INSTITUTO SUPERIOR TÉCNICO ADI – Aprendizagem e Decisão Inteligente MEIC 2016-2017 – 2º Semestre Grupo 27 – Alameda

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Homework 2 - Markov Decision Problems

Exercise 1

NOTA: (1,2) significa "lobo na posição 1, lebre na posição 2", etc.

b)

$$P_{up} = P_{down} =$$

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