Supplementary Material

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***Abstract*—Supplementary material of the paper entitled: A CMA-ES with Deb's Tournament Selection and Adaptive Penalty Method for Solving Constrained Engineering Optimization Problems**

I. Classical Engineering Problems

This section describes some of the classic mechanical engineering optimization problems used in this work. For each problem, a specific number of objective function evaluations were used, which will be described in the details of each one. In order not to extend this work, the images of the problems can be consulted in the references.

A. The Tension/Compression Spring design

This problem aims to minimize the volume V of a coil spring under a constant tension/compression load. There are three design variables to be considered: The number of active coils of the spring ($N = x_1 \in [2, 15]$, the winding diameter ($D = x_2 \in [0.25, 1.3]$) and the wire diameter ($d = x_3 \in [0.05, 2]$). The objective function is written as [1].

$$V = (x_1 + 2)x_2x_3^2 \tag{1}$$

$$g_1(x) = 1 - \frac{x_3^2 x_1}{71785x_3^4} \leq 0 \tag{2}$$

$$g_2(x) = \frac{4x_2^2 - x_3x_2}{12566(x_2x_3^3 - x_3^4)} + \frac{1}{5108x_3^2} - 1 \leq 0 \tag{3}$$

$$g_3(x) = 1 - \frac{140.45x_3}{x_2^2 x_1} \leq 0 \tag{4}$$

$$g_4(x) = \frac{{}_x2 + x_3}{1.5} \leq 0 \tag{5}$$

Where

$$2 \leq x_1 \leq 15 \qquad 0.25 \leq x_2 \leq 1.3 \qquad 0.005 \leq x_3 \leq 2$$

B. The Speed Reducer design

The goal is to minimize the weight W of the speed reducer [1]. The design variables are the face width ($b = x_1 \in [2.6, 3.6]$), the module of teeh ($m = x_2 \in [0.7, 0.8]$), the number of teeth on ($n = x_3 \in [17, 28]$), the length of the shaft 1 between the bearings ($l_1 = x_4 \in [7.3, 8.3]$), the length of the shaft 2 between the bearings ($l_2 = x_5 \in [7.8, 8.3]$), o the diameter of the shaft 1($d_1 = x_6 \in [2.9, 3.9]$) and, finally, the diameter of the shaft 2 ($d_2 = x_7 \in [5.0, 5.5]$). The constraints include limitations on the bending and surface stress of the gear teeth, transverse deflections of the shafts 1 and 2 generated by the transmitted force, and the stress in the shafts 1 and 2. The weight to be minimized and the constraints are given by:

$$W = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2) \tag{6}$$

$$g_1(x) = 27x_1^{-1}x_2^{-2}x_3^{-1} \leq 1 \tag{7}$$

$$g_2(x) = 397.5x_1^{-1}x_2^{-2}x_3^{-2} \leq 1 \tag{8}$$

$$g_3(x) = 1.93x_2^{-1}x_3^{-1}x_4^3x_6^{-4} \leq 1 \tag{9}$$

$$g_4(x) = 1.93x_2^{-1}x_3^{-1}x_5^3x_7^{-4} \leq 1 \tag{10}$$

$$g_5(x) = \frac{1}{0.1x_6^3}\left[\left(\frac{745x_4}{x_2x_3}\right)^2 + \{16.9\}10^6\right]^{0.5} \leq 1100 \tag{11}$$

$$g_6(x) = \frac{1}{0.1x_7^3}\left[\left(\frac{745x_5}{x_2x_3}\right)^2 + \{157.5\}10^6\right]^{0.5} \leq 850 \tag{12}$$

$$g_7(x) = x_2x_3 \leq 40 \tag{13}$$

$$g_8(x) = \frac{x_1}{x_2} \geq 5 \tag{14}$$

$$g_9(x) = \frac{x_1}{x_2} \leq 12 \tag{15}$$

$$g_{10}(x) = (1.5x_6 + 1.9)x_4^{-1} \leq 1 \tag{16}$$

$$g_{11}(x) = (1.1x_7 + 1.9)x_5^{-1} \leq 1 \tag{17}$$

where

$$\begin{array}{lll} 2.6 \leq x_1 \leq 3.6 & 0.7 \leq x_2 \leq 0.8 & 17 \leq x_3 \leq 28.0 \\ 7.3 \leq x_4 \leq 8.3 & 7.8 \leq x_5 \leq 8.3 & 2.9 \leq x_6 \leq 3.9 \\ 5.0 \leq x_7 \leq 5.5 & & \end{array}$$

C. The Welded Beam design

The objective of this problem is to minimize the cost $C(h, l, t, b)$ of the beam [1], where $h \in [0.125, 10]$ and $0.1 \leq l, t, b \leq 10$. The cost and constraints are given as:

$$C(h, l, t, b) = 1.10471h^2l + 0.04811tb(14.0 + l) \tag{18}$$

$$g_1(\tau) = 13600 - \tau \geq 0 \tag{19}$$

$$g_2(\sigma) = 30000 - \sigma \geq 0 \tag{20}$$

$$g_3(b, h) = b - h \geq 0 \tag{21}$$

$$g_4(P_c) = P_c - 6000 \geq 0 \tag{22}$$

$$g_5(\delta) = 0.25 - \delta \geq 0 \tag{23}$$

The expressions for τ, σ, P_c and δ are given by:

$$\tau = \sqrt{(\tau')^2 + (\tau'')^2 + l\tau'\tau''/\alpha} \qquad \tau' = \frac{6000}{\sqrt{2}ht}$$
$$\alpha = \sqrt{0.25(l^2 + (h+t)^2)} \qquad \sigma = \frac{504000}{t^2 b}$$
$$P_c = 64743.022(1 - 0.0282346t)tb^3 \qquad \delta = \frac{2.1952}{t^3 b}$$
$$\tau'' = \frac{6000(14 + 0.5l)\alpha}{2(0.707hl(l^2/12 + 0.25(h+t)^2))}$$

D. The Pressure Vessel design

The goal is to minimize the weight W of a cylindrical pressure vessel with two spherical heads as shown in [1]. This problem has four design variables: the thickness of the pressure vessel(T_s), the thickness of the head (T_h), the inner radius of the vessel (R) and, finally, the length of the cylindrical component (L). The model has two continuous variables (R e L) e two discrete (T_s e T_h). The design variables and the weight are given by:

$$W(T_s, T_h, R, L) = 0.6224 T_s T_h R + \tag{24}$$
$$1.7781 T_h R^2 + 3.1661 T_s^2 R$$
$$g_1(T_s, R) = T_s - 0.0193R \geq 0 \tag{25}$$
$$g_2(T_h, R) = T_h - 0.00954R \geq 0 \tag{26}$$
$$g_3(R, L) = \pi R^2 L + 4/3\pi R^3 - 1296000 \geq 0 \tag{27}$$
$$g_4(L) = -L + 240 \geq 0 \tag{28}$$

where

$$0.00625 \leq T_s, T_h \leq 5 \quad \text{in constants steps of 0.0625}$$
$$10 \leq R, L \leq 200$$

E. The Cantilever Beam design

This problem has the objective of minimizing the volume V of a cantilever beam where the load $P = 50000$N. Ten design variables corresponding to the width (B_i) and the height (H_i) of the rectangular cross-section of each of the five constants steps. The figure can be seen in the [2]. The variables B_1 and H_1 are integer, B_2 and B_3 assume discrete values to be chosen from the set $[2.4, 2.6, 2.8, 3.1]$, H_2 and H_3 are discrete and chosen from the set $[45.0, 50.0, 55.0, 60.0]$ and finally B_4, H_4, B_5 and H_5 are continuous. The Young's modulus of the material is equal to 200 GPa and the variables are given in centimeters. The volume of the beam and the constraints are written as:

$$V(H_i, B_i) = 100 \sum_{i=1}^{5} H_i B_i \tag{29}$$
$$g_i(H_i, B_i) = \sigma \leq 14000 N/cm^2, \quad i = 1, \ldots, 5 \tag{30}$$
$$g_{i+5}(H_i, B_i) = \frac{H_i}{B_i} \leq 20, \quad i = 1, \ldots, 5 \tag{31}$$
$$g_{11}(H_i, B_i) = \delta \leq 2.7cm \tag{32}$$

where δ is the tip deflection of the beam in the vertical direction.

II. Structures Problems

This section describes some structural optimization problems involving trusses. As with classical engineering problems, for each one, a specific number of objective function evaluations were used. The images of the structures can be found in the references.

A. The 10-bar Truss design

This is a well-known problem [3], that aims to minimize the weight W of a 10-bar truss. The objective function is to find the set design variables, that represents the cross-sectional areas of the bars ($A_i, \quad i = 1, \ldots, 10$), that minimizes the weight of the structure, according to the following equation:

$$W(a) = \sum_{i=1}^{n} \rho A_i L_i$$

The constraints involve the stress in each member and the displacements at the nodes. The allowable stress is limited to the allowable stress $\overline{\sigma}$ is limited to $\pm 25ksi$ and the displacements are limited to $2in$ in x and y directions. The density of the material is $0.1lb/in^3$, Young's modulus is $E = 10^4 ksi$, and vertical downward loads of $P = 100kips$ are applied at nodes 2 and 4.

Two cases are analyzed. One with continuous variables and the other with discrete variables. For the fist one, the cross-sectional areas are range from $0.1in^2$ a $33.50in^2$. For the discrete case, the cross-sectional areas are chosen from a set S containing 32 options: 1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.93, 3.13, 3.38, 3.47, 3.55, 3.63, 3.88, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.97, 11.50, 13.50, 14.20, 15.50, 16.90, 18.80, 19.90, 22.00, 26.50, 30.00, 33.50.

B. The 25-bar Truss design

This is also a well known problem that aims to minimize a structure of 25 bar. The material has a density of $0.1lb/in^3$ and Young's modulus equal to $10^4 ksi$. Only the discrete case was considered, where the value of the cross-section areas are chosen from the set S with 30 options (in^2): 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.1, 3.2. The constraints require that the maximum stress in the bars remain in the range of $\pm 40ksi$ and the maximum displacements at nodes 1 and 2 be limited to $0.35in$, in both x and y directions. The loading data is listed in table I In order to maintain the symmetry of the structure, the truss bars are grouped and each group has a single area. The grouping can be seen in table II.

TABLE I: Loading data for the 25-bar truss.

Node	F_x	F_y	F_z
1	1	-10	-10
2	0	-10	-10
3	0.5	0	0
6	0.6	0	0

TABLE II: Member grouping for the 25-bar truss.

Group	Connectivities
A_1	1-2
A_2	1-4, 2-3, 1-5, 2-6
A_3	2-5, 2-4, 1-3, 1-6
A_4	3-6, 4-5
A_5	3-4, 5-6
A_6	3-10, 6-7, 4-9, 5-8
A_7	3-8, 4-7, 6-9, 5-10
A_8	3-7, 4-8, 5-9, 6-10

C. The 60-bar Truss design

Once again, the 60-bar truss problem aims to minimize the weight W of the structure. The material has Young's modulus is $E = 10^4 ksi$ and a density of $0.1lb/in^3$. The inner and outer radius is $90in$ and $100in$, respectively. There are 198 contraints, where 180 refer to allowable stress ($\rho_i = 60ksi$, $1 \leq i \leq 60$) and 18 refer to displacements given as three constraints, along the x and y directions, of magnitude 1.75 at node 4, 2.25 at node 13 and 2.75 at node 19. The load conditions is given in table III. The cross-sectional areas of the bars are assumed to be continuous and vary from $0.5in^2$ to $5in^2$. As with the 25-bar truss, a grouping is performed, which can be consulted in table IV.

TABLE III: Loading data for the 60-bar truss.

Load	Node	F_x	F_y
1	1.0	-10.0	0
	7.0	9.0	0
3	15.0	-8.0	3.0
	18.0	-8.0	3.0
6	22.0	-20.0	10.0

TABLE IV: Member grouping for the 60-bar truss.

Group	Bars	Group	Bars
A_1	49 a 60	A_{14}	25 e 37
A_2	1 e 13	A_{15}	26 e 38
A_3	2 e 14	A_{16}	27 e 39
A_4	3 e 15	A_{17}	28 e 40
A_5	4 e 16	A_{18}	29 e 41
A_6	5 e 17	A_{19}	30 e 42
A_7	6 e 18	A_{20}	31 e 43
A_8	7 e 19	A_{21}	32 e 44
A_9	8 e 20	A_{22}	33 e 45
A_{10}	9 e 21	A_{23}	34 e 46
A_{11}	10 e 22	A_{24}	35 e 47
A_{12}	11 e 23	A_{25}	36 e 48
A_{13}	12 e 24		

D. The 72-bar Truss design

It is a problem that aims to minimize the weight of a structure composed of 72 bars, where the design variables are the cross-sectional areas of the bars, which can vary from continuously de $0.1in^2$ a $5in^2$. The 72 design variables are linked in sixteen groups detailed in table VI Again, constraints involve a maximum allowed displacement value, which is 0.25 at the nodes 1 to 16 along the x and y directions, and maximum allowable stress in each bar restricted to the range of $\pm 25ksi$. The material has Young's modulus of $E = 10^4 ksi$ and a density of $0.1lb/in^3$. The load cases are defined in table V.

TABLE V: Loading data for the 72-bar truss.

Loading	Node	F_x	F_y	F_z
1	1	5.0	5.0	-5.0
	7	0.0	0.0	-5.0
2	2	0.0	0.0	-5.0
	3	0.0	0.0	-5.0
	4	0.0	0.0	-5.0

TABLE VI: Member grouping for the 72-bar truss.

Group	Bars
A_1	1, 2, 3 e 4
A_2	5, 6, 7, 8, 9, 10, 11 e 12
A_3	13, 14, 15 e 16
A_4	17 e 18
A_5	19, 20, 21 e 22
A_6	23, 24, 25, 26, 27, 28, 29 e 30
A_7	31, 32, 33 e 34
A_8	35 e 36
A_9	37, 38, 39 e 40
A_{10}	41, 42, 43, 44, 45, 46, 47 e 48
A_{11}	49, 50, 51 e 52
A_{12}	53 e 54
A_{13}	55, 56, 57 e 58
A_{14}	59, 60, 61, 62, 63, 64, 65 e 66
A_{15}	67, 68, 69 e 70
A_{16}	71 e 72

III. Detailed Analysis of CMA-ES + DEB Variants

This section presents the results of the experiments using the CMA-ES and its variant along with Deb's tournament selection. For each problem, a different number of objective function evaluations was used, based on the literature.

A. The Tension/Compression Spring design

Table VII shows that the results of all variants are very close, however, the Superlinear and Linear manage to have the best averages, standard deviations, and worst case.

TABLE VII: Results for tension/compression spring using 36000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + DEB	**0.0127**	**0.0127**	0.0128	3.20e-04	0.0144	30
CMA-ES Linear + DEB	**0.0127**	**0.0127**	**0.0127**	**1.50e-07**	**0.0127**	30
CMA-ES SL + DEB	**0.0127**	**0.0127**	**0.0127**	**2.07e-06**	**0.0127**	30

B. The Speed Reducer design

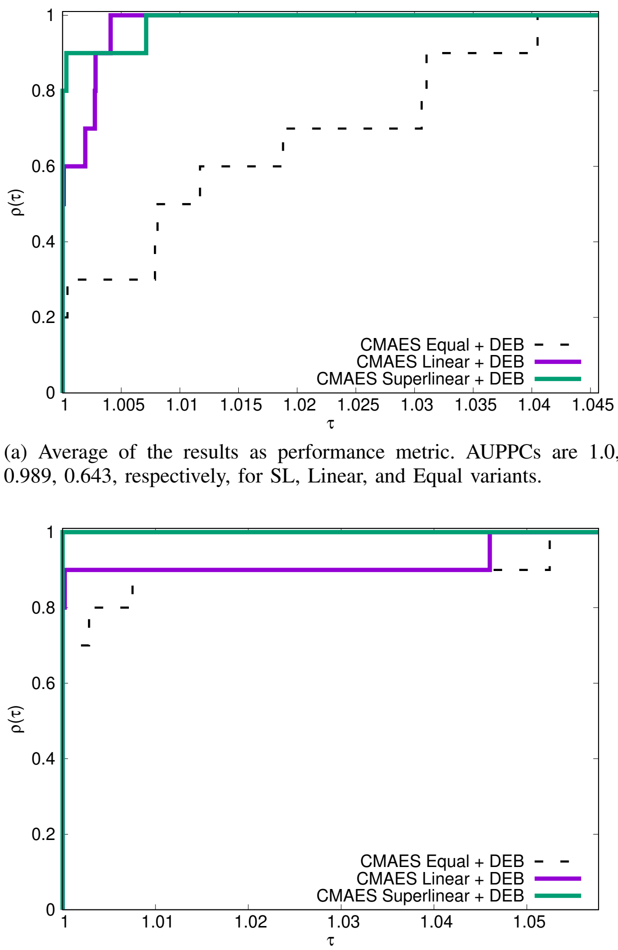
Analyzing the table VIII all variants obtain equal values for better and median, however, the variant with SL obtained the best mean and standard deviation, while the best worst case was obtained by the variant with Equal.

C. The Welded Beam design

Observing the table IX it is clear that all variants achieved very similar results, varying only in the standard deviation, and yet the difference is tiny, the order of 10^{-9}.

TABLE VIII: Results for speed reducer using 36000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + DEB	**2996.3482**	**2996.3482**	3155.1980	4.86e+02	**4664.4562**	30
CMA-ES Linear + DEB	**2996.3482**	**2996.3482**	3068.8518	3.97e+02	5171.4565	30
CMA-ES SL + DEB	**2996.3482**	**2996.3482**	**3060.2519**	**3.50e+02**	4913.4616	30



(a) Average of the results as performance metric. AUPPCs are 1.0, 0.989, 0.643, respectively, for SL, Linear, and Equal variants.

(b) Best results as performance metric. AUPPCs are 1.0, 0.912, 0.880, respectively, for SL, Linear, and Equal variants.

Fig. 1: PPs of CMA-ES with Deb's tournament selection.

TABLE IX: Results for welded beam using 320000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + DEB	**2.3811**	**2.3811**	**2.3811**	**1.56e-09**	**2.3811**	30
CMA-ES Linear + DEB	**2.3811**	**2.3811**	**2.3811**	**5.75e-15**	**2.3811**	30
CMA-ES SL + DEB	**2.3811**	**2.3811**	**2.3811**	**2.90e-13**	**2.3811**	30

D. The Pressure Vessel design

It can be seen from the table X that the variant with SL variant obtained the best results in almost all metrics used, except in the standard deviation.

E. The Cantilever Beam design

In this problem, the variant with Linear achieved the best results, since it obtained the best median and average, followed by the variant with SL, which obtained the best standard deviation and worst case. The results can be seen in table XI.

F. The 10-bar Truss design

In the continuous case of the 10-bar truss, it is noticed that variant with SL achieved the best results, which can be seen in table XII.

For the discrete case, there is a greater difference between the results, which can be seen in table XIII. The best one was the variant with Linear, which achieved the best average, standard deviation, and worst case, followed by the SL, which obtained the best median. Both achieved the best value of the objective function.

G. The 25-bar Truss design

In the 25-bar truss discrete case, there is a greater fluctuation in the results, and the variant with SL stood out since it achieved the best average, standard deviation, and worst case. The variant with Linear followed obtained the best median. Both variants achieved the best result of the objective function. The results are displayed in the XIV.

H. The 60-bar Truss design

Observing the table XV that the variant with SL was the best, which was superior in all metrics used.

I. The 72-bar Truss design

Finally, for the 72-bar truss continuous case, all variants achieved very similar results, with a small difference, the order of 10^{-5}, in the standard deviation. The results can be found in the table XVI.

IV. Detailed Analysis of CMA-ES + APM Variants

Similar to the proposal with the CMA-ES + DEB variants, this section will be presented the results from the experiments using the APM technique.

A. The Tension/Compression Spring

In this problem, the variant with SL and Linear obtained better results, due to the standard deviation and worst case. One can notice that the variant with SL did not find a feasible solution in one independent run. The results are shown in XVII.

B. The Speed Reducer design

In this problem, the variant with SL achieved better results in all metrics. The results can be found in table XVIII.

TABLE X: Results for pressure vessel using 80000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + DEB	6410.0868	6820.4101	6901.1248	**3.26e+02**	7888.3713	30
CMA-ES Linear + DEB	6370.7797	6796.0035	6840.0041	3.44e+02	**7544.4925**	30
CMA-ES SL + DEB	**6090.5262**	**6771.5969**	**6821.2264**	3.67e+02	**7544.4925**	30

TABLE XI: Results for cantilever beam using 35000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + DEB	64578.1989	69702.9750	69498.4514	2.88e+03	77259.5659	30
CMA-ES Linear + DEB	**64578.1940**	**68644.0942**	**68940.2830**	3.65e+03	83673.3723	30
CMA-ES SL + DEB	**64578.1940**	69425.6862	68962.6198	**2.44e+03**	**72831.0029**	30

TABLE XII: Results for 10-bar truss, continuous case, using 280000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + DEB	**5060.85**	**5060.85**	5065.60	7.37e+00	**5076.67**	30
CMA-ES Linear + DEB	**5060.85**	**5060.85**	5064.02	6.43e+00	**5076.67**	30
CMA-ES SL + DEB	**5060.85**	**5060.85**	**5063.49**	**5.99e+00**	**5076.67**	30

TABLE XIII: Results for 10-bar truss, discrete case, using 90000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + DEB	5532.12	5650.99	5838.69	4.30e+02	7488.02	30
CMA-ES Linear + DEB	**5490.74**	5575.55	**5611.47**	**1.48e+02**	**6258.74**	30
CMA-ES SL + DEB	**5490.74**	**5548.83**	5651.44	2.55e+02	6542.66	30

TABLE XIV: Results for 25-bar truss, discrete case, using 20000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + DEB	**499.25**	520.17	520.59	1.83e+01	564.58	30
CMA-ES Linear + DEB	**499.25**	**502.77**	507.20	8.43e+00	526.15	30
CMA-ES SL + DEB	**499.25**	504.49	**505.13**	**6.20e+00**	**525.58**	30

TABLE XV: Results for 60-bar truss, continuous case, using 12000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + DEB	310.00	313.25	316.63	7.85e+00	340.70	30
CMA-ES Linear + DEB	309.19	310.83	311.39	2.54e+00	322.26	30
CMA-ES SL + DEB	**309.12**	**310.17**	**310.79**	**2.01e+00**	**318.35**	30

C. The Welded Beam design

For this problem, table XIX shows that variants with SL and Linear obtained better results than the variant with Equal. Comparing SL and Linear variants, there is a negligible difference between the standard deviations.

D. The Pressure Vessel design

In this problem, the variant with Equal obtained the best value of median and average. One can notice that the variant with SL obtained the best standard deviation and worst case. Both achieved the best result of the objective function. The results are shown in table XX.

E. The Cantilever Beam design

Table XXI, shows that the variant with Linear, which obtained the best average, median and worst case. All three variants achieved the best value of the objective function.

F. The 10-bar Truss design

For the continuous case of this problem, the variant with SL was the best, which, despite having achieved the best result of the objective function, median and worst case along with the others, obtained the best average and standard deviation. The results are presented in table XXII.

In the discrete case, the variant with SL achieved the best result in all metrics. The results are presented in table XXIII.

G. The 25-bar Truss design

For this problem, the variant with SL was the best one, since it achieved better results in three metrics: average, standard deviation, and worst case. The results are shown in table XXIV.

H. The 60-bar Truss design

For this problem, the variants with Equal and SL achieved the best value of the objective function, however, the variant with SL was superior, since it obtained the better results in the other metrics. The results can be seen in table XXV.

I. The 72-bar Truss design

In this problem, table XXVI shows that the performance of all three variants was very similar. The variants with SL and Linear were better because both obtained the best worst case.

V. Design Variables

This section presents the design variables for each problem.

References

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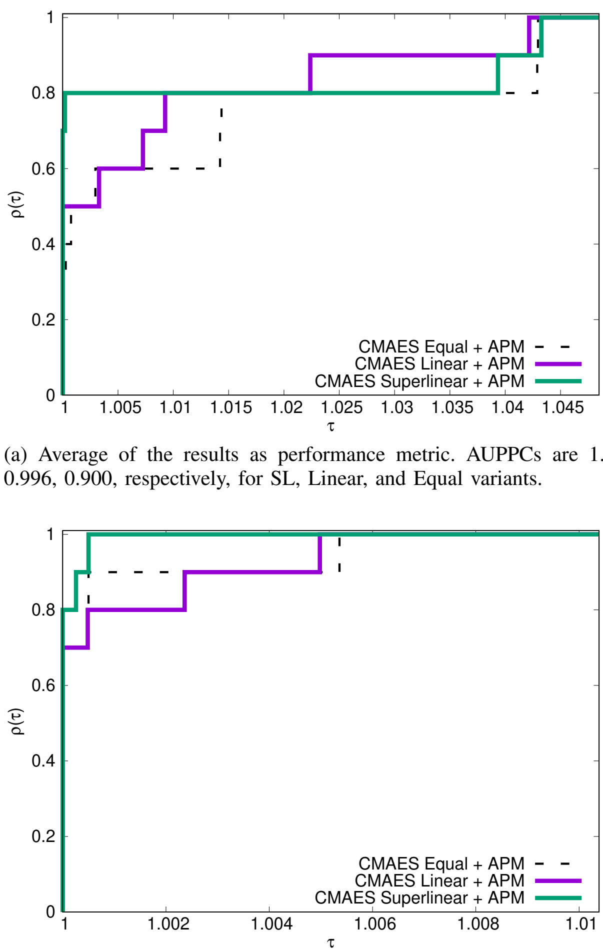
TABLE XVI: Results for 72-bar truss, continuous case, using 35000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + DEB	**379.61**	**379.61**	**379.61**	**4.70e-05**	**379.61**	30
CMA-ES Linear + DEB	**379.61**	**379.61**	**379.61**	**2.38e-06**	**379.61**	30
CMA-ES SL + DEB	**379.61**	**379.61**	**379.61**	**5.07e-06**	**379.61**	30



(a) Average of the results as performance metric. AUPPCs are 1.0, 0.996, 0.900, respectively, for SL, Linear, and Equal variants.

(b) Best results as performance metric. AUPPCs are 1.0, 0.903, 0.866, respectively, for SL, Equal, and Linear variants.

Fig. 2: PPs of CMA-ES with APM.

TABLE XVII: Results for tension/compression spring using 36000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + APM	**0.0127**	**0.0127**	**0.0127**	1.03e-04	0.0130	30
CMA-ES Linear + APM	**0.0127**	**0.0127**	**0.0127**	**1.24e-05**	**0.0127**	30
CMA-ES SL + APM	**0.0127**	**0.0127**	**0.0127**	**4.87e-07**	**0.0127**	29

TABLE XVIII: Results for speed reducer using 36000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + APM	**2996.3482**	**2996.3482**	3162.4248	4.95e+02	5171.4565	30
CMA-ES Linear + APM	**2996.3482**	**2996.3482**	3060.2519	3.50e+02	4913.4616	30
CMA-ES SL + APM	**2996.3482**	**2996.3482**	**3032.1568**	**1.09e+02**	**3363.8734**	30

TABLE XIX: Results for welded beam using 320000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + APM	**2.3811**	**2.3811**	2.3829	9.58e-03	2.4336	30
CMA-ES Linear + APM	**2.3811**	**2.3811**	**2.3811**	**4.28e-11**	**2.3811**	30
CMA-ES SL + APM	**2.3811**	**2.3811**	**2.3811**	**4.09e-13**	**2.3811**	30

TABLE XX: Results for pressure vessel using 80000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + APM	**6059.7143**	**6796.8871**	**6917.5290**	9.01e+02	10325.0567	30
CMA-ES Linear + APM	6089.8929	6821.1634	7209.2307	1.29e+03	11960.9106	30
CMA-ES SL + APM	**6059.7143**	7122.6938	7189.7360	**6.75e+02**	**8952.7206**	30

TABLE XXI: Results for cantilever beam using 35000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + APM	**64578.1940**	70210.6510	70767.9537	**4.55e+03**	82808.8314	30
CMA-ES Linear + APM	**64578.1940**	**68992.7800**	**70559.2827**	5.66e+03	**82596.3676**	30
CMA-ES SL + APM	**64578.1940**	73218.5084	73612.3966	6.64e+03	92056.9735	30

TABLE XXII: Results for 10-bar truss, continuous case, using 280000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + APM	**5060.85**	**5060.85**	5063.84	6.15e+00	**5076.67**	30
CMA-ES Linear + APM	**5060.85**	**5060.85**	**5062.44**	**4.83e+00**	**5076.67**	30
CMA-ES SL + APM	**5060.85**	**5060.85**	5063.49	5.99e+00	**5076.67**	30

TABLE XXIII: Results for 10-bar truss, discrete case, using 90000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + APM	5527.84	5700.19	5838.91	3.16e+02	6551.16	30
CMA-ES Linear + APM	5511.36	5624.41	5724.06	2.79e+02	6621.31	30
CMA-ES SL + APM	**5498.37**	**5573.37**	**5598.75**	**7.92e+01**	**5785.60**	30

TABLE XXIV: Results for 25-bar truss, discrete case, using 20000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + APM	499.70	509.77	511.59	1.16e+01	549.23	30
CMA-ES Linear + APM	**499.45**	**502.86**	508.01	9.86e+00	534.74	30
CMA-ES SL + APM	499.70	503.09	**504.35**	**4.99e+00**	**521.80**	30

TABLE XXV: Results for 60-bar truss, continuous case, using 12000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + APM	**309.11**	312.65	314.48	5.52e+00	336.15	30
CMA-ES Linear + APM	309.26	310.45	311.09	1.95e+00	317.96	30
CMA-ES SL + APM	309.19	**309.96**	**310.07**	**6.95e-01**	**311.87**	30

TABLE XXVI: Results for 72-bar truss, continuous case, using 35000 objective function evaluations.

	Best	Median	Average	St.Dev	Worst	fr
CMA-ES Equal + APM	**379.61**	**379.61**	**379.61**	**3.41e-04**	379.62	30
CMA-ES Linear + APM	**379.61**	**379.61**	**379.61**	**1.63e-05**	**379.61**	30
CMA-ES SL + APM	**379.61**	**379.61**	**379.61**	**3.20e-05**	**379.61**	30

TABLE XXVII: Final design variables and objective function for the best solutions found for the tension/compression spring.

	CMA-ES SL + APM	CMA-ES SL + DEB	GA + APM	PSO + APM
x_1	10.7679	10.2354	11.781000	11.327051
x_2	0.3565	0.3566	0.348607	0.356086
x_3	0.0517	0.0517	0.051350	0.051663
Obj. Func	0.0127	0.0127	0.012667	0.012666

TABLE XXVIII: Final design variables and objective function for the best solutions found for the speed reducer.

	CMA-ES SL + APM	CMA-ES SL + DEB	GA + APM	PSO + APM
x_1	3.5	3.5	3.5	3.500003
x_2	0.7	0.7	0.7	0.700000
x_3	17	17	17	17.000000
x_4	7.3	7.3	7.3	7.300185
x_5	7.8	7.8	7.800002	7.800145
x_6	3.3489	3.3463	3.350215	3.350221
x_7	5.2864	5.2859	5.286684	5.286684
Obj. Func	2996.3482	2996.3482	2996.348509	2996.356314

TABLE XXIX: Final design variables and objective function for the best solutions found for the welded beam.

	CMA-ES SL + APM	CMA-ES SL + DEB	GA + APM	PSO + APM
x_1	0.2444	0.2444	0.244369	0.244367
x_2	6.2186	6.2186	6.218615	6.218653
x_3	8.2915	8.2915	8.291472	8.291492
x_4	0.2444	0.2444	0.244369	0.244370
Obj. Func	2.3811	2.3811	2.381134	2.381146

TABLE XXX: Final design variables and objective function for the best solutions found for the pressure vessel.

	CMA-ES SL + APM	CMA-ES SL + DEB	GA + APM	PSO + APM
x_1	0.8355	0.9037	-	13.292847
x_2	0.4653	0.4137	-	6.733947
x_3	42.0984	45.3368	-	42.098445
x_4	11.9886	10	-	?
Obj. Func	6059.7143	6090.5262	6059.7305	6059.714360

TABLE XXXI: Final design variables and objective function for the best solutions found for the cantilever beam.

	CMA-ES SL + APM	CMA-ES SL + DEB	GA + APM	PSO + APM
x_1	2.5014	2.5001	-	3.242513
x_2	46.5917	50.3331	-	60.721308
x_3	2.9501	2.95	-	2.854063
x_4	52.5001	52.5149	-	1.763154
x_5	2.532	2.504	-	1.350839
x_6	47.5069	47.5001	-	1.332820
x_7	2.2046	2.2046	-	2.280889
x_8	37.7582	44.0911	-	45.617713
x_9	1.7498	1.7498	-	1.749767
x_{10}	33.622	34.4687	-	34.995138
Obj. Func	64578.194	64578.194	64599.9803	64578.225361

TABLE XXXII: Final design variables and objective function for the best solutions found for the 10-bar truss, continuous case.

	CMA-ES SL + APM	CMA-ES SL + DEB	PSO + APM
1	30.5218	30.5218	30.633433
2	0.1	0.1	0.100000
3	23.1999	23.1999	23.041744
4	14.7332	14.7332	15.197787
5	0.1	0.1	0.100000
6	0.1	0.1	0.554800
7	7.4572	7.4572	7.464924
8	20.9506	20.9506	20.970234
9	20.8359	20.8359	21.636972
10	0.1	0.1	0.100000
Obj. Func	5060.8537	5060.8537	5060.947307

TABLE XXXIII: Final design variables and objective function for the best solutions found for the 10-bar truss, discrete case.

	CMA-ES SL + APM	CMA-ES SL + DEB	PSO + APM
1	23.6337	22.7698	30.999442
2	0.1	0.1	0.214494
3	19.3194	19.9722	28.288087
4	10.928	5.3114	24.951834
5	0.1	0.1	0.001618
6	0.1	0.1	0.440247
7	6.7996	5.4044	20.079916
8	16.0853	7.9172	27.867945
9	19.4257	8.2391	28.27785
10	0.1	0.1	0.1
Obj. Func	5498.3746	5490.7379	5509.717373

TABLE XXXIV: Final design variables and objective function for the best solutions found for the 25-bar truss.

	CMA-ES SL + APM	CMA-ES SL + DEB	PSO + APM
A_1	0.1	0.1	0.100000
A_2	0.3701	0.3185	0.300000
A_3	2.506	2.1695	3.400000
A_4	0.1	0.1	0.100000
A_5	0.4128	0.3773	2.100000
A_6	0.8529	0.8554	1.000000
A_7	0.2908	0.3631	0.500000
A_8	3.1546	3.0597	3.400000
Obj. Func	499.6997	499.2508	484.854179

TABLE XXXV: Final design variables and objective function for the best solutions found for the 60-bar truss.

	CMA-ES SL + APM	CMA-ES SL + DEB	PSO + APM
A_1	1.1337	1.1368	1.168084
A_2	1.9853	1.9895	2.102395
A_3	0.5	0.5	0.503932
A_4	1.5281	1.4177	2.000652
A_5	1.4458	1.451	1.932965
A_6	0.5302	0.5348	0.573366
A_7	1.8051	1.8015	1.893816
A_8	1.7643	1.7525	1.925602
A_9	0.9556	0.9262	1.060730
A_{10}	1.5817	1.5996	1.764067
A_{11}	1.6358	1.6006	1.615935
A_{12}	0.5	0.5	0.506942
A_{13}	1.9954	2.0044	2.129686
A_{14}	1.2303	1.219	1.264224
A_{15}	1.0018	1.0042	1.154981
A_{16}	0.567	0.5687	0.508007
A_{17}	0.6296	0.6342	0.709542
A_{18}	0.9287	0.9273	0.989056
A_{19}	1.1107	1.0998	1.140337
A_{20}	1.1291	1.137	1.139156
A_{21}	0.9433	0.9261	1.072108
A_{22}	1.0409	1.0395	1.077415
A_{23}	0.5819	0.5	0.623711
A_{24}	1.0019	1.0058	1.090640
A_{25}	1.2342	1.2391	1.259983
Obj. Func	309.1862	309.1212	311.717427

TABLE XXXVI: Final design variables and objective function for the best solutions found for the 72-bar truss.

	CMA-ES SL + APM	CMA-ES SL + DEB	PSO + APM
A_1	0.1565	0.1565	0.1568
A_2	0.5454	0.5455	0.544394
A_3	0.4103	0.4102	0.415662
A_4	0.5696	0.5696	0.553284
A_5	0.5225	0.5234	0.514888
A_6	0.5171	0.517	0.517994
A_7	0.1	0.1	0.1
A_8	0.1	0.1	0.100405
A_9	1.2676	1.2681	1.266802
A_{10}	0.5116	0.5116	0.512055
A_{11}	0.1	0.1	0.10064
A_{12}	0.1	0.1	0.1
A_{13}	1.8859	1.8858	1.893667
A_{14}	0.5122	0.5122	0.515792
A_{15}	0.1	0.1	0.100124
A_{16}	0.1	0.1	0.100031
Obj. Func	379.6148	379.6148	379.652909