I a) $\sigma^1 = \text{sign}(w \cdot u) =$

$= \text{sign}(1.1 + 1.1 + 1.1) = +1$

no error occurred

$\sigma^2 = \text{sign}(w \cdot u) = \text{sign}(1.1 + 1.2 + 1.2) = +1$

no error occurred

$\sigma^3 = \text{sign}(w \cdot u) = \text{sign}(1.1 + 1.0 + 1(-1)) = +1$

error occurred.

$w = [1\ 1\ 1]^T + (-1-1)(1\ 0\ -1)^T =$

$= (-1\ 1\ 3)$

$\sigma^4 = \text{sign}(w u) = \text{sign}(-1.1 + 1(-1) + 3.0) = -1$

concluded the first epoch without convergence

$\sigma^1 = \text{sign}(w \cdot u) = \text{sign}(-1.1 + 1.1 + 3.1) = +1$

no error occurred

$\sigma^2 = \text{sign}(w \cdot u) = \text{sign}(-1.1 + 1.2 + 3.2) = +1$

no error occurred

$\sigma^3 = \text{sign}(w \cdot u) = \text{sign}(-1.1 + 1.0 + 3(-1)) = -1$

no error occurred

$\sigma^4 = \text{sign}(w \cdot u) = \text{sign}(-1.1 + 1.(-1) + 3.0) = -1$

no error occurred

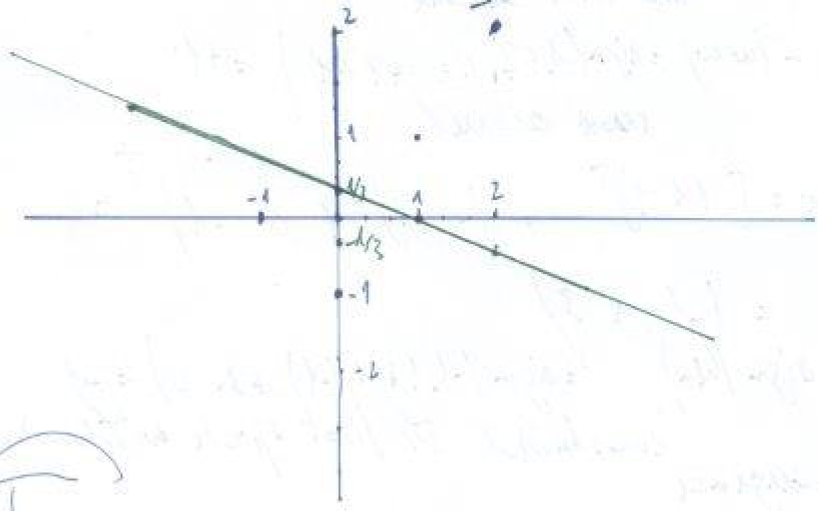
convergence after 2° epoch

(5)

$$w_0 u_0 + w_1 u_1 + w_2 u_2 = 0$$

$$-1 + u_1 + 3u_2 = 0$$

$$u_2 = \frac{1 - u_1}{3}$$




II

~~Gain(F1) =~~

$$I(C) = -\frac{3}{5}\log\frac{3}{5} + -\frac{2}{5}\log\frac{2}{5} = 1{,}58$$

~~$G(F_1) = I(C) - E(F_1) =$~~

~~$= 1{,}58 - \frac{3}{5}\left[-\frac{2}{3}\log\frac{2}{3} - \frac{1}{3}\log\frac{1}{3}\right] - \frac{2}{5}\left[1\log\left(\frac{1}{2}\right)\right]$~~

$= 0{,}98$ ~~1,08~~ 0,98

$G(F_2) = I(C) - E(F_2)$

$= 1{,}58 - \frac{2}{5}\left(\log - \frac{1}{2}\log\frac{1}{2}\right) - \frac{3}{5}\left(-\log\frac{1}{3}\right)$

$= 0{,}5938$

~~$G(F_3) = I(C) - E(F_3) =$~~

$G(F_1) = I(C) - E(F_1) = 1{,}58 - \left[\frac{3}{5}\left(-\frac{2}{3}\log\left(\frac{2}{3}\right) - \frac{1}{3}\log\left(\frac{1}{3}\right)\right) + \frac{2}{5}\left(-1\log\frac{1}{2}\right)\right] =$

$= 0{,}98$

$G(F_3) = I(C) - E(F_3) = 1{,}58 - \left[\frac{2}{5}\left(\log\frac{1}{2}\right) + \frac{3}{5}\left(\log\frac{1}{3}\right)\right]$

$= 0{,}13$

$G(F_4) = I(C) - E(F_4) = 1{,}58 - \left[\frac{1}{5}\log 1 + \frac{2}{5}\log(1) + \frac{2}{5}\left(\frac{1}{2}\log\frac{1}{2}\right)\right] =$

$G(F_4) = 1{,}08$

~~First the root~~ $F_4$ in the root of the Decision Tree

Given that is the biggest Gain

3)

$I(C) = -\frac{1}{2}\log\frac{1}{2}\log\frac{1}{2}) = 1$

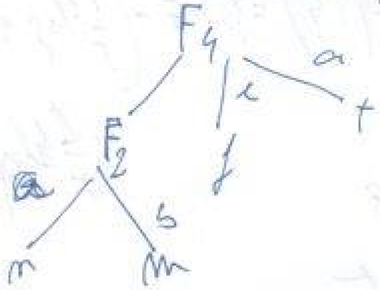
$E(F_1) = -\frac{1}{2}\log(\frac{1}{2}\log\frac{1}{2} = 1$

$G(F_1) = I(C) - E(F_1) = 0$

$G(F_2) = I(C) - E(F_2) = 1 - \frac{1}{2}(\log - \frac{1}{2}\log 1) - \frac{1}{2}\log 1 = 1$

$G(F_3) = I(C) - E(F_3) = 1 - \frac{1}{2}(-\log 1) - \frac{1}{2}(-\log 1)$

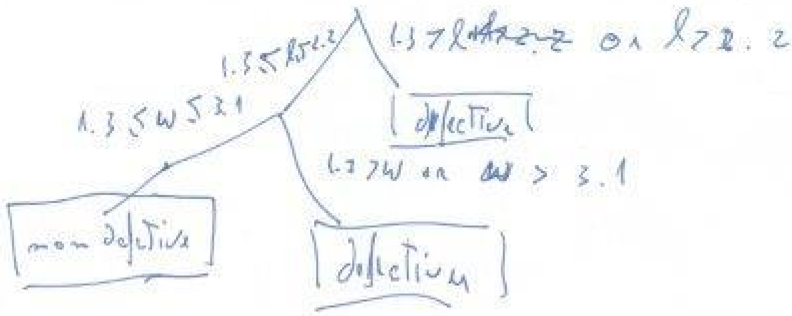
$= 1$



III (b) Both

A single perceptron by design can only draw a linear separation line. More complex rules need more layers. In this case, at least 2 layers would be needed.

And also Decision is usefull to classify the pieces as defective and non defective pieces



IV)

a) Querry vector: $x = [5 ; 10]^T$

$p(C=A) = \frac{1}{2} = 0{,}5$

$p(C=B) = \frac{1}{2} = 0{,}5$

| | $p(x_1 \mid C=A)$ | $p(x_1 \mid C=B)$ |
|---|---|---|
| $\mu$ | 3,75 | 42,5 |
| $\sigma$ | 4,7871 | 9,5743 |

$x_1 | C=A$: $\mu = \frac{0+0+10+5}{4} = \frac{15}{4} \approx 3{,}75$;

$\sigma^2 = \frac{(0-3{,}75)^2 + (0-3{,}75)^2 + (10-3{,}75)^2 + (5-3{,}75)^2}{4-1} \approx \frac{68{,}75}{3} \approx$

$= 22{,}9167$; $\sigma = \sqrt{22{,}9167} \approx 4{,}7871$

$x_1 | C=B$: $\mu = \frac{30+40+50+50}{4} = \frac{170}{4} = 42{,}5$;

$\sigma^2 = \frac{(30-42{,}5)^2 + (40-42{,}5)^2 + (50-42{,}5)^2 + (50-42{,}5)^2}{4-1} =$

$= \frac{275}{3} \approx 91{,}6667$; $\sigma = \sqrt{91{,}6667} \approx 9{,}5743$

$x_2 | C=A$: $\mu = \frac{10+20+10+20}{4} = \frac{60}{4} = 15$

$\sigma^2 = \frac{(10-15)^2 + (20-15)^2 + (10-15)^2 + (20-15)^2}{4-1} = \frac{100}{3} \approx 33{,}3333$

$\sigma = \sqrt{33{,}3333} \approx 5{,}7735$

$x_2 | C=B$: $\mu = \frac{30+40+30+50}{4} = \frac{150}{4} = 37{,}5$

$\sigma^2 = \frac{(30-37{,}5)^2 + (40-37{,}5)^2 + (30-37{,}5)^2 + (50-37{,}5)^2}{4-1} = \frac{275}{3} \approx$

$\approx 91{,}6667$; $\sigma = \sqrt{91{,}6667} \approx 9{,}5743$

| | $p(x_2 \mid C=A)$ | $p(x_2 \mid C=B)$ |
|---|---|---|
| $\mu$ | 15 | 37,5 |
| $\sigma$ | 5,7735 | 9,5743 |

$p(C=A \mid x_1=5; x_2=10) = \frac{p(C=A)\, p(x_1=5 \mid C=A)\, p(x_2=10 \mid C=A)}{p(x_1=5; x_2=10)} =$

$$= \frac{p(C=A)\, N(5 \mid \mu=3{,}75; \sigma=4{,}7871)\, N(10 \mid \mu=15; \sigma=5{,}7735)}{p(x_1=5; x_2=10)};$$

$$N(5 \mid \mu=3{,}75; \sigma=4{,}7871) = \frac{1}{4{,}7871\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{5-3{,}75}{4{,}7871}\right)^2} \approx$$

$$\approx 0{,}0805438$$

$$N(10 \mid \mu=15; \sigma=5{,}7735) = \frac{1}{5{,}7735\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{10-15}{5{,}7735}\right)^2} \approx$$

$$\approx 0{,}0474909$$

$$p(C=A \mid x_1=5; x_2=10) = \frac{0{,}5 \times 0{,}0805438 \times 0{,}0474909}{p(x_1=5; x_2=10)} \approx$$

$$\approx \frac{0{,}0019}{p(x_1=5; x_2=10)} = 0{,}0019$$

$$p(C=B \mid x_1=5; x_2=10) = \frac{p(C=B)\, p(x_1=5 \mid C=B)\, p(x_2=10 \mid C=B)}{p(x_1=5; x_2=10)} =$$

$$= \frac{p(C=B)\, N(5 \mid \mu=42{,}5; \sigma=9{,}5743)\, N(10 \mid \mu=37{,}5; \sigma=9{,}5743)}{p(x_1=5; x_2=10)}$$

$$N(5 \mid \mu=42{,}5; \sigma=9{,}5743) = \frac{1}{9{,}5743\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{5-42{,}5}{9{,}5743}\right)^2} \approx$$

$$\approx 0{,}0000194351$$

$$N(10 \mid \mu=37{,}5; \sigma=9{,}5743) = \frac{1}{9{,}5743\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{10-37{,}5}{9{,}5743}\right)^2} \approx$$

$$\approx 0{,}000673518$$

$$p(C=B \mid x_1=5; x_2=10) \approx \frac{0{,}5 \times 0{,}0000194351 \times 0{,}000673518}{p(x_1=5; x_2=10)} \approx$$

$$\approx \frac{0{,}000000006545}{p(x_1=5; x_2=10)} = 6{,}5449 \times 10^{-9}$$

$$p(C=A \mid x_1=5; x_2=10) \approx 0{,}0019 > 6{,}5449 \times 10^{-9} =$$

$$= p(C=B \mid x_1=5; x_2=10)$$

So, it chooses Class A

b) C=A: $\sigma^2_{x_1} = \frac{1}{n-1} \sum_{i=1}^{n} (x_{1i} - \mu_{x_1})^2 \Leftrightarrow$

$\Leftrightarrow \sigma^2_{x_1} = \frac{(0-3,75)^2 + (0-3,75)^2 + (10-3,75)^2 + (5-3,75)^2}{4-1} = \frac{68,75}{3} \approx$

$\approx 22,92$

$\sigma(x_1, x_2) = \frac{1}{n-1} \sum_{i=1}^{n} (x_{1i} - \mu_{x_1})(x_{2i} - \mu_{x_2}) \Leftrightarrow$

$\Leftrightarrow \sigma(x_1, x_2) = [(0-3,75)(10-15) + (0-3,75)(20-15) + (10-3,75)(10-15) +$
$+ (5-3,75)(20-15)] \times \frac{1}{4-1} \approx -8,3333$

$\sigma^2_{x_2} = \frac{(10-15)^2 + (20-15)^2 + (10-15)^2 + (20-15)^2}{4-1} \approx 33,3333$

C=B: $\sigma^2_{x_1} = \frac{(30-42,5)^2 + (40-42,5)^2 + (50-42,5)^2 + (50-42,5)^2}{4-1} \approx$

$\approx 91,6667$

$\sigma(x_1, x_2) = [(30-42,5)(30-37,5) + (40-42,5)(40-37,5) +$
$+ (50-42,5)(30-37,5) + (50-42,5)(50-37,5)] \times \frac{1}{4-1} \approx 41,6667$

$\sigma^2_{x_2} = \frac{(30-37,5)^2 + (40-37,5)^2 + (30-37,5)^2 + (50-37,5)^2}{4-1} =$

$= \frac{275}{3} \approx 91,6667$

| | $p(x_1, x_2 \mid C=A)$ | $p(x_1, x_2 \mid C=B)$ |
|---|---|---|
| $\mu$ | $\begin{bmatrix} 3,75 \\ 15 \end{bmatrix}$ | $\begin{bmatrix} 42,5 \\ 37,5 \end{bmatrix}$ |
| $\Sigma$ | $\begin{bmatrix} 22,92 & -8,3333 \\ -8,3333 & 33,3333 \end{bmatrix}$ | $\begin{bmatrix} 91,6667 & 41,6667 \\ 41,6667 & 91,6667 \end{bmatrix}$ |

C=A:

$p(x \mid \mu; \Sigma) = N(x \mid \mu; \Sigma) = \frac{1}{(2\pi)^{D/2}} \cdot \frac{1}{|\Sigma|^{1/2}} e^{[-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)]}$

$D=2$; $|\Sigma| = \begin{vmatrix} 22,92 & -8,3333 \\ -8,3333 & 33,3333 \end{vmatrix} \approx 694,5553$

$$\Sigma^{-1} \approx \begin{bmatrix} 0,0480 & 0,0120 \\ 0,0120 & 0,0330 \end{bmatrix}; \quad N\left(\begin{bmatrix}5\\10\end{bmatrix} \middle| \mu = \begin{bmatrix}3,75\\15\end{bmatrix}; \Sigma = \begin{bmatrix} 22,92 & -8,3333 \\ -8,3333 & 33,3333\end{bmatrix}\right)$$

$$(x-\mu)^T = \left(\begin{bmatrix}5\\10\end{bmatrix} - \begin{bmatrix}3,75\\15\end{bmatrix}\right)^T = \begin{bmatrix}1,25\\-5\end{bmatrix}^T = [1,25 \quad -5];$$

$$N\left(\begin{bmatrix}5\\10\end{bmatrix} \middle| \mu = \begin{bmatrix}3,75\\15\end{bmatrix}; \Sigma = \begin{bmatrix} 22,92 & -8,3333 \\ -8,3333 & 33,3333\end{bmatrix}\right) = \frac{e^{-\frac{1}{2}[1,25 \quad -5]\begin{bmatrix} 0,0480 & 0,0120 \\ 0,0120 & 0,0330 \end{bmatrix}\begin{bmatrix}1,25\\-5\end{bmatrix}}}{(2\pi)^{2/2} \times 694,5553^{1/2}} =$$

$$= \frac{1}{2\pi\sqrt{694,5553}} e^{-\frac{1}{2}[1,25 \quad -5]\begin{bmatrix}0\\-0,15\end{bmatrix}} \approx \frac{1}{165,5897} e^{-\frac{1}{2}[0,75]} \approx$$

$$\approx \frac{e^{-0,375}}{165,5897} \approx 0,0042; \quad p(C=A \mid x_1=5; x_2=10) =$$

$$= \frac{p(C=A)\, p(x_1=5; x_2=10 \mid C=A)}{p(x_1=5; x_2=10)} =$$

$$= \frac{p(C=A)\, N\left(\begin{bmatrix}5\\10\end{bmatrix} \middle| \mu = \begin{bmatrix}3,75\\15\end{bmatrix}; \Sigma = \begin{bmatrix} 22,92 & -8,3333 \\ -8,3333 & 33,3333\end{bmatrix}\right)}{p(x_1=5; x_2=10)} \approx \frac{0,5 \times 0,0042}{p(x_1=5; x_2=10)} =$$

$$= 0,5 \times 0,0042 = 0,0021;$$

C=B: $|\Sigma| = \begin{vmatrix} 91,6667 & 41,6667 \\ 41,6667 & 91,6667 \end{vmatrix} \approx 6666,67;$

$$(x-\mu)^T = \left(\begin{bmatrix}5\\10\end{bmatrix} - \begin{bmatrix}42,5\\37,5\end{bmatrix}\right)^T = \begin{bmatrix}-37,5\\-27,5\end{bmatrix}^T = [-37,5 \quad -27,5];$$

$$\Sigma^{-1} = \begin{bmatrix} 91,6667 & 41,6667 \\ 41,6667 & 91,6667 \end{bmatrix}^{-1} \approx \begin{bmatrix} 0,0137 & -0,0062 \\ -0,0062 & 0,0137 \end{bmatrix};$$

$$N\left(\begin{bmatrix}5\\10\end{bmatrix} \middle| \mu = \begin{bmatrix}42,5\\37,5\end{bmatrix}; \Sigma = \begin{bmatrix} 91,6667 & 41,6667 \\ 41,6667 & 91,6667\end{bmatrix}\right) =$$

$$= \frac{e^{-\frac{1}{2}[-37,5 \quad -27,5]\begin{bmatrix} 0,0137 & -0,0062 \\ -0,0062 & 0,0137 \end{bmatrix}\begin{bmatrix}-37,5\\-27,5\end{bmatrix}}}{2\pi\sqrt{6666,67}} \approx \frac{e^{-\frac{1}{2}[-37,5 \quad -27,5]\begin{bmatrix}-0,3432\\-0,1442\end{bmatrix}}}{513,0201} \approx$$

$$\approx \frac{e^{-\frac{1}{2}[16,83875]}}{513,0201} \approx \frac{e^{-8,419375}}{513,0201} \approx 4,2991 \times 10^{-7}$$

$$p(C=B \mid x_1=5; x_2=10) = \frac{p(C=B)\, N\left(\begin{bmatrix}5\\10\end{bmatrix} \middle| \mu = \begin{bmatrix}42,5\\37,5\end{bmatrix}; \Sigma = \begin{bmatrix} 91,6667 & 41,6667 \\ 41,6667 & 91,6667\end{bmatrix}\right)}{p(x_1=5; x_2=10)} =$$

$$= \frac{0,5 \times 4,2991 \times 10^{-7}}{p(x_1=5; x_2=10)} = 0,5 \times 4,2991 \times 10^{-7} \approx 2,14955 \times 10^{-7}$$

$$P(C=A \mid x_1=5; x_2=10) = \boxed{0{,}0021 > 2{,}14955 \times 10^{-7}} = P(C=B \mid x_1=5; x_2=10)$$

$\Downarrow$

So, it classifies as $\boxed{\text{Class A}}$