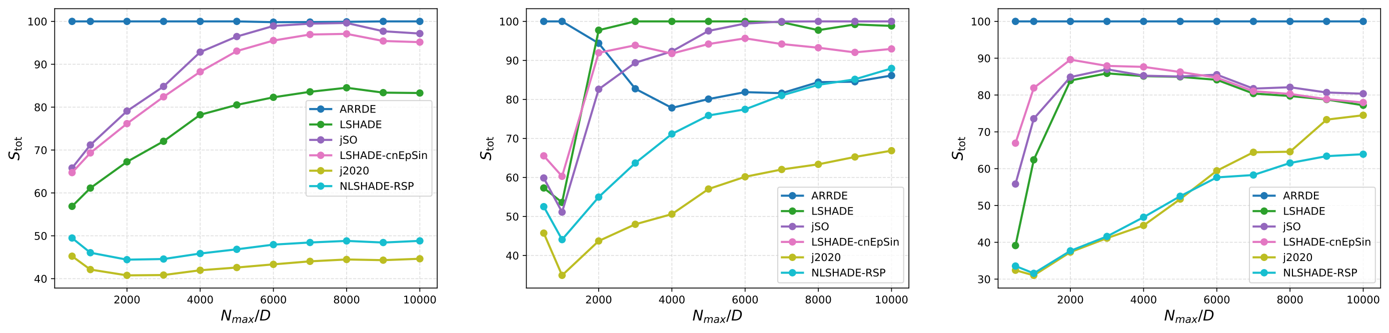
While CEC2011 represents the case of very high dimensionality under relatively restricted evaluation budgets, the CEC2019 100-Digit Challenge presents the opposite setting. It focuses on low-dimensional problems (seven functions at 10D and the remaining ones at 9D, 16D, and 18D) but allows an extremely large evaluation budget. In the original competition, many participating algorithms used $N_{\max} > 10^9$. In our experiments, we adopt $N_{\max} = 10^8$, which, although very large compared to the other CEC suites, is still far from the effectively unlimited budgets used by some competition entries. Such a generous budget largely eliminates stagnation caused by evaluation limits and allows algorithms to explore the search space extensively. As observed earlier for CEC2020, settings with abundant evaluations tend to reward algorithms with strong exploratory and global-search behavior. The results in Table 8 show that ARRDE performs exceptionally well under these conditions.

According to our accuracy-based metric $\mathcal{E}$, ARRDE attains the lowest error among all competitors, achieving a value of 0.009 (rank 1). j2020 obtains errors of the same order (0.009) but is slightly worse on several functions, placing it in second position. ARRDE also achieves the best rank-based score R and the highest combined score $S = 100$. The W/T/L statistics of the *other* algorithms relative to ARRDE indicate that ARRDE wins more often than it loses across the ten problems, consistent with its leading position in both $\mathcal{E}$ and S.

A notable difference emerges when comparing our ranking with the official competition ranking S_{2019}. As discussed previously, the CEC2019 scoring rule computes the average number of correct digits using only the 25 best solutions found by the algorithm across all runs. Under this metric, ARRDE achieves a score of 83 (corresponding to an average of 8.3 correct digits), while j2020 attains the highest score of 87. This discrepancy is driven primarily by function F8, where j2020 occasionally finds the full 10 correct digits in rare runs, thereby boosting its digit-accuracy score. Consequently, j2020 ranks first under S_{2019} even though ARRDE wins more often overall.

4.3. CEC2017, CEC2020, CEC2022 with Varying N_{max}

Our results on CEC2011, CEC2017, CEC2019, CEC2020, and CEC2022 show that ARRDE performs strongly across all suites when evaluated under the official $N_{\max}$ prescribed by each



(a) CEC2017 (b) CEC2020 (c) CEC2022

Figure 1: Combined score $S_{\rm tot}$ as a function of normalized evaluation budget $N_{\max}/D$ for the CEC2017, CEC2020, and CEC2022 benchmark suites.

competition. A natural question, however, is how ARRDE behaves under different evaluation budgets. As shown in [9], the ranking of algorithms can change drastically when $N_{\max}$ departs from the value specified by the competition. Since robustness with respect to varying budgets is crucial in practical optimization—particularly when objective evaluations are expensive—we additionally examine the performance of ARRDE across a broad range of evaluation limits. In many real-world applications, even $N_{\max} = 50{,}000$ (as used in CEC2011) is prohibitively large, underscoring the importance of understanding algorithmic behavior under smaller budgets. To this end, we evaluate ARRDE and all competing algorithms under varying limits of the form $N_{\max} = 500D, 1000D, \ldots, 10{,}000D$.

Figure 1 summarizes the behaviour of the combined score S as a function of the normalized evaluation budget $N_{\max}/D$ for CEC2017, CEC2020, and CEC2022. For CEC2017 [Fig. 1(a)], ARRDE remains essentially at the maximum value $S = 100$ across the entire range of $N_{\max}/D$. The gap between ARRDE and the next-best algorithms (jSO and LSHADE-cnEpSin) gradually narrows as the budget increases, but the relative order among algorithms is perfectly preserved compared with Table 2. Thus, for this suite, increasing the evaluation budget does not change the ranking, only the magnitude of the separation.

The CEC2020 curves in Fig. 1(b) display a very different pattern. ARRDE is the top performer for $N_{\max}/D < 2000$, but its S score decreases as the budget grows, reaching a minimum around $N_{\max}/D \approx 4000$ before slowly improving again. In contrast, LSHADE, LSHADE-cnEpSin, and jSO exhibit a sharp rise in performance for $1000 < N_{\max}/D < 3000$, after which their scores plateau. j2020 and NLSHADE-RSP show steady improvement once $N_{\max}/D > 1000$. From the detailed results in Table 4, we know that ARRDE, j2020, and NLSHADE-RSP are the strongest algorithms at the official CEC2020 settings, where $N_{\max}$ is very large (often exceeding $100{,}000D$). The S–curves explain this shift: LSHADE, jSO, and LSHADE-cnEpSin improve for moderate budgets but fail to scale their performance further, while ARRDE, j2020, and NLSHADE-RSP continue to benefit from large evaluation budgets. This suite thus clearly illustrates that ranking is highly dependent on the evaluation budget.

The behaviour on CEC2022 [Fig. 1(c)] resembles that of CEC2017. ARRDE again maintains $S = 100$ across all budget levels, and the gap to the other algorithms remains consistently large, indicating that ARRDE dominates this suite regardless of $N_{\max}/D$. LSHADE, jSO, and LSHADE-cnEpSin converge to very similar scores when $N_{\max}/D > 2000$, preserving the relative ranking found in Table 5 for the official (large) budgets. The distinctive behaviour in this suite comes from j2020: its score remains relatively low for $N_{\max}/D < 5000$, then increases steadily and approaches its ranking from Table 5 at large budgets. Thus, aside from the low-budget region where j2020 underperforms, the ranking structure is again stable.

Overall, these S–curves show that ARRDE is the least sensitive to evaluation budgets across all three benchmark suites: it reaches near-optimal performance even at small budgets and maintains it as the budget grows. In contrast, several competing algorithms exhibit strong budget dependence, sometimes performing well only at small or very large $N_{\max}/D$. These results further reinforce ARRDE's robustness and its ability to generalize across heterogeneous problem types and evaluation regimes.

5. Conclusion

This work introduced the Adaptive Restart–Refine Differential Evolution (ARRDE) algorithm, a DE variant designed for robustness across heterogeneous problem types and evaluation budgets. ARRDE combines nonlinear population-size reduction with an adaptive restart–refine mechanism, enabling a dynamic balance between exploration and exploitation without requiring extensive parameter tuning.

To evaluate robustness broadly, we tested ARRDE on five benchmark suites—CEC2011, CEC2017, CEC2019, CEC2020, and CEC2022—spanning real-world problems, hybrid and composition functions, and both low- and high-dimensional settings. Using both Friedman-rank–based and relative-error–based scoring metrics, we observe that several competing algorithms exhibit strong but highly suite-specific behavior (e.g., jSO and LSHADE-cnEpSin on CEC2017, j2020 and NLSHADE-RSP on CEC2020), with performance dropping sharply outside their preferred environments. In contrast, ARRDE performs consistently well across all five suites, ranking first on every benchmark suite considered here.

We also examined performance as the normalized evaluation budget $N_{\max}/D$ varies from 500 to 10 000. The combined-score curves $S(N_{\max}/D)$ show that algorithm rankings can shift with the budget—most notably on CEC2020. ARRDE, however, remains highly stable on CEC2017 and CEC2022, maintaining $S = 100$ across all budgets, and the relative ordering of the competing algorithms is generally preserved, with only the performance gaps changing. On CEC2020, ARRDE is not the top performer within this budget range, but it still performs better than j2020 and NLSHADE-RSP. These results indicate that ARRDE's performance is comparatively stable across a wide range of evaluation budgets.

In summary, ARRDE provides strong and consistent performance under a broad range of benchmark conditions and evaluation regimes. While some specialized algorithms excel under narrow or suite-specific scenarios, ARRDE offers the most reliable general-purpose behavior among the methods tested here.

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Appendix A. Extended Error Statistics for CEC2017, CEC2019, CEC2020, CEC2022, and CEC2011 Benchmark Suites

Table A.9: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2017 benchmark suite at dimension 10. Each value is computed over 51 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F1	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F3	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F4	mean	**0.000e+00**	**0.000e+00**	2.157e-04	1.755e-01	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	1.391e-03	2.220e-01	**0.000e+00**	**0.000e+00**
	best	9.950e-01	**0.000e+00**	9.950e-01	9.950e-01	**0.000e+00**	**0.000e+00**
F5	mean	2.306e+00	1.951e+00	3.556e+00	3.305e+00	**1.697e+00**	1.855e+00
	std	9.967e-01	8.803e-01	1.048e+00	1.521e+00	8.655e-01	**8.128e-01**
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F6	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	1.079e+01	**8.863e+00**	9.552e+00	1.177e+01	1.096e+01	1.090e+01
F7	mean	1.214e+01	1.223e+01	1.444e+01	1.502e+01	1.203e+01	**1.196e+01**
	std	6.178e-01	9.684e-01	1.579e+00	1.970e+00	**5.637e-01**	6.447e-01
	best	9.950e-01	**0.000e+00**	1.990e+00	9.950e-01	**0.000e+00**	**0.000e+00**
F8	mean	2.499e+00	1.814e+00	3.834e+00	3.360e+00	**1.717e+00**	1.894e+00
	std	**8.215e-01**	9.389e-01	1.072e+00	1.605e+00	9.669e-01	8.419e-01
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F9	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	4.500e-01	1.900e-01	1.900e-01	6.830e+00	**6.000e-02**	3.000e-01
F10	mean	4.575e+01	1.047e+02	9.930e+01	1.096e+02	5.605e+01	**3.186e+01**
	std	5.806e+01	1.039e+02	7.540e+01	7.931e+01	5.931e+01	**4.670e+01**
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F11	mean	4.373e-02	**0.000e+00**	6.022e-01	1.805e+00	**0.000e+00**	**0.000e+00**
	std	2.352e-01	**0.000e+00**	5.582e-01	1.646e+00	**0.000e+00**	**0.000e+00**
	best	**0.000e+00**	**0.000e+00**	6.100e-01	2.500e-01	**0.000e+00**	**0.000e+00**
F12	mean	2.124e+01	1.480e+01	1.444e+02	7.617e+01	**2.673e-01**	9.710e+01
	std	4.528e+01	3.839e+01	2.213e+02	6.776e+01	**1.809e-01**	6.845e+01
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	9.900e-01	**0.000e+00**	**0.000e+00**
F13	mean	2.403e+00	3.584e+00	5.042e+00	6.531e+00	**6.149e-01**	4.289e+00
	std	2.464e+00	2.067e+00	2.537e+00	2.398e+00	**1.499e+00**	2.392e+00
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F14	mean	3.031e-01	4.300e-01	2.333e-01	7.325e-01	**3.882e-02**	**3.882e-02**
	std	6.472e-01	1.831e+00	4.197e-01	6.486e-01	**1.922e-01**	**1.922e-01**
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F15	mean	1.747e-01	1.596e-01	**1.298e-01**	1.154e+00	3.216e-01	2.676e-01
	std	1.993e-01	**1.707e-01**	2.770e-01	7.781e-01	2.027e-01	2.186e-01
	best	1.200e-01	2.000e-02	2.000e-02	2.000e-02	2.000e-02	**1.000e-02**
F16	mean	**3.653e-01**	5.969e-01	4.971e-01	8.234e+00	5.420e-01	5.639e-01
	std	**1.467e-01**	3.502e-01	1.752e-01	2.021e+01	2.484e-01	2.720e-01
	best	**0.000e+00**	1.200e-01	**0.000e+00**	**0.000e+00**	2.000e-02	1.000e-02
F17	mean	**1.471e-01**	3.131e+00	2.025e-01	4.904e-01	4.349e-01	1.931e-01
	std	**1.738e-01**	5.288e+00	3.313e-01	5.376e-01	3.787e-01	2.527e-01
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F18	mean	2.743e-01	6.308e-01	4.278e-01	6.849e-01	**2.592e-01**	2.731e+00
	std	2.037e-01	2.754e+00	3.330e-01	5.906e-01	**1.989e-01**	6.415e+00
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F19	mean	**7.843e-03**	2.510e-02	2.706e-02	1.353e-01	1.137e-02	4.627e-02
	std	**1.016e-02**	2.782e-02	2.412e-02	3.283e-01	1.329e-02	2.072e-01
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F20	mean	**0.000e+00**	1.859e-01	**0.000e+00**	1.039e-02	6.686e-02	9.725e-02
	std	**0.000e+00**	2.650e-01	**0.000e+00**	5.220e-02	1.415e-01	1.680e-01
	best	1.000e+02	1.000e+02	**0.000e+00**	**0.000e+00**	1.000e+02	1.000e+02
F21	mean	1.262e+02	1.047e+02	**9.412e+01**	9.721e+01	1.243e+02	1.465e+02
	std	4.326e+01	2.011e+01	2.353e+01	**1.968e+01**	4.381e+01	5.130e+01
	best	1.000e+02	**0.000e+00**	**0.000e+00**	**0.000e+00**	1.000e+02	1.000e+02
F22	mean	1.000e+02	7.768e+01	**3.631e+01**	5.694e+01	1.000e+02	1.000e+02
	std	8.232e-02	3.197e+01	2.599e+01	3.899e+01	**0.000e+00**	5.546e-02
	best	3.000e+02	**0.000e+00**	**0.000e+00**	3.042e+02	3.000e+02	3.000e+02
F23	mean	3.020e+02	2.837e+02	**2.409e+02**	3.085e+02	3.005e+02	3.019e+02
	std	1.650e+00	7.095e+01	1.264e+02	2.573e+00	**1.083e+00**	1.603e+00
	best	1.000e+02	2.673e+01	**0.000e+00**	**0.000e+00**	1.000e+02	1.000e+02
F24	mean	2.810e+02	9.856e+01	**7.963e+01**	1.267e+02	2.321e+02	3.112e+02
	std	9.232e+01	**1.016e+01**	3.276e+01	5.532e+01	1.107e+02	6.163e+01
	best	3.977e+02	**1.000e+02**	1.000e+02	**1.000e+02**	3.977e+02	3.977e+02
F25	mean	4.114e+02	**2.424e+02**	3.861e+02	3.003e+02	4.068e+02	4.238e+02
	std	2.083e+01	1.219e+02	5.779e+01	1.353e+02	**1.804e+01**	2.252e+01
	best	3.000e+02	**0.000e+00**	**0.000e+00**	**0.000e+00**	3.000e+02	3.000e+02
F26	mean	3.000e+02	1.100e+02	**3.725e+01**	1.665e+02	3.000e+02	3.000e+02
	std	**0.000e+00**	9.981e+01	8.156e+01	1.219e+02	**0.000e+00**	**0.000e+00**
	best	3.873e+02	3.887e+02	**2.120e+00**	3.890e+02	3.890e+02	3.890e+02
F27	mean	3.935e+02	3.891e+02	**3.837e+02**	3.899e+02	3.931e+02	3.905e+02
	std	1.560e+00	**1.945e-01**	5.399e+01	1.075e+00	1.634e+00	1.958e+00
	best	3.000e+02	**0.000e+00**	**0.000e+00**	**0.000e+00**	3.000e+02	3.000e+02
F28	mean	3.056e+02	2.622e+02	**1.400e+02**	2.858e+02	3.056e+02	3.941e+02
	std	**3.934e+01**	9.619e+01	1.406e+02	7.346e+01	**3.934e+01**	1.377e+02
	best	2.285e+02	**1.723e+02**	1.826e+02	2.312e+02	2.277e+02	2.259e+02
F29	mean	2.364e+02	2.309e+02	2.439e+02	2.463e+02	2.372e+02	**2.282e+02**
	std	3.994e+00	8.836e+00	1.278e+01	8.712e+00	3.452e+00	**1.507e+00**
	best	**3.945e+02**	**3.945e+02**	4.545e+02	4.123e+02	**3.945e+02**	**3.945e+02**
F30	mean	4.044e+02	3.955e+02	6.743e+02	6.335e+02	**3.945e+02**	6.451e+04
	std	2.018e+01	6.674e+00	1.898e+02	1.643e+02	**5.368e-02**	2.197e+05

Table A.10: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2017 benchmark suite at dimension 30. Each value is computed over 51 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F1	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	**0.000e+00**	**0.000e+00**	7.166e+00	3.671e+00	**0.000e+00**	**0.000e+00**
F3	mean	**0.000e+00**	**0.000e+00**	9.713e+01	1.064e+03	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	9.247e+01	3.242e+03	**0.000e+00**	**0.000e+00**
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	5.856e+01	**0.000e+00**	5.856e+01
F4	mean	5.440e+01	3.319e+01	**2.325e+00**	8.398e+01	5.861e+01	5.856e+01
	std	1.594e+01	2.775e+01	5.435e+00	9.811e+00	8.596e+00	**7.105e-15**
	best	**3.050e+00**	3.980e+00	3.788e+01	3.961e+01	4.976e+00	6.187e+00
F5	mean	**6.892e+00**	1.018e+01	6.233e+01	6.030e+01	9.349e+00	1.192e+01
	std	**1.472e+00**	3.663e+00	1.006e+01	1.106e+01	1.799e+00	2.357e+00
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F6	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	**3.437e+01**	3.511e+01	6.132e+01	7.395e+01	3.467e+01	3.818e+01
F7	mean	**3.760e+01**	3.931e+01	8.810e+01	9.970e+01	3.845e+01	4.279e+01
	std	**1.416e+00**	2.944e+00	8.461e+00	1.273e+01	1.695e+00	2.031e+00
	best	3.112e+00	**2.985e+00**	3.589e+01	4.611e+01	5.970e+00	7.736e+00
F8	mean	**7.613e+00**	1.009e+01	6.531e+01	6.520e+01	1.022e+01	1.238e+01
	std	**1.461e+00**	3.201e+00	9.575e+00	1.110e+01	1.977e+00	2.304e+00
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F9	mean	**0.000e+00**	**0.000e+00**	1.529e+01	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	1.903e+01	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	1.005e+03	**6.015e+02**	1.191e+03	3.098e+03	9.870e+02	8.503e+02
F10	mean	1.552e+03	2.090e+03	1.859e+03	3.756e+03	1.593e+03	**1.450e+03**
	std	2.107e+02	5.675e+02	2.745e+02	3.936e+02	**1.950e+02**	2.165e+02
	best	9.900e-01	**0.000e+00**	1.068e+01	1.283e+01	**0.000e+00**	1.390e+00
F11	mean	1.881e+01	7.422e+00	2.229e+01	5.383e+01	**3.325e+00**	1.562e+01
	std	2.460e+01	1.408e+01	1.023e+01	2.830e+01	**2.149e+00**	2.312e+01
	best	3.075e+02	2.066e+01	5.380e+03	6.182e+03	**2.100e-01**	7.660e+00
F12	mean	1.099e+03	4.299e+02	1.395e+04	4.624e+04	**1.070e+02**	3.827e+02
	std	3.776e+02	2.438e+02	7.450e+03	3.137e+04	**7.218e+01**	1.790e+02
	best	1.990e+00	9.900e-01	3.752e+01	9.968e+01	**0.000e+00**	**0.000e+00**
F13	mean	1.620e+01	1.496e+01	1.121e+02	3.276e+02	**1.328e+01**	1.762e+01
	std	6.459e+00	**3.443e+00**	7.951e+01	2.776e+02	6.368e+00	1.075e+01
	best	2.002e+01	2.001e+01	1.016e+01	3.487e+01	2.032e+01	**3.510e+00**
F14	mean	2.215e+01	2.210e+01	3.909e+01	5.006e+01	2.273e+01	**2.125e+01**
	std	1.095e+00	**1.061e+00**	9.184e+00	7.993e+00	1.331e+00	3.809e+00
	best	4.100e-01	**3.100e-01**	4.530e+00	1.972e+01	3.400e-01	4.400e-01
F15	mean	2.867e+00	**1.008e+00**	1.185e+01	4.467e+01	1.054e+00	2.606e+00
	std	1.773e+00	7.847e-01	9.226e+00	2.142e+01	**6.348e-01**	1.631e+00
	best	1.466e+01	**2.340e+00**	1.251e+02	4.984e+01	4.580e+00	8.450e+00
F16	mean	1.126e+02	3.724e+01	3.703e+02	4.072e+02	1.697e+02	**3.054e+01**
	std	9.295e+01	5.818e+01	1.136e+02	1.283e+02	9.391e+01	**3.389e+01**
	best	1.783e+01	2.498e+01	2.073e+01	6.816e+01	1.715e+01	**1.676e+01**
F17	mean	3.112e+01	4.382e+01	5.634e+01	1.213e+02	3.307e+01	**3.008e+01**
	std	5.915e+00	8.013e+00	1.475e+01	3.350e+01	6.894e+00	**5.216e+00**
	best	3.410e+00	**5.500e-01**	2.240e+02	6.335e+01	2.042e+01	2.023e+01
F18	mean	2.182e+01	**2.027e+01**	6.837e+03	2.609e+02	2.083e+01	2.133e+01
	std	2.879e+00	2.819e+00	4.595e+03	3.904e+02	**3.999e-01**	1.008e+00
	best	2.800e+00	**1.890e+00**	6.120e+00	1.653e+01	2.050e+00	2.820e+00
F19	mean	5.774e+00	**4.532e+00**	1.174e+01	2.557e+01	4.668e+00	5.567e+00
	std	1.798e+00	**1.608e+00**	2.793e+00	4.709e+00	1.854e+00	1.683e+00
	best	1.723e+01	1.435e+01	1.226e+01	3.581e+01	**1.191e+01**	1.962e+01
F20	mean	3.765e+01	3.578e+01	9.365e+01	9.402e+01	3.364e+01	**3.248e+01**
	std	6.965e+00	1.233e+01	6.107e+01	5.175e+01	1.941e+01	**5.962e+00**
	best	2.045e+02	2.029e+02	**1.001e+02**	1.267e+02	2.055e+02	2.070e+02
F21	mean	2.076e+02	2.102e+02	**1.511e+02**	2.623e+02	2.099e+02	2.127e+02
	std	**1.486e+00**	3.903e+00	7.106e+01	2.090e+01	1.827e+00	2.241e+00
	best	**1.000e+02**	**1.000e+02**	**1.000e+02**	**1.000e+02**	**1.000e+02**	**1.000e+02**
F22	mean	**1.000e+02**	**1.000e+02**	**1.000e+02**	**1.000e+02**	**1.000e+02**	**1.000e+02**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	3.363e+02	3.452e+02	**1.556e+02**	3.900e+02	3.389e+02	3.470e+02
F23	mean	3.465e+02	3.579e+02	3.801e+02	4.146e+02	**3.454e+02**	3.564e+02
	std	3.275e+00	6.723e+00	7.017e+01	1.148e+01	**3.135e+00**	3.232e+00
	best	4.174e+02	4.230e+02	**1.020e+02**	4.666e+02	4.121e+02	4.237e+02
F24	mean	4.225e+02	4.288e+02	4.565e+02	4.857e+02	**4.218e+02**	4.294e+02
	std	**1.808e+00**	2.529e+00	1.619e+02	1.083e+01	3.143e+00	2.601e+00
	best	3.867e+02	**3.834e+02**	3.868e+02	**3.834e+02**	3.867e+02	3.867e+02
F25	mean	3.868e+02	**3.854e+02**	3.870e+02	3.869e+02	3.867e+02	3.867e+02
	std	2.637e-02	1.587e+00	8.788e-02	7.496e-01	1.200e-02	**6.944e-03**
	best	8.191e+02	**2.000e+02**	**2.000e+02**	1.493e+03	**2.000e+02**	8.073e+02
F26	mean	8.897e+02	5.074e+02	**2.098e+02**	1.716e+03	8.883e+02	9.544e+02
	std	4.222e+01	3.353e+02	**2.974e+01**	1.145e+02	1.044e+02	4.227e+01
	best	4.890e+02	**4.694e+02**	5.029e+02	4.914e+02	4.707e+02	4.927e+02
F27	mean	5.090e+02	4.924e+02	5.152e+02	5.034e+02	**4.864e+02**	5.049e+02
	std	6.093e+00	**4.850e+00**	5.525e+00	5.864e+00	8.012e+00	5.262e+00
	best	**3.000e+02**	**3.000e+02**	**3.000e+02**	**3.000e+02**	**3.000e+02**	**3.000e+02**
F28	mean	3.300e+02	**3.000e+02**	3.040e+02	3.963e+02	**3.000e+02**	3.173e+02
	std	4.890e+01	**0.000e+00**	1.780e+01	5.553e+01	**0.000e+00**	4.005e+01
	best	4.188e+02	4.227e+02	3.822e+02	4.386e+02	3.991e+02	**3.672e+02**
F29	mean	4.348e+02	4.507e+02	4.831e+02	5.817e+02	4.363e+02	**4.324e+02**
	std	**6.989e+00**	1.473e+01	4.083e+01	5.760e+01	8.565e+00	1.133e+01
	best	1.972e+03	1.941e+03	2.655e+03	2.329e+03	1.956e+03	**1.941e+03**
F30	mean	2.080e+03	**1.967e+03**	3.167e+03	3.817e+03	1.987e+03	1.974e+03
	std	8.766e+01	2.323e+01	2.205e+02	9.750e+02	**1.720e+01**	4.027e+01

Table A.11: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2017 benchmark suite at dimension 50. Each value is computed over 51 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	2.870e-01	2.700e-02	**0.000e+00**	**0.000e+00**
F1	mean	**0.000e+00**	**0.000e+00**	2.203e+02	3.557e+03	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	3.301e+02	4.226e+03	**0.000e+00**	**0.000e+00**
	best	**0.000e+00**	**0.000e+00**	4.629e+03	8.629e+03	**0.000e+00**	**0.000e+00**
F3	mean	**0.000e+00**	**0.000e+00**	9.344e+03	4.485e+04	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	2.780e+03	3.808e+04	**0.000e+00**	**0.000e+00**
	best	**0.000e+00**	**0.000e+00**	7.129e+01	2.851e+01	**0.000e+00**	8.048e+00
F4	mean	7.932e+01	**2.982e+01**	7.470e+01	8.814e+01	5.559e+01	4.962e+01
	std	4.372e+01	2.232e+01	**1.290e+00**	5.478e+01	4.261e+01	4.108e+01
	best	1.029e+01	**6.965e+00**	1.235e+02	1.092e+02	1.492e+01	9.406e+00
F5	mean	**1.470e+01**	1.586e+01	1.642e+02	1.456e+02	2.114e+01	2.554e+01
	std	**2.139e+00**	4.407e+00	1.571e+01	2.085e+01	3.268e+00	6.745e+00
	best	**0.000e+00**	**0.000e+00**	2.000e-03	**0.000e+00**	**0.000e+00**	**0.000e+00**
F6	mean	1.373e-04	**0.000e+00**	1.255e-02	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	8.406e-04	**0.000e+00**	7.875e-03	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	6.015e+01	**5.844e+01**	1.406e+02	1.577e+02	5.960e+01	6.486e+01
F7	mean	**6.355e+01**	6.441e+01	1.717e+02	2.220e+02	6.579e+01	7.617e+01
	std	**1.687e+00**	3.653e+00	1.572e+01	3.215e+01	2.737e+00	6.581e+00
	best	**7.811e+00**	8.955e+00	1.140e+02	1.105e+02	1.395e+01	1.373e+01
F8	mean	**1.364e+01**	1.633e+01	1.623e+02	1.514e+02	2.073e+01	2.722e+01
	std	**2.198e+00**	3.826e+00	1.486e+01	2.558e+01	2.678e+00	6.364e+00
	best	**0.000e+00**	**0.000e+00**	2.971e+02	**0.000e+00**	**0.000e+00**	**0.000e+00**
F9	mean	**0.000e+00**	**0.000e+00**	1.878e+03	8.902e-03	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	1.396e+03	6.295e-02	**0.000e+00**	**0.000e+00**
	best	2.382e+03	2.381e+03	2.588e+03	6.657e+03	**2.122e+03**	2.151e+03
F10	mean	3.283e+03	3.798e+03	3.704e+03	7.770e+03	3.248e+03	**3.205e+03**
	std	**2.845e+02**	8.006e+02	3.566e+02	6.609e+02	3.488e+02	3.271e+02
	best	3.319e+01	**1.926e+01**	4.694e+01	6.137e+01	2.125e+01	2.213e+01
F11	mean	5.967e+01	2.682e+01	6.084e+01	9.537e+01	**2.631e+01**	2.716e+01
	std	1.110e+01	4.262e+00	9.988e+00	2.027e+01	2.455e+00	**1.999e+00**
	best	1.119e+03	1.055e+03	2.215e+04	2.930e+05	6.922e+02	**4.744e+02**
F12	mean	2.290e+03	2.126e+03	1.283e+05	2.717e+06	1.570e+03	**1.289e+03**
	std	5.003e+02	5.374e+02	5.234e+04	1.857e+06	4.469e+02	**3.814e+02**
	best	2.168e+01	6.590e+00	3.213e+02	2.501e+02	**5.600e+00**	9.610e+00
F13	mean	6.655e+01	3.281e+01	6.326e+02	8.084e+03	**2.271e+01**	7.673e+01
	std	2.922e+01	**1.655e+01**	2.486e+02	8.381e+03	1.668e+01	3.998e+01
	best	2.412e+01	**2.001e+01**	7.255e+01	1.340e+02	2.315e+01	2.312e+01
F14	mean	2.991e+01	**2.328e+01**	4.990e+02	2.190e+02	2.764e+01	2.675e+01
	std	3.927e+00	**1.803e+00**	5.168e+02	6.350e+01	2.612e+00	2.003e+00
	best	2.401e+01	1.834e+01	5.025e+01	2.228e+02	**1.787e+01**	1.912e+01
F15	mean	4.710e+01	**2.259e+01**	1.240e+02	7.801e+02	2.262e+01	2.547e+01
	std	1.423e+01	2.526e+00	4.730e+01	6.677e+02	**2.155e+00**	3.590e+00
	best	1.850e+02	**1.269e+02**	5.983e+02	6.628e+02	1.317e+02	1.419e+02
F16	mean	4.210e+02	**2.999e+02**	8.869e+02	1.109e+03	4.452e+02	3.134e+02
	std	1.167e+02	**1.151e+02**	1.292e+02	2.042e+02	1.368e+02	1.187e+02
	best	1.387e+02	**5.330e+01**	3.137e+02	4.393e+02	2.156e+02	9.345e+01
F17	mean	2.735e+02	2.510e+02	5.784e+02	7.996e+02	3.672e+02	**1.993e+02**
	std	6.861e+01	**6.359e+01**	1.386e+02	1.581e+02	9.183e+01	6.648e+01
	best	2.540e+01	2.185e+01	9.169e+03	3.484e+03	2.065e+01	**2.039e+01**
F18	mean	5.515e+01	2.544e+01	2.309e+04	2.893e+04	**2.325e+01**	2.494e+01
	std	1.984e+01	2.402e+00	1.018e+04	1.504e+04	**1.305e+00**	1.972e+00
	best	1.555e+01	**6.960e+00**	1.906e+01	1.048e+02	8.440e+00	1.238e+01
F19	mean	3.474e+01	1.370e+01	4.529e+01	2.789e+02	**1.279e+01**	1.733e+01
	std	1.305e+01	2.844e+00	1.414e+01	1.722e+02	**2.315e+00**	2.550e+00
	best	8.650e+01	**4.114e+01**	8.489e+01	3.417e+02	5.781e+01	7.430e+01
F20	mean	1.971e+02	**1.021e+02**	3.614e+02	5.726e+02	1.742e+02	1.127e+02
	std	8.101e+01	6.212e+01	1.174e+02	1.295e+02	8.522e+01	**2.286e+01**
	best	2.107e+02	2.084e+02	**1.592e+02**	3.139e+02	2.138e+02	2.148e+02
F21	mean	**2.149e+02**	2.160e+02	3.629e+02	3.582e+02	2.220e+02	2.296e+02
	std	**2.294e+00**	4.243e+00	3.292e+01	2.117e+01	3.708e+00	6.725e+00
	best	**1.000e+02**	**1.000e+02**	1.001e+02	**1.000e+02**	**1.000e+02**	**1.000e+02**
F22	mean	6.318e+02	7.702e+02	2.623e+03	7.633e+03	**1.831e+02**	1.458e+03
	std	1.268e+03	1.561e+03	2.201e+03	1.481e+03	**5.874e+02**	1.666e+03
	best	4.158e+02	4.305e+02	5.379e+02	5.253e+02	**4.056e+02**	4.224e+02
F23	mean	4.291e+02	4.413e+02	6.068e+02	5.980e+02	**4.231e+02**	4.402e+02
	std	6.588e+00	**5.189e+00**	2.506e+01	2.740e+01	7.954e+00	7.319e+00
	best	4.971e+02	5.052e+02	**1.552e+02**	6.007e+02	4.896e+02	5.044e+02
F24	mean	5.047e+02	5.115e+02	6.820e+02	6.575e+02	**4.996e+02**	5.158e+02
	std	4.398e+00	**2.965e+00**	1.078e+02	2.164e+01	5.104e+00	7.202e+00
	best	4.802e+02	**4.283e+02**	5.281e+02	4.609e+02	4.551e+02	4.773e+02
F25	mean	4.944e+02	4.865e+02	5.598e+02	5.059e+02	4.889e+02	**4.805e+02**
	std	2.868e+01	2.139e+01	2.308e+01	3.821e+01	1.817e+01	**2.385e+00**
	best	9.162e+02	**3.000e+02**	3.001e+02	2.159e+03	9.711e+02	9.821e+02
F26	mean	1.128e+03	**1.089e+03**	1.332e+03	2.815e+03	1.155e+03	1.233e+03
	std	**6.970e+01**	2.807e+02	1.202e+03	2.712e+02	7.290e+01	9.028e+01
	best	5.204e+02	4.932e+02	5.944e+02	5.118e+02	**4.915e+02**	5.037e+02
F27	mean	5.407e+02	5.135e+02	6.468e+02	5.589e+02	**5.035e+02**	5.246e+02
	std	1.294e+01	7.438e+00	2.338e+01	3.083e+01	**6.607e+00**	1.015e+01
	best	**4.588e+02**	**4.588e+02**	4.850e+02	**4.588e+02**	**4.588e+02**	**4.588e+02**
F28	mean	4.972e+02	4.591e+02	5.084e+02	4.915e+02	**4.588e+02**	4.608e+02
	std	2.009e+01	2.113e+00	6.362e+00	2.215e+01	**5.684e-14**	9.480e+00
	best	**3.258e+02**	3.362e+02	4.617e+02	5.986e+02	3.398e+02	3.294e+02
F29	mean	3.544e+02	3.801e+02	7.201e+02	9.088e+02	3.712e+02	**3.513e+02**
	std	**9.726e+00**	2.512e+01	1.084e+02	1.564e+02	1.461e+01	1.069e+01
	best	5.794e+05	**5.794e+05**	6.027e+05	6.035e+05	5.794e+05	**5.794e+05**
F30	mean	6.129e+05	6.040e+05	6.926e+05	7.854e+05	**5.888e+05**	6.598e+05
	std	3.029e+04	3.140e+04	4.717e+04	1.359e+05	**1.581e+04**	8.375e+04

Table A.12: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2017 benchmark suite at dimension 100. Each value is computed over 51 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	2.543e+02	6.810e-01	**0.000e+00**	**0.000e+00**
F1	mean	**0.000e+00**	**0.000e+00**	3.031e+03	7.020e+03	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	1.139e+03	9.174e+03	**0.000e+00**	**0.000e+00**
	best	**0.000e+00**	**0.000e+00**	6.391e+04	7.489e+04	**0.000e+00**	**0.000e+00**
F3	mean	**0.000e+00**	**0.000e+00**	8.939e+04	2.773e+05	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	1.203e+04	1.233e+05	**0.000e+00**	**0.000e+00**
	best	8.640e+01	7.030e-01	1.516e+02	1.974e+02	**1.000e-03**	1.902e+02
F4	mean	1.834e+02	**1.360e+02**	1.921e+02	2.227e+02	1.360e+02	2.042e+02
	std	3.151e+01	5.449e+01	3.297e+01	1.361e+01	6.356e+01	**9.729e+00**
	best	3.169e+01	**1.890e+01**	4.210e+02	3.809e+02	4.158e+01	4.731e+01
F5	mean	4.035e+01	**3.036e+01**	5.249e+02	5.087e+02	5.502e+01	6.047e+01
	std	**4.712e+00**	4.771e+00	3.985e+01	6.316e+01	7.435e+00	1.256e+01
	best	**0.000e+00**	**0.000e+00**	1.450e-01	**0.000e+00**	**0.000e+00**	**0.000e+00**
F6	mean	2.902e-03	1.745e-03	3.500e-01	**0.000e+00**	3.922e-04	**0.000e+00**
	std	2.107e-03	2.333e-03	9.690e-02	**0.000e+00**	1.156e-03	**0.000e+00**
	best	1.265e+02	**1.194e+02**	3.224e+02	5.555e+02	1.286e+02	1.467e+02
F7	mean	1.369e+02	**1.290e+02**	3.861e+02	6.886e+02	1.442e+02	1.652e+02
	std	5.098e+00	**4.641e+00**	2.897e+01	7.861e+01	7.657e+00	6.717e+00
	best	3.035e+01	**1.691e+01**	4.020e+02	3.702e+02	4.337e+01	3.792e+01
F8	mean	4.170e+01	**3.045e+01**	5.363e+02	5.068e+02	5.505e+01	5.474e+01
	std	**4.679e+00**	5.453e+00	4.375e+01	7.050e+01	6.239e+00	6.765e+00
	best	**0.000e+00**	**0.000e+00**	8.608e+03	**0.000e+00**	**0.000e+00**	**0.000e+00**
F9	mean	2.197e-01	1.420e-02	2.379e+04	**0.000e+00**	4.071e-02	**0.000e+00**
	std	2.891e-01	6.570e-02	4.293e+03	**0.000e+00**	9.546e-02	**0.000e+00**
	best	8.983e+03	8.477e+03	9.457e+03	1.836e+04	**8.246e+03**	8.867e+03
F10	mean	1.047e+04	1.090e+04	1.092e+04	2.091e+04	**9.713e+03**	1.064e+04
	std	5.306e+02	1.488e+03	6.261e+02	9.709e+02	5.962e+02	**4.938e+02**
	best	3.331e+02	5.350e+01	3.408e+02	1.101e+03	5.474e+01	**2.265e+01**
F11	mean	5.119e+02	1.070e+02	4.118e+02	3.056e+03	1.091e+02	**5.415e+01**
	std	7.940e+01	**2.694e+01**	4.371e+01	1.736e+03	3.281e+01	3.566e+01
	best	1.187e+04	6.864e+03	2.588e+05	1.211e+07	7.507e+03	**3.196e+03**
F12	mean	2.682e+04	1.574e+04	7.283e+05	5.300e+07	1.826e+04	**4.546e+03**
	std	9.360e+03	5.791e+03	3.095e+05	2.466e+07	7.762e+03	**6.353e+02**
	best	4.306e+02	8.263e+01	2.826e+03	3.270e+02	6.319e+01	**4.928e+01**
F13	mean	1.502e+03	1.439e+02	3.661e+03	4.426e+03	1.538e+02	**1.123e+02**
	std	7.798e+02	3.986e+01	4.197e+02	4.829e+03	3.970e+01	**3.572e+01**
	best	1.901e+02	4.507e+01	2.280e+04	3.183e+03	3.502e+01	**3.400e+01**
F14	mean	2.549e+02	6.790e+01	7.509e+04	5.329e+04	5.118e+01	**4.810e+01**
	std	3.051e+01	9.761e+00	2.747e+04	8.079e+04	7.607e+00	**6.102e+00**
	best	1.703e+02	1.354e+02	2.169e+02	9.355e+02	7.688e+01	**4.619e+01**
F15	mean	2.429e+02	2.026e+02	3.815e+02	7.130e+03	1.415e+02	**8.859e+01**
	std	4.551e+01	3.338e+01	9.357e+01	5.810e+03	3.694e+01	**2.575e+01**
	best	1.060e+03	**5.381e+02**	2.134e+03	3.432e+03	1.238e+03	7.939e+02
F16	mean	1.664e+03	**1.185e+03**	2.764e+03	4.423e+03	1.939e+03	1.263e+03
	std	2.642e+02	3.201e+02	2.656e+02	4.827e+02	2.628e+02	**2.287e+02**
	best	6.833e+02	**3.928e+02**	1.217e+03	2.030e+03	9.065e+02	3.938e+02
F17	mean	1.132e+03	9.698e+02	1.780e+03	2.949e+03	1.343e+03	**9.573e+02**
	std	**1.850e+02**	2.484e+02	2.487e+02	3.518e+02	1.965e+02	1.876e+02
	best	1.416e+02	1.343e+02	9.720e+04	9.954e+04	1.113e+02	**4.115e+01**
F18	mean	2.085e+02	2.254e+02	1.782e+05	1.901e+05	1.491e+02	**7.381e+01**
	std	4.005e+01	4.878e+01	6.032e+04	5.390e+04	2.261e+01	**1.890e+01**
	best	1.204e+02	9.494e+01	1.960e+02	1.599e+02	5.437e+01	**3.948e+01**
F19	mean	1.758e+02	1.567e+02	5.124e+02	6.111e+03	9.506e+01	**5.390e+01**
	std	2.156e+01	2.331e+01	1.939e+02	7.999e+03	1.920e+01	**6.788e+00**
	best	1.038e+03	**5.736e+02**	1.156e+03	1.545e+03	9.236e+02	7.460e+02
F20	mean	1.609e+03	1.217e+03	1.768e+03	2.610e+03	1.479e+03	**1.117e+03**
	std	1.939e+02	2.670e+02	2.673e+02	3.161e+02	2.355e+02	**1.531e+02**
	best	2.490e+02	**2.458e+02**	6.243e+02	6.547e+02	2.613e+02	2.688e+02
F21	mean	2.672e+02	**2.565e+02**	7.110e+02	7.553e+02	2.766e+02	2.793e+02
	std	6.359e+00	**4.931e+00**	3.598e+01	5.935e+01	7.273e+00	6.711e+00
	best	1.085e+04	8.461e+03	1.027e+02	2.006e+04	**1.025e+02**	9.264e+03
F22	mean	1.179e+04	1.117e+04	1.220e+04	2.200e+04	**1.045e+04**	1.046e+04
	std	**4.785e+02**	1.302e+03	1.792e+03	9.411e+02	2.655e+03	6.387e+02
	best	5.849e+02	**5.489e+02**	7.373e+02	8.758e+02	5.577e+02	5.784e+02
F23	mean	6.026e+02	**5.706e+02**	7.809e+02	9.786e+02	5.901e+02	6.023e+02
	std	9.733e+00	**8.014e+00**	2.299e+01	5.084e+01	1.462e+01	9.486e+00
	best	8.921e+02	8.958e+02	1.187e+03	1.266e+03	**8.428e+02**	8.919e+02
F24	mean	9.132e+02	9.102e+02	1.251e+03	1.370e+03	**8.746e+02**	9.223e+02
	std	1.049e+01	**7.879e+00**	2.866e+01	6.158e+01	1.511e+01	1.505e+01
	best	6.979e+02	6.015e+02	7.937e+02	6.393e+02	5.773e+02	**5.770e+02**
F25	mean	7.477e+02	7.043e+02	8.567e+02	7.742e+02	7.349e+02	**6.804e+02**
	std	2.858e+01	4.434e+01	**1.695e+01**	4.667e+01	4.338e+01	4.895e+01
	best	3.139e+03	3.012e+03	6.142e+03	7.139e+03	**2.856e+03**	2.901e+03
F26	mean	3.298e+03	3.293e+03	7.113e+03	8.344e+03	3.172e+03	**3.165e+03**
	std	**8.946e+01**	1.094e+02	3.590e+02	6.810e+02	1.289e+02	1.355e+02
	best	6.296e+02	5.560e+02	7.927e+02	6.106e+02	5.567e+02	**5.540e+02**
F27	mean	6.634e+02	5.847e+02	8.461e+02	6.895e+02	**5.785e+02**	5.897e+02
	std	1.642e+01	1.181e+01	2.367e+01	3.781e+01	**9.177e+00**	1.404e+01
	best	**4.782e+02**	**4.782e+02**	5.889e+02	4.879e+02	**4.782e+02**	**4.782e+02**
F28	mean	5.353e+02	5.190e+02	6.398e+02	5.682e+02	5.257e+02	**5.149e+02**
	std	3.131e+01	2.212e+01	3.193e+01	3.020e+01	2.455e+01	**1.896e+01**
	best	9.592e+02	**8.351e+02**	2.514e+03	2.966e+03	9.138e+02	9.033e+02
F29	mean	1.296e+03	1.184e+03	3.118e+03	3.789e+03	1.228e+03	**1.165e+03**
	std	1.892e+02	1.853e+02	2.537e+02	2.832e+02	1.743e+02	**1.375e+02**
	best	2.226e+03	2.123e+03	7.494e+03	6.053e+03	2.123e+03	**2.117e+03**
F30	mean	2.606e+03	**2.299e+03**	8.974e+03	1.490e+04	2.341e+03	2.408e+03
	std	1.656e+02	1.071e+02	7.703e+02	6.074e+03	**9.543e+01**	1.376e+02

Table A.13: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2019 benchmark suite under $N_{max} = 1e + 8$ evaluation budget. Each value is computed over 50 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
F1	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F2	best	**0.000e+00**	**0.000e+00**	2.669e-07	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	**0.000e+00**	**0.000e+00**	1.174e-01	**3.137e-13**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	3.733e-01	**5.420e-13**	**0.000e+00**	**0.000e+00**
F3	best	**0.000e+00**	9.000e-12	9.000e-12	2.000e-12	9.000e-12	**1.000e-12**
	mean	**6.431e-12**	9.000e-12	9.000e-12	2.417e-02	9.000e-12	7.853e-11
	std	5.647e-12	**0.000e+00**	**1.539e-17**	9.625e-02	**0.000e+00**	8.873e-11
F4	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	1.717e+00	**0.000e+00**	**0.000e+00**	**0.000e+00**	3.121e-01	1.151e+00
	std	8.143e-01	**0.000e+00**	**0.000e+00**	**0.000e+00**	4.617e-01	6.645e-01
F5	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F6	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F7	best	1.874e-01	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	1.249e-01
	mean	3.058e+01	3.674e-03	1.018e+01	**0.000e+00**	2.602e+00	2.354e+01
	std	4.629e+01	1.470e-02	3.192e+01	**0.000e+00**	1.641e+01	4.448e+01
F8	best	2.891e-01	3.886e-02	7.773e-02	**0.000e+00**	9.716e-02	1.522e-01
	mean	7.601e-01	8.770e-02	2.944e-01	**3.935e-02**	1.462e-01	3.876e-01
	std	2.888e-01	**2.403e-02**	2.198e-01	3.729e-02	3.971e-02	1.655e-01
F9	best	3.465e-03	**1.820e-03**	9.944e-03	1.883e-02	1.916e-03	2.825e-03
	mean	1.538e-02	**5.029e-03**	2.425e-02	3.208e-02	7.508e-03	1.188e-02
	std	6.041e-03	**1.610e-03**	7.133e-03	8.791e-03	3.295e-03	4.240e-03
F10	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	3.985e-08	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	2.817e-07	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**

Table A.14: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2020 benchmark suite at dimension 5. Each value is computed over 51 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
F1	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F2	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	4.420e-01	1.596e+00	**1.206e-01**	1.160e+01	5.117e+00	1.412e-01
	std	1.298e+00	2.630e+00	1.352e-01	1.198e+01	2.297e+01	**9.825e-02**
F3	best	**0.000e+00**	6.130e-01	**0.000e+00**	9.950e-01	**0.000e+00**	5.148e+00
	mean	4.999e+00	4.520e+00	**2.933e+00**	4.332e+00	5.167e+00	5.232e+00
	std	1.170e+00	1.396e+00	2.500e+00	1.617e+00	7.430e-01	**1.224e-01**
F4	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	6.098e-02	6.765e-02	**9.804e-04**	9.216e-02	8.176e-02	6.961e-02
	std	3.158e-02	5.772e-02	**2.974e-03**	6.708e-02	3.719e-02	3.657e-02
F5	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	9.333e-02	8.510e-02	9.247e-01
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	2.473e-01	2.134e-01	4.773e+00
F6	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F7	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	1.626e+01
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	4.076e+01
F8	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	mean	3.928e+00	6.257e-01	**0.000e+00**	1.244e+00	1.529e+01	2.101e+00
	std	1.944e+01	2.615e+00	**0.000e+00**	3.154e+00	3.443e+01	1.395e+01
F9	best	1.000e+02	**0.000e+00**	**0.000e+00**	**0.000e+00**	1.000e+02	1.000e+02
	mean	1.000e+02	9.680e+00	**1.319e+00**	5.099e+01	1.000e+02	1.000e+02
	std	**0.000e+00**	2.086e+01	5.354e+00	5.376e+01	**0.000e+00**	**0.000e+00**
F10	best	3.000e+02	**0.000e+00**	**0.000e+00**	**0.000e+00**	3.000e+02	3.000e+02
	mean	3.418e+02	1.752e+02	2.745e+02	**1.510e+02**	3.446e+02	3.455e+02
	std	1.526e+01	9.172e+01	9.598e+01	1.010e+02	1.115e+01	**9.195e+00**

Table A.15: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2020 benchmark suite at dimension 10. Each value is computed over 51 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F1	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	2.500e-01	**6.000e-02**	**6.000e-02**	**6.000e-02**	1.900e-01	1.900e-01
F2	mean	5.927e+00	2.708e+00	**2.093e+00**	2.439e+00	7.689e+00	3.636e+00
	std	4.516e+00	**2.807e+00**	3.550e+00	4.128e+00	1.638e+01	3.089e+00
	best	1.037e+01	7.220e-01	**0.000e+00**	**0.000e+00**	1.072e+01	1.079e+01
F3	mean	1.181e+01	9.955e+00	**7.981e+00**	9.167e+00	1.182e+01	1.157e+01
	std	5.676e-01	2.398e+00	4.279e+00	3.119e+00	5.693e-01	**3.795e-01**
	best	1.000e-01	**0.000e+00**	**0.000e+00**	**0.000e+00**	1.000e-01	1.000e-01
F4	mean	1.504e-01	2.110e-01	**3.922e-04**	9.412e-02	1.678e-01	1.469e-01
	std	2.512e-02	5.457e-02	**1.941e-03**	8.598e-02	3.005e-02	2.128e-02
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F5	mean	2.627e-01	**1.606e-01**	1.836e+00	5.120e-01	2.751e-01	1.955e+01
	std	1.837e-01	**1.475e-01**	4.105e+00	1.570e+00	2.758e-01	2.978e+01
	best	**0.000e+00**	**0.000e+00**	2.000e-02	1.700e-01	1.000e-02	**0.000e+00**
F6	mean	1.631e-01	1.757e-01	9.569e-02	1.009e+00	**6.000e-02**	3.324e-01
	std	1.230e-01	1.551e-01	**6.140e-02**	2.149e+00	6.553e-02	2.592e-01
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F7	mean	1.510e-01	5.784e-02	**7.647e-03**	5.314e-02	1.608e-02	5.551e-01
	std	2.004e-01	1.114e-01	**4.291e-02**	1.283e-01	7.104e-02	2.951e-01
	best	1.000e+02	**0.000e+00**	**0.000e+00**	**0.000e+00**	1.000e+02	1.000e+02
F8	mean	1.000e+02	1.970e+01	1.255e+01	**8.245e+00**	1.000e+02	1.000e+02
	std	**0.000e+00**	1.479e+01	2.274e+01	2.359e+01	**0.000e+00**	**0.000e+00**
	best	1.000e+02	**0.000e+00**	**0.000e+00**	**0.000e+00**	1.000e+02	1.000e+02
F9	mean	2.489e+02	**6.151e+01**	6.726e+01	1.025e+02	2.092e+02	2.746e+02
	std	1.068e+02	4.805e+01	4.499e+01	**4.464e+01**	1.075e+02	9.688e+01
	best	3.977e+02	**1.000e+02**	**1.000e+02**	**1.000e+02**	3.977e+02	3.977e+02
F10	mean	4.166e+02	**1.079e+02**	3.861e+02	1.216e+02	4.023e+02	4.247e+02
	std	2.235e+01	2.688e+01	5.779e+01	6.909e+01	**1.354e+01**	2.240e+01

Table A.16: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2020 benchmark suite at dimension 15. Each value is computed over 51 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F1	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	9.000e-02	1.200e-01	4.000e-02	**0.000e+00**	1.200e-01	1.300e-01
F2	mean	5.552e+00	5.650e+00	1.697e+00	**2.533e-01**	5.891e+01	9.740e+00
	std	4.359e+00	8.719e+00	3.643e+00	**7.284e-01**	5.489e+01	1.789e+01
	best	1.557e+01	1.557e+01	**0.000e+00**	**0.000e+00**	1.557e+01	1.557e+01
F3	mean	1.609e+01	1.565e+01	1.483e+01	**9.771e+00**	1.636e+01	1.615e+01
	std	4.133e-01	**1.254e-01**	3.190e+00	7.413e+00	4.311e-01	2.983e-01
	best	1.500e-01	1.700e-01	**0.000e+00**	**0.000e+00**	2.000e-01	1.500e-01
F4	mean	2.347e-01	3.175e-01	**2.941e-03**	1.620e-01	2.788e-01	2.373e-01
	std	3.363e-02	4.902e-02	**8.470e-03**	1.055e-01	3.479e-02	3.543e-02
	best	1.000e-02	**0.000e+00**	3.100e-01	1.700e-01	6.200e-01	3.100e-01
F5	mean	1.729e+00	**2.494e-01**	2.279e+01	1.249e+01	3.100e+00	6.240e+01
	std	1.877e+00	**1.787e-01**	2.460e+01	1.050e+01	2.105e+00	6.311e+01
	best	4.000e-02	4.000e-02	**0.000e+00**	4.000e-02	6.000e-02	1.000e-02
F6	mean	2.286e-01	4.680e-01	**2.120e-01**	7.347e+00	4.141e-01	3.802e-01
	std	1.736e-01	2.681e-01	**1.202e-01**	1.011e+01	3.028e-01	2.939e-01
	best	**1.000e-02**	5.000e-02	5.000e-02	5.000e-02	4.000e-02	2.100e-01
F7	mean	6.980e-01	5.525e-01	**4.584e-01**	6.335e-01	6.951e-01	2.637e+01
	std	2.006e-01	**1.456e-01**	2.245e-01	2.656e-01	2.067e-01	4.887e+01
	best	1.000e+02	**0.000e+00**	**0.000e+00**	**0.000e+00**	1.000e+02	1.000e+02
F8	mean	1.000e+02	4.673e+01	**1.997e+01**	7.843e+01	1.000e+02	1.000e+02
	std	**0.000e+00**	2.405e+01	3.195e+01	4.113e+01	**0.000e+00**	**0.000e+00**
	best	3.000e+02	1.000e+02	**0.000e+00**	1.000e+02	1.000e+02	3.890e+02
F9	mean	3.673e+02	1.000e+02	**9.804e+01**	1.934e+02	3.599e+02	3.897e+02
	std	3.062e+01	**0.000e+00**	1.386e+01	1.090e+02	4.666e+01	1.973e-01
	best	4.000e+02	**1.000e+02**	4.000e+02	1.001e+02	4.000e+02	4.000e+02
F10	mean	4.000e+02	**2.643e+02**	4.000e+02	3.882e+02	4.000e+02	4.000e+02
	std	**0.000e+00**	1.084e+02	**0.000e+00**	5.822e+01	**0.000e+00**	**0.000e+00**

Table A.17: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2020 benchmark suite at dimension 20. Each value is computed over 51 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F1	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	9.000e-02	3.000e-02	**0.000e+00**	**0.000e+00**	6.000e-02	9.000e-02
F2	mean	1.646e+00	3.839e-01	3.518e-01	**2.412e-02**	1.222e+00	1.483e+00
	std	1.090e+00	5.569e-01	8.652e-01	**2.378e-02**	1.227e+00	9.828e-01
	best	2.039e+01	2.039e+01	2.039e+01	**0.000e+00**	2.039e+01	2.039e+01
F3	mean	2.079e+01	2.040e+01	2.039e+01	**1.619e+01**	2.142e+01	2.118e+01
	std	4.886e-01	3.501e-02	**0.000e+00**	7.897e+00	4.321e-01	4.635e-01
	best	2.300e-01	2.300e-01	**0.000e+00**	3.000e-02	2.600e-01	2.300e-01
F4	mean	3.206e-01	3.855e-01	**3.922e-04**	1.912e-01	3.639e-01	3.214e-01
	std	3.643e-02	5.274e-02	**2.773e-03**	8.255e-02	4.770e-02	3.156e-02
	best	2.100e-01	**0.000e+00**	1.000e-01	1.310e+00	1.310e+00	1.200e+00
F5	mean	2.312e+01	**8.533e-01**	1.081e+02	7.492e+01	7.627e+00	1.158e+02
	std	4.626e+01	**7.739e-01**	1.284e+02	6.873e+01	4.511e+00	8.041e+01
	best	1.200e-01	3.100e-01	**6.000e-02**	1.000e-01	**6.000e-02**	1.000e-01
F6	mean	2.702e-01	5.169e-01	**1.578e-01**	3.363e-01	9.216e-01	3.312e-01
	std	8.183e-02	1.386e-01	**5.902e-02**	1.004e-01	4.146e-01	2.650e-01
	best	5.000e-01	4.000e-01	1.700e-01	**1.000e-02**	**1.000e-02**	2.600e-01
F7	mean	1.957e+00	6.518e-01	2.874e+01	7.785e+00	**1.535e-01**	3.058e+01
	std	4.420e+00	**1.292e-01**	4.406e+01	1.541e+01	1.584e-01	4.976e+01
	best	1.000e+02	**0.000e+00**	4.441e+01	4.357e+01	1.000e+02	1.000e+02
F8	mean	1.000e+02	**6.604e+01**	8.892e+01	9.889e+01	1.000e+02	1.000e+02
	std	**0.000e+00**	2.544e+01	1.609e+01	7.824e+00	**0.000e+00**	**0.000e+00**
	best	3.892e+02	**0.000e+00**	1.000e+02	2.000e+02	3.000e+02	3.952e+02
F9	mean	3.967e+02	**9.608e+01**	1.000e+02	3.973e+02	3.872e+02	4.014e+02
	std	2.310e+00	1.941e+01	**1.386e-02**	5.766e+01	1.861e+01	1.742e+00
	best	4.137e+02	**3.000e+02**	3.990e+02	3.990e+02	4.137e+02	4.137e+02
F10	mean	4.137e+02	**3.972e+02**	4.036e+02	4.005e+02	4.137e+02	4.137e+02
	std	6.056e-03	1.375e+01	6.532e+00	3.520e+00	**1.386e-03**	5.083e-03

Table A.18: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2022 benchmark suite at dimension 10. Each value is computed over 51 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F1	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F2	mean	2.686e+00	**0.000e+00**	**0.000e+00**	9.804e-05	1.564e-01	7.291e+00
	std	3.480e+00	**0.000e+00**	**0.000e+00**	6.932e-04	7.739e-01	2.463e+00
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F3	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	9.950e-01	**0.000e+00**	4.975e+00	1.990e+00	**0.000e+00**	**0.000e+00**
F4	mean	2.088e+00	**1.307e+00**	1.288e+01	6.020e+00	2.166e+00	1.600e+00
	std	7.174e-01	**6.382e-01**	3.365e+00	2.359e+00	8.966e-01	8.829e-01
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F5	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	1.408e-01	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	9.956e-01	**0.000e+00**	**0.000e+00**
	best	**0.000e+00**	3.000e-02	2.000e-02	3.000e-02	1.000e-02	**0.000e+00**
F6	mean	3.245e-01	3.398e-01	3.296e-01	5.639e-01	**1.986e-01**	3.227e-01
	std	1.593e-01	1.383e-01	2.272e-01	4.735e-01	**1.328e-01**	1.593e-01
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F7	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	1.235e-02	3.647e-02
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	8.595e-02	1.459e-01
	best	8.000e-02	**0.000e+00**	**0.000e+00**	**0.000e+00**	2.000e-02	**0.000e+00**
F8	mean	9.896e-01	**1.024e-01**	1.269e-01	3.947e-01	2.855e-01	1.317e+00
	std	1.503e+00	**1.562e-01**	1.714e-01	9.600e-01	2.650e-01	4.701e+00
	best	2.293e+02	**0.000e+00**	2.293e+02	1.000e+02	2.293e+02	2.293e+02
F9	mean	2.293e+02	2.186e+02	2.293e+02	**2.169e+02**	2.293e+02	2.293e+02
	std	**2.842e-14**	4.469e+01	**2.842e-14**	3.769e+01	**2.842e-14**	**2.842e-14**
	best	1.002e+02	2.500e-01	**0.000e+00**	**0.000e+00**	1.001e+02	1.001e+02
F10	mean	1.002e+02	6.223e+01	9.451e-02	**7.627e-02**	1.002e+02	1.002e+02
	std	2.696e-02	4.292e+01	4.808e-01	7.043e-02	**2.169e-02**	3.030e-02
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F11	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	1.649e+02	**1.586e+02**	1.595e+02	**1.586e+02**	1.627e+02	1.627e+02
F12	mean	1.649e+02	1.607e+02	1.641e+02	**1.603e+02**	1.648e+02	1.646e+02
	std	**4.953e-03**	1.340e+00	9.524e-01	1.014e+00	5.231e-01	5.295e-01

Table A.19: Error statistics (best, mean, and standard deviation) for all algorithms on the CEC2022 benchmark suite at dimension 20. Each value is computed over 51 independent runs. Boldface denotes the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F1	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	4.489e+01	**0.000e+00**	**0.000e+00**	2.400e-02	4.489e+01	4.489e+01
F2	mean	4.892e+01	**1.580e-01**	4.450e+01	5.673e+00	4.621e+01	4.867e+01
	std	8.131e-01	**7.823e-01**	1.308e+01	1.318e+01	1.944e+00	1.246e+00
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F3	mean	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	2.985e+00	**9.950e-01**	4.080e+01	1.095e+01	3.980e+00	2.985e+00
F4	mean	4.897e+00	**3.005e+00**	6.751e+01	1.992e+01	6.985e+00	6.029e+00
	std	**8.996e-01**	9.550e-01	1.313e+01	4.507e+00	1.358e+00	1.172e+00
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
F5	mean	**0.000e+00**	**0.000e+00**	2.574e-01	**0.000e+00**	**0.000e+00**	**0.000e+00**
	std	**0.000e+00**	**0.000e+00**	1.297e+00	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	8.000e-02	1.000e-01	1.500e-01	1.250e+00	3.300e-01	**2.000e-02**
F6	mean	5.175e-01	**4.337e-01**	1.270e+01	4.390e+00	4.792e-01	1.341e+00
	std	3.341e-01	1.238e-01	9.256e+00	1.989e+00	**3.693e-02**	1.456e+00
	best	6.100e-01	**0.000e+00**	9.900e-01	3.100e-01	**0.000e+00**	**0.000e+00**
F7	mean	3.694e+00	3.879e+00	1.023e+01	1.200e+01	**3.487e+00**	6.404e+00
	std	**2.952e+00**	6.559e+00	7.064e+00	8.136e+00	4.593e+00	8.778e+00
	best	4.230e+00	1.900e-01	1.227e+01	1.933e+01	6.900e-01	**1.800e-01**
F8	mean	1.816e+01	**1.421e+01**	2.035e+01	2.164e+01	1.717e+01	1.561e+01
	std	3.930e+00	8.227e+00	1.203e+00	**4.738e-01**	5.453e+00	8.018e+00
	best	**1.808e+02**	**1.808e+02**	**1.808e+02**	**1.808e+02**	**1.808e+02**	**1.808e+02**
F9	mean	**1.808e+02**	**1.808e+02**	**1.808e+02**	**1.808e+02**	**1.808e+02**	**1.808e+02**
	std	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**
	best	1.002e+02	6.920e+00	**0.000e+00**	**0.000e+00**	1.002e+02	1.002e+02
F10	mean	1.003e+02	8.189e+01	**0.000e+00**	2.247e-01	1.002e+02	1.024e+02
	std	2.935e-02	3.507e+01	**0.000e+00**	5.932e-01	2.225e-02	1.558e+01
	best	3.000e+02	**0.000e+00**	3.000e+02	**0.000e+00**	3.000e+02	3.000e+02
F11	mean	3.000e+02	**1.765e+01**	3.000e+02	2.882e+02	3.059e+02	3.000e+02
	std	**0.000e+00**	7.059e+01	**0.000e+00**	5.823e+01	2.353e+01	**0.000e+00**
	best	2.312e+02	**2.289e+02**	2.342e+02	2.318e+02	2.307e+02	2.310e+02
F12	mean	2.347e+02	**2.313e+02**	2.428e+02	2.359e+02	2.323e+02	2.344e+02
	std	2.738e+00	**5.361e-01**	3.710e+00	2.595e+00	9.209e-01	2.707e+00

Table A.20: Statistics (best, mean, and standard deviation of the objective values) for all algorithms on the CEC2011 benchmark suite under an evaluation budget of $N_{\max}$ = 50,000. Each value is computed over 51 independent runs. Boldface indicates the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	2.260e-12	**0.000e+00**	**0.000e+00**
F1	mean	**2.369e-01**	1.819e+00	4.392e+00	6.987e+00	1.118e+00	1.770e+00
	std	**1.626e+00**	3.945e+00	5.375e+00	5.989e+00	3.393e+00	4.324e+00
	best	-2.315e+01	**-2.838e+01**	-2.578e+01	-1.314e+01	-2.177e+01	-2.238e+01
F2	mean	-2.043e+01	**-2.608e+01**	-2.128e+01	-9.412e+00	-1.848e+01	-2.038e+01
	std	1.507e+00	**1.197e+00**	1.808e+00	1.535e+00	1.237e+00	1.244e+00
	best	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**
F3	mean	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**
	std	**6.939e-18**	**6.939e-18**	**6.939e-18**	**6.939e-18**	**6.939e-18**	**6.939e-18**
	best	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**
F4	mean	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**
	std	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**
	best	-3.567e+01	**-3.685e+01**	-3.630e+01	-3.143e+01	-3.283e+01	-3.465e+01
F5	mean	-3.364e+01	-3.373e+01	**-3.466e+01**	-2.786e+01	-2.807e+01	-3.281e+01
	std	9.244e-01	1.898e+00	**8.367e-01**	2.127e+00	2.409e+00	1.052e+00
	best	-2.894e+01	-2.743e+01	**-2.916e+01**	-2.585e+01	-2.223e+01	-2.868e+01
F6	mean	-2.577e+01	-2.197e+01	**-2.627e+01**	-2.118e+01	-6.649e+00	-2.393e+01
	std	1.793e+00	2.121e+00	2.178e+00	**1.654e+00**	5.969e+00	2.197e+00
	best	1.037e+00	**6.706e-01**	8.873e-01	1.201e+00	8.979e-01	9.842e-01
F7	mean	1.259e+00	**9.426e-01**	1.176e+00	1.543e+00	1.373e+00	1.247e+00
	std	1.015e-01	1.256e-01	1.123e-01	1.769e-01	1.226e-01	**9.629e-02**
	best	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**
F8	mean	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	2.970e+05	**1.294e+03**	6.190e+05	1.446e+05	2.313e+05	7.953e+03
F9	mean	4.228e+05	**2.606e+03**	8.501e+05	1.801e+05	3.125e+05	1.686e+04
	std	4.554e+04	**6.834e+02**	8.058e+04	1.677e+04	4.384e+04	5.932e+03
	best	-8.211e+00	-8.211e+00	-8.204e+00	-8.184e+00	**-8.212e+00**	-8.211e+00
F10	mean	-8.207e+00	-8.206e+00	-8.085e+00	-7.758e+00	-8.208e+00	**-8.208e+00**
	std	3.225e-03	4.186e-03	1.243e-01	3.366e-01	**1.700e-03**	1.986e-03
	best	6.035e+04	**5.102e+04**	4.389e+05	2.632e+06	5.150e+04	5.922e+04
F11	mean	8.627e+04	**5.229e+04**	6.852e+05	1.528e+07	5.378e+04	9.417e+04
	std	1.904e+04	**5.999e+02**	1.452e+05	9.573e+06	6.167e+03	2.583e+04
	best	1.266e+06	**1.075e+06**	2.789e+06	3.033e+06	1.099e+06	1.241e+06
F12	mean	1.381e+06	**1.079e+06**	3.582e+06	3.985e+06	1.185e+06	1.389e+06
	std	5.295e+04	**2.065e+03**	2.776e+05	3.495e+05	5.275e+04	8.909e+04
	best	**1.544e+04**	**1.544e+04**	1.544e+04	1.544e+04	**1.544e+04**	**1.544e+04**
F13	mean	**1.544e+04**	1.545e+04	1.545e+04	1.545e+04	**1.544e+04**	1.545e+04
	std	**3.638e-12**	4.093e+00	1.851e+00	5.067e+00	**3.638e-12**	1.302e+01
	best	1.806e+04	1.803e+04	1.851e+04	1.829e+04	**1.802e+04**	1.803e+04
F14	mean	1.812e+04	1.813e+04	1.869e+04	1.851e+04	1.811e+04	**1.810e+04**
	std	4.839e+01	5.943e+01	1.046e+02	9.838e+01	4.649e+01	**3.909e+01**
	best	3.275e+04	3.275e+04	3.294e+04	3.282e+04	3.275e+04	**3.274e+04**
F15	mean	3.277e+04	3.278e+04	3.305e+04	3.295e+04	3.277e+04	**3.275e+04**
	std	1.470e+01	2.535e+01	4.565e+01	6.252e+01	1.108e+01	**3.748e+00**
	best	1.248e+05	1.247e+05	1.317e+05	1.339e+05	1.248e+05	**1.240e+05**
F16	mean	1.267e+05	1.263e+05	1.358e+05	1.427e+05	1.265e+05	**1.252e+05**
	std	7.285e+02	8.029e+02	1.958e+03	4.081e+03	6.816e+02	**5.563e+02**
	best	1.921e+06	**1.858e+06**	1.924e+06	3.374e+06	1.887e+06	1.896e+06
F17	mean	1.942e+06	**1.890e+06**	1.998e+06	3.676e+07	1.909e+06	1.923e+06
	std	2.342e+04	1.205e+04	1.453e+05	2.600e+07	**8.978e+03**	9.085e+03
	best	9.398e+05	**9.338e+05**	9.401e+05	2.345e+06	9.382e+05	9.338e+05
F18	mean	9.448e+05	**9.370e+05**	1.140e+06	3.948e+06	9.423e+05	9.395e+05
	std	2.691e+03	2.099e+03	2.777e+05	6.941e+05	1.887e+03	**1.559e+03**
	best	9.818e+05	**9.347e+05**	1.197e+06	2.876e+06	9.420e+05	9.400e+05
F19	mean	1.090e+06	**9.415e+05**	1.805e+06	4.576e+06	9.810e+05	9.549e+05
	std	5.973e+04	**2.903e+03**	3.283e+05	8.192e+05	3.077e+04	2.335e+04
	best	9.507e+05	**9.421e+05**	9.491e+05	2.356e+06	9.481e+05	9.449e+05
F20	mean	9.553e+05	**9.451e+05**	1.125e+06	4.001e+06	9.512e+05	9.483e+05
	std	2.501e+03	1.935e+03	2.498e+05	6.912e+05	1.852e+03	**1.599e+03**
	best	2.885e+01	**2.272e+01**	2.857e+01	2.953e+01	2.978e+01	2.930e+01
F21	mean	3.357e+01	**3.050e+01**	3.321e+01	3.762e+01	3.648e+01	3.519e+01
	std	2.130e+00	2.854e+00	2.136e+00	2.667e+00	2.867e+00	**1.795e+00**
	best	1.169e+01	1.152e+01	1.209e+01	1.218e+01	1.154e+01	**1.149e+01**
F22	mean	1.252e+01	1.231e+01	1.369e+01	1.588e+01	**1.212e+01**	1.259e+01
	std	7.364e-01	1.582e+00	1.086e+00	2.366e+00	**4.556e-01**	1.281e+00

Table A.21: Statistics (best, mean, and standard deviation of the objective values) for all algorithms on the CEC2011 benchmark suite under an evaluation budget of $N_{\max}$ = 100,000. Each value is computed over 51 independent runs. Boldface indicates the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**7.547e-14**	**0.000e+00**	**0.000e+00**
F1	mean	**1.443e-03**	1.650e-01	3.322e+00	1.119e+00	2.158e-01	1.273e+00
	std	**7.122e-03**	1.167e+00	4.974e+00	3.121e+00	1.526e+00	3.552e+00
	best	-2.666e+01	**-2.842e+01**	-2.750e+01	-1.697e+01	-2.637e+01	-2.585e+01
F2	mean	-2.498e+01	**-2.699e+01**	-2.438e+01	-1.284e+01	-2.416e+01	-2.455e+01
	std	7.485e-01	7.950e-01	1.498e+00	1.934e+00	1.218e+00	**6.701e-01**
	best	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**
F3	mean	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**
	std	**6.939e-18**	**6.939e-18**	**6.939e-18**	**6.939e-18**	**6.939e-18**	**6.939e-18**
	best	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**
F4	mean	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**
	std	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**
	best	-3.675e+01	-3.685e+01	**-3.686e+01**	-3.398e+01	-3.671e+01	-3.676e+01
F5	mean	**-3.583e+01**	-3.453e+01	-3.572e+01	-3.243e+01	-3.417e+01	-3.542e+01
	std	**7.636e-01**	1.162e+00	8.432e-01	1.397e+00	1.054e+00	7.762e-01
	best	-2.908e+01	**-2.917e+01**	-2.917e+01	-2.819e+01	-2.884e+01	-2.916e+01
F6	mean	**-2.827e+01**	-2.370e+01	-2.825e+01	-2.499e+01	-2.289e+01	-2.644e+01
	std	**6.986e-01**	2.371e+00	8.017e-01	1.742e+00	5.620e+00	2.534e+00
	best	8.491e-01	**5.438e-01**	7.507e-01	7.349e-01	8.485e-01	8.629e-01
F7	mean	1.125e+00	**8.360e-01**	1.051e+00	1.244e+00	1.165e+00	1.114e+00
	std	1.007e-01	1.174e-01	1.280e-01	1.340e-01	1.521e-01	**8.655e-02**
	best	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**
F8	mean	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	2.306e+04	**1.307e+03**	1.449e+05	8.064e+04	1.097e+04	1.551e+03
F9	mean	5.788e+04	**1.992e+03**	2.739e+05	1.082e+05	3.784e+04	2.292e+03
	std	1.685e+04	**3.401e+02**	5.773e+04	1.041e+04	1.544e+04	4.423e+02
	best	**-8.212e+00**	-8.211e+00	-8.209e+00	-8.212e+00	-8.211e+00	-8.212e+00
F10	mean	-8.209e+00	-8.209e+00	-8.156e+00	-7.898e+00	-8.210e+00	**-8.210e+00**
	std	1.742e-03	1.227e-03	5.081e-02	2.378e-01	**9.636e-04**	1.100e-03
	best	5.120e+04	5.134e+04	5.236e+04	1.429e+06	**5.086e+04**	5.088e+04
F11	mean	**5.197e+04**	5.213e+04	7.114e+04	2.222e+06	5.198e+04	5.219e+04
	std	**3.586e+02**	4.715e+02	2.693e+04	4.810e+05	5.215e+02	5.890e+02
	best	1.076e+06	**1.073e+06**	1.428e+06	1.922e+06	1.073e+06	1.074e+06
F12	mean	1.079e+06	1.077e+06	1.830e+06	2.209e+06	1.078e+06	**1.077e+06**
	std	2.782e+03	1.577e+03	1.865e+05	1.605e+05	8.229e+03	**1.375e+03**
	best	**1.544e+04**	**1.544e+04**	1.544e+04	1.544e+04	**1.544e+04**	**1.544e+04**
F13	mean	**1.544e+04**	1.544e+04	1.545e+04	1.545e+04	**1.544e+04**	1.545e+04
	std	**3.638e-12**	1.687e+00	8.446e-01	3.397e+00	**3.638e-12**	1.077e+01
	best	1.808e+04	**1.802e+04**	1.845e+04	1.816e+04	1.808e+04	1.802e+04
F14	mean	1.810e+04	1.811e+04	1.868e+04	1.835e+04	**1.810e+04**	1.810e+04
	std	3.282e+01	4.389e+01	1.194e+02	8.105e+01	**2.448e+01**	3.465e+01
	best	3.274e+04	**3.273e+04**	3.291e+04	3.278e+04	3.274e+04	3.274e+04
F15	mean	3.274e+04	3.275e+04	3.303e+04	3.290e+04	3.274e+04	**3.274e+04**
	std	2.384e+00	1.407e+01	3.921e+01	5.080e+01	**1.807e+00**	6.017e+00
	best	1.234e+05	1.236e+05	1.296e+05	1.322e+05	1.238e+05	**1.228e+05**
F16	mean	1.248e+05	1.249e+05	1.338e+05	1.372e+05	1.249e+05	**1.238e+05**
	std	**5.411e+02**	6.374e+02	1.664e+03	2.818e+03	5.910e+02	5.798e+02
	best	1.869e+06	**1.825e+06**	1.921e+06	2.010e+06	1.843e+06	1.838e+06
F17	mean	1.896e+06	**1.859e+06**	1.933e+06	5.219e+06	1.872e+06	1.864e+06
	std	1.106e+04	1.144e+04	**7.402e+03**	3.086e+06	1.149e+04	1.083e+04
	best	9.341e+05	9.297e+05	9.390e+05	1.885e+06	9.341e+05	**9.286e+05**
F18	mean	9.373e+05	9.331e+05	9.641e+05	3.056e+06	9.370e+05	**9.314e+05**
	std	1.195e+03	1.834e+03	6.855e+04	5.090e+05	1.419e+03	**1.171e+03**
	best	9.385e+05	**9.341e+05**	1.147e+06	2.218e+06	9.387e+05	9.371e+05
F19	mean	9.416e+05	9.400e+05	1.397e+06	3.642e+06	9.421e+05	**9.397e+05**
	std	1.292e+03	1.949e+03	1.909e+05	6.964e+05	1.401e+03	**1.267e+03**
	best	9.423e+05	**9.362e+05**	9.477e+05	1.896e+06	9.396e+05	9.363e+05
F20	mean	9.453e+05	9.406e+05	9.868e+05	3.054e+06	9.447e+05	**9.393e+05**
	std	**1.364e+03**	1.702e+03	8.314e+04	5.209e+05	1.656e+03	1.503e+03
	best	2.524e+01	**2.225e+01**	2.396e+01	2.964e+01	2.527e+01	2.683e+01
F21	mean	3.036e+01	**2.900e+01**	2.952e+01	3.458e+01	3.116e+01	3.138e+01
	std	1.917e+00	2.902e+00	2.725e+00	2.410e+00	2.722e+00	**1.892e+00**
	best	1.154e+01	**1.149e+01**	1.176e+01	1.176e+01	1.154e+01	1.149e+01
F22	mean	1.194e+01	1.199e+01	1.285e+01	1.473e+01	**1.176e+01**	1.229e+01
	std	4.706e-01	9.395e-01	7.128e-01	1.919e+00	**3.746e-01**	1.325e+00

Table A.22: Statistics (best, mean, and standard deviation of the objective values) for all algorithms on the CEC2011 benchmark suite under an evaluation budget of $N_{\max}$ = 150,000. Each value is computed over 51 independent runs. Boldface indicates the best result for each statistic.

Function	Statistic	LSHADE	ARRDE	NLSHADE-RSP	j2020	jSO	LSHADE-cnEpSin
	best	**0.000e+00**	**0.000e+00**	**0.000e+00**	**7.369e-14**	**0.000e+00**	**0.000e+00**
F1	mean	5.079e-05	1.650e-01	2.495e+00	9.922e-01	**7.340e-29**	1.524e+00
	std	2.343e-04	1.167e+00	4.545e+00	3.031e+00	**5.190e-28**	3.835e+00
	best	-2.773e+01	**-2.842e+01**	-2.773e+01	-1.949e+01	-2.751e+01	-2.711e+01
F2	mean	-2.642e+01	**-2.727e+01**	-2.570e+01	-1.476e+01	-2.607e+01	-2.595e+01
	std	**5.310e-01**	6.508e-01	1.182e+00	2.015e+00	6.359e-01	5.352e-01
	best	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**
F3	mean	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**	**-4.049e-02**
	std	**6.939e-18**	**6.939e-18**	**6.939e-18**	**6.939e-18**	**6.939e-18**	**6.939e-18**
	best	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**
F4	mean	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**	**1.339e+02**
	std	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**	**2.842e-14**
	best	-3.685e+01	-3.685e+01	**-3.689e+01**	-3.499e+01	-3.684e+01	-3.681e+01
F5	mean	**-3.642e+01**	-3.436e+01	-3.617e+01	-3.380e+01	-3.558e+01	-3.630e+01
	std	**4.205e-01**	1.391e+00	6.858e-01	6.107e-01	8.836e-01	6.216e-01
	best	-2.916e+01	**-2.917e+01**	**-2.917e+01**	-2.904e+01	-2.917e+01	-2.916e+01
F6	mean	**-2.893e+01**	-2.384e+01	-2.881e+01	-2.707e+01	-2.808e+01	-2.647e+01
	std	**4.302e-01**	2.342e+00	6.131e-01	1.413e+00	1.163e+00	3.508e+00
	best	8.523e-01	**5.000e-01**	6.361e-01	6.849e-01	7.553e-01	8.313e-01
F7	mean	1.067e+00	**7.775e-01**	9.611e-01	1.137e+00	1.090e+00	1.062e+00
	std	**8.326e-02**	1.538e-01	1.447e-01	1.300e-01	1.220e-01	8.965e-02
	best	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**
F8	mean	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**	**2.200e+02**
	std	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**	**0.000e+00**
	best	2.581e+03	8.352e+02	2.595e+04	7.138e+04	1.262e+03	**7.688e+02**
F9	mean	6.634e+03	**1.471e+03**	6.885e+04	8.620e+04	3.550e+03	1.547e+03
	std	3.582e+03	**3.459e+02**	2.698e+04	7.444e+03	2.780e+03	3.814e+02
	best	-8.212e+00	-8.212e+00	-8.211e+00	-8.212e+00	-8.212e+00	**-8.212e+00**
F10	mean	**-8.210e+00**	-8.209e+00	-8.156e+00	-7.983e+00	-8.210e+00	-8.210e+00
	std	1.149e-03	1.150e-03	4.981e-02	1.678e-01	**8.572e-04**	1.626e-03
	best	5.119e+04	**5.064e+04**	5.145e+04	5.283e+05	5.105e+04	5.120e+04
F11	mean	5.199e+04	**5.170e+04**	5.227e+04	1.492e+06	5.208e+04	5.209e+04
	std	**3.998e+02**	4.827e+02	5.122e+02	3.457e+05	4.852e+02	4.816e+02
	best	1.072e+06	1.073e+06	1.081e+06	1.547e+06	1.072e+06	**1.070e+06**
F12	mean	1.075e+06	1.075e+06	1.251e+06	1.741e+06	1.077e+06	**1.074e+06**
	std	**1.054e+03**	1.361e+03	1.161e+05	9.837e+04	1.798e+03	1.302e+03
	best	**1.544e+04**	**1.544e+04**	**1.544e+04**	1.544e+04	**1.544e+04**	**1.544e+04**
F13	mean	**1.544e+04**	1.544e+04	1.545e+04	1.545e+04	**1.544e+04**	1.545e+04
	std	**3.638e-12**	1.802e-01	8.963e-01	1.842e+00	**3.638e-12**	1.188e+01
	best	1.808e+04	**1.802e+04**	1.843e+04	1.812e+04	1.808e+04	1.806e+04
F14	mean	1.810e+04	**1.810e+04**	1.868e+04	1.827e+04	1.810e+04	1.810e+04
	std	3.034e+01	4.421e+01	1.089e+02	7.549e+01	**2.485e+01**	3.866e+01
	best	3.274e+04	**3.273e+04**	3.293e+04	3.279e+04	3.274e+04	3.274e+04
F15	mean	3.274e+04	3.274e+04	3.302e+04	3.287e+04	**3.274e+04**	3.274e+04
	std	6.143e+00	1.015e+01	4.250e+01	3.936e+01	**5.931e-01**	5.901e+00
	best	1.225e+05	1.230e+05	1.306e+05	1.302e+05	1.230e+05	**1.221e+05**
F16	mean	1.240e+05	1.242e+05	1.334e+05	1.348e+05	1.238e+05	**1.233e+05**
	std	5.745e+02	5.578e+02	1.392e+03	3.175e+03	**4.615e+02**	5.484e+02
	best	1.835e+06	1.818e+06	1.917e+06	2.000e+06	1.828e+06	**1.801e+06**
F17	mean	1.863e+06	1.840e+06	1.933e+06	3.159e+06	1.848e+06	**1.822e+06**
	std	1.051e+04	1.174e+04	**7.986e+03**	9.250e+05	9.362e+03	1.013e+04
	best	9.307e+05	9.287e+05	9.376e+05	1.654e+06	9.315e+05	**9.269e+05**
F18	mean	9.336e+05	9.309e+05	9.470e+05	2.690e+06	9.345e+05	**9.289e+05**
	std	1.339e+03	1.193e+03	2.533e+04	5.065e+05	1.359e+03	**9.885e+02**
	best	9.372e+05	9.356e+05	1.086e+06	1.615e+06	9.382e+05	**9.351e+05**
F19	mean	9.398e+05	9.385e+05	1.255e+06	3.372e+06	9.407e+05	**9.379e+05**
	std	1.144e+03	1.658e+03	9.522e+04	5.879e+05	1.337e+03	**1.089e+03**
	best	9.387e+05	9.342e+05	9.464e+05	1.505e+06	9.374e+05	**9.334e+05**
F20	mean	9.414e+05	9.385e+05	9.556e+05	2.642e+06	9.412e+05	**9.367e+05**
	std	**1.244e+03**	1.864e+03	2.277e+04	4.687e+05	1.615e+03	1.584e+03
	best	2.406e+01	**2.194e+01**	2.282e+01	2.830e+01	2.264e+01	2.432e+01
F21	mean	2.918e+01	2.904e+01	2.883e+01	3.246e+01	**2.864e+01**	3.038e+01
	std	**1.973e+00**	2.487e+00	2.538e+00	2.176e+00	2.526e+00	2.150e+00
	best	1.149e+01	**1.149e+01**	1.182e+01	1.162e+01	1.150e+01	**1.149e+01**
F22	mean	1.170e+01	1.172e+01	1.281e+01	1.400e+01	**1.161e+01**	1.263e+01
	std	3.563e-01	4.309e-01	7.462e-01	1.485e+00	**1.822e-01**	1.579e+00