Bayesian Persuasion without Commitment

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Abstract

We introduce a model of persuasion in which a sender without any commitment power privately gathers information about an unknown state of the world and then chooses what to verifiably disclose to a receiver. The receiver does not know how many experiments the sender is able to run, and may therefore be uncertain as to whether the sender disclosed all of her information. Despite this challenge, we show that, under general conditions, the sender is able to achieve the same payoff as in the full-commitment Bayesian persuasion case.

1   Introduction

A central question in the theory of information design is how a sender can influence the action of a receiver by strategically disclosing information about an uncertain state. While much of the literature assumes that the sender can commit to a signal structure ex ante, in many environments commitment is implausible: the sender acquires information privately and can choose, after the fact, what to reveal. This paper studies such a setting in which the sender has no commitment power and the receiver reasons both about what the sender revealed and what she may have hidden.

We consider a model in which the sender designs and executes a set of experiments to learn about the underlying state. The total number of experiments the sender can run is determined exogenously by Nature. The sender strategically selects which experiments to

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perform and observes the realization of each. All of the sender's information acquisition is private: the receiver does not observe the number of experiments, their design, or their outcomes. After running the experiments, without commitment, the sender chooses which outcomes, if any, to reveal to the receiver (along with the design of the experiment from which it arose). The receiver, understanding both the sender's discretion over experiment choice and the possibility of selective disclosure, updates beliefs accordingly and selects an action. We identify a simple condition under which, in the sender-optimal equilibrium, the sender does as well as if she had full commitment power (that is, there is an equilibrium that implements the Bayesian persuasion solution).

As a motivating example, consider a corporation seeking to influence a regulatory decision on a major infrastructure project. To support its position, the firm can commission independent studies on various aspects such as economic impact, environmental effects, and job creation, but faces practical constraints like limited funding, time pressure, or expert availability, which are beyond its control and vary across projects. The firm strategically chooses which types of studies to conduct, observes the results, and then selectively discloses any subset of the findings to the regulator. Aware of the firm's discretion over both the design and disclosure of studies, the regulator updates its beliefs about the project and makes a policy decision accordingly.

As another example, consider a pharmaceutical company preparing a drug approval application. It may conduct clinical trials targeting different populations or treatment variations, but institutional constraints such as limited patient availability, recruitment delays, or regulatory approvals restrict how many trials can be executed. The company strategically selects which trials to run, observes the outcomes, and includes only a subset of trials and their results in its submission. Aware of the potential for selective disclosure, the health authority evaluates the submitted evidence with caution before making an approval decision.

We introduce a model in which the sender can perform a limited number of simultaneous, conditionally independent experiments to learn about an unknown state. The number of experiments is determined randomly by Nature according to a known distribution and is observed only by the sender. Each experiment produces a signal about the state, and the sender observes all outcomes privately. After observing the results, the sender can choose to disclose any subset of the outcomes to a receiver. Each disclosed

signal is labeled with the experiment from which it originated, ensuring that the receiver can correctly interpret the information contained in its outcome. The receiver, aware of the sender's discretion over both experiment selection and selective disclosure, forms a belief about the state and chooses an action to maximize expected utility.

We show, under general conditions, the sender can achieve the Bayesian persuasion benchmark in equilibrium. This result holds despite the absence of commitment and despite the receiver's awareness that the sender may be withholding information. The conditions needed for this result are that (i) the is always able to run at least one experiment, and (ii) there exists a "punishing" action (possibly mixed) that the sender does not prefer in any state to any of the actions chosen in the Bayesian persuasion solution. The latter condition is guaranteed to hold when the sender has state-independent preferences (also known as "transparent motives").

The Bayesian persuasion solution can be achieved in a relatively simple way when the sender has transparent motives. In that case, it suffices to consider a single experiment whose outcomes correspond one-to-one with actions used in the Bayesian persuasion solution. The sender simply repeats the same experiment as many times as she is able to, and always reveals only one outcome—the one associated with her most preferred action among all of the outcomes she observed. We show that there always exists an experiment for which this strategy generates the same distribution of posterior beliefs for the receiver as in Bayesian persuasion.

With state-dependent preferences, the sender's disclosure decisions can depend in more complicated ways on the set of outcomes she observes. Moreover, the sender's information may run counter to her interests; for example, with a binary state, it could be that the sender wants to induce a low belief in the receiver precisely when her own belief is high. In this case, it suffices to consider two kinds of strategies for the sender: for any given number k of experiments that she can run, she either runs k identical experiments or she runs $k - 1$ identical experiments along with one experiment that fully reveals the state. Either way, the sender discloses exactly one realized outcome from the repeated experiment; if she learns the state, she uses it only to inform her choice of which outcome to disclose. We employ a fixed point argument to show that there exists an experiment that generates the Bayesian persuasion solution when the sender chooses optimally from this class of strategies for each k. The resulting experiment can differ

substantially from that in the standard one-shot optimal Bayesian persuasion policy.

It is sometimes possible to obtain the Bayesian persuasion solution even if there is a positive probability that the sender cannot run any experiments. We provide a tight characterization for the case of transparent motives. To prevent deviations by the sender, it must be that disclosing nothing leads to the worst Bayesian persuasion action, $\underline{a}$, for the sender. The Bayesian persuasion solution can be attained in equilibrium if the probability that the sender cannot run any experiments is low enough that the posterior associated with $\underline{a}$ could arise by combining the prior belief when the sender runs no experiments with the posterior obtained conditional on running experiments and disclosing the worst-case outcome.

The original model of Bayesian persuasion (Kamenica and Gentzkow, 2011) features a sender who can choose any experiment and fully commits to disclosing its outcome. In an extension, they show that the sender can achieve the Bayesian persuasion payoff if the choice of experiment is public and signals are verifiable. More recently, a number of papers have relaxed the full commitment assumption in various ways. The Bayesian persuasion payoff can also be obtained (under some conditions) if the sender sends cheap talk messages but has incentives to maintain a reputation (Best and Quigley, 2024; Mathevet, Pearce, and Stacchetti, 2019). In other models, the sender cannot typically obtain the Bayesian persuasion payoffs. Lipnowski and Ravid (2020) analyze cheap talk environments in which the sender has no commitment power and messages are not verifiable. Lipnowski, Ravid, and Shishkin (2022) studies intermediate cases in which, with some probability, the sender can freely change the message observed by the receiver. In Koessler and Skreta (2023), the sender commits to the experiment only after learning the state of the world. In contrast to these models, in our model, the sender discloses verifiable information and is uninformed at the time of choosing the experiment.

Dai, Fudenberg, and Pei (2025) independently developed a model that, like ours, features a receiver who is uncertain about how many experiments the sender can run, the outcomes of which can be verifiably disclosed. In their model, unlike ours, the sender also commits to a public experiment and acquires information sequentially. In the case of transparent motives, under a stronger notion of equilibrium than we employ, they identify conditions under which the Bayesian persuasion payoff is not attainable for the sender. Lou (2023) similarly considers a sender who can run multiple sequential experiments and

disclose a subset of them. Unlike the present paper and Dai, Fudenberg, and Pei (2025), in his model, a sender who discloses the outcome of one experiment must also disclose the outcomes of all previous experiments.

Beginning with Glazer and Rubinstein (2004), a series of papers have analyzed games in which a sender with a private type can disclose exogenous verifiable evidence to a receiver, and have identified conditions under which the *receiver* cannot benefit from commitment to a mechanism (Glazer and Rubinstein, 2006; Sher, 2011; Hart, Kremer, and Perry, 2017; Ben-Porath, Dekel, and Lipman, 2019). In contrast, we focus on the role of commitment to a disclosure policy by a *sender* who is uninformed ex ante and optimally chooses what evidence to look for.

2 Model

There is an unknown state of the world $\omega \in \Omega$, where $\Omega = \{\omega_1, \ldots, \omega_n\}$ is a finite set. The state is realized according to a common prior probability $p \in \Delta(\Omega)$. We assume without loss of generality that $p_\omega > 0$ for every $\omega \in \Omega$. The receiver has a finite action set A, and the receiver's utility is given by a function $u : A \times \Omega \to \mathbb{R}$, the sender's utility is given by $\tilde{v} : A \times \Omega \to \mathbb{R}$. When the sender has state-independent preferences, we sometimes omit the ω argument from $\tilde{v}$.

The sender can acquire information about the state by choosing a sequence of conditionally independent experiments and observing their realizations. The number of independent experiments that the sender can choose is determined at random according to some distribution $\nu \in \Delta(\mathbb{N}_+)$; we write ν_k for the probability ν assigns to $k \in \mathbb{N}_+$ and refer to k as the *type* of the sender. Information acquisition is not observed by the receiver. Let S be a measurable set containing at least n elements. An *experiment* is a mapping $\pi : \Omega \to \Delta(S)$. Together with the prior p, an experiment induces a probability distribution $\mathbf{P}_\pi \in \Delta(\Omega \times S)$. Let Π denote the set of all experiments.

The interaction between the sender and receiver takes place as follows. First, a number $k \in \mathbb{N}_+$ of experiments that the sender can conduct is drawn with probability ν_k and is observed only by the sender. The sender then chooses k experiments $\pi_1, \ldots, \pi_k \in \Pi$. Thereafter, the state $\omega \in \Omega$ is realized, and the experiments are conducted independently given ω. The state is not directly observed by the sender; only the outcomes

$(s_1, \ldots, s_k)$ of the k experiments are observed. The sender can then choose any subset $T \subseteq \{(\pi_1, s_1), \ldots, (\pi_k, s_k)\}$ (with repetition allowed) to reveal to the receiver. The receiver observes the subset T and chooses an action $a \in A$.

Formally, a strategy for the sender consists, for each $k \in \operatorname{supp} \nu$, of an element of Π^k indicating which experiments to run together with a mapping $\tau_k : S^k \longrightarrow 2^{\{1,\ldots,k\}}$ indicating which outcomes to disclose. Let $\tau = (\tau_k)_{k \in \operatorname{supp} \nu}$ denote the sender's strategy.

For the receiver, let Δ^* be the set of all subsets $\{(\pi_1, s_1), \ldots, (\pi_\ell, s_\ell)\}$ of $\Pi \times S$ of size ℓ for any finite $\ell \leq \sup \operatorname{supp} \nu$ (including $\ell = 0$). A belief system for the receiver is a mapping $\rho : \Delta^* \to \Delta(\Omega)$ that assigns a posterior belief $\rho(T)$ to each $T \in \Delta^*$.[1] A strategy for the receiver is a mapping $\sigma : \Delta^* \longrightarrow \Delta(A)$ that specifies an action—possibly mixed—for each possible set of disclosed information.

Note that the sender's strategy τ induces a belief distribution $\mathbb{P}_\tau \in \Delta(\Delta^* \times \Omega)$.

Definition 1. A profile $(\tau, (\rho, \sigma))$ is a *weak perfect Bayesian equilibrium* (wPBE) of the game if τ_k is a best reply to (ρ, σ) for every k, $\rho(T) = \mathbb{P}_\tau(\cdot|T)$ for almost every T (with respect to τ), and $\sigma(T)$ is optimal given the belief $\rho(T)$ for almost every T.

3 Main Result

Note that, in any wPBE, the distribution of posterior beliefs for the receiver must be Bayes plausible. Thus the maximal expected utility that the sender can achieve in any wPBE is bounded above by the Bayesian persuasion utility. We ask whether the sender can achieve this utility in our setting.

Definition 2. Given a subset of actions $B \subseteq A$, the receiver has a *punishing action* with respect to B if there exists a posterior belief $q \in \Delta(\Omega)$ and a distribution over actions $x \in \Delta(A)$ such that x is a best reply for the receiver to the belief q and $\sum_{a \in A} x(a)\tilde{v}(a, \omega) \leq \tilde{v}(b, \omega)$ for every $b \in B$ and $\omega \in \Omega$.

Thus a punishing action exists with respect to B if the action distribution x is worse for the sender than any $b \in B$ at every state $\omega \in \Omega$. For example, if the sender has transparent motives, then for every B, there exists a punishing action with respect to B (namely, the sender's least preferred action in B).

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[1] We omit beliefs about elements of the history other than the state ω because they are not relevant for equilibrium.

We henceforth assume for simplicity that there exists a unique Bayesian persuasion policy. By the revelation principle, we can identify this policy with a direct recommendation policy $P \in \Delta(A \times \Omega)$. Let $\tilde{A}$ be the set of actions that are played with positive probability under P.

Theorem 1. *If the receiver has a punishing action with respect to $\tilde{A}$, then there exists a wPBE in which the sender achieves the Bayesian persuasion utility.*

Proofs are in the Appendix.

Unlike the standard persuasion argument to establish the existence of the sender-optimal equilibrium with a Bayesian persuasion policy, we use a fixed point argument.

We first illustrate the theorem with an example.

Example 1. Consider the classic product adoption example where the state space $\Omega = \{\omega_0, \omega_1\}$ represents the quality of a product which can be low ($\omega = \omega_0$) or high ($\omega = \omega_1$), with equal prior probability. The action set is $A = \{a_0, a_1\}$. Action a_0 represents a non-adoption action that yields a payoff of 0 to both sender and receiver. Action a_1 represents adoption and yields a payoff of 1 to the sender but is risky for the receiver, with $u(a_1\omega_0) = -3$ and $u(a_1, \omega_1) = 1$. Thus, the receiver would like to adopt the product if and only if she assigns probability at least $3/4$ to state ω_1. The sender's indirect utility is depicted in Figure 1.
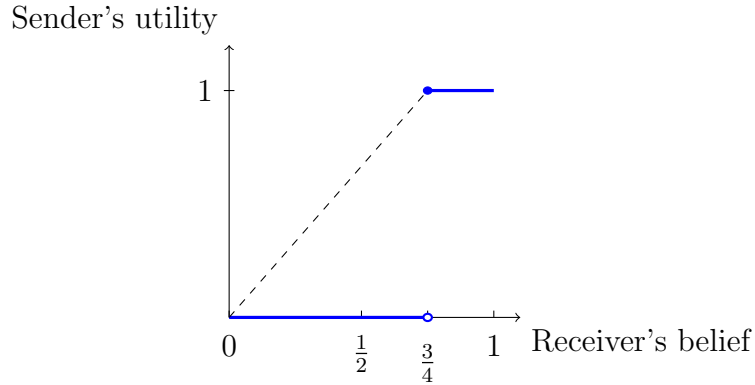

Figure 1: The sender's indirect utility for Example 1. The dashed line depicts the concavification associated with the Bayesian persuasion solution.

The Bayesian persuasion utility is implemented by the distribution of posteriors $\frac{1}{3}\delta_0 + \frac{2}{3}\delta_{\frac{3}{4}}$, where δ_q denotes the distribution placing unit mass on q. Suppose the number of experiments available to the sender is distributed geometrically with parameter r, that

is, that $\nu_k = (1-r)^{k-1}r$ for each k. We will illustrate how to generate a wPBE that implements the Bayesian persuasion distribution.

Consider the experiment F that generates the posterior distribution

$$(1-\alpha)\delta_0 + \alpha\delta_{\frac{r+2}{2r+2}},$$

where $\alpha = \frac{2r+2}{2r+4}$. This posterior distribution is achieved by an experiment $F : \Omega \to \Delta(S)$ for $S = \{s_0, s_1\}$ with $F(\omega_1)[s_1] = 1$ and $F(\omega_0)[s_1] = \frac{r}{r+2}$.

Consider the following strategy τ for the sender. For each k, repeat the experiment F k times. If there is at least one realized s_1 signal, reveal $\{(F, s_1)\}$ to the receiver; otherwise, reveal $\{(F, s_0)\}$ to the receiver.

We first claim that, on the equilibrium path, the receiver's beliefs in a wPBE must be $\rho(\{F, s_1\}) = 3/4$ if $\{(F, s_1)\}$ is revealed and $\rho(\{F, s_0\}) = 0$ if $\{(F, s_0)\}$ is revealed. The latter follows immediately from the construction of F; for the former, we must show that

$$P_\tau(\omega_1 | \{(F, s_1)\}) = \frac{3}{4}.$$

To see this, note that the signal s_1 is realized with probability one given ω_1. Therefore, $P_\tau(\{(F, s_1)\} | \omega_1) = 1$. By definition,

$$P_\tau(\{(F, s_0)\} | \omega_0) = \sum_{k=1}^{\infty} p(1-p)^{k-1}\left(1 - \frac{p}{p+2}\right)^k = \frac{2}{3}.$$

Therefore, $P_\tau(\omega_1 | \{(F, s_1)\}) = 3/4$, as desired. It is straightforward to verify that the sender has no profitable deviation.

To interpret the construction, notice that the positive signal in the experiment has a posterior $\frac{r+2}{2r+2} > \frac{3}{4}$. Thus, in order to compensate for the possibility that the sender can run multiple experiments, to persuade the receiver to take action a_1, a high realization from any one experiment must provide stronger evidence in favor of the high state than does the high signal in the Bayesian persuasion solution.

3.1 Outline of Proof of Theorem 1

It turns out that the equilibrium construction in Example 1 applies more generally as long as the sender has transparent motives (in which case a punishing action always exists). In that case, as we show in the appendix, there exists an experiment $F : \Omega \to \Delta(S)$ where each signal $s_j \in S = \{s_1, \ldots, s_m\}$ corresponds to an action a_j such that $\tilde{v}(a_j) \le \tilde{v}(a_i)$

for $j < i$. The strategy for the sender of type k is to draw k conditionally i.i.d. signals $\{s^1, \ldots, s^k\}$ according to F and then disclose $\{(F, s_i)\}$ where $i = \arg\max\{j : s_j = s^l \text{ for some } l\}$. The receiver then plays the corresponding action a_i.

Outside of the transparent motives case, the sender's preferences over actions may depend on her posterior belief, making it no longer possible to order actions by the sender's preference. The sender's optimal disclosure strategy may therefore depend in complicated ways on the realized profile of signals. To overcome this difficulty, we use a fixed point argument. The following example illustrates the basic logic.

Example 2. Let $\Omega = \{\omega_0, \omega_1\}$. The prior is $1/2$, where we identify the prior and posterior with the probability of ω_1. The receiver's action set is $A = \{a_0, a_1, a_2\}$. Action a_0 is optimal for the receiver for posteriors in $[0, 1/3]$, action a_1 is optimal for posteriors in $[1/3, 2/3]$, and action a_2 is optimal for posteriors in $[2/3, 1]$. The sender's utility is $\tilde{v}(a_0, \omega_0) = 0$, $\tilde{v}(a_0, \omega_1) = 3/2$, $\tilde{v}(a_1, \omega) = 0$ for each ω, $\tilde{v}(a_2, \omega_1) = 0$, and $\tilde{v}(a_2, \omega_0) = 3/2$. Note that a_1 is a punishing action. The sender's indirect utility is depicted in Figure 2.
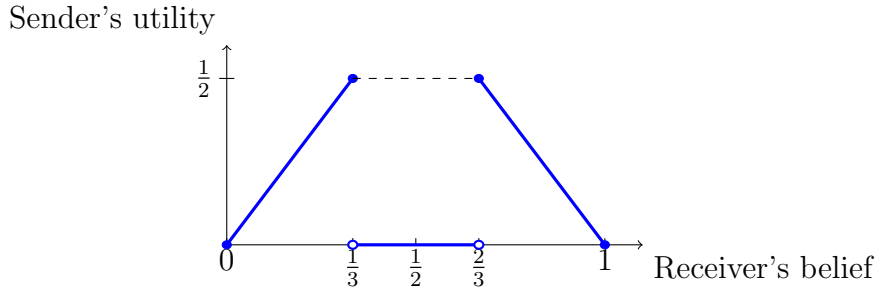

Figure 2: The sender's indirect utility for Example 2.

Notice that the sender's interests are opposed to the receiver's in the sense that, if they had the same belief, they would never agree on the optimal action: whenever the receiver prefers action a_2 to action a_0, a sender with the same belief would prefer a_0 to a_2, and vice versa. Thus, for example, a sender with belief above $1/2$ would ideally like to induce a belief of at most $1/3$ in the receiver. One might expect this conflict to create difficulties in persuading the receiver, yet Theorem 1 indicates that the Bayesian persuasion payoff is achievable for the sender.

Let $\nu = (\nu_k)_{k \geq 1}$ be any distribution of types for the sender. Consider any experiment $F : \Omega \to \{s_1, s_2\}$. Let $x = F(\omega_1)[s_1]$ and $y = F(\omega_0)[s_1]$.

Suppose the receiver plays action a_0 if the sender discloses $\{(F, s_1)\}$, plays action a_2

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if the sender discloses $\{(F, s_2)\}$, and plays action a_1 otherwise. It follows that whenever the sender assigns posterior probability greater than $1/2$ to ω_1, she will disclose $\{(F, s_1)\}$ whenever she has received at least one signal equal to s_1; similarly, whenever she assigns probability less than $1/2$ to ω_1 she will disclose $\{(F, s_2)\}$ whenever it is feasible to do so. This disclosure strategy generates a probability distribution $P \in \Delta(\Omega \times A)$ where, for example, $P(a_0|\omega_1)$ is the conditional probability that action a_0 is played in state ω_1.

Given this disclosure strategy, define a mapping by

$$T(x, y) = \left(x + \alpha^* \left(\frac{2}{3} - P(a_0|\omega_1) \right), y + \alpha^* \left(\frac{1}{3} - P(a_0|\omega_0) \right) \right)$$

for $\alpha^* > 0$. If this mapping has a fixed point, it means that the corresponding experiment together with the above strategies lead to $P(a_0|\omega_1) = 2/3$ and $P(a_0|\omega_0) = 1/3$, as in the Bayesian persuasion solution. It is straightforward to verify that this forms a wPBE.

A couple of modifications are needed to ensure that $T(x, y)$ has a fixed point. First, we need the mapping to be continuous. By allowing the sender to choose any mixture of optimal disclosures when she is indifferent, we obtain a correspondence satisfying the required continuity property. Second, in some cases, at least some types of the sender may prefer to deviate and run one experiment that fully reveals the state; the result of this fully revealing experiment is never disclosed to the receiver, but can increase the sender's utility because it enables her to tailor her disclosure choice according to the state. To accommodate this possibility, we simply allow for the option to run one fully revealing experiment as part of the sender's optimal strategy.

A final challenge is to ensure that T maps binary experiments to binary experiments, i.e., that its image is contained in $[0, 1]^2$. We prove that this is indeed the case when α^* is sufficiently small.

4   Examples

In this section, we analyze three examples. The first shows how the equilibrium experiment disclosed to the receiver can be qualitatively different from the one used in the Bayesian persuasion solution. The second shows that the conclusion of Theorem 1 can fail if there is no punishing action. The third shows that the conclusion of Theorem 1 can fail if there is a positive probability that the sender cannot run any experiments. Section 5 examines this last issue in detail for the case of transparent motives.

Example 3. Let $\Omega = \{\omega_0, \omega_1\}$ with prior $p = 1/2$. Suppose the sender's type is equal to 3 with probability one. There are 4 actions $A = \{a_1, a_2, a_3, a_4\}$. The sender's utility is $u(a_1, \omega) = u(a_4, \omega) = -1/2$ for each ω, $u(a_2, \omega_0) = u(a_3, \omega_1) = 1/2$, and $u(a_2, \omega_1) = u(a_3, \omega_0) = -1/2$. The sender's indirect utility is depicted in Figure 3.
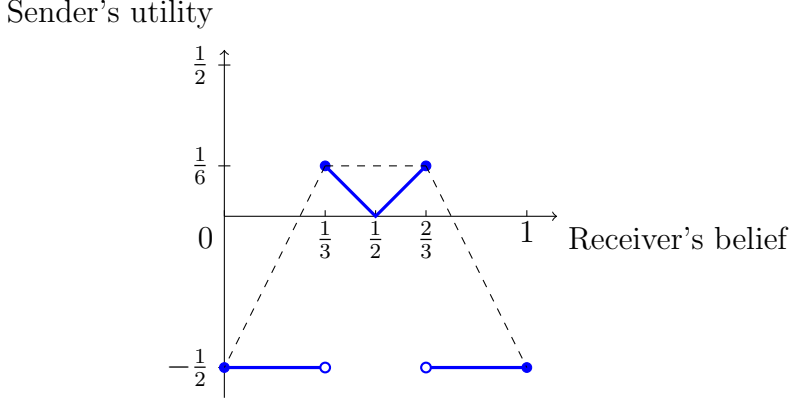

Sender's utility

Receiver's belief

Figure 3: The sender's indirect utility for Example 3. The dashed line depicts the concavification. The distribution of posteriors that implements the Bayesian persuasion policy is $\frac{1}{2}\delta_{\frac{1}{3}} + \frac{1}{2}\delta_{\frac{2}{3}}$.

We first try to construct an equilibrium of the following form. The sender runs 3 conditionally i.i.d. symmetric binary experiments $F : \Omega \to \{s_2, s_3\}$ and discloses $\{(F, s_i)\}$ if and only if signal s_i occurs a majority of the time (i.e., either 2 or 3 times). After observing disclosure $\{(F, s_i)\}$, the receiver plays action a_i. These strategies result in the Bayesian persuasion solution if $F(\omega_1)[s_3] = F(\omega_0)[s_2] = q$ where $q \approx 0.613$ is the solution of $q^3 + \binom{3}{2}(q^2(1-q)) = 2/3$. For such an experiment, the sender's disclosure strategy is to reveal the majority signal and the receiver's strategy is a best reply to the sender's since $P_F(\omega_1 \mid \text{majority of } s_2) = 1/3$ and $P_F(\omega_1 \mid \text{majority of } s_3) = 2/3$.

We claim, however, that the sender has a profitable deviation. Consider instead using one of the three experiments to learn the state while running the other two experiments as above according to F. The sender then discloses $\{(F, s_3)\}$ in state ω_1 if she has received at least one s_3 signal, and discloses $\{(F, s_2)\}$ in state ω_0 if she has received at least one s_2 signal. This is indeed a profitable deviation since the conditional probability of playing action a_3 in state ω_1 is at least $3/4$ (and similarly for a_2 in state ω_0), which exceeds the corresponding probability of $2/3$ in the above strategy profile.

Given that the sender learns the state, in equilibrium, the receiver accounts for the

likelihood that disclosure of $\{(F, s_3)\}$ means the sender learned the state is ω_1. This effect tends to increase the receiver's belief following disclosure of $\{(F, s_3)\}$. How, then, can the sender's strategy ensure that this belief is not greater than $2/3$? Perhaps counterintuitively, the resolution involves reversing the direction of the signals so that s_3 is relatively *more* likely in state ω_0 than in state ω_1.

For the exact construction, take $F(\omega_1)[s_2] = F(\omega_0)[s_3] = \frac{1}{\sqrt{3}} \approx 0.577$. The sender's strategy is as follows. She runs two experiments according to F and one that fully reveals the state. She then discloses $\{(F, s_3)\}$ in state ω_1 whenever doing so is feasible, and discloses $\{(F, s_2)\}$ in state ω_0 whenever feasible. The receiver then plays action a_3 if and only if he observes signal s_3. Thus the receiver plays a_3 despite the fact that the disclosed signal, if taken at face value, suggests that the state is more likely to be ω_0.

To see that this is indeed a wPBE, we need to verify first that the posterior distribution generated by the sender's strategy is the desired one. Indeed, the probability of revealing signal s_3 at state ω_1 is $1 - (1/\sqrt{3})^2 = 2/3$, as desired, and similarly for s_2. The sender's disclosure strategy, given her information, maximizes her utility by construction. The only deviation that needs to be considered is that the sender could draw 3 signals according to F instead of learning the state and reveal the majority signal. Under this deviation, the probability of revealing signal s_3 in state ω_1 is

$$\left(1 - \frac{1}{\sqrt{3}}\right)^3 + 3\left(\frac{1}{\sqrt{3}}\right)^2\left(1 - \frac{1}{\sqrt{3}}\right) < \frac{2}{3}.$$

Therefore, the sender has no profitable deviation.

Example 4. This example shows that if there is no punishing action, the conclusion of Theorem 1 may not hold.

Let $\Omega = \{\omega_0, \omega_1\}$ and $A = \{a_0, a_1\}$. The prior, which we identify with the probability of state ω_1, is $1/2$. Action a_0 is optimal for the receiver if and only if she assigns posterior probability at most $2/3$ to state ω_1 and action a_1 is optimal if and only if she assigns probability at least $2/3$ to state ω_1. The utility of the sender is $\tilde{v}(a_0, \omega) = 1$ for each $\omega \in \Omega$, $\tilde{v}(a_1, \omega_1) = 2/3 + \varepsilon$, and $\tilde{v}(a_1, \omega_0) = 5/3 + \varepsilon$ for some $\varepsilon > 0$. The indirect utility of the sender is depicted in Figure 4. As is evident from the graph, the Bayesian persuasion policy induces a posterior distribution of $\frac{1}{4}\delta_0 + \frac{3}{4}\delta_{\frac{2}{3}}$.

Suppose the distribution of sender types k is given by $\nu_1 = \nu_2 = \frac{1}{2}$. We claim that, for sufficiently small ε, the Bayesian persuasion policy cannot be obtained in any equilibrium.
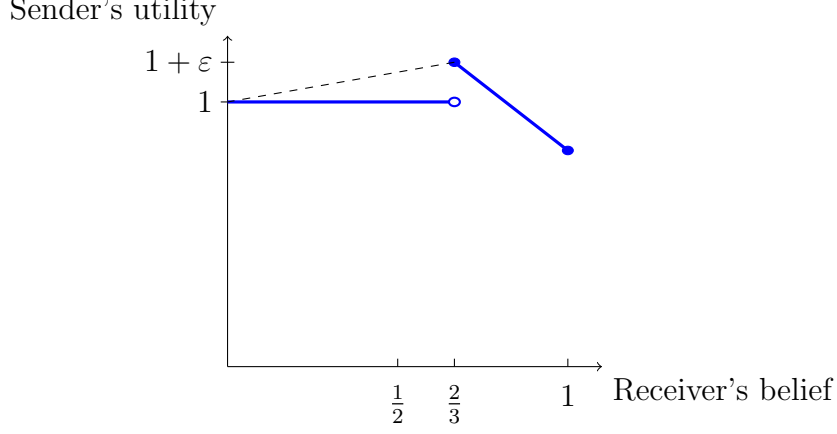
Figure 4: The sender's indirect utility for Example 4. The dashed line depicts the concavification.

We relegate the formal proof of this claim to the appendix and provide only some intuition here.

Suppose there was such an equilibrium. Then is must be that whenever the receiver chooses action a_0, his belief is equal to 0. It follows that, in equilibrium, the sender's belief is also always equal to 0 when the receiver chooses action a_0, and this belief for the sender must arise with probability at least $1/4$.

Now consider the action played by the receiver if the sender discloses nothing. Since a sender with belief 0 strictly prefers action a_1 to action a_0, and a_0 is played with positive probability when the sender has belief 0, it must be that the receiver plays a_0 with probability one following no disclosure.

Notice that each type k must induce the receiver to play a_1 with positive probability (in which case the receiver has belief $2/3$). It is without loss to assume that type $k = 1$ runs a binary experiment F generating a low signal leading to posterior 0 and a high signal leading to a positive posterior. The optimal response for type $k = 2$ is to run F twice. But when ε is small, if both experiments generate high signals, then the sender's belief exceeds $2/3 + \varepsilon$, which implies that the sender strictly prefers action a_0 to action a_1. Since the sender can induce a_0 by not disclosing anything, it follows that a_0 is chosen after two high signals, contradicting that the sender's belief is 0 whenever a_0 is chosen.

Example 5. We next ask whether our analysis is robust with respect to ignorance, i.e., whether the conclusion of Theorem 1 holds if ν assigns some small probability to $k = 0$. Note that when $k = 0$, only the empty evidence $\varnothing$ is available for the sender to disclose.
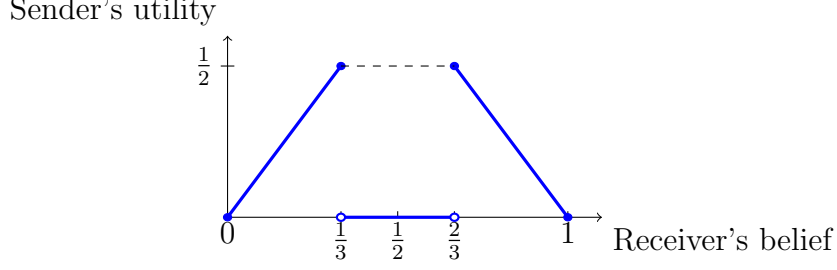
Figure 5: The sender's indirect utility for Example 5. The dashed line depicts the concavification.

Let $\Omega = \{\omega_0, \omega_1\}$ with prior $1/2$. The receiver's action set is $A = \{a_0, a_1, a_2\}$. Action a_0 is optimal for the receiver for posteriors in $[0, 1/3]$, action a_1 is optimal for posteriors in $[1/3, 2/3]$, and action a_2 is optimal for posteriors in $[2/3, 1]$. The sender's utility is $\tilde{v}(a_0, \omega_0) = 0$, $\tilde{v}(a_0, \omega_1) = 3/2$, $\tilde{v}(a_1, \omega_0) = \tilde{v}(a_1, \omega_1) = 0$, $\tilde{v}(a_2, \omega_0) = 3/2$, and $\tilde{v}(a_2, \omega_1) = 0$. The sender's indirect utility is depicted in Figure 5.

Let ν be any distribution of k that contains 0 in its support. Suppose for contradiction that there exists a wPBE implementing the Bayesian persuasion policy. Since the evidence $\varnothing$ is on the equilibrium path, the receiver's belief after observing $\varnothing$ must be either $1/3$ or $2/3$, and he must correspondingly play either action a_0 or a_2 with probability one. Assume without loss of generality that he plays action a_0. Note that whenever the sender's posterior belief lies above $1/2$, she strictly prefers action a_0 to action a_2. Therefore, it must be that the probability of ω_1 conditional on a_2 is at most $1/2$, for otherwise the sender would have a profitable deviation to disclosing $\varnothing$ instead of every disclosure that leads to a_2. This contradicts that the probability of ω_1 conditional on a_2 in the Bayesian persuasion solution is $2/3$.

5   Transparent Motives

One case of particular interest is when the sender has transparent motives, which ensures the existence of a punishing action. In this section, we examine this case in more detail. As we show in the appendix, under the assumption of transparent motives, one can prove Theorem 1 in a more constructive way without relying on a fixed point argument (beyond the Intermediate Value Theorem).

The transparent motives case also differs from the general case with respect to robust-

ness to ignorance: as long as the posterior associated with the sender's least preferred action among those played in the Bayesian persuasion policy is interior, the equilibrium is robust to ignorance. This result is a consequence of Theorem 2 below.

Recall that the state space is $\Omega = \{\omega_1, \ldots, \omega_n\}$ and the prior is $p \in \Delta(\Omega)$. The distribution of types of the sender is $\nu \in \Delta(\mathbb{N})$ (in particular, the support of ν may now include 0). The sender's utility from action $a \in A = \{a_1, \ldots, a_m\}$ is $\tilde{v}(a)$. Order the actions so that $\tilde{v}(a_i) \leq \tilde{v}(a_j)$ for $i \leq j$. Let $P \in \Delta(A \times \Omega)$ be the Bayesian persuasion policy. Suppose without loss of generality that P induces a belief distribution of the form

$$\sum_{i=1}^{m} \alpha_i \delta_{q^i},$$

where $q_j^i = P(\omega_j|a_i)$ and $\alpha_i = P(a_i \times \Omega)$.

For any $q \in \Delta(\Omega) \setminus \{p\}$, let $\beta(q)$ be the maximal $\beta \in \mathbb{R}_+$ such that $p + \beta(q-p) \in \Delta(\Omega)$. Note that $\beta(q) \geq 1$ and $\beta(q) > 1$ if $q \in \text{int}(\Delta(\Omega))$. Let $\alpha(q) = 1 - \frac{1}{\beta(q)} \in [0,1]$, where we define $\alpha(p) = 1$. Note that $\alpha(q)p + (1 - \alpha(q))(p + \beta(q)(q - p)) = q$. In addition, by construction, $\alpha(q)$ is the largest $\alpha \in [0,1]$ for which there exists $q_* \in \Delta(\Omega)$ such that $\alpha p + (1 - \alpha)q_* = q$.

Theorem 2. *There exists a wPBE in which the sender achieves the Bayesian persuasion utility if and only if $\nu_0 \leq \alpha(q^1)\alpha_1$.*

As noted above, this result implies that if $q^1 \in \text{int}(\Delta(\Omega))$, then as long as the probability that $k = 0$ is small enough, there is wPBE that implement the Bayesian persuasion solution.

The idea behind Theorem 2 is that, to prevent deviations by types $k \geq 1$ whose realized signals lead to action a^1, it must be that disclosing nothing also leads to action a^1. The posterior associated with a^1 must therefore result from a combination of the prior p with weight in proportion ν_0 and posteriors for types $k \geq 1$ with weights in proportion to their likelihood coupled with the likelihood that their experiments lead to realizations associated with a^1. The condition that $\nu_0 \leq \alpha(q^1)\alpha_1$ ensures that the weight given to the prior is low enough for their to exist posteriors that, when combined with the prior, lead to the posterior associated with a^1.

A Proof of Theorem 1

Let $\tilde{A} = \{a_1, \ldots, a_m\}$ be the set of actions played with positive probability, where $m > 1$ (for $m = 1$, the result is trivial). Without loss of generality, we can assume that $m \leq n$. For any state $\omega_i \in \Omega$ and action $a_j \in \tilde{A}$, let

$$Q_{ij}^* = P(a_j | \omega_i)$$

be the conditional probability of a recommendation of action a_j given state ω_i.

Consider a single-agent decision problem $\Gamma_k(Q)$ for the sender. Let $M_{n \times m}$ denote the set of $n \times m$ matrices with real-valued entries. The decision problem is defined by a row-stochastic matrix $Q = \{q_{ij}\}_{1 \leq i \leq n, 1 \leq j \leq m} \in M_{n \times m}$, a natural number $k \geq 1$, and a set $S = \{s_1, \ldots, s_m\}$ of m signals—one for each action $a_j \in \tilde{A}$.

In the first stage, if $k > 1$, the sender chooses between two levels of information acquisition: obtaining k signals or $k - 1$ signals. If $k = 1$, the sender has no choice and must select $k = 1$. After this choice, the state $\omega_i \in \Omega$ is realized according to the prior p. Conditional on the realized state, the signal-generating matrix Q determines the distribution of signals, where $P_Q(s_j \mid \omega_i) = q_{ij}$.

If the sender chooses to acquire k signals, then k conditionally i.i.d. draws $s^1, \ldots, s^k$ are generated, with each signal s_j occurring with probability q_{ij} in state ω_i. If the sender chooses $k - 1$, then $k - 1$ conditionally i.i.d. signal draws are generated in the same way; in this case, however, the sender also observes the realized state ω_i in addition to the $k - 1$ signals.

In the second stage, after observing his information, the sender selects one of the signals she has received and reveals it. Her utility depends on the revealed signal: if she reveals s^l such that $s^l = s_j$, her payoff is $\tilde{v}(a_j, \omega_i)$.

Formally, for $k > 1$, a pure strategy $\psi \in \Psi_k$ for the sender in problem $\Gamma_k(Q)$ is an element of $\{k - 1, k\} \times \Lambda_k$, where the first component corresponds to the sender's choice of information acquisition, and $\lambda \in \Lambda_k$ specifies which signal to reveal based on the sender's information. In particular, when the sender has k signals $(s^1, \ldots, s^k)$, λ maps this vector to a revealed signal $s \in \{s^1, \ldots, s^k\}$. When the sender has $k - 1$ signals $(s^1, \ldots, s^{k-1})$ and additionally observes the state $\omega_i \in \Omega$, λ maps $(s^1, \ldots, s^{k-1}, \omega_i)$ to a signal $s \in \{s^1, \ldots, s^{k-1}\}$. For $k = 1$, the sender has no choice, as she possesses only one signal, which she must reveal.

We note that any strategy ψ induces a probability distribution P^ψ over $\Omega \times S$, where $P^\psi(s_j|\omega_i)$ denotes the conditional probability that the sender reveals signal s_j in state ω_i. This distribution gives rise to a row-stochastic matrix $Q^\psi = \{q_{ij}^\psi\}_{1 \leq i \leq n,\, 1 \leq j \leq m}$, where $q_{ij}^\psi = P^\psi(s_j \mid \omega_i)$.

We next establish the following lemma.

Lemma 1. *Let $\tilde{\Psi}_k$ be the set of all mixed strategies in $\Gamma_k(Q)$ that maximize the sender's utility. Then the set*

$$C_{\Gamma_k}(Q) = \{\, Q^{\tilde{\psi}} : \tilde{\psi} \in \tilde{\Psi}_k \,\}$$

is a convex and compact subset of $\mathbb{R}^{n \cdot m}$.

Proof. Note that the set of pure strategies Ψ_k in Γ_k is finite. Let Ψ_k' be the set of all pure utility-maximizing strategies. Clearly $\tilde{\Psi}_k$ is the set of all convex combinations of elements in Ψ_k'. Therefore $\tilde{\Psi}_k$ is a convex and compact set in $\Delta(\Psi_k)$. The mapping $\tilde{\psi} \to Q^{\tilde{\psi}}$ is linear on $\Delta(\Psi_k)$ and therefore $C_{\Gamma_k}(Q)$ is compact and convex as a linear image of the compact convex set $\tilde{\Psi}_k$. $\qquad\square$

Now consider a compound decision problem $\Gamma_\nu(Q)$. In this problem, the value k is drawn according to the distribution ν and the sender continues as in $\Gamma_k(Q)$. Let K be the support of ν. A pure strategy $\psi = (\psi_k)_{k \in K}$ in $\Gamma_\nu(Q)$ is just a concatenation of pure strategies ψ_k in $\Gamma_k(Q)$ for every $k \in K$. Note that a strategy $\psi = (\psi_k)_{k \in K}$ in $\Gamma_\nu(Q)$ is utility maximizing if and only if each ψ_k is utility maximizing in $\Gamma_k(Q)$. Therefore, extending the definition of $C_{\Gamma_k}(Q)$ to $C_{\Gamma_\nu}(Q)$ in the obvious way, we have

$$C_{\Gamma_\nu}(Q) = \sum_k \nu_k C_{\Gamma_k}(Q).$$

In particular, $C_{\Gamma_\nu}(Q)$ is convex and compact.

Let $\mathcal{Q}$ be the set of all row-stochastic matrices $Q \in M_{n \times m}$ such that $Q_{ij} = 0$ whenever $Q_{ij}^* = 0$. Note that $\mathcal{Q}$ is convex and compact.

For $\alpha > 0$, define a correspondence $T_\alpha : \mathcal{Q} \to M_{n \times m}$ by

$$T_\alpha(Q) = \{Q + \alpha(Q^* - Q') \mid Q' \in C_{\Gamma_\nu}(Q)\}.$$

Note that, by Lemma 1, T_α is a convex and compact-valued correspondence.

Lemma 2. *There exists $\alpha^* > 0$ for which the range of T_{α^*} is contained in $\mathcal{Q}$.*

Proof. For $q \in [0, 1]$, let $\Pi(q) = \mathbb{E}_\nu \left[1 - (1 - q)^k \right]$; when $q = Q(s|\omega)$, since $1 - (1 - q)^k$ is the probability of getting at least one s signal out of k signals, $\Pi(q)$ is the maximal conditional probability of revealing a signal s in state ω. Note that Π is continuous, strictly increasing, and satisfies $\Pi(0) = 0$ and $\Pi(1) = 1$.

Let $a_{\min} = \min\{Q^*_{ij} : Q^*_{ij} > 0\} \in (0, 1]$, and $q_0 = \Pi^{-1}(a_{\min}) \in (0, 1]$. We claim that $\alpha^* = \min\{\frac{q_0}{1 - a_{\min}}, 1\}$ satisfies the desired properties. First note that $\alpha^* \in (0, 1]$.

Fixing any $Q \in \mathcal{Q}$ and any $Q' \in C_{\Gamma_\nu}(Q)$, let

$$\tilde{Q} = Q + \alpha^*(Q^* - Q').$$

Consider a given (i, j). Write

$$q = Q_{ij}, \quad a = Q^*_{ij}, \quad q' = Q'_{ij}, \quad \text{and} \quad \tilde{q} = q + \alpha^*(a - q').$$

We claim that $\tilde{q}$ is nonnegative. If $q = 0$, then signal s_j never appears in state ω_i, and hence $q' = 0$. Therefore, $\tilde{q} = 0$. In particular, if $a = 0$, then by the definition of $\mathcal{Q}$, we have $q = 0$ and therefore $\tilde{q} = 0$.

Now suppose $q \in (0, q_0]$. Recall that q' represents the probability of revealing signal s_j in state ω_i. Since $\Pi(q)$ is the maximal probability that s_j can be revealed given q, we have $q' \leq \Pi(q) \leq \Pi(q_0) = a_{\min} \leq a$, which implies $a - q' \geq 0$ and thus $\tilde{q} \geq q \geq 0$.

If $q > q_0$, then

$$\tilde{q} = q - \alpha^*(q' - a) \geq q - \alpha^*(1 - a_{\min}) \geq q_0 - \alpha^*(1 - a_{\min}) \geq q_0 - q_0 = 0,$$

where the final inequality follows from the definition of α^*. This completes the proof of the claim that $\tilde{q}$ is nonnegative.

The fact that $\sum_j \tilde{Q}_{ij} = 1$ for every i is immediate from the definition of $\tilde{Q}$ and the corresponding property of Q, Q^*, and Q'.

We have shown that for every (i, j), $\tilde{q}_{ij} \geq 0$, the row sums of $\tilde{Q}$ equal one, and $\tilde{Q}_{ij} = 0$ whenever $Q^*_{ij} = 0$. Therefore, $\tilde{Q} \in \mathcal{Q}$. $\square$

Next, we show that T_{α^*} has a fixed point.

Lemma 3. *There exist $\alpha^* > 0$ and $Q \in \mathcal{Q}$ such that $Q \in T_{\alpha^*}(Q)$.*

Proof. We use the Kakutani fixed point theorem. Let α^* be as in Lemma 2. Since T_{α^*} is convex and compact-valued and, by Lemma 2, has range contained in $\mathcal{Q}$, all that remains is to show that the graph of T_{α^*} is closed.

Let $(Q_n)_n$ be a sequence in $\mathcal{Q}$ that converges to Q and let $Z_n \in T_\alpha(Q_n)$ for each n with $Z_n \to Z$. We must show that $Z \in T_\alpha(Q)$.

For each n, let $R_n \in C_{\Gamma_\nu}(Q_n)$ be such that

$$Z_n = Q_n + \alpha^* \left(Q^* - R_n\right).$$

Note that, by definition, $C_{\Gamma_\nu}(Q_n) = \sum_k \nu_k C_{\Gamma_k}(Q_n)$. Therefore, for each n and each k in the support of ν, there exists $R_n^k \in C_{\Gamma_k}(Q_n)$ such that $R_n = \sum_k \nu_k R_n^k$. Since $C_{\Gamma_k}(Q_n) \subset \mathcal{Q}$ and $\mathcal{Q}$ is compact, by using a diagonal argument, we can assume that for each k, the sequence $(R_n^k)_n$ converges to some R_*^k. In particular, we have $Z = Q + \alpha^*(Q^* - \sum_k \nu_k R_*^k)$.

Since C_{Γ_k} is a closed correspondence we have $R_k^* \in C_{\Gamma_k}(Q)$. Hence

$$Z \in Q + \alpha^* \left(Q^* - \sum_k \nu_k C_{\Gamma_k}(Q) \right).$$

Therefore, $Z \in T_\alpha(Q)$ as desired. □

As a corollary of the existence of a fixed point, we have

Corollary 1. *There exists a matrix $Q \in \mathcal{Q}$ such that $Q^* \in C_{\Gamma_\nu}(Q)$.*

Proof. Let Q be the fixed point of T_{α^*}. By definition there exists $Q' \in C_{\Gamma_\nu}(Q)$ such that

$$Q = Q + \alpha^*(Q^* - Q').$$

The corollary follows since $\alpha^* > 0$. □

Proof of Theorem 1. Let $Q \in \mathcal{Q}$ be such that $Q^* \in C_{\Gamma_\nu}(Q)$. Note that any pure strategy $\psi = (\psi_k)_{k \in K}$ in $\Gamma_\nu(Q)$ induces a strategy for the sender in our evidence game as follows. Let $S = \{s_1, \ldots, s_m\}$ and $\pi : \Omega \to S$ be the experiment that induces Q; that is, let $\pi(\omega_i)[s_j] = q_{ij}$ for every $1 \le i \le n$ and $1 \le j \le m$. The strategy induced by ψ is defined naturally as follows: for every realized k, if ψ_k chooses to acquire k signals, then the sender acquires k experiments $\pi_1, \ldots, \pi_k$, all equal to π. After the k signals $s^1, \ldots, s^k$ are realized, the sender reveals to the receiver a unique realization (π, s) where the signal $s \in \{s^1, \ldots, s^k\}$ is chosen according to the corresponding λ. If ψ_k acquires $k - 1$ signals, then the sender chooses $k - 1$ experiments $\pi_1, \ldots, \pi_{k-1}$, all equal to π, and uses the extra experiment to choose $\tilde{\pi}$, which is the full information experiment. That is, $\tilde{S} = \{t_1, \ldots, t_n\}$ and $\tilde{\pi}(\omega_i)[t_j] = \delta_{ij}$. After the $k - 1$ signals $s^1, \ldots, s^{k-1}$ are realized and

the state $\omega_i = t_i$ is revealed, the sender chooses to reveal the signal $s \in \{s^1, \ldots, s^{k-1}\}$ that is induced by λ with respect to $(s^1, \ldots, s^{k-1}, \omega_i)$. Note that this construction readily extends to mixed strategies $\tilde{\psi}$. Since $Q^* \in C_{\Gamma_\nu}(Q)$ by construction, there exists an optimal strategy $\tilde{\psi} = (\tilde{\psi}_k)_{k \in K}$ in $\Gamma_\nu(Q)$ such that $Q^{\tilde{\psi}} = Q^*$.

Let the sender's strategy be the one that is induced by $\tilde{\psi}$. Define an assessment (ρ, σ) for the receiver as follows.[2] Let $\{p^j\}_{j=1}^m \subset \Delta(\Omega)$ be the posteriors induced by the Bayesian persuasion policy, where $p^j = P(\cdot \mid a_j)$ is the conditional probability over Ω given that action a_j is recommended by the sender. Define the receiver's assessment following a disclosure $\{(\pi, s)\}$ such that $s = s_j$ to be $\rho(\{(\pi, s_j)\}) = p^j$ and $\sigma(\{(\pi, s_j)\}) = \delta_{a_j}$. That is, if the receiver observes the experiment π with a corresponding signal s_j, then his posterior belief is p^j and he plays action a_j with probability one. For any other $T \in \Delta^*$ let $\rho(T) = q$ and $\sigma(T) = x$, where $x \in \Delta(A)$ is the punishing action with respect to $\tilde{A}$, and $q \in \Delta(\Omega)$ is a posterior belief for which x is optimal for the receiver.

We claim that the above profile forms wPBE. First consider the receiver. Let $\tilde{P} \in \Delta(\Omega \times S)$ be the probability distribution induced by the sender's strategy; namely, $\tilde{P}(\omega_i, s_j)$ is the probability that the realized state is ω_i and the revealed signal is s_j. By construction, $\tilde{P}(s_j \mid \omega_i) = Q^*_{ij}$. Since the marginal of $\tilde{P}$ on Ω is the prior p, it must be that $\tilde{P}$ induces the same probability distribution as the Bayesian persuasion policy $P \in \Delta(\Omega \times A)$ in the sense that $\tilde{P}(\omega_i, s_j) = P(\omega_i, a_j)$. Therefore, the belief described by ρ is the correct belief on the equilibrium path. Moreover, σ is optimal given ρ by construction.

We claim that the sender's strategy is optimal given the receiver's strategy. It is enough to show that it is optimal conditional on every realized $k \in K$. Note that, by construction, the sender's strategy is optimal subject to the constraint that the only available experiments to her are the experiment π defined above and the full information experiment $\tilde{\pi}$. By the construction of the receiver's strategy and the definition of a punishing action, revealing evidence that differs from $\{(\pi, s)\}$ is never a profitable deviation. Thus, the only possible deviation that needs to be considered is that one or more of the k experiments is conducted according to $\pi' \neq \pi$ but is revealed to the receiver with probability zero. Therefore, the sender only uses the other experiment to acquire further information about the state. Replacing any such experiments with $\tilde{\pi}$ makes the sender

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[2]Recall that an assessment (ρ, σ) comprises a belief system ρ together with a strategy σ.

better informed and thus can only improve her utility. Therefore, the sender's strategy maximizes her expected utility given the receiver's strategy. □

B   Proof for Example 4

Proposition 1. *There exists $\varepsilon > 0$ for which there is no wPBE implementing the Bayesian persuasion policy in Example 4.*

Proof. Assume by way of contradiction that such a wPBE (G, ϕ) exists for every $\varepsilon > 0$. Note that on the equilibrium path the receiver can have only two posterior beliefs about state ω_1: 0 and 2/3. Note also that, on the equilibrium path, conditional on the receiver assigning probability 0 to state ω_1, the sender does as well. In particular, there is a positive probability that, after observing the realized signals, the sender assigns probability 0 to state ω_1.

Let $\varnothing$ denote the empty evidence. We claim that $\sigma(\varnothing)$ is a_0 with probability one. Note that whenever the sender's belief about state ω_1 is 0, she strictly prefers action 1 to action 0. In that case, if $\sigma(\varnothing)$ is not deterministically a_0, the sender has a profitable deviation to $\varnothing$.

We claim that the posterior belief of the sender can never lie strictly above $2/3 + \varepsilon$ in equilibrium. To see this, note that above this value, the utility of action a_1 lies below 1. Since the sender's belief is not 0, it must be that, on the equilibrium path, what she reveals induces the receiver to have belief 2/3 and to choose action a_1. But then the sender prefers to deviate to disclosing $\varnothing$. Revealing a signal on the equilibrium path that induces the receiver to have belief 2/3 leads to action a_1 by the receiver. In this case, the sender can deviate and play $\varnothing$.

Next, we claim that the posterior distribution induced by the single experiment type $k = 1$ uses takes the form $\alpha_\varepsilon \delta_0 + (1 - \alpha_\varepsilon) F_\varepsilon$, where α_ε converges to $1/4$ and $F_\varepsilon \in \Delta((0, 1])$ converges in probability to $2/3$ as $\varepsilon \to 0$. Without loss of generality, any experiment of type $k = 1$ generates a posterior distribution $\alpha_\varepsilon \delta_0 + (1 - \alpha_\varepsilon) F_\varepsilon$ (allowing for α_ε to be 0). Since the support of the posterior distribution for $k = 1$ has supremum no greater than $2/3 + \varepsilon$ and has an expectation of $1/2$, α_ε can be at most $1/4$ as $\varepsilon \to 0$.

Suppose α_ε lies strictly below $1/4 - \delta$ for some $\delta > 0$ and arbitrarily small $\varepsilon > 0$. Then, since $(1 - \alpha_\varepsilon) \mathbb{E}(F_\varepsilon) = 1/2$, F_ε must have an expectation $\mathbb{E}(F_\varepsilon)$ that lies strictly

below $2/3 - \eta$ for some $\eta > 0$. Note that conditional on any posterior $q > 0$ for type $k = 1$, the receiver must play action a_1. Therefore, the expected payoff for type $k = 1$ is

$$\alpha_\varepsilon + (1 - \alpha_\varepsilon) \left(-\mathbb{E}(F_\varepsilon) + \frac{5}{3} + \varepsilon \right).$$

This expression lies strictly above

$$\frac{1}{4} + \frac{3}{4} \left(-\left(\frac{2}{3} - \eta \right) + \frac{5}{3} + \varepsilon \right),$$

which in turn is strictly larger than the Bayesian persuasion expected payoff of

$$\frac{1}{4} + \frac{3}{4} \left(-\frac{2}{3} + \frac{5}{3} + \varepsilon \right).$$

Since the expected utility of type $k = 2$ must be at least as large as that of type $k = 1$, it follows that the sender's ex ante expected utility exceeds that under Bayesian persuasion, which is impossible. Therefore, $\alpha_\varepsilon \to 0$ as $\varepsilon \to 0$.

Finally, by similar reasoning, for every $\delta > 0$, the probability that the experiment of type $k = 1$ leads to a posterior that lies in the interval $[2/3 - \delta, 2/3 + \varepsilon]$ approaches $3/4$ as $\varepsilon \to 0$. Otherwise, the probability that the posterior lies in $(0, 2/3 - \delta]$ is at least some $\eta > 0$ for arbitrarily small $\varepsilon > 0$. In this case, the expected utility of type $k = 1$ is at least

$$\frac{1}{4} + \eta \left(-\left(\frac{2}{3} - \delta \right) + \frac{5}{3} + \varepsilon \right) + \left(\frac{3}{4} - \eta \right) \left(-\left(\frac{2}{3} + \varepsilon \right) + \frac{5}{3} + \varepsilon \right).$$

Again, this expected payoff lies strictly above the Bayesian persuasion expected payoff for sufficiently small ε, and we have a contradiction.

The preceding argument implies that the experiment chosen by type $k = 1$ generates a posterior distribution that lies arbitrarily close to the Bayesian persuasion distribution as $\varepsilon \to 0$, and gives an expected utility for the sender arbitrarily close to the Bayesian persuasion utility.

Now consider type $k = 2$. We claim that there exists $\delta > 0$ such that the expected utility of type $k = 2$ exceeds the Bayesian persuasion utility by at least δ when $\varepsilon > 0$ is sufficiently small. To see this, consider the following strategy for type $k = 2$: choose the same experiment as type $k = 1$ twice. If the realized signals correspond to a posterior of 0 together with another posterior $q \in (0, 2/3 + \varepsilon]$, reveal the signal that led to q together with its experiment. Otherwise, reveal either signal realization together with its experiment. Note that the event of drawing both signals corresponding to 0 and q in the

two experiments has a probability bounded below by some $\eta > 0$. Using this strategy guarantees an expected utility of at least

$$\left(\frac{1}{4} - \eta\right) + \eta\left(\frac{5}{3} + \varepsilon\right) + \frac{3}{4}\left(-\left(\frac{2}{3} + \varepsilon\right) + \frac{5}{3} + \varepsilon\right),$$

which is strictly greater than the Bayesian persuasion utility when ε is sufficiently small. Therefore, the sender's ex ante expected utility is strictly greater than the Bayesian persuasion utility, a contradiction. $\qquad\square$

C Proof of Theorem 2

In order to prove Theorem 2, we allow the number of samples k to depend on the realized state as well. Thus, in this case, we have $\nu \in \Delta(\Omega \times \mathbb{N})$; we write ν_ω^k for $\nu(k \mid \omega)$. Given an experiment $\pi \in \Pi$, let $\mathbb{P}_{\pi,\nu} \in \Delta\left(\Omega \times \left(\cup_{k=1}^\infty S^k\right)\right)$ be the distribution that is generated by π and ν when k is chosen according to ν given ω and thereafter k conditionally i.i.d. signals are drawn according to π.

Lemma 4. *Consider a distribution $\gamma\delta_{q^L} + (1 - \gamma)\delta_{q^H} \in \Delta(\Delta(\Omega))$ with a barycenter equal to the prior: $\gamma q^L + (1 - \gamma)q^H = p$. There exists a binary signal $\pi \in \Delta(\Omega \times S')$, where $S' = \{s_H, s_L\}$, such generating the signal k times conditionally i.i.d., where k is determined according to $\nu \in \Delta(\Omega \times \mathbb{N})$, gives rise to signals $s^1, \ldots, s^k$ such that $\mathbb{P}_{\pi,\nu}(\cdot \mid s^1 = \cdots = s^k = s_L) = q^L$ and $\mathbb{P}_{\pi,\nu}(s^1 = \cdots = s^k = s_L) = \gamma$.*

Proof. Consider a binary signal on S' that generates a posterior distribution $\alpha\delta_{p^L} + (1 - \alpha)\delta_{p^H} \in \Delta(\Delta(\Omega))$, where p^L is the posterior distribution given s_L and p^H is the posterior distribution given s_H. We claim that such a binary signal has the desired property if and only if

$$\sum_{\omega \in \Omega} \sum_{k=1}^n \nu_\omega^k p_\omega \left(\frac{\alpha p_\omega^L}{p_\omega}\right)^k = \gamma \tag{1}$$

and

$$\frac{p_\omega \sum_{k=1}^n \nu_\omega^k \left(\frac{\alpha p_\omega^L}{p_\omega}\right)^k}{\sum_{\omega' \in \Omega} p_{\omega'} \sum_{k=1}^n \nu_{\omega'}^k \left(\frac{\alpha p_{\omega'}^L}{p_{\omega'}}\right)^k} = q_\omega^L \tag{2}$$

for every $\omega \in \Omega$. To understand these two conditions, note that, by Bayes' rule, the conditional probability of signal s_L given state ω is $\alpha p_\omega^L / p_\omega$. Thus, (1) states that the

total probability of receiving only s_L signals is γ, and (2) states that the conditional probability of state ω given the event that $s^i = s_L$ for all $i = 1, \ldots, k$ is q^L_ω, as desired.

To see that (1) and (2) have a solution, note that they are equivalent to requiring that for every $\omega \in \Omega$,

$$p_\omega \sum_{k=1}^{n} \nu^k_\omega \left(\frac{\alpha p^L_\omega}{p_\omega} \right)^k = \gamma q^L_\omega. \tag{3}$$

Note that, for every ω, the left-hand side of (3) is strictly increasing as a function of $x_\omega = \alpha p^L_\omega / p_\omega$, which is the conditional probability of the low signal given state ω. Therefore, (3) has a unique solution x'_ω for each $\omega \in \Omega$. Since $\gamma q^L_\omega \leq p_\omega$, we have that $x'_\omega \leq 1$. It follows that $\alpha = \sum_\omega x'_\omega p_\omega \leq 1$. Consequently, $p^L_\omega = x'_\omega p_\omega / \sum_{\omega'} (x'_{\omega'} p_{\omega'})$ is uniquely determined and corresponds to $p^L \in \Delta(\Omega)$, as desired. $\qquad \square$

The following proposition generalizes Lemma 4.

Proposition 2. *Consider a belief distribution $\gamma_1 q^1 + \cdots + \gamma_l q^l \in \Delta(\Delta(\Omega))$ with barycenter p. There exists a signal set $S = \{s_1, \ldots s_l\}$ and an experiment π such that, for each $j = 1, \ldots, l$, $\mathbb{P}_{\pi,\nu}(\cdot \mid \max\{i \mid s^m = s_i \text{ for some } m \in \{1, \ldots, k\}\} = j) = q^j$ and $\mathbb{P}_{\pi,\nu}(\max\{i \mid s^m = s_i \text{ for some } m \in \{1, \ldots, k\}\} = j) = \gamma_j$.*

Proof. Lemma 4 proves the proposition for the case $l = 2$. We prove the proposition by induction on l. Assume the result holds for $l - 1 \geq 2$ and let $\gamma_1 q^1 + \cdots + \gamma_l q^l \in \Delta(\Delta(\Omega))$ be a belief distribution with barycenter p. Define a new distribution $\gamma'_1 q^1_* + \cdots + \gamma'_{l-1} q^{l-1}_* \in \Delta(\Delta(\Omega))$ as follows: (i) for each $i = 1, \ldots, l - 2$, let $\gamma'_i = \gamma_i$ and $q^i_* = q^i$, and (ii) let $\gamma'_{l-1} = \gamma_{l-1} + \gamma_l$ and $q^{l-1}_* = (\gamma_{l-1} q^{l-1} + \gamma_l q^l)/(\gamma_{l-1} + \gamma_l)$.

By the induction hypothesis, there is a signal set $S' = \{s'_1, \ldots, s'_{l-1}\}$ and an experiment π' that implements $\gamma'_1 q^1_* + \cdots + \gamma'_{l-1} q^{l-1}_*$. That is,

$$\mathbb{P}_{\pi',\nu}(\cdot \mid \max\{i \mid s^m = s'_i \text{ for some } m \in \{1, \ldots, k\}\} = j) = q^j_* \text{ and}$$

$$\mathbb{P}_{\pi',\nu}(\max\{i \mid s^m = s'_i \text{ for some } m \in \{1, \ldots, k\}\} = j) = \gamma'_j.$$

Let L be the event that $\max\{i \mid s^m = s_i \text{ for some } m \in \{1, \ldots, k\}\} = l - 1$. By construction, $\mathbb{P}_{\pi',\nu}(\cdot \mid L) = q^{l-1}_*$. Let $\tilde{\nu} \in \Delta(\Omega \times \mathbb{N})$ be the joint conditional distribution of the variables k and ω given L. Since $q^{l-1}_* = (\gamma_{l-1} q^{l-1} + \gamma_l q^l)/(\gamma_{l-1} + \gamma_l)$, by Lemma 4, we can define a signal set $\tilde{S} = \{\tilde{s}_1, \tilde{s}_2\}$ and an experiment $\tilde{\pi}$ for which the marginal of $\mathbb{P}_{\tilde{\pi},\tilde{\nu}}$ on Ω is q^{l-1}_* and $\mathbb{P}_{\tilde{\pi},\tilde{\nu}}(\cdot \mid s^i = \tilde{s}_1 \text{ for all } i = 1, \ldots, k) = q^{l-1}$ and $\mathbb{P}_{\tilde{\pi},\tilde{\nu}}(s^i = \tilde{s}_1 \text{ for all } i = 1, \ldots, k) = \gamma_{l-1}/(\gamma_{l-1} + \gamma_l)$.

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Let $S = \{s_1, \ldots, s_l\}$. Define an experiment π using the experiments π' and $\tilde{\pi}$ as follows: $\pi(s_j|\omega) = \pi'(s'_j|\omega)$ for every $j = 1, \ldots, l-2$, $\pi(s_{l-1}|\omega) = \pi'(s'_{l-1}|\omega)\tilde{\pi}(\tilde{s}_1|\omega)$, and $\pi(s_l|\omega) = \pi'(s'_{l-1}|\omega)\tilde{\pi}(\tilde{s}_2|\omega)$. It follows by construction that π has the desired properties with respect to ν and $\gamma_1 q^1 + \cdots + \gamma_l q^l$. $\qquad\square$

We next provide an alternative proof of Theorem 1 for the transparent motives case where the sender's utility is state independent.

Alternative Proof of Theorem 1 under Transparent Motives. Let $\alpha_1 q^1 + \cdots + \alpha_l q^l \in \Delta(\Delta(\Omega))$ be the optimal policy of the sender. Without loss of generality, we can assume that each posterior q^i corresponds to a different optimal action a^i of the receiver, and that $u(a^i) < u(a^j)$ for $i < j$. Let π be the experiment that implements the distribution according to Proposition 2 with $S = \{s_1, \ldots, s_l\}$. Consider the strategy for the sender that, for each type k, runs the experiment π k times and reveals to the receiver the highest of the realized signals she observes, that is, she reveals $\{(\pi, s^j)\}$, where $j = \max\{i \mid s^m = s_i \text{ for some } m \in \{1, \ldots, k\}\}$.

The assessment (ρ, σ) of the receiver is as follows. On the equilibrium path, the belief is given by $\rho(\{\pi, s_j\}) = q^j$ for each j and the strategy is $\sigma(\{\pi, s_j\}) = a^j$. Let $a_* \in A$ be the worst action for the sender that is rationalizable for the receiver and let $q_* \in \Delta(\Omega)$ be a belief for which a_* is optimal. For any $T \in \Delta^*$ that is off the equilibrium path, let $\rho(T) = q_*$ and $\sigma(T) = a_*$. It follows by construction that this profile forms a weak perfect Bayesian equilibrium. $\qquad\square$

Proof of Theorem 2. Let $\alpha_1 \delta_{q^1} + \cdots + \alpha_l \delta_{q^l}$ be the Bayesian persuasion policy. Assume first that the condition in the statement of the theorem holds. Let $\delta = \nu_0/\alpha_1$. Note that $\delta \leq \beta(q^1)$. Therefore, by definition, we can find q_* such that $\delta p + (1-\delta)q_* = q^1$.

Consider the posterior distribution $\alpha'_1 q^1_* + \cdots + \alpha'_l q^l_*$ where $q^1_* = q_*$, $\alpha'_1 = (\alpha_1 - \nu_0)/(1 - \nu_0)$, and for $i = 1, \ldots, l$, $q^i_* = q^i$ and $\alpha'_i = \alpha_i/(1 - \nu_0)$. Note first that

$$\sum_{i=1}^l \alpha'_i = \frac{\alpha_1 - \nu_0 + \sum_{i=2}^l \alpha_i}{1 - \nu_0} = \frac{1 - \nu_0}{1 - \nu_0} = 1.$$

In addition, by construction, $(1-\delta)q_* = q^1 - \delta p$. Hence, multiplying by α_1, we have

$(\alpha_1 - \nu_0)q_* = \alpha_1 q^1 - \nu_0 p$. Therefore,

$$
\begin{aligned}
\sum_{i=1}^{l} \alpha_i' q_*^i &= \frac{(\alpha_1 - \nu_0)q_*^1 + \sum_{i=2}^{l} \alpha_i q^i}{1 - \nu_0} \\
&= \frac{\alpha_1 q^1 - \nu_0 p + \sum_{i=2}^{l} \alpha_i q^i}{1 - \nu_0} \\
&= \frac{p - p\nu_0}{1 - \nu_0} \\
&= p.
\end{aligned}
$$

Thus, the distribution $\alpha_1' q_*^1 + \cdots + \alpha_l' q_*^l$ has barycenter p.

Let $\nu' \in \Delta(\mathbb{N})$ be such that $\nu_k' = \nu_k/(1 - \nu_0)$ for every $k \geq 1$. Let π be the experiment constructed in the alternative proof of Theorem 1 (for the transparent motives case) that implements $\alpha_1' q_*^1 + \cdots + \alpha_l' q_*^l$ with respect to ν'. That is, $S = \{s_1, \ldots, s_l\}$ and $\pi : \Omega \to S$ satisfies

$$
\mathbb{P}_{\pi,\nu'}(\cdot \mid \max\{i \mid s^m = s_i \text{ for some } m \in \{1, \ldots, k\}\} = j) = q_*^j
$$

$$
\text{and} \quad \mathbb{P}_{\pi,\nu'}(\max\{i \mid s^m = s_i \text{ for some } m \in \{1, \ldots, k\}\} = j) = \alpha_j'
$$

for every $j = 1, \ldots, l$.

Suppose the sender plays as follows. For any $k \geq 1$, she generates k k conditionally i.i.d. signals according to π. Let $j = \max\{i \mid s^m = s_i \text{ for some } m \in \{1, \ldots, k\}\}$ be the index of the highest signal she observes. If $j > 1$, she reveals $\{(\pi, s_j)\}$ to the receiver; if $j = 1$, she reveals $\varnothing$. Suppose the receiver plays action a^j upon observing $T = \{(\pi, s_j)\}$ and plays a^1 following any other $T \in \Delta^*$. We claim that these strategies are part of a wPBE.

Note that if the sender implements this strategy, then the resulting posterior distribution of the receiver following the evidence $\varnothing$ is

$$
\frac{\nu_0 p + (1 - \nu_0)\alpha_1' q_*^1}{\nu_0 + (1 - \nu_0)\alpha_1'}. \tag{4}
$$

To see this, note that $\varnothing$ is revealed if either $k = 0$ or if $k > 0$ and $s^m = s_1$ for all $m = 1, \ldots, k$. The former holds with probability ν_0 and induces a posterior p while the latter holds with probability $(1 - \nu_0)\alpha_1'$ and induces a posterior q_*^1. Since $\alpha_1' = (\alpha_1 - \nu_0)/(1 - \nu_0)$, the posterior distribution in equation (4) equals $(\nu_0 p + (\alpha_1 - \nu_0)q_*)/\alpha_1 = q^1$.

In addition, revealing a signal s_j for $j > 1$ generates a posterior of q^j and occurs with probability $(1 - \nu_0)\alpha_j' = \alpha_j$. This induces the desired distribution of posteriors. If

the receiver's off-equilibrium-path belief is always equal to q^1, the receiver's strategy is optimal and so is the sender's.

Conversely, consider a wPBE that implements the Bayesian persuasion policy and assume $\nu_0 > 0$. (If $\nu_0 = 0$, the result holds by Theorem 1.) We first claim that the action that is played when $\varnothing$ is revealed must be the lowest action a^1. First note that since $\nu_0 > 0$, the action played following $\varnothing$ must be one of the Bayesian persuasion actions. If some action $a^j \neq a_1$ is played following $\varnothing$, then for any disclosure leading to a^1, the sender would prefer to deviate to disclosing $\varnothing$.

Let α_1' be the probability of sending $\varnothing$ conditional on $k > 0$, and let p_* be the conditional probability over $\Delta(\Omega)$ given the event that the sender reveals $\varnothing$ and $k > 0$. By definition, it must be that $\nu_0 + (1 - \nu_0)\alpha_1' = \alpha_1$ and

$$\frac{\nu_0}{\nu_0 + (1 - \nu_0)\alpha_1'}p + \frac{(1 - \nu_0)\alpha_1'}{\nu_0 + (1 - \nu_0)\alpha_1'}p_* = q^1.$$

By definition of $\beta(q^1)$, we must have

$$\frac{\nu_0}{\nu_0 + (1 - \nu_0)\alpha_1'} \leq \beta(q^1).$$

Since $\alpha_1' = (\alpha_1 - \nu_0)/(1 - \nu_0)$, we have that

$$\beta(q^1) \geq \frac{\nu_0}{\nu_0 + (1 - \nu_0)\alpha_1'} = \frac{\nu_0}{\nu_0 + (1 - \nu_0)\frac{\alpha_1 - \nu_0}{1 - \nu_0}} = \frac{\nu_0}{\nu_0 + \alpha_1 - \nu_0} = \frac{\nu_0}{\alpha_1},$$

and therefore, $\nu_0 \leq \beta(q^1)\alpha_1$. □

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