

Impact of Cosmic Ray Distribution on the Growth and Saturation of Bell Instability

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ABSTRACT

Cosmic rays (CRs) streaming in weakly magnetized plasmas can drive large-amplitude magnetic fluctuations via nonresonant streaming instability (NRSI), or Bell instability. Using one-dimensional kinetic simulations, we investigate how mono-energetic and power-law CR momentum distributions influence the growth and saturation of NRSI. The linear growth is governed solely by the CR current and is largely insensitive to the CR distribution. However, the saturation depends strongly on the CR distribution and is achieved through CR isotropization, which quenches the driving current. Mono-energetic CRs effectively amplify the magnetic field and isotropize. For power-law distributions, the lowest-energy CRs dominate current relaxation and magnetic growth, while the highest-energy CRs remain weakly scattered, limiting their contribution to saturation. In the absence of low-energy CRs, high-energy particles amplify magnetic fields effectively and isotropize. We provide a modified saturation prescription accounting for these effects and propose a layered CR-confinement scenario upstream of astrophysical shocks, relevant to particle acceleration to high energies.

Keywords: Plasma astrophysics (1261) — Plasma physics (2089) — Cosmic rays (329) — Magnetic fields (994) — Shocks (2086)

1. INTRODUCTION

A central challenge in high-energy astrophysics is to understand the acceleration, propagation, and confinement of cosmic rays (CRs). The diffusive shock acceleration (DSA) at a strong shock provides a robust framework for particle acceleration where the maximum energy of CRs relies critically on their confinement (W. I. Axford et al. 1977; A. R. Bell 1978; R. D. Blandford & J. P. Ostriker 1978; G. F. Krymskii 1977). Without sufficient electromagnetic turbulence, CRs escape too quickly due to their large Larmor radii, limiting their maximum energy (A. M. Hillas 1984). Identifying the physical origins of such turbulence and its dependence on plasma parameters is essential to complete our understanding of CR acceleration and its backreaction on the background plasma.

Nonthermal X-ray and radio observations of supernova remnants (SNRs) reveal magnetic fields of several hundred microgauss in their vicinity, far exceeding those typically found in the interstellar medium (ISM) (E. G. Berezhko et al. 2003; J. Vink & J. M. Laming 2003; H. J. Völk et al. 2005). This suggests that CR-accelerating

sites are influenced by local magnetic field amplification, either due to preexisting turbulence or self-generated instabilities driven by the CRs themselves. The latter, also known as CR streaming instabilities, are expected to arise naturally due to the relative drift between CRs and the thermal plasma upstream of SNR shocks (J. McKenzie & H. Völk 1982; A. Bykov et al. 2013; E. G. Zweibel 2013).

The long-known CR resonant streaming instability is responsible for CR scattering and magnetic field amplification in the interstellar and intracluster medium (I. Lerche 1967; R. Kulsrud & W. P. Pearce 1969; D. G. Wentzel 1969). However, it is generally limited to modest amplification levels, with $\delta B/B_0 \lesssim 1$ (X. N. Bai et al. 2019; C. Holcomb & A. Spitkovsky 2019). A major advance came with the finding of CR-driven *nonresonant instabilities*, capable of amplifying magnetic fields well beyond background levels (A. R. Bell & S. G. Lucek 2001; A. R. Bell 2004). Upstream plasma in strong shocks, such as SNRs, provides ideal conditions for significant NRSI-driven magnetic field amplification.

Although both resonant and nonresonant streaming instabilities (hereafter, RSI and NRSI, respectively) generate electromagnetic turbulence, their mechanisms differ significantly. The RSI occurs when the transverse

fluctuations grow by resonantly interacting with the gyromotion of the CRs. In contrast, NRSI is driven by a strong super-Alfvénic CR current, which grows transverse magnetic fluctuations via the $\mathbf{J} \times \mathbf{B}$ force.

The NRSI (Bell instability) has been studied extensively using analytical approaches, and kinetic, hybrid, and magnetohydrodynamic (MHD) simulations (J. Niemiec et al. 2008; V. Zirakashvili & V. Ptuskin 2008; A. Bret 2009; M. A. Riquelme & A. Spitkovsky 2009; E. Amato & P. Blasi 2009; L. Gargaté et al. 2010; B. Reville & A. Bell 2013; J. H. Matthews et al. 2017; M. S. Weidl et al. 2019). These studies explored key aspects, including magnetic field saturation (A. R. Bell 2004; S. Gupta et al. 2021; G. Zacharek et al. 2024), effects of background plasma temperature (E. G. Zweibel & J. E. Everett 2010; A. Marret et al. 2021), and the differences between proton and lepton-driven NRSIs (S. Gupta et al. 2021), using *mono-energetic* CRs. However, a systematic study of how NRSI depends on the shape of the CR distribution is still lacking (C. Haggerty et al. 2019), particularly the *power-law* momentum distribution $f(p) \propto p^{-4}$ expected from DSA.

Uncovering the influence of different CR distributions during the linear growth and nonlinear saturation phases of NRSI is the central focus of the present work. We use first-principles kinetic plasma simulations to address the following questions.

- Does the linear regime of the instability depend on the distribution of CRs?
- How does the saturated magnetic field depend on changes to the minimum momentum (p_{\min}) or the maximum momentum (p_{\max}) of the power-law CR distribution?
- Does the saturation differ between a mono-energetic and a power-law CR distribution with similar bulk properties?

The paper is organized as follows. Section 2 goes through the analytical approaches for NRSI. Section 3 outlines our numerical setup of kinetic simulations, and Section 4 shows the results. We discuss the astrophysical implications of our findings in Section 5, before concluding in Section 6.

2. ANALYTIC THEORY

We analytically study the linear growth (Section 2.1) and saturation (Section 2.2) of NRSI, comparing them for mono-energetic and power-law CR distributions.

2.1. Linear Growth

We derive the linear dispersion relations for NRSI following standard methods. We review the fluid approach (Section 2.1.1) and examine the impact of CR

distribution functions with the kinetic approach (Section 2.1.2). We show that the fastest-growing NRSI modes are largely insensitive to the CR distributions (Section 2.1.3).

2.1.1. Fluid approach

We consider a plasma with electrons and ions having number densities n_e and n_i , respectively, and CRs with number density n_{cr} , much smaller than n_e , n_i . The charge-neutrality of plasma requires

$$n_{\text{cr}} + n_i = n_e. \quad (1)$$

The CRs drift along the background magnetic field ($B_0 \hat{\mathbf{x}}$) with a drift speed of v_d . To maintain current neutrality in the plasma, the background electrons quickly develop a return current (E. Amato & P. Blasi 2009; S. Gupta et al. 2024), with a drift speed

$$v_e = \frac{n_{\text{cr}}}{n_e} v_d. \quad (2)$$

Treating CRs as unperturbed during the linear phase, the NRSI growth rate is

$$\gamma_{\text{fluid}}(k) = v_{A0} k \left[\frac{k_u}{k} - 1 \right]^{1/2} \quad (3)$$

(S. Gupta et al. 2021). Here, $v_{A0} = B_0 / (4\pi m_i n_0)^{1/2}$ is the Alfvén speed, $n_0 = n_e \approx n_i$ is background plasma density, k is parallel wavenumber, and

$$k_u = \frac{v_e}{v_{A0}^2} \omega_{ci} = \frac{n_{\text{cr}}}{n_e} \frac{v_d}{v_{A0}^2} \omega_{ci}, \quad (4)$$

with $\omega_{ci} = eB_0/m_i c$, the cyclotron frequency of background ion. Modes with $k < k_u$ are unstable.

From Equation 3, the fastest-growing mode k_{fast} and its growth rate γ_{fast} are evaluated as

$$k_{\text{fast}} d_i = \frac{1}{2} \frac{n_{\text{cr}}}{n_e} \frac{v_d}{v_{A0}} = \frac{\gamma_{\text{fast}}}{\omega_{ci}}, \quad (5)$$

where $d_i = v_{A0}/\omega_{ci}$ is the skin depth of background ions. When $k_{\text{fast}} (= k_u/2)$ is much larger than the inverse of the Larmor radius of even the lowest-energy CR particle ($k_{\text{fast}} \gg 1/r_{L,0}$), the instability becomes non-resonant.

2.1.2. Kinetic approach

We define the CR distribution functions as

$$f'_{\text{cr}}(p, \mu) = \frac{n_{\text{cr}}}{2\pi} g'(p, \mu), \quad (6)$$

where g' is normalized such that

$$\int_0^\infty dp \int_{-1}^1 d\mu [2\pi p^2 f'_{\text{cr}}(p, \mu)] = n_{\text{cr}}, \quad (7)$$

and $\mu = p_x/p$ is the pitch-angle, p_x being the x -component (background field direction) of momentum p . The apostrophe ('') denotes the CR-frame, the frame in which the CR distribution function is conveniently specified and not necessarily the frame in which the CR bulk velocity vanishes.

We consider two CR momentum distributions: mono-energetic (ME) and power-law (PL). The ME CRs are defined with

$$g'_{\text{ME}}(p, \mu) = \left[\frac{1}{p^2} \delta(p - p'_0) \right] \left[\frac{\Theta(\mu - \mu'_{\min})}{1 - \mu'_{\min}} \right], \quad (8)$$

where the delta (δ) and Heaviside (Θ) functions are used. The PL CRs are specified by

$$g'_{\text{PL}}(p, \mu) = \left[\frac{p^{-4}}{(p'_{\min}^{-1} - p'_{\max}^{-1})} \right] \left[\frac{\Theta(\mu - \mu'_{\min})}{1 - \mu'_{\min}} \right], \quad (9)$$

with momentum limits $p'_{\min} \leq p' \leq p'_{\max}$.

The parameter μ'_{\min} determines the pitch-angle distributions of CRs: $\mu'_{\min} = -1$ corresponds to *isotropic* or spherical distributions, while $\mu'_{\min} = 0$ yields a forward *cone* or hemispherical distribution (i.e. all the CRs have positive p_x). The isotropic CRs require additional boost velocity $v_b \hat{x}$ to drift along $B_0 \hat{x}$ relative to the background plasma. The cone CRs naturally have a net drift and do not require additional boost to generate a current. The cone distribution qualitatively represents CRs streaming in the ISM with limited scattering.

Using these CR distributions, we derive NRSI growth rates by linearizing the Vlasov equation, following the approach in E. Amato & P. Blasi (2009). Appendix A contains a brief summary of the dispersion relations.

2.1.3. Linear growth rates for different CR distributions

We compare the NRSI growth rates obtained from the kinetic approach for different CR distributions with the fluid dispersion, which, by definition, is agnostic to CR distributions. Figure 1 shows these growth rates, where we have used the parameters $v_{A0} = 0.02c$, $n_{\text{cr}}/n_e = 0.01$, $p'_0 = p'_{\min} \approx 5m_i c$, and $p'_{\max} \approx 400m_i c$. For isotropic and cone CR distributions, the drift speeds are $v_d \approx 0.3c$ and $v_d \approx 0.5c$, respectively. We normalize the wavenumbers k and growth rates γ with k_{fast} and γ_{fast} (as Equation 5). Note that we use the same parameters later in our simulations (see Section 3).

For all cases in Figure 1, the maximum growth rate and its wavenumber closely match the fluid approach, implying that the fastest-growing NRSI modes depend only on the net CR current. However, γ for mono-energetic CRs drop sharply below $k \lesssim 1/r'_{L,0}$ ($r'_{L,0} = p'_0 c / e B_0$ is the Larmor radius), where CRs become resonant and their kinetic response suppresses NRSI, unlike

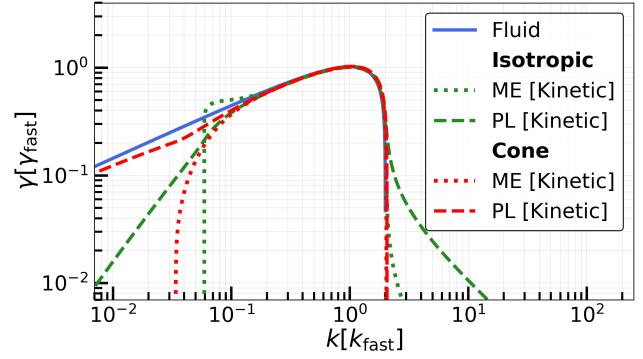


Figure 1. Analytic growth rates of NRSI derived for mono-energetic (ME, dotted) and power-law (PL, dashed) momentum distributions of CRs. The blue curve represents the growth rate obtained from the fluid approach (Equation 3), while all other curves are obtained from the kinetic approach (Equations A2 and A5). The maximum growth rates and the corresponding wavenumbers remain almost the same; they depend on the bulk CR current.

the fluid derivation. In contrast, for power-law CRs, γ does not plummet for $k < 1/r'_{L,\min}$ (here, $r'_{L,\min} = r'_{L,0}$ as $p'_{\min} = p'_0$), as the high-energy CRs are still non-resonant even when the lowest-energy CRs are resonant.

These results show that the linear growth of NRSI, particularly the dominant modes, is effectively insensitive to the CR distribution. This highlights a unique characteristic of NRSI, distinguishing it from RSI, where details of the CR distributions play a significant role (see e.g., C. Holcomb & A. Spitkovsky 2019).

2.2. Saturation

We now examine the saturation of NRSI. Being a non-linear process, deriving an exact formula for the saturated magnetic field from first principles is challenging. Recent kinetic studies introduced an anisotropy parameter, ξ , to explain and predict the saturated magnetic field for mono-energetic isotropic CRs (S. Gupta et al. 2021; G. Zacharek et al. 2024). We extend this framework to other CR distributions.

The anisotropy parameter ξ is defined as

$$\xi = \frac{1}{2} \frac{P_{\text{cr,ani0}}}{P_{B0}} \equiv \frac{n_{\text{cr}} p_d v_d}{n_0 m_i v_{A0}^2}. \quad (10)$$

Here, $P_{\text{cr,ani0}} = n_{\text{cr}} p_d v_d$ is the initial anisotropic CR momentum flux¹, $P_{B0} = B_0^2 / 8\pi$ is the initial magnetic

¹ It is equivalent to the excess anisotropy pressure $P_{\text{cr},xx} - \frac{1}{2}(P_{\text{cr},yy} + P_{\text{cr},zz})$, where $P_{\text{cr},ij} \equiv \langle p_{\text{cr},i} v_{\text{cr},j} \rangle$ calculated in the plasma ion rest frame. Usually, pressure is evaluated in the comoving frame of the species. However, for NRSI, the free energy is in the CR anisotropy pressure in the plasma-ion rest frame (A. Achterberg 2013), which yields the product $\langle p_{\text{cr},x} \rangle \langle v_{\text{cr},x} \rangle = p_d v_d$.

energy density; p_d is the mean x -momentum, and v_d is the drift speed of CRs, along $B_0\hat{x}$.

We evaluate v_d in the plasma ion rest frame. Starting from the CR distributions $g'(p, \mu)$ in the CR frame, we calculate the average x -component of their velocities in the plasma ion rest frame by applying the boost speed v_b with the CRs. This yields

$$v_d = \int_0^c dv \int_{-1}^1 d\mu \left(\frac{\mu v + v_b}{1 + \mu v v_b / c^2} \right) v^2 g'_v(v, \mu), \quad (11)$$

where $g'_v(v, \mu)v^2 dv d\mu \equiv g'(p, \mu)p^2 dp d\mu$. Using Equation 11, an approximate expression of v_d for relativistic CRs with $v \approx c$ is

$$\frac{v_d}{c} \approx \begin{cases} \frac{c}{v_b} - \left(\frac{\frac{c^2}{v_b^2} - 1}{1 - \mu'_{\min}} \right) \ln \left| \frac{1 + \frac{v_b}{c}}{1 + \frac{v_b \mu'_{\min}}{c}} \right| & (v_b \neq 0), \\ \frac{1}{2} (1 + \mu'_{\min}) & (v_b = 0). \end{cases} \quad (12)$$

For the isotropic and cone CR distributions, $v_d \approx 2v_b/3$ and $v_d \approx 0.5c$, respectively.

Similar to the drift speed, we calculate the mean momentum p_d from the Lorentz transformation of the CR momenta using the expression

$$p_d = \int_0^\infty dp \int_{-1}^1 d\mu \gamma_b (\mu p + E v_b / c^2) p^2 g'(p, \mu), \quad (13)$$

where $\gamma_b = \sqrt{1 - (v_b/c)^2}$ is the Lorentz factor for the boost speed v_b and $E = \sqrt{p^2 c^2 + m_i^2 c^4}$ is the energy. For relativistic CRs with $p \gg m_i c$, $E \approx pc$; for mono-energetic (ME) and power-law (PL) CRs, Equation 13 is approximated as

$$p_d \approx \gamma_b \left[\frac{1 + \mu'_{\min}}{2} + \frac{v_b}{c} \right] \times \begin{cases} p'_0, & \text{ME}, \\ \frac{\ln |p'_{\max}/p'_{\min}|}{p'_{\min} - p'_{\max}}, & \text{PL}. \end{cases} \quad (14)$$

Using Equation 11 for v_d and Equation 13 for p_d , we calculate ξ from Equation 10 for any general CR distribution function boosted with speed v_b .

NRSI saturates when the transverse magnetic field B_\perp satisfies the condition

$$\left(\frac{B_\perp}{B_0} \right)_{\text{sat}}^2 \approx \frac{1}{4} \frac{P_{\text{cr,ani0}}}{P_{B0}} \implies \left(\frac{B_\perp}{B_0} \right)_{\text{sat}} \approx \sqrt{\frac{\xi}{2}}. \quad (15)$$

The factor of 1/4 arises because the free energy in CR anisotropy pressure is distributed among eight components: three pressure components of plasma ions, three pressure components of plasma electrons (jointly result

Table 1. Details of grid, initial field, and particles

Parameters	Values
Box size ($N_x \times N_y$)	$300,000 \times 2$
{for benchmark runs}	$1,000,000 \times 2$
Cell width (Δx)	$0.2d_e$
Time step of iteration (Δt)	$0.09\omega_{pe}^{-1}$
Alfvén speed ($v_{A0} = B_0/\sqrt{4\pi m_i n_0}$)	$0.02c$
Particle-per-cell (each species)	25
Mass ratio (m_i/m_e)	25
Ion thermal speed ($v_{\text{th},i} = \sqrt{k_B T_i/m_i}$)	$0.008c$
Temperature ratio (T_i/T_e)	1

Note: Plasma frequency and skin depth of background electrons are ω_{pe} and d_e . Changing the box size, Δx , Δt , particle-per-cell, and mass ratio yields consistent results.

in plasma heating), and two transverse components of magnetic pressure (G. Zacharek et al. 2024). Equation 15 gives the maximum possible value of the saturated field for NRSI for any general CR distribution.

3. NUMERICAL SETUP

We use a publicly available version of the electromagnetic particle-in-cell code, Tristan-MP (A. Spitskovsky 2005), modified to investigate the CR streaming instability (S. Gupta et al. 2021; E. Lichko et al. 2025). Our simulations are spatially in 1D, with velocities and fields having all three components.

We initialize the grids with a magnetic field $\mathbf{B} = B_0\hat{x}$, the background plasma species (ions and electrons) with thermal distributions, and the CR ions with mono-energetic or power-law momentum distributions. For isotropic distributions, we boost each CR particle with a velocity $v_b\hat{x}$, unlike the cone distributions, where we do not boost the CRs. We boost the thermal electrons to balance the CR current (see Equation 2). Starting from this initial setup, we solve Maxwell's equations to evolve the electromagnetic fields and the equation of motion to evolve the particles' positions at each iteration, under periodic boundary conditions. Thus, the system evolves self-consistently. Table 1 contains the parameters of our numerical setup.

We use an artificial ion-to-electron mass ratio m_i/m_e to reduce their scale separation, which has almost no effect in the saturation of the magnetic field (see, e.g., Appendix A in S. Gupta et al. 2024). We implement the effective number densities of the species by adjusting their weights, keeping their charge-to-mass ratios intact. By fixing $n_e = 1$, we fix a n_{cr}/n_e for a particular simulation, and set n_i/n_e as per Equation 1. We perform

Table 2. Details of parameters for different runs

Sl. No.	Runs	v_d [c]	p_d [$m_i c$]	ξ	$k_{\text{fast}} d_i$ ($\times 10^{-2}$)
I1	MEI:5*	0.281	2.18	15.35	7.03
I2	PLI:5-30	0.278	4.66	32.36	6.95
I3	PLI:5-80	0.278	6.38	44.35	6.95
I4	PLI:5-400*	0.278	9.55	66.32	6.94
I5	PLI:10-400	0.276	16.46	113.71	6.91
I6	PLI:20-400	0.276	27.51	189.77	6.90
I7	MEI:10.7	0.277	4.67	32.36	6.93
I8	MEI:14.7	0.276	6.42	44.35	6.91
I9	MEI:22	0.276	9.61	66.32	6.90
C1	MEC:5*	0.490	2.45	30.00	12.25
C2	PLC:5-30	0.496	5.30	65.76	12.40
C3	PLC:5-80	0.496	7.29	90.43	12.41
C4	PLC:5-400*	0.497	10.92	135.53	12.41
C5	PLC:10-400	0.499	18.85	235.17	12.48
C6	PLC:20-400	0.500	31.51	393.65	12.49
C7	MEC:10.7	0.498	5.33	66.33	12.45
C8	MEC:14.7	0.499	7.33	91.45	12.47
C9	MEC:22	0.499	10.96	137.30	12.49
I10	PLI:50-400:10 $^{-3}$	0.276	51.86	35.76	0.69
C10	PLC:50-400:10 $^{-3}$	0.500	59.40	74.25	1.25

Note: In the labels of the runs, ‘ME’ and ‘PL’ refer to mono-energetic and power-law CR momentum distributions, and ‘I’ and ‘C’ denote isotropic or cone pitch-angle distributions, respectively. The numbers following the colon (:) are p'_0 for mono-energetic CRs, or $p'_{\min} - p'_{\max}$ for power-law CRs. The CR number fraction $n_{\text{cr}}/n_0 = 10^{-2}$ for all these runs, except I10 and C10, for which $n_{\text{cr}}/n_0 = 10^{-3}$ (mentioned in the label). The boost speeds for isotropic and cone CRs are $\mathbf{v}_b = 0.4c\hat{\mathbf{x}}$ and $\mathbf{v}_b = \mathbf{0}$, respectively. Remaining columns show drift speed v_d , mean momentum p_d , anisotropy parameter ξ , and fastest growing mode $k_{\text{fast}} d_i$ ($= \gamma_{\text{fast}}/\omega_{ci}$). Runs marked with (*) are the benchmark runs; movies for them are provided ([Click here](#)).

simulations, detailed in Table 2, to primarily answer the questions listed in Section 1.

4. RESULTS

We study the linear regime (Section 4.1) and the non-linear regime (Section 4.2), particularly the saturation, of NRSI. We focus on how these regimes differ between mono-energetic and power-law CR distributions.

4.1. Linear Growth: Mono-energetic vs. Power-law

We investigate the spatial profiles of the magnetic fields (Section 4.1.1) and their Fourier components (Section 4.1.2). In the linear phase, the instability is largely independent of the CR distributions.

4.1.1. Spatial Profiles of Magnetic Field Components

We show the magnetic field components B_x , B_y , and B_z for the benchmark isotropic runs MEI:5 and PLI:5-400 (I1 and I4 in Table 2). The left panels of Figure 2 show the profiles at $t \approx 7\gamma_{\text{fast}}^{-1}$, a time during the linear growth. Since the entire box is large (to capture the Larmor radius — hence, the resonant mode — of the highest-energy CR, $r'_{L,p'=400m_i c} \approx 1.95 \times 10^4 d_i$), we zoom into $x \in [300, 310]\lambda_{\text{fast}}$. The profiles of B_y and B_z at $t \approx 7\gamma_{\text{fast}}^{-1}$ show their growth, as B_y/B_0 and $B_z/B_0 \gtrsim 1$. The dominant wavelengths $\sim \lambda_{\text{fast}}$ are due to NRSI. The profiles are similar for both runs.

When driven by CR ions, NRSI causes the transverse components to orient anticlockwise along the background field. Thus, $B_y(x)$ lags behind $B_z(x)$ by a phase difference $\Delta\phi \sim \pi/2$, and we identify the components as *right-handed* (R-handed). In contrast, *left-handed* (L-handed) RSI causes the transverse components to orient clockwise, and $B_y(x)$ leads $B_z(x)$ with $\Delta\phi \sim -\pi/2$ ([S. Gupta et al. 2021](#)). In the left panels of Figure 2, B_y lags behind B_z by $\Delta\phi \sim \pi/2$, due to R-handed NRSI.

4.1.2. Fourier Mode Analysis

We perform Fourier analysis of the transverse field components to identify different NRSI modes. $\tilde{B}_y(k)$ and $\tilde{B}_z(k)$ are the Fourier transforms of $B_y(x)$ and $B_z(x)$, and we use them to calculate *helicity* of the modes using Equations B8 and B9 (see Appendix B for details). Helicity is *equivalent* to the phase difference in the k -space, $\Delta\phi(k) = \phi_y(k) - \phi_z(k)$. The positive and negative values of helicity, with $\Delta\phi(k) \sim \pi/2$ and $-\pi/2$, signify R-handed NRSI and L-handed RSI, respectively.

The right panels of Figure 2 show the helicities for the benchmark runs. We indicate k_{fast} , the upper cutoff of NRSI $k_u = 2k_{\text{fast}}$, and the mode resonant with CRs $k = 1/r'_{L,0} = 1/r'_{L,\min}$. For both runs, at $t \lesssim 10\gamma_{\text{fast}}^{-1}$, the R-handed NRSI with $k \sim k_{\text{fast}}$ is prominent; modes with $1/r'_{L,0} \lesssim k \lesssim k_u$ show positive helicity. Later at $t \gtrsim 10\gamma_{\text{fast}}^{-1}$, for MEI:5, $k \lesssim 1/r'_{L,0}$ (e.g., $k \in (1, 4) \times 10^{-3} d_i^{-1}$) show negative helicity due to RSI. In contrast, for PLI:5-400, these modes show positive helicity due to NRSI, consistent with the analytical dispersion relation (see Figure 1). A few modes with $k > k_u$ showing positive helicity at $t \gtrsim 10\gamma_{\text{fast}}^{-1}$ is a nonlinear phenomenon due to $B_\perp/B_0 \gtrsim 1$.

In Figure 3, we perform the diagnostics for the benchmark cone runs, MEC:5 and PLC:5-400 (C1 and C4 in Table 2), and obtain similar results.

4.1.3. Evolution of Transverse Magnetic Fields

Figure 4 shows the evolution of the transverse magnetic fields $B_\perp = \sqrt{B_y^2 + B_z^2}$ for the runs with isotropic

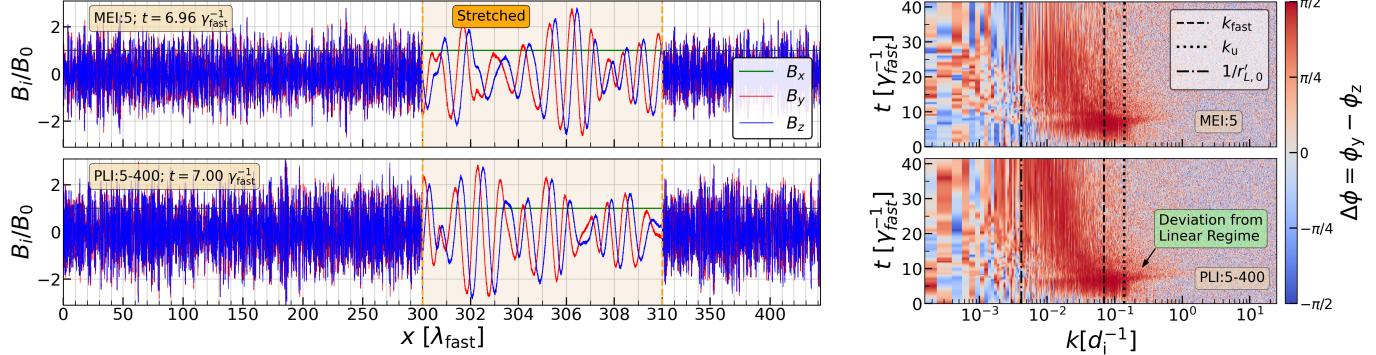


Figure 2. Diagnostics of the magnetic fields for a mono-energetic (upper panels) and a power-law (lower panels) isotropic CR distributions. [Left panels] The spatial profiles of the magnetic field components, normalized to the initial magnetic field B_0 , at $t \approx 7\gamma_{\text{fast}}^{-1}$. The x -axes are normalized to the wavelengths of the fastest-growing modes ($\lambda_{\text{fast}} = 2\pi/k_{\text{fast}} \approx 90d_i$). From the shaded zoomed-in part, the dominant wavelengths, almost equal to λ_{fast} , are prominent. [Right panels] Time evolution of helicities $\Delta\phi(k)$. The positive and negative values signify R-handed NRSI and L-handed RSI, respectively.

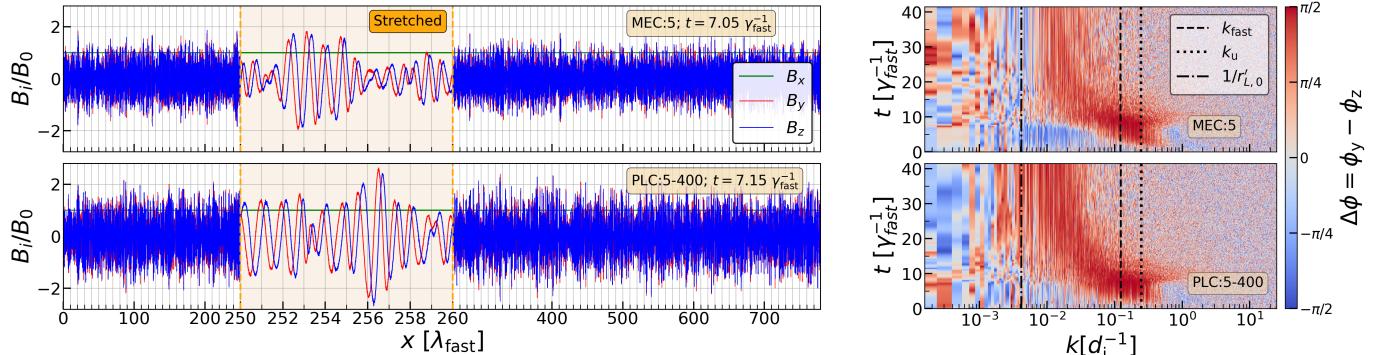


Figure 3. Similar to Figure 2, except that the CRs initially have cone distributions. [Left panels] The spatial profiles of the magnetic field components at $t \approx 7\gamma_{\text{fast}}^{-1}$. The x -axes are normalized to $\lambda_{\text{fast}} = 2\pi/k_{\text{fast}} \approx 50d_i$. In the shaded zoomed-in part, the dominant wavelengths $\sim \lambda_{\text{fast}}$ are prominent. [Right panels] Time evolution of helicities $\Delta\phi(k)$.

(I1-I9) and cone (C1-C9) CRs in Table 2. In the linear regime ($B_\perp/B_0 \lesssim 1$), B_\perp grows exponentially for all runs, consistent with the analytical predictions. In the nonlinear regime ($B_\perp/B_0 \gtrsim 1$) at time $t \sim 10\gamma_{\text{fast}}^{-1}$, B_\perp stops growing and the instability saturates. The saturated magnetic fields differ for the runs.

4.2. Saturation: Mono-energetic vs. Power-law

We test the existing saturation prescription and model the saturated magnetic fields (Section 4.2.1). We investigate why saturation levels differ across CR distributions with similar ξ (Section 4.2.2), and examine CR isotropization (Section 4.2.3).

4.2.1. Modeling the Saturated Magnetic Field

We take the maximum values of B_\perp/B_0 (e.g. in Figure 4) as the saturated magnetic field $(B_\perp/B_0)_{\text{sat}}$. Figure 5 shows $(B_\perp/B_0)_{\text{sat}}$ versus ξ for our runs.

First, we investigate the CR distributions with different ξ 's. We consider the pink symbols: triangles for mono-energetic CRs, and squares and diamonds for

power-law CRs. For mono-energetic CRs, as p'_0 increases, ξ increases (see Equations 14 and 10), and $(B_\perp/B_0)_{\text{sat}}$ increases accordingly. For power-law CRs, MEI:5 and PLI:5-30/80/400¹, as p'_{\max} increases, ξ increases (see Equation 14). However, $(B_\perp/B_0)_{\text{sat}}$ increases from MEI:5 to PLI:5-30, and it stops increasing significantly in PLI:5-80 and PLI:5-400. In contrast, for the runs PLI:5/10/20-400 with increase in p'_{\min} , $(B_\perp/B_0)_{\text{sat}}$ increases prominently. Thus, for power-law CRs with a wide energy range and a fixed n_{cr} , $(B_\perp/B_0)_{\text{sat}}$ is sensitive to a change in p'_{\min} , but does not depend strongly on a change in p'_{\max} .

Next, we investigate the CR distributions with similar ξ 's. For them, $(B_\perp/B_0)_{\text{sat}}$'s are not similar, unlike what is predicted in Equation 15. $(B_\perp/B_0)_{\text{sat}}$'s are similar when the difference between p'_{\min} and p'_{\max} for the power-law CRs is smaller, e.g., MEI:10.7 and

¹ MEI:5 is a special instance of power-law CRs with $p'_{\min} = p'_{\max} = 5m_i c$; p'_{\max} increases gradually for these runs.

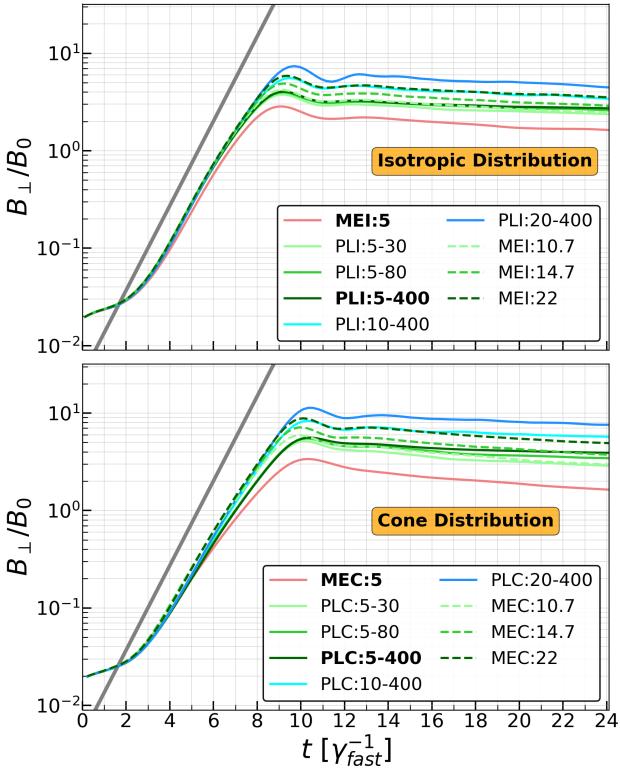


Figure 4. Time evolution of the transverse magnetic fields B_{\perp} for the runs in Table 2. Their linear growth shows good agreement with the analytical prediction (grey solid lines). However, the saturated magnetic fields are different.

PLI:5-30 with $\xi \sim 32$. When this difference is larger, $(B_{\perp}/B_0)_{\text{sat}}$ for the power-law CRs is smaller than the mono-energetic CRs, e.g., MEI:22 and PLI:5-400 with $\xi \sim 66$. We observe similar results across all the runs, including the cone CRs.

The mono-energetic CRs show good agreement with Equation 15, unlike the power-law CRs, for which ξ overestimates $(B_{\perp}/B_0)_{\text{sat}}$. We provide a fitting function to model their saturated fields. For the power-law CRs with a fixed p'_{\min} and an increasing p'_{\max} (e.g. MEI:5 and PLI:5-30/80/400), $(B_{\perp}/B_0)_{\text{sat}}$ vs ξ show a ‘tanh’ dependence in log-log space. This motivates a fitting function for power-law (PL) CRs with anisotropy parameter $\xi[\text{PL}:p'_{\min} - p'_{\max}]$, given by

$$\left(\frac{B_{\perp}}{B_0}\right)_{\text{sat,PL}} = \left(\frac{B_{\perp}}{B_0}\right)_{\text{sat,ME}} \exp\left(0.4 \frac{\tilde{\xi}_{\text{PL}}^3 - 1}{\tilde{\xi}_{\text{PL}}^3 + 1}\right). \quad (16)$$

Here, $(B_{\perp}/B_0)_{\text{sat,ME}}$ and $\tilde{\xi}_{\text{PL}}$ are defined as

$$\begin{aligned} \left(\frac{B_{\perp}}{B_0}\right)_{\text{sat,ME}} &= \sqrt{\frac{\xi[\text{ME}:p'_{\min}]}{2}} \quad \text{and} \\ \tilde{\xi}_{\text{PL}} &= \frac{\xi[\text{PL}:p'_{\min} - p'_{\max}]}{\xi[\text{ME}:p'_{\min}]} \end{aligned} \quad (17)$$

$\xi[\text{ME}:p'_{\min}]$ is anisotropy parameter ξ calculated for mono-energetic (ME) CRs with momentum p'_{\min} and number density n_{cr} . The fitting function indicates that the saturated magnetic field depends on both the anisotropy parameter and the minimum momentum of the power-law CRs. Also, for $p'_{\max} \gg p'_{\min}$, $\tilde{\xi}_{\text{PL}}^3 \gg 1$; hence, Equation 16 yields $(B_{\perp}/B_0)_{\text{sat,PL}} \approx \exp(0.4) \times \sqrt{\xi[\text{ME}:p'_{\min}]/2}$. This is the maximum saturated magnetic field for power-law CRs (with $f'_{\text{cr}}(p) \propto p^{-4}$) with a wide energy range. Equation 16 also applies to mono-energetic CRs, for which $p'_{\min} = p'_{\max} = p'_0$ and $\tilde{\xi}_{\text{PL}} = 1$; hence, Equation 16 reduces to Equation 15.

The inset in Figure 5 shows the normalized saturated magnetic field $(B_{\perp}/B_0)_{\text{sat,PL}} / (B_{\perp}/B_0)_{\text{sat,ME}}$ versus the normalized anisotropy parameter $\tilde{\xi}_{\text{PL}}$. The runs align well with the fit, i.e. Equation 16.

4.2.2. Mono-energetic vs Power-law CRs with similar ξ

For the similar ξ (or similar initial CR anisotropy pressure; see Equation 10), the saturated magnetic field for the mono-energetic CRs is larger than the power-law CRs (see Section 4.2.1). The saturation of NRSI is caused by the isotropization of CRs, through which the free energy in the CR anisotropy pressure gets transferred to the transverse magnetic fields and the background plasma. We show that the ineffective isotropization of higher-energy CRs for a power-law distribution leads to a smaller saturated field compared to mono-energetic CRs. We consider two runs, MEI:22 and PLI:5-400, with $\xi \approx 66$ (I9 and I4 in Table 2), and analyze the differences in the evolution of their momentum distributions and pressure components.

Figure 6 shows the momentum distributions for the two runs in the plasma-ion rest frame. For the plasma ions and electrons, we observe the emergence of nonthermal tails and the shifts in their thermal peaks, due to plasma heating. For the CRs, the entire distribution for MEI:22 changes over time. In contrast, for PLI:5-400, CRs with the lowest momenta are affected, without any significant change in the high-energy CRs.

Figure 7 shows longitudinal (P_{xx}) and transverse ($P_{\perp\perp} = (P_{yy} + P_{zz})/2$) pressure components for plasma species and CRs, along with the magnetic pressure, for the two runs. We compute the pressure component $P_{s,ij} = \langle p_{s,i} v_{s,j} \rangle$ of species s in the plasma-ion rest frame. The pressures of plasma ions and electrons evolve similarly in both runs. The heating of plasma species, as well as the saturated magnetic pressure, is larger for MEI:22 than PLI:5-400 at $t \sim 9\gamma_{\text{fast}}^{-1}$, when the overall saturation is reached. The increase in magnetic and background plasma pressures results from the equipartition of the CR anisotropy pressure $P_{\text{cr},xx} - P_{\text{cr},\perp\perp}$. For

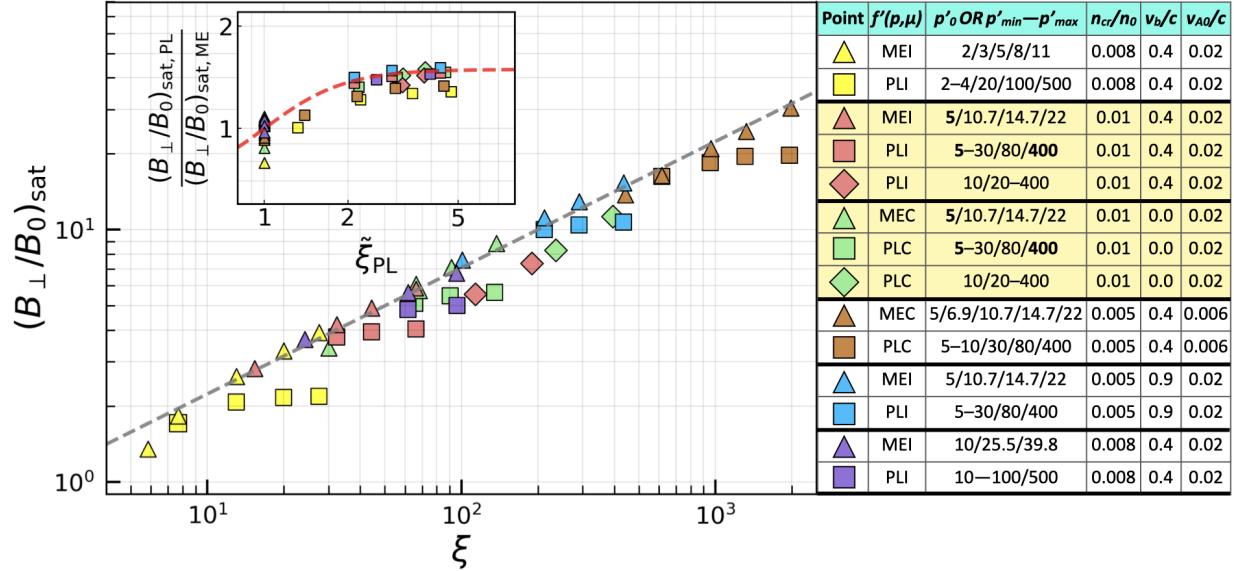


Figure 5. The saturated magnetic field $(B_{\perp}/B_0)_{\text{sat}}$ versus the anisotropy parameter ξ . The shapes and colors represent different simulations (see the table on the right; see Table 2 for the nomenclature of the runs). Momenta separated by ‘/’ represent different runs (e.g. PLI:5-30/80 implies PLI:5-30 and PLI:5-80). For the points with the same colors and shapes, larger ξ ’s correspond to larger momenta (e.g. for PLI:5-30/80, ξ for PLI:5-80 is larger than PLI:5-30). The grey dashed line represents the prediction, i.e. Equation 15. The runs with mono-energetic CRs mostly follow this prediction, unlike those with power-law CRs. Inset: Modelling of $(B_{\perp}/B_0)_{\text{sat}}$ using Equation 16 (red dashed line).

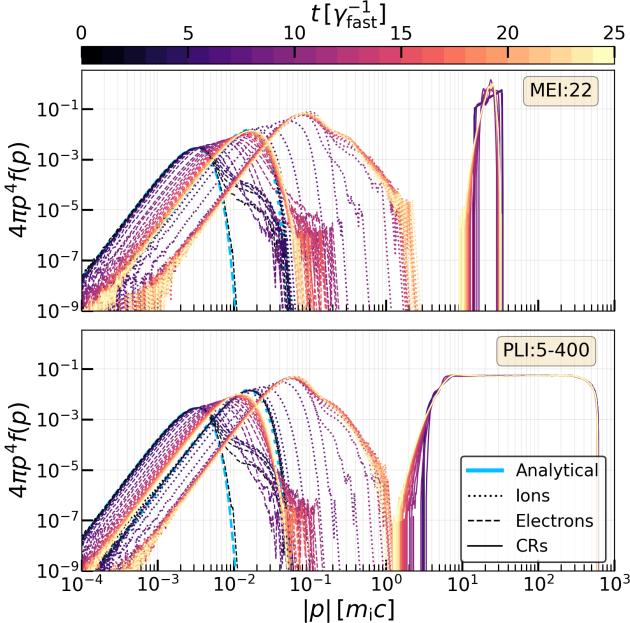


Figure 6. The momentum distributions of plasma ions (dotted lines), plasma electrons (dashed lines), and CRs (solid lines) in the plasma ion rest frame, for mono-energetic (top) and power-law (bottom) CRs with similar $\xi \approx 66$. The colors represent distributions at different times. Heating of plasma species (nonthermal tails and shifts in thermal peaks) and changes in CR distributions are observed at later times. The higher-energy CRs in the power-law distribution are essentially undisturbed.

MEI:22, CRs isotropize efficiently; thus, $P_{\text{cr},xx} \approx P_{\text{cr},\perp\perp}$ at $t \sim 9\gamma_{\text{fast}}^{-1}$, maximally amplifying the magnetic and background plasma pressures. In contrast, for PLI:5-400 at $t \sim 9\gamma_{\text{fast}}^{-1}$, the CR pressure components remain anisotropic, particularly due to the slower isotropization of the high-energy CRs (see Section 4.2.3); it leads to a smaller magnetic and background plasma pressures. Later at $t \gtrsim 9\gamma_{\text{fast}}^{-1}$, the CR pressure components of the high-energy particles continue to isotropize slowly. Their residual anisotropy pressure contributes to the gradual heating of the background plasma instead of feeding the transverse fluctuations, due to the continuous damping of the magnetic energy.

The transverse field growth is stopped, as the net CR drift does not remain super-Alfvénic after the overall saturation. In the bottom panel of Figure 7, $v_d/v_A \sim 1$ at $t \gtrsim 9\gamma_{\text{fast}}^{-1}$ for both runs. Even if the high-energy CRs maintain a super-Alfvénic drift, the isotropization of more numerous lower-energy CRs causes the overall drift relaxation.

4.2.3. Isotropization of CRs

We discuss how the isotropization of mono-energetic CRs differs from the power-law CRs. We use our benchmark runs MEI:5 and PLI:5-400 (I1 and I4 in Table 2), and analyze the CR distributions in momenta (p) and direction cosines ($\mu = p_x/p$) in plasma ion rest frame. Figure 8 shows the CR distributions for the two runs. For MEI:5, the anisotropic distribution at $t \approx 3\gamma_{\text{fast}}^{-1}$ (more

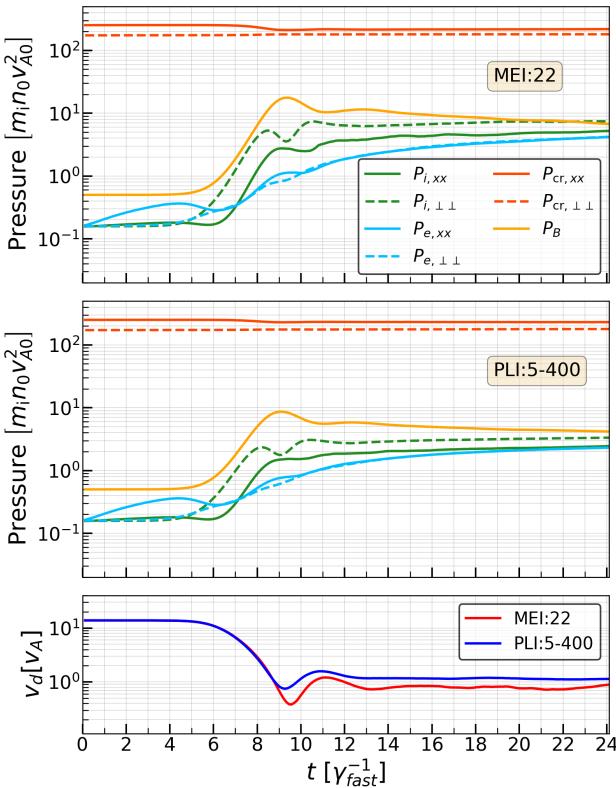


Figure 7. [Top two panels] Evolution of the longitudinal (P_{xx} , solid lines) and transverse ($P_{\perp\perp}$, dashed lines) pressure of plasma ions (green), plasma electrons (cyan), and CRs (red) in plasma ion rest frame, for the mono-energetic (top) and power-law (middle) CRs with a similar $\xi \approx 66$. The net magnetic pressure (orange) is $P_B = B^2/8\pi$, where $B^2 = B_0^2 + B_{\perp}^2$. After saturation, isotropization of CR pressure components for MEI:22 is apparent, whereas they remain anisotropic for PLI:5-400. [Bottom panel] CR drift v_d in plasma ion rest frame, normalized to the Alfvén speed v_A . After saturation, sub-Alfvénic CR drifts cease further magnetic field growth.

particles have positive μ) becomes mostly isotropic during saturation at $t \approx 10\gamma_{\text{fast}}^{-1}$. For PLI:5-400, we observe the anisotropy at $t \approx 3\gamma_{\text{fast}}^{-1}$ (more particles with a particular momentum have positive μ ; three contours of $4\pi p^2 f_{\text{cr}}$ show the anisotropy). At $t \approx 10\gamma_{\text{fast}}^{-1}$, the lower-energy CRs isotropize, and the higher-energy CRs remain unaffected (limiting their contribution to the saturated field; see Section 4.2.3). However, later at $t \approx 42\gamma_{\text{fast}}^{-1}$, the higher-energy CRs slowly isotropize. We will show in Section 4.2.4 that, in the absence of the lower-energy CRs, the higher-energy ones isotropize and effectively contribute to the saturated field as well.

We estimate an effective cutoff momentum p'_{eff} for power-law CRs with $p'_{\min} - p'_{\max}$, such that the CRs with momenta $p'_{\min} - p'_{\text{eff}}$ isotropize and contribute to

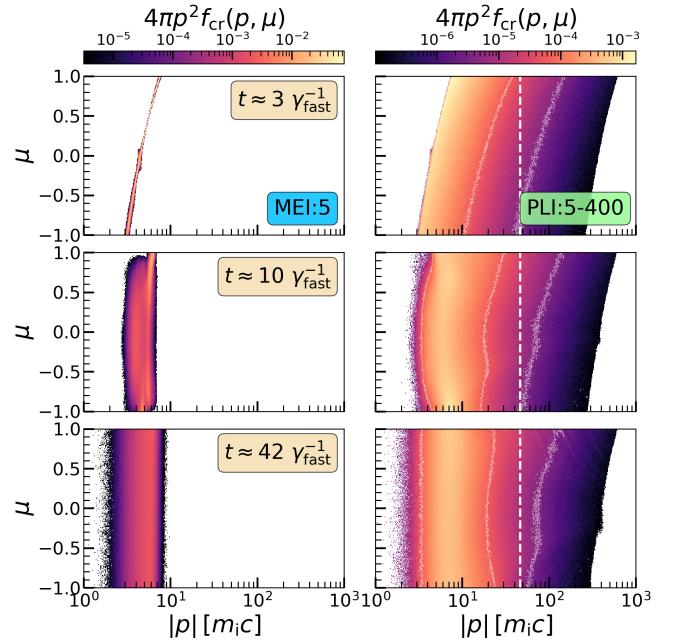


Figure 8. CR distributions $f_{\text{cr}}(p, \mu)$ for the runs with mono-energetic (left) and power-law (right) CRs at different times. The mono-energetic CRs isotropize at late times. For the power-law case, lower-energy CRs isotropize with time, unlike the higher-energy CRs. It is indicated with three white contours: $4\pi p^2 f_{\text{cr}} \approx 1.6 \times 10^{-3}$ (left), 6.2×10^{-5} (middle), 4×10^{-6} (right). The white dashed line marks $p'_{\text{eff}} \approx 9.3p'_{\min}$ (Equation 18), a cutoff above which the CRs do not isotropize effectively till overall saturation.

the saturated field, and above p'_{eff} do not. Therefore, the saturated field would be $\sqrt{\xi[\text{PL}:p'_{\min} - p'_{\text{eff}}]/2}$ (following Equation 15). Equating this with $\exp(0.4) \times \sqrt{\xi[\text{ME}:p'_{\min}]/2}$ (maximum saturated field for power-law CRs, see the corollary below Equation 16), we get

$$p'_{\text{eff}} \approx \exp(\exp(0.8))p'_{\min} \implies p'_{\text{eff}} \approx 9.3p'_{\min}. \quad (18)$$

It also implies that the saturated field for power-law CRs is $\sim \exp(0.4) \times \sqrt{\xi[\text{ME}:p'_{\min}]/2}$ when $p'_{\max} \gtrsim p'_{\text{eff}}$, otherwise $\sqrt{\xi[\text{PL}:p'_{\min} - p'_{\max}]/2}$. The white dashed lines in Figure 8 mark $p \approx p'_{\text{eff}}$. Till saturation (right middle box), CRs with $p \gtrsim p'_{\text{eff}}$ do not isotropize effectively. This cut-off momentum in Equation 18 is only valid for $f'_{\text{cr}}(p) \propto p^{-4}$; it decreases for $f'_{\text{cr}}(p) \propto p^{-5}$ and increases for $f'_{\text{cr}}(p) \propto p^{-3}$ (see Appendix C).

4.2.4. High-energy CRs with $p'_{\text{eff}} \leq p' \leq p'_{\max}$ in absence of lower-energy CRs

We show that, in the absence of the lower-energy CRs, the higher-energy ones with $p' \gtrsim p'_{\text{eff}}$ drive NRSI, contribute to its saturated field, and isotropize. Note that this study is different from Section 4.2.1, where we varied p'_{\min} or p'_{\max} by keeping n_{cr} fixed, and found that

saturation is more sensitive to p'_{\min} . Unlike that, here we vary both n_{cr} and p'_{\min} , essentially studying NRSI by higher-energy CRs in absence of lower-energy ones.

We consider two power-law CR distributions: the full spectrum with $p'_{\min} \leq p' \leq p'_{\max}$ and number density $n_{\text{cr}}[p'_{\min} - p'_{\max}]$, and the higher-energy CRs with $p'_{\text{eff}} \leq p' \leq p'_{\max}$ and number density $n_{\text{cr}}[p'_{\text{eff}} - p'_{\max}] \sim 0.1 \times n_{\text{cr}}[p'_{\min} - p'_{\max}]$ (as $p'_{\text{eff}} \sim 10p'_{\min}$, see Equation 18). The growth rate of NRSI is $\gamma_{\text{fast}} \propto n_{\text{cr}}v_{dpd}$ (Equation 5), where the drift speed v_d is not sensitive to p'_{\min} (see Equation 12). Thus, the growth of NRSI for the higher-energy CRs is slower by a factor of 10 compared to the whole spectrum. However, the anisotropy parameter $\xi \propto n_{\text{cr}}v_{dpd}$ (Equation 10). For $p'_{\max} \gg p'_{\min}$, particularly for $p'_{\max} \gtrsim 10p'_{\text{eff}} (\sim 100p'_{\min})$, the saturated fields for the high-energy CRs and the whole spectra are $\exp(0.4) \times \sqrt{\xi[\text{ME}:p'_{\text{eff}}]/2}$ and $\exp(0.4) \times \sqrt{\xi[\text{ME}:p'_{\min}]/2}$ (see the corollary below Equation 18), respectively; these saturated fields are similar as $\xi[\text{ME}:p'_{\min}] \approx \xi[\text{ME}:p'_{\text{eff}}]$ (the number densities compensate the momenta).

Now, we compare the above analytical expectations with simulations. We consider our benchmark runs PLI:5-400 and PLC:5-400 (I4 and C4 in Table 2), where $p'_{\min} \sim 5m_i c$, $p'_{\max} \sim 400m_i c$, and $n_{\text{cr}}/n_0 = 10^{-2}$. The higher-energy tails for these runs will have $p'_{\text{eff}} \sim 50m_i c$, $p'_{\max} \sim 400m_i c$, and $n_{\text{cr}}/n_0 \sim 10^{-3}$. The simulations with these higher-energy CRs are PLI:50-400:10⁻³ and PLC:50-400:10⁻³ (I10 and C10 in Table 2). The top panel of Figure 9 shows the growth of B_{\perp} for these runs, which are similar. The saturated magnetic fields for the higher-energy CRs and the whole spectrum are also similar, as shown in the inset. The bottom panel of Figure 9 shows that CR distributions for PLI:50-400:10⁻³ in plasma-ion rest frame isotropize effectively after saturation. Thus, in the absence of the lower-energy CRs, the higher-energy CRs contribute to the saturated field.

5. ASTROPHYSICAL IMPLICATIONS

In astrophysical shocks (e.g., SNRs) or the ISM plasmas, the CR distributions can deviate substantially from commonly assumed idealized forms, such as isotropic mono-energetic or power-law ($f'_{\text{cr}}(p) \propto p^{-4}$) populations. The linear growth of current-driven NRSI is independent of the CR distribution (see Section 4.1). Therefore, any CR spectrum drifting through ISM plasma satisfying Bell's criterion $r_{L,\min}^{f-1} < k_{\text{fast}} < d_i^{-1}$ drives NRSI, making this instability effective for the magnetic field amplification and scattering/self-confinement of CRs.

However, as expected from DSA, CRs naturally follow a power-law momentum distribution with $f'_{\text{cr}}(p) \propto p^{-4}$. These CRs are self-confined in the upstream

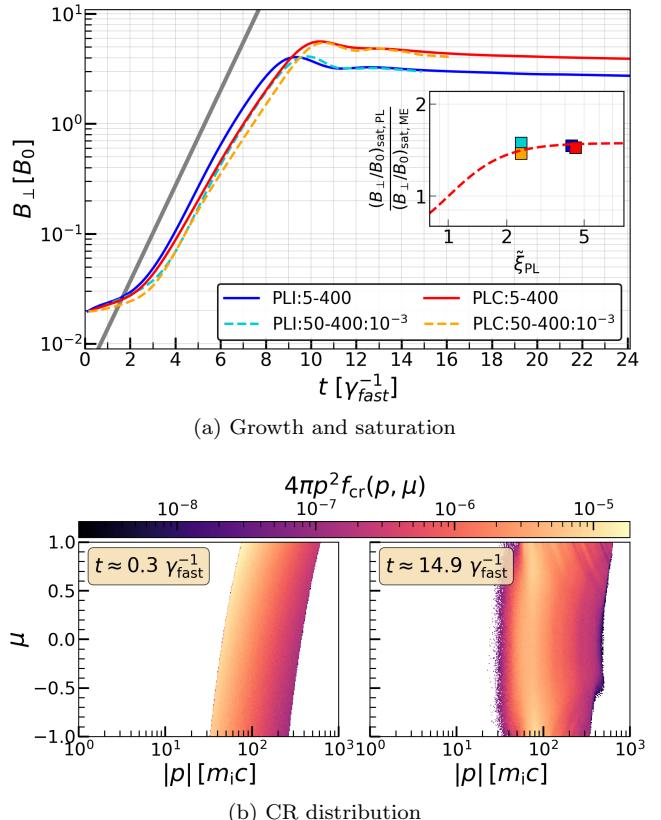


Figure 9. [Top panel] Transverse magnetic fields B_{\perp} for runs I4, C4, I10, and C10 in Table 2. Inset: $(B_{\perp}/B_0)_{\text{sat}}$ versus ξ , normalized as Equation 16 (red dashed line) for the runs with squares of respective colors. [Bottom panel] CR distributions for PLI:50-400:10⁻³. During saturation, roughly all the higher-energy CRs isotropize, efficiently contributing to the saturated field (in absence of the lower-energy ones).

plasma through isotropization via NRSI in the plasma-rest frame. In the shock rest frame, the CRs, along with the upstream plasma, are carried back to the shock front and further injected into DSA, reaching PeV energies.

For power-law distributions with $p'_{\min} \leq p' \leq p'_{\max}$ and $p'_{\max} \gg p'_{\min}$, the CRs with momentum $p' \lesssim 10p'_{\min}$ isotropize and contribute to the saturation of NRSI (Section 4.2.3). The higher-energy CRs, in the presence of the lower-energy ones, do not isotropize effectively, and may escape a plasma region before getting confined, potentially hindering their acceleration to higher energies.

Our results suggest a layered picture of the upstream plasma for CR confinement and magnetic field amplification. Lower-energy CRs are self-confined closer to the shock. Higher-energy CRs, leaking further ahead, enter an unperturbed plasma region and act as fresh drivers of NRSI. In the absence of the lower-energy CRs, the higher-energy ones can drive NRSI, contribute to its saturation, and isotropize (Section 4.2.4). Thus, the power-

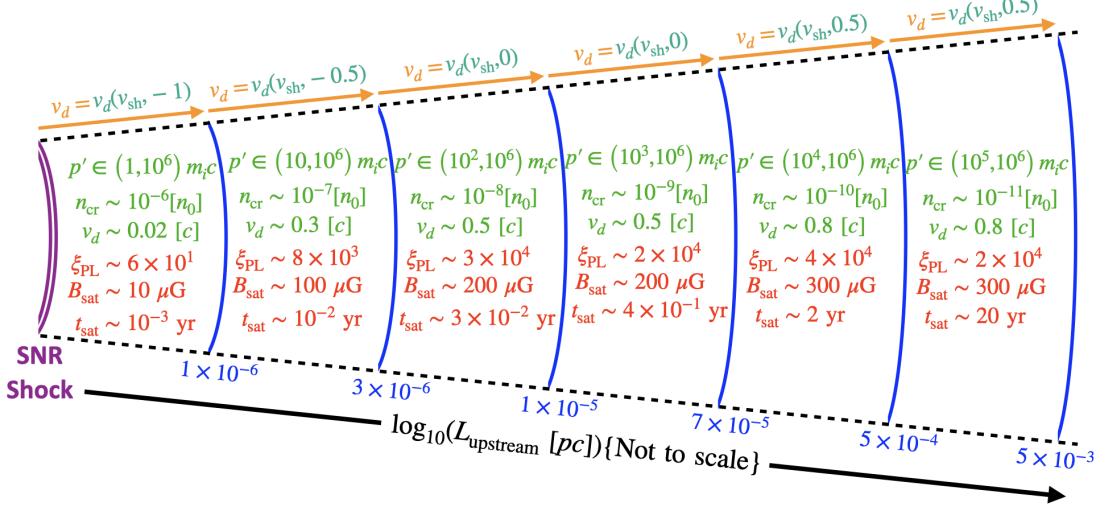


Figure 10. Schematic illustration of CR self-confinement via NRSI in a layered upstream ISM plasma ahead of an SNR shock. CRs with $f'_{\text{cr}}(p) \propto p^{-4}$, produced at the shock, enter an initially unperturbed upstream region and amplify magnetic fluctuations through NRSI. CRs with momenta $p' \in (p'_{\text{min}}, 10p'_{\text{min}})$ isotropize in each layer, where p'_{min} is the minimum momentum in that layer (as $p'_{\text{eff}} \sim 10p'_{\text{min}}$ in Equation 18; see Section 4.2.3 for details). Higher-energy particles escape to the next unperturbed region, where they become fresh drivers of NRSI with p'_{min} increased by a factor of 10 and reduced number density n_{cr} . The CR drift speed $v_d = v_d(v_b, \mu'_{\text{min}})$ depends on the boost speed v_b and minimum pitch angle μ'_{min} (see Equation 12). In all layers, v_b equals the shock speed v_{sh} (transforming to upstream plasma rest frame). In successive layers, we assume that μ'_{min} increases, implying more forward-beamed distributions in farther layers, hence larger saturated magnetic fields B_{sat} . After the instability saturates at time $t_{\text{sat}} \sim 10\gamma_{\text{fast}}^{-1}$ in each layer, CRs of higher momenta diffuse to the next layer over a distance comparable to their Larmor radii, i.e. $\Delta L_{\text{upstream}} \sim 10p'_{\text{min}}c/eB_{\text{sat}}$.

law CRs amplify magnetic fields over successive layers; each decade of CR momenta gets confined in each layer.

There are two limitations of this model. Firstly, the higher-energy CRs in the successive upstream layers have a smaller number density, leading to significantly slower growth of NRSI. Secondly, the saturated magnetic fields, similar in all upstream layers¹, are small. To resolve these issues, we use different pitch-angle distributions of CRs in the upstream layers, which is reasonable for more anisotropic higher-energy CRs in the farther layers. This anisotropy produces a larger CR drift speed v_d and a larger mean momentum p_d . Thus, the growth is faster, and the saturated field is larger.

We estimate the saturated magnetic fields and saturation timescales in our schematic model. We use typical values for the ISM plasma, i.e. the number density $n_0 = 1 \text{ cm}^{-3}$, and the magnetic field $B_{\text{ISM}} \sim 3 \mu\text{G}$. The shock speed is $v_{\text{sh}} \sim 0.03c$ for an SNR in the free-expansion or the Sedov-Taylor phase. For CRs, we take $p'_{\text{min}} \sim 1m_i c \sim \text{GeV}/c$ and $p'_{\text{max}} \sim 10^6m_i c \sim \text{PeV}/c$. The CR number density is $n_{\text{cr}} \sim 4 \times 10^{-6}n_0$; we assume that 10% of the shock kinetic energy den-

sity ($0.5\rho_{\text{ISM}}v_{\text{sh}}^2$) converts to the CR energy density ($U_{\text{cr}} = c \int_{p'_{\text{min}}}^{p'_{\text{max}}} p f'_{\text{cr}} d^3p$). Figure 10 shows an illustration of this model and the estimates.

We consider the minimum pitch angle μ'_{min} as a free parameter. For the first layer, $\mu'_{\text{min}} = -1$, i.e. CRs are isotropic. In the successive layers, μ'_{min} gradually increases. We calculate the growth rate γ_{fast} and the saturated magnetic field (B_{\perp})_{sat,PL} using Equations 5 and 16, respectively; net saturated magnetic field is $B_{\text{sat}} = \sqrt{B_{\text{ISM}}^2 + (B_{\perp})_{\text{sat,PL}}^2}$.

Our model shows that CRs up to PeV energies can be confined within a lengthscale of $L_{\text{upstream}} \sim 5 \times 10^{-3} \text{ pc}$, much smaller than the SNR itself (e.g. radius $R_{\text{SNR}} \sim 0.1 \text{ pc}$). The confinement timescale (similar to the saturation timescale) is $\sim 20 \text{ yr}$, much shorter than the Sedov-Taylor phase of an SNR ($\sim 1000 \text{ yr}$). The magnetic field amplification up to $\sim 300 \mu\text{G}$, significantly larger than the ISM magnetic field, is also possible.

Although motivated by our results, the model is simplistic and speculative. Testing this scenario requires more realistic setups, allowing for particle escape and re-seeding of instabilities in fresh plasma. Such models are essential for assessing whether SNR shocks can truly sustain the conditions required for PeV acceleration and account for the observed Galactic CR spectrum up to the ‘knee’ and beyond.

¹ The saturated magnetic fields for the higher-energy CRs with $10p'_{\text{min}} \leq p' \leq p'_{\text{max}}$ and the whole spectrum with $p'_{\text{min}} \leq p' \leq p'_{\text{max}}$ are similar (see Section 4.2.4).

6. CONCLUSIONS

In this work, we have used different distributions of cosmic rays (CRs) to study the linear growth and nonlinear saturation of the nonresonant streaming instability (NRSI) using 1D particle-in-cell simulations. We show that, during saturation, the CRs with a power-law distribution, having a wide range of energies, behave differently from the CRs with a mono-energetic distribution. The following is a summary of our key results:

1. The linear growth of NRSI depends on the net CR current through the background plasma; the fastest-growing modes and their growth rates are nearly the same across different CR distributions.
2. Saturation of NRSI occurs through the isotropization of CRs in the background plasma, which ceases the CR current. Thus, the excess anisotropy pressure of CRs (difference between parallel and transverse CR pressure as measured in the plasma rest frame) gets transferred to the transverse magnetic pressure and the background plasma (resulting in plasma heating).
3. Saturation of NRSI depends on the CR distributions. Mono-energetic CRs isotropize and contribute to the saturation. In contrast, for the power-law CRs, only the lowest-energy particles contribute to the saturation; the isotropization of the highest-energy CRs is inefficient, limiting their contribution to the saturated field. However, in the absence of the low-energy CRs, the high-energy ones effectively amplify the transverse fields and isotropize.
4. We tested the effectiveness of the anisotropy parameter ξ , the ratio of CR momentum flux to the magnetic pressure (Equation 10), which predicts the saturated magnetic field (Equation 15). For mono-energetic CRs, ξ accurately estimates the saturated field, consistent with the previous find-

ings (S. Gupta et al. 2021; G. Zacharek et al. 2024). However, for power-law CRs extending across a wide momentum range, the usual ξ overestimates the saturated field, as it does not account for the negligible contribution of the high-energy CRs. We provide a generalized saturation model for power-law CRs with $f'_{\text{cr}}(p) \propto p^{-4}$ (Equation 16) based on the fact that only the CRs with $p' \lesssim 10p'_{\min}$ contribute to the saturation (Equation 18).

Based on these findings, we propose a plausible model for CR confinement via NRSI in the ISM plasma upstream of an SNR shock (see Figure 10). Power-law CRs enter the unperturbed ISM and excite NRSI. The lowest-energy CRs get confined closer to the shock; the higher-energy CRs escape and enter the subsequent unperturbed ISM. Thus, the magnetic field is amplified and the CRs are confined over successive layers in the upstream ISM. We estimate the amplified magnetic fields and the length and timescales of CR confinement, suitable for an SNR. Future realistic simulations will be crucial to test the viability of this distribution-dependent confinement in astrophysical environments.

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Facilities: Param-Pravega Supercomputing Facility (National Supercomputing Mission, Indian Institute of Science)

APPENDIX

A. KINETIC DISPERSION OF NRSI

We derive the kinetic dispersions similar to E. Amato & P. Blasi (2009). By linearizing the Vlasov equation, we get the dispersion relation for parallel-propagating

transverse waves in a composite plasma, given by

$$\frac{k^2 c^2}{\omega^2} = 1 + \sum_s \frac{4\pi^2 q_s^2}{\omega} \int_0^\infty dp \int_{-1}^1 d\mu \frac{p^2 v(p) (1 - \mu^2)}{\omega - kv(p)\mu - \Omega_s} \times \left[\frac{\partial f_s}{\partial p} + \left(\frac{kv(p)}{\omega} - \mu \right) \frac{1}{p} \frac{\partial f_s}{\partial \mu} \right], \quad (\text{A1})$$

where q_s and $\Omega_s = \frac{q_s B_0}{\gamma m_s c}$ are the charge and relativistic gyrofrequency of species s , respectively. Replacing f_s

with the distributions of cold ions and electrons, and CRs, we evaluate the imaginary solutions of ω , which provide the growth rates (γ) of RSI and NRSI.

The growth rates of NRSI for isotropic (I) CRs are

$$\gamma_I(k) = \frac{1}{\sqrt{2}} \sqrt{-\Theta_I(k) + \sqrt{\Theta_I(k)^2 + (\alpha(k)Y_I(k))^2}}, \quad (A2)$$

where $\Theta_I(k) = k^2 v_{A0}^2 + \alpha(k)(1 + X_I(k))$ and $\alpha(k) = -\frac{n_{cr}}{n_0} k v_{ci}$. For mono-energetic (ME) CRs, $X_I(k)$ and $Y_I(k)$ are

$$X_{MEI}(k) = -\frac{1}{2} \frac{p'_0}{(kr'_{L,0})^2} \ln \left| \frac{1 + kr'_{L,0}}{1 - kr'_{L,0}} \right| \quad (A3a)$$

$$Y_{MEI}(k) = -\frac{\pi}{2} \frac{p'_0}{(kr'_{L,0})^2} \times \begin{cases} 0, & (kr'_{L,0} < 1) \\ 1, & (kr'_{L,0} \geq 1) \end{cases}. \quad (A3b)$$

For power-law (PL) CRs, $X_I(k)$ and $Y_I(k)$ are

$$X_{PLI}(k) = \frac{\mathcal{A}}{4} \left[\left(\frac{1}{s^4} - \frac{1}{s^2} \right) (s^2 - 1) \ln \left| \frac{1+s}{1-s} \right| + 2 \left(\frac{1}{s^3} + \frac{1}{s} \right) \right]_{kr'_{L,min}}^{kr'_{L,max}} \quad (A4a)$$

$$Y_{PLI}(k) = \frac{\pi \mathcal{A}}{4} \left[\frac{2}{s^2} - \frac{1}{s^4} \right]_{\text{Max}[1, kr'_{L,min}]}^{kr'_{L,max}} \quad (A4b)$$

where $\mathcal{A} = \left(\frac{1}{kr'_{L,min}} - \frac{1}{kr'_{L,max}} \right)^{-1}$ and $r'_L = \frac{p'c}{eB_0}$ is the Larmor radius.

Comparing the expressions of $X_{PLI}(k)$ and $Y_{PLI}(k)$ with those of $I_1^+(k)$ and $I_2(k)$ (their Equations 24 and 25) in E. Amato & P. Blasi (2009), we observe that the terms with $1/s^3$ and $1/s^4$ are absent. This causes the growth rates of RSI and NRSI to be almost similar for $k < 1/r'_{L,min}$ (see their Figure 1). However, these terms cannot be neglected for $k < 1/r'_{L,min}$, as they cause the growth rates of RSI to be larger than NRSI; this has important consequences for CR scattering.

The growth rates of NRSI for cone (C) CRs are

$$\gamma_C(k) = \frac{1}{2} \left[\Phi(k) + \Im \left\{ \sqrt{\Theta_{C1}(k)^2 + 4\Theta_{C2}(k)} \right\} \right], \quad (A5)$$

where $\Theta_{C1}(k) = \frac{n_{cr}}{n_0} \omega_{ci} (1 - X_C(k)) + j\Phi(k)$, $\Theta_{C2}(k) = \alpha(k) + k^2 v_{A0}^2$, $\Phi(k) = \frac{n_{cr}}{n_0} \omega_{ci} Y_C(k)$. For mono-energetic

Table 3. Runs with different power-law indices

Sl. No.	Runs	v_d [c]	p_d [m;c]	ξ	$k_{\text{fast}} d_i$ ($\times 10^{-2}$)
A1	PLI:5-400: p^{-5}	0.279	4.25	29.63	6.97
A2	PLI:5-400: p^{-3}	0.276	39.18	270.8	6.91

Note: The labels of the runs and the parameters are the same as those given in Table 2. Additionally, in the labels, the CR distribution $f'_{\text{cr}}(p) \propto p^{-5}$ and p^{-3} are mentioned. Movies for these runs are provided ([Click here](#)).

(ME) CRs, $X_C(k)$ and $Y_C(k)$ are

$$X_{MEC}(k) = \frac{p'_0}{(kr'_{L,0})^2} \ln |1 - kr'_{L,0}| \quad (A6a)$$

$$Y_{MEC}(k) = -\frac{\pi p'_0}{(kr'_{L,0})^2} \times \begin{cases} 0 & ; \text{ if } kr'_{L,0} < 1 \\ 1 & ; \text{ if } kr'_{L,0} \geq 1 \end{cases}. \quad (A6b)$$

For power-law (PL) CRs, $X_C(k)$ and $Y_C(k)$ are

$$X_{PLC}(k) = \frac{\mathcal{A}}{2} \left[\left(1 - \frac{2}{s^2} + \frac{1}{s^4} \right) \ln |1 - s| - \ln |s| + \left(\frac{1}{s} + \frac{1}{2s^2} + \frac{1}{s^3} \right) \right]_{kr'_{L,min}}^{kr'_{L,max}} \quad (A7a)$$

$$Y_{PLC}(k) = \frac{\pi \mathcal{A}}{2} \left[\frac{2}{s^2} - \frac{1}{s^4} \right]_{\text{Max}[1, kr'_{L,min}]}^{kr'_{L,max}}. \quad (A7b)$$

B. HELICITY OF UNSTABLE MODES

We calculate the helicity $\Delta\phi(k)$ using the formula

$$\Delta\phi(k) = \tan^{-1} \left[\frac{V(k)}{\{Q^2(k) + U^2(k)\}^{1/2}} \right]. \quad (B8)$$

We calculate the Stokes parameters $Q(k)$, $U(k)$, and $V(k)$, using the Fourier transforms of the transverse magnetic field components, as

$$Q(k) = \Re \left([\tilde{B}_y(k)\tilde{B}_y^*(k) - \tilde{B}_z(k)\tilde{B}_z^*(k)] \right) \quad (B9a)$$

$$U(k) = \Re \left([\tilde{B}_y(k)\tilde{B}_z^*(k) + \tilde{B}_z(k)\tilde{B}_y^*(k)] \right) \quad (B9b)$$

$$V(k) = \Re \left([\tilde{B}_y(k)\tilde{B}_z(k) - \tilde{B}_z(k)\tilde{B}_y(k)] / j \right). \quad (B9c)$$

Here, $\tilde{B}_i(k)$ is the Fourier transform of $B_i(x)$ along the x -direction and $\tilde{B}_i^*(k)$ is its complex conjugate.

C. DIFFERENT POWER-LAW INDICES

DSA in a strong shock produces a power-law distribution of CRs with $f'_{\text{cr}}(p) \propto p^{-4}$. However, the power-law

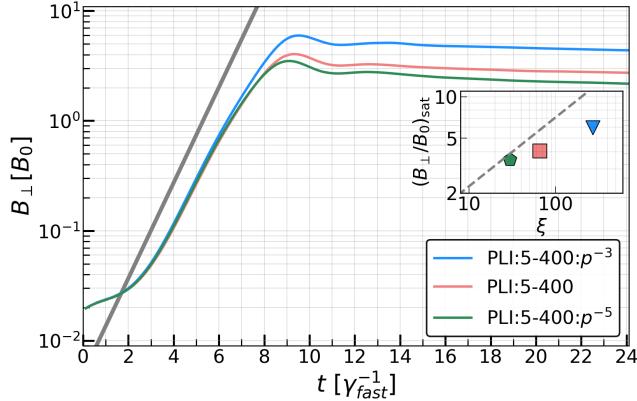


Figure 11. Evolution of transverse magnetic field B_{\perp} for the runs PLI:5-400: p^{-3} , PLI:5-400(p^{-4}), and PLI:5-400: p^{-5} . Inset: Saturated magnetic field $(B_{\perp}/B_0)_{\text{sat}}$ versus anisotropy parameter ξ for these runs (green pentagon for p^{-5} , pink square p^{-4} , blue nabla for p^{-3}). The grey dashed line represents Equation 15.

index can differ when the shock is not highly supersonic or is modified by CR transport. We investigate how a change in the power-law index affects NRSI. We consider isotropic CRs with $g'(p) \propto p^{-5}$ and p^{-3} (see Equation 6). We perform the runs PLI:5-400: p^{-5} and PLI:5-400: p^{-3} (A1 and A2 in Table 3) with these CR distributions. We also consider PLI:5-400 (I4 in Table 2), for which $f'_{\text{cr}}(p) \propto p^{-4}$.

We begin with the linear regime. For a fixed n_{cr}/n_0 , k_{fast} and γ_{fast} (Equation 5) depend on drift speed v_d . For relativistic CRs, v_d depends only on μ'_{\min} and v_b (Equation 12). Thus, the linear regime is independent of the power-law indices (cf. k_{fast} of A1, A2, and I4). Figure 11 shows the evolution of transverse magnetic fields $B_{\perp} = \sqrt{B_y^2 + B_z^2}$, normalized to the initial magnetic field B_0 , for these runs. The exponential growths for PLI:5-400: p^{-5} and PLI:5-400: p^{-3} are similar to PLI:5-400. The other diagnostics are similar to Figure 2.

We test the saturation prediction in Equation 15 for these runs. We evaluate CR momentum drift p_d (Equations 13) and anisotropy parameter ξ (Equation 10). The inset of Figure 11 shows $(B_{\perp}/B_0)_{\text{sat}}$ versus ξ . As the distribution becomes steeper, ξ decreases, resulting in a smaller $(B_{\perp}/B_0)_{\text{sat}}$. However, $(B_{\perp}/B_0)_{\text{sat}}$ for $f'_{\text{cr}}(p) \propto p^{-3}$ deviates more from the prediction than $f'_{\text{cr}}(p) \propto p^{-5}$. This is due to numerous higher-energy CRs in PLI:5-400: p^{-3} , which take longer to isotropize and therefore do not contribute effectively to the saturated field. In other words, the quantity p'_{eff} (Equation 18) is smaller for $f'_{\text{cr}}(p) \propto p^{-5}$ and larger for $f'_{\text{cr}}(p) \propto p^{-3}$, compared to $f'_{\text{cr}}(p) \propto p^{-4}$.

REFERENCES

- Achterberg, A. 2013, MNRAS, 436, 705, doi: [10.1093/mnras/stt1608](https://doi.org/10.1093/mnras/stt1608)
- Amato, E., & Blasi, P. 2009, MNRAS, 392, 1591, doi: [10.1111/j.1365-2966.2008.14200.x](https://doi.org/10.1111/j.1365-2966.2008.14200.x)
- Axford, W. I., Leer, E., & Skadron, G. 1977, Proc. 15th Int. Cosmic Ray Conf., 11, 132. <https://ui.adsabs.harvard.edu/abs/1977ICRC...11..132A>
- Bai, X. N., Ostriker, E. C., Plotnikov, I., & Stone, J. M. 2019, ApJ, 876, 60, doi: [10.3847/1538-4357/ab1648](https://doi.org/10.3847/1538-4357/ab1648)
- Bell, A. R. 1978, MNRAS, 182, 147, doi: [10.1093/mnras/182.2.147](https://doi.org/10.1093/mnras/182.2.147)
- Bell, A. R. 2004, MNRAS, 353, 550, doi: [10.1111/j.1365-2966.2004.08097.x](https://doi.org/10.1111/j.1365-2966.2004.08097.x)
- Bell, A. R., & Lucek, S. G. 2001, MNRAS, 321, 433, doi: [10.1046/j.1365-8711.2001.04063.x](https://doi.org/10.1046/j.1365-8711.2001.04063.x)
- Berezhko, E. G., Ksenofontov, L. T., & Völk, H. J. 2003, A&A, 412, L11, doi: [10.1051/0004-6361:20031667](https://doi.org/10.1051/0004-6361:20031667)
- Blandford, R. D., & Ostriker, J. P. 1978, ApJ, 221, L29, doi: [10.1086/182658](https://doi.org/10.1086/182658)
- Bret, A. 2009, ApJ, 699, 990, doi: [10.1088/0004-637X/699/2/990](https://doi.org/10.1088/0004-637X/699/2/990)
- Bykov, A., Brandenburg, A., Malkov, M., & Osipov, S. 2013, Space Science Reviews, 178, 201, doi: [10.1007/s11214-013-9988-3](https://doi.org/10.1007/s11214-013-9988-3)
- Gargaté, L., Fonseca, R., Niemiec, J., et al. 2010, ApJ Letters, 711, L127, doi: [10.1088/2041-8205/711/2/L127](https://doi.org/10.1088/2041-8205/711/2/L127)
- Gupta, S., Caprioli, D., & Haggerty, C. C. 2021, ApJ, 923, 208, doi: [10.3847/1538-4357/ac23cf](https://doi.org/10.3847/1538-4357/ac23cf)
- Gupta, S., Caprioli, D., & Spitkovsky, A. 2024, ApJ, 968, 17, doi: [10.3847/1538-4357/ad3e75](https://doi.org/10.3847/1538-4357/ad3e75)
- Gupta, S., Caprioli, D., & Spitkovsky, A. 2024, ApJ, 976, 10, doi: [10.3847/1538-4357/ad7c4c](https://doi.org/10.3847/1538-4357/ad7c4c)
- Haggerty, C., Caprioli, D., & Zweibel, E. 2019, ICRC, 36, 279, doi: [10.22323/1.358.0279](https://doi.org/10.22323/1.358.0279)
- Hillas, A. M. 1984, Annual Review of Astronomy and Astrophysics, 22, 425, doi: [10.1146/annurev.aa.22.090184.002233](https://doi.org/10.1146/annurev.aa.22.090184.002233)
- Holcomb, C., & Spitkovsky, A. 2019, ApJ, 882, 3, doi: [10.3847/1538-4357/ab328a](https://doi.org/10.3847/1538-4357/ab328a)
- Krymskii, G. F. 1977, Sov. Phys. Dokl., 22, 327
- Kulsrud, R., & Pearce, W. P. 1969, ApJ, vol. 156, p. 445, 156, 445

- Lerche, I. 1967, ApJ, vol. 147, p. 689, 147, 689,
doi: [10.1086/149045](https://doi.org/10.1086/149045)
- Lichko, E., Caprioli, D., Schroer, B., & Gupta, S. 2025,
ApJ, 980, 240, doi: [10.3847/1538-4357/adadf5](https://doi.org/10.3847/1538-4357/adadf5)
- Marret, A., Ciardi, A., Smets, R., & Fuchs, J. 2021,
MNRAS, 500, 2302, doi: [10.1093/mnras/staa3465](https://doi.org/10.1093/mnras/staa3465)
- Matthews, J. H., Bell, A. R., Blundell, K. M., & Araudo,
A. T. 2017, MNRAS, 469, 1849,
doi: [10.1093/mnras/stx905](https://doi.org/10.1093/mnras/stx905)
- McKenzie, J., & Völk, H. 1982, A & A, 116, 191
- Niemiec, J., Pohl, M., Stroman, T., & Nishikawa, K.-I.
2008, ApJ, 684, 1174, doi: [10.1086/590054](https://doi.org/10.1086/590054)
- Reville, B., & Bell, A. 2013, MNRAS, 430, 2873,
doi: [10.1093/mnras/stt100](https://doi.org/10.1093/mnras/stt100)
- Riquelme, M. A., & Spitkovsky, A. 2009, ApJ, 694, 626,
doi: [10.1088/0004-637X/694/1/626](https://doi.org/10.1088/0004-637X/694/1/626)
- Spitkovsky, A. 2005, AIP Conference Proceedings, 801, 345,
doi: [10.1063/1.2141897](https://doi.org/10.1063/1.2141897)
- Vink, J., & Laming, J. M. 2003, ApJ, 584, 758,
doi: [10.1086/345832](https://doi.org/10.1086/345832)
- Völk, H. J., Berezhko, E. G., & Ksenofontov, L. T. 2005, A
& A, 433, 229, doi: [10.1051/0004-6361:20042015](https://doi.org/10.1051/0004-6361:20042015)
- Weidl, M. S., Winske, D., & Niemann, C. 2019, ApJ, 872,
48, doi: [10.3847/1538-4357/aafad0](https://doi.org/10.3847/1538-4357/aafad0)
- Wentzel, D. G. 1969, ApJ, vol. 156, p. 303, 156, 303,
doi: [10.1086/149965](https://doi.org/10.1086/149965)
- Zacharekcas, G., Caprioli, D., Haggerty, C., Gupta, S., &
Schroer, B. 2024, ApJ, 967, 71,
doi: [10.3847/1538-4357/ad3960](https://doi.org/10.3847/1538-4357/ad3960)
- Zirakashvili, V., & Ptuskin, V. 2008, ApJ, 678, 939,
doi: [10.1086/529580](https://doi.org/10.1086/529580)
- Zweibel, E. G. 2013, Physics of Plasmas, 20, 055501,
doi: [10.1063/1.4807033](https://doi.org/10.1063/1.4807033)
- Zweibel, E. G., & Everett, J. E. 2010, ApJ, 709, 1412,
doi: [10.1088/0004-637X/709/2/1412](https://doi.org/10.1088/0004-637X/709/2/1412)