Homework 4 – POMDP

Group 27

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a)

$$\boldsymbol{w}^* = \underset{\boldsymbol{w}\in\mathbb{R}^{P+1}}{\operatorname{argmin}} \{-\log \mathbb{P}[\mathcal{D}; \boldsymbol{w}]\} = \underset{\boldsymbol{w}\in\mathbb{R}^{P+1}}{\operatorname{argmin}} \left\{-\log \prod_{n=1}^{N} \mathbb{P}[a_n \mid \boldsymbol{x}_n; \boldsymbol{w}]\right\}. \qquad (2)$$

$$\ell(\mathcal{D}; \pi) \stackrel{\text{def}}{=} \sum_{n=1}^{N} [a_n \log \pi(1 \mid \boldsymbol{x}_n; \boldsymbol{w}) + (1 - a_n) \log(1 - \pi(1 \mid \boldsymbol{x}_n; \boldsymbol{w}))].$$

$$L(\mathcal{D}) = \prod_{n=1}^{N} \pi(a_n \mid x_n)$$

$$\log L(\mathcal{D}) = \sum_{n=1}^{N} (a_n \log \pi(1 \mid x_n) + (1 - a_n) \log(1 - \pi(1 \mid x_n)))$$

Therefore:

$$\omega^* = \underset{\omega\in\mathbb{R}^{P+1}}{\arg\min}\{-logL(D;\pi)\} = \underset{\omega\in\mathbb{R}^{P+1}}{\arg\min}\{-l(D;\pi)\}$$

$$\omega^* = \underset{\omega\in\mathbb{R}^{P+1}}{\arg\max}\, l(D;\pi)$$

b)

$$l(D;\pi)' = \sum_{n=1}^{N} [a_n (\log \pi (1|x_n; w))' + (1-a_n) \log (1-\pi(1|x_n; w))]$$

$$\log \pi (1|x_n; w)' = \frac{x_n}{e^{x_n w}+1}$$

$$\log (1-\pi(1|x_n; w))' = -\frac{x_n e^{x_n w}}{e^{x_n w}+1}$$

$$l(D;\pi)' = \sum_{n=1}^{N} \left[a_n \left(\frac{x_n}{e^{w x_n}+1}\right) - \frac{x_n e^{x_n w}}{e^{x_n w}+1} + a_n \left(\frac{x_n e^{x_n w}}{e^{x_n w}+1}\right)\right]$$

$$l(D;\pi)' = \sum_{n=1}^{N} \left[x_n \left(a_n \overbrace{\left(\frac{1}{e^{x_n w}+1} + \frac{e^{x_n w}}{e^{x_n w}+1}\right)}^{1} - \frac{e^{x_n w}}{e^{x_n w}+1}\right)\right]$$

$$l(D;\pi)' = \sum_{n=1}^{N} \left[x_n \left(a_n - \left(\frac{e^{x_n w}}{e^{x_n w}+1} + 1 - 1\right)\right)\right]$$

$$l(D;\pi)' = \sum_{n=1}^{N} \left[x_n \left(a_n + \left(\frac{e^{x_n w}}{e^{x_n w}+1} + \frac{1}{e^{x_n w}+1} - \frac{1}{e^{x_n w}+1}\right)\right)\right]$$

$$l(D;\pi)' = \sum_{n=1}^{N} \left[x_n \left(a_n - \left(+1 - \frac{1}{e^{x_n w}+1}\right)\right)\right]$$

$$l(D;\pi)' = \sum_{n=1}^{N} [x_n (a_n - \pi(1|x_n; w))]$$

c)

$$\frac{\partial}{\partial w}\left[\sum_{n=1}^{N} x_n\left(a_n - \pi(1|x_n; w)\right)\right] =$$

$$= \sum_{n=1}^{N} \frac{\partial}{\partial w}\left(x_n\left(a_n - \pi(1|x_n; w)\right)\right) =$$

$$= -\sum_{n=1}^{N} x_n \frac{\partial}{\partial w}\left(\pi(1|x_n; w)\right) =$$

$$= -\sum_{n=1}^{N} x_n \left[\frac{x_n^T e^{wx}}{(1+e^{wx})^2}\right] =$$

$$= -\sum_{n=1}^{N} x_n x_n^T \pi(1|x_n; w) \frac{e^{wx}}{1+e^{wx}} =$$

$$= -\sum_{n=1}^{N} x_n x_n^T \pi(1|x_n; w) \frac{1+e^{wx}-1}{1+e^{wx}} =$$

$$= \underbrace{-\sum_{n=1}^{N} x_n x_n^T \pi(1|x_n; w)\left(1-\pi(1|x_n; w)\right)}_{H}$$