UNIVERSIDADE DE AVEIRO

DEPARTAMENTO DE ELECTRÓNICA, TELECOMUNICAÇÕES E INFORMÁTICA

Information and Coding (2025/26)

Problems

1. A signal has values in $[0, 1]$. We want to represente this signal with only eight levels, using quantization. Calculate the maximum error that is introduced by this operation and indicate the quantization levels.

2. Determine the number of bits needed to represent, without compression, 10 minutes of an audio signal, sampled at 16 kHz and with 8 bits per sample.

3. Determine the number of bits needed to represent, without compression, an image with 512×512 pixels and 256 gray levels.

4. Determine the number of non-compressed video minutes that can be stored in a DVD9 ($\approx$ 8.5 Gbyte of capacity). Consider RGB (24 bits per pixel) videos, with 720×576 pixels of spatial resolution and 25 frames per second of temporal resolution.

5. A webcam acquires YUV video (12 bits per pixel) at 15 frames per second, with 320×240 pixels of spatial resolution. Determine the compression ratio needed to be able to transmit this video through a link operating at 128 kbps.

6. Without compression, how many bytes of storage would require a photo with 10×15 cm scanned at 1200 ppi [1] and 24 bits per pixel?

7. Suppose you have made a film with a video camera and later viewed it at twice the speed. If $x(t)$ represents the video signal you recorded, what should be the representation of the signal corresponding to the double speed instance?

8. Draw the signal $\displaystyle\sum_{k=0}^{\infty} \delta(n-k)$ and compare it with the signal $u(n)$.

9. Show that a system defined by the relation $H[x(t)] = x(t)\cos(2\pi f t)$ (AM modulator) is linear but not time invariant.

10. Give an example that shows that the digital system defined by $H[x(n)] = y(n) = x(n) + 2y(n-1)$ is not stable.

[1] Pixels per inch (one inch is 2.54 cm).

11. Consider a digital system defined by $H[x(n)] = x(n) + x(n-1)$. Is it:
    (a) Linear?
    (b) Time invariante?
    (c) Causal?
    (d) Stable?
12. A system responds to $x(n)$ with $y(n) = H[x(n)] = x(0)x(n)$. Is the system:
    (a) Linear?
    (b) Time invariant?
13. Calculate the response of the linear and time invariant digital system with impulse response $h(n) = 2\delta(n) + \delta(n-1) - \delta(n-2)$ to signal $x(n) = \delta(n) + \delta(n-1) - \delta(n-2) - \delta(n-3)$.
14. Is the code $\{00, 11, 0101, 111, 1010, 100100, 0110\}$ uniquely decodable?
15. Is the (ternary) code $\{00, 012, 0110, 0112, 100, 201, 212, 22\}$ uniquely decodable?
16. Find a probability distribution $\{p_1, p_2, p_3, p_4\}$ such that there are two optimal codes that assign different lengths $\{l_1, l_2, l_3, l_4\}$ to the four symbols.
17. Explain why every compression method will expand some of its inputs, instead of compressing them.
18. Consider a Golomb code with parameter $m = 5$ (note that m is not a power of two).
    (a) According to this code, give, justifying, a sequence of bits that represents the integers $a = 12$ and $b = 13$ as efficiently as possible.
    (b) Indicate the optimal probability distribution for this code, i.e., the values of $P(n), n \in \mathbb{N}_0$.
19. Consider a coding system for non-negative integer numbers, where a certain value n is represented by n "0" bits followed by a "1" bit. For what probability distribution, $P(n), n \in \mathbb{N}_0$, is this coding system maximally efficient?
20. Consider the following table (incomplete) of correspondences between symbols σ_i, probabilities p_i, and codewords:

$$\begin{array}{lcr}
\sigma_1(p_1 = 0.30) & \longrightarrow & 00 \\
\sigma_2(p_2 = 0.30) & \longrightarrow & ? \\
\sigma_3(p_3 = 0.15) & \longrightarrow & 110 \\
\sigma_4(p_4 = 0.15) & \longrightarrow & ? \\
\sigma_5(p_5 = 0.10) & \longrightarrow & 111
\end{array}$$

    (a) Complete the table in order to obtain a Huffman code.
    (b) Calculate the redundancy of the code, considering the first order entropy of the information source.

21. Consider the following table of correspondences between symbols σ_i, probabilities p_i, and codewords:

$$\begin{array}{lcr} \sigma_1(p_1 = 0.1) & \longrightarrow & 000 \\ \sigma_2(p_2 = 0.2) & \longrightarrow & 01 \\ \sigma_3(p_3 = 0.2) & \longrightarrow & 10 \\ \sigma_4(p_4 = 0.5) & \longrightarrow & 1 \end{array}$$

   (a) Assuming that the symbols occur independently, calculate the entropy of this information source.
   (b) This variable-length code is not built correctly. Why?
   (c) Show that it is not possible to build a prefix-free code with this set of codeword lengths.
   (d) Propose a variable-length code appropriate for the given probability distribution.
   (e) Calculate the redundancy (in relation to the entropy of the source) of the code built in the previous question.

22. Explain, briefly, the principles of one of the coding algorithms based on dictionaries, pointing out the main advantages and disadvantages.

23. Explain, briefly, the working principle of an arithmetic encoder.

24. Give the code sequence produced by a LZ78 encoder, if the input sequence is:

    zzxyxyxxxyxzxxxyxxzzxyxx

    Also, provide the final state of the dictionary.

25. Give the code sequence produced by a LZ77 encoder if the input sequence is:

    zzxyxyxxxyxzxxxyxxzzxyxx

    Consider that the input buffer has size 4 and the dictionary window has size 12.

26. Give the code sequence produced by a LZW encoder if the input sequence is:

    zzxyxyxxxyxzxxxyxxzzxyxx

    Consider that the input alphabet is $\Sigma = \{$x, y, z$\}$.

27. Taking into consideration the principles of arithmetic coding, and that $P(0) = P(1) = 0.5$, indicate a value in the $[0, 1)$ interval that represents all messages that start with the binary sequence "1011".

28. Consider that a certain second-order finite-context model (i.e., that uses the two previously occurred symbols for conditioning the probability of occurrence of the next symbol) has already observed the following binary sequence:

    100101101111010100110000

    Indicate an estimate for the probability given by this model that the next symbol will be "1".