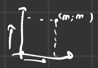
Ficha 3

MD

1. $\binom{164}{5} \times \binom{66}{5} = 8,308 \times 10^{15}$

2.

a) $\binom{2m}{m}$ → porque ele tem de escolher um dos lados



b) $\binom{3+2}{3} \times \binom{2+3}{2} = 100$

3.

a) $\binom{8+5-1}{8} = \binom{12}{8} = 495$

b) $5^8 = 390\,625$

4. Colocando uma carta em cada envelope, sobram 8 cartas para 12 envelopes

$\binom{8+12-1}{8} = \binom{19}{8} = 75582$

5. $5^7 \times \binom{6+5-1}{6} = 16406250$

6. 1 0 1 _ _ _ _ _ _ _ _ _ 0 0 0 1

Logo, faltam 2 uns e 7 zeros

$\binom{9}{2} \times \binom{7}{7} = 36$

8. $5^6 \times \binom{4+5-1}{4} = 1093750$

9. $A_{57,20} = 2,944 \times 10^{33}$

10.

a) $A_{57,12} = 3,3879 \times 10^{20}$

b)

$\binom{12+12}{12} = \binom{24}{12} = 2704156$

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a) $\dfrac{14!}{3! \times 2! \times 2! \times 3!} = 605404800$

b) $605404800 - \dfrac{12!}{2! \times 2! \times 3!} = 585446400$

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a) Sendo $(1+x)^m = \sum_{k=0}^{m} \binom{m}{k} x^k$, então quando $x=2$ $\quad 3^m = \sum_{k=0}^{m} \binom{m}{k} 2^k$

b)

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a) $(x^2 + \frac{y}{x} + 2z)^6 = \sum_{m=6} \binom{6}{m_1\, m_2\, m_3} (x^2)^{m_1} \left(\frac{y}{x}\right)^{m_2} (2z)^{m_3}$

Para obter xy^3z damos que $m_1 = 2$; $m_2 = 3$; $m_3 = 1$

$\binom{6}{2\,3\,1} (x^2)^2 \left(\frac{y}{x}\right)^3 (2z)^1 = \dfrac{6!}{2! \times 3! \times 1} x^4 \dfrac{y^3}{x^3} 2z = 120\, xy^3z$

b) $(a+b)^4 = \sum_{m=4} \binom{4}{m_1\, m_2} a^{m_1} b^{m_2} = \binom{4}{4\,0} a^4b^0 + \binom{4}{3\,1} a^3b + \binom{4}{2\,2} a^2b^2 + \binom{4}{1\,3} ab^3 + \binom{4}{0\,4} a^0b^4$

$= \dfrac{4!}{4!\,0!} a^4b^0 + \dfrac{4!}{3! \times 1} a^3b + \dfrac{4!}{2!\,2!} a^2b^2 + \dfrac{4!}{1!\,3!} ab^3 + \dfrac{4!}{0! \times 4!} a^0b^4$

$= a^4b^0 + 4a^3b + 6a^2b^2 + 4ab^3 + a^0b^4$

Se $a = 1$ e $b = 4$, $5^4 = (1+4)^4 = 4^0 + 4 \times 4^1 + 6 \times 4^2 + 4 \times 4^3 + 4^4$

$\therefore$ Logo, $c_0 = 1$, $c_1 = 4$, $c_2 = 6$, $c_3 = 4$ e $c_4 = 1$

c) sendo $\sum_{k=0}^{m} \binom{m}{k} = 32 \Rightarrow 2^m = 32 \Leftrightarrow m = 5$

$(x^3 + \sqrt{x})^5 = \sum_{k=0}^{5} \binom{5}{k} (x^3)^k (\sqrt{5})^{5-k} = \sum_{k=0}^{5} \binom{5}{k} x^{3k} x^{\frac{5-k}{2}} = \sum_{k=0}^{5} \binom{5}{k} x^{\frac{5k+5}{2}}$

$\frac{5k+5}{2} = 10$

$\Leftrightarrow 5k + 5 = 20$

$\Leftrightarrow k = 3$

$\therefore$ Logo $\binom{5}{3} = 10$

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$(x_1 - x_2 + 2x_3 - 2x_4)^8 = \sum_{m=8} \binom{8}{m_1\, m_2\, m_3\, m_4} x^{m_1} (-x_2)^{m_2} (2x_3)^{m_3} (-2x_4)^{m_4}$

Para obter $x_1^2 x_2^3 x_3 x_4^2$, $m_1 = 2$, $m_2 = 3$, $m_3 = 1$, $m_4 = 2$

$\binom{8}{2\,3\,1\,2} x_1^2 (-x_2)^3 (2x_3)(-2x_4)^2 = \dfrac{8!}{2!\,3!\,2!} x_1^2 - x_2^3\, 2x_3\, 4x_4^2 = -13440\, x_1^2 x_2^3 x_3 x_4^2$

15 Temos que $2^m = 256 \Leftrightarrow m = 8$

$\left(\frac{m}{2}\right)! = \left(\frac{8}{2}\right)! = (4)! = 24$

16 $(a+b)^m = \sum_{k=0}^{m} \binom{m}{k} a^k b^{m-k}$

Logo, em $a^2 b^{m-2} \Rightarrow k = 2$ $\binom{m}{2} = 28 \Leftrightarrow \frac{m!}{(m-2)!\,2!} = 28 \Leftrightarrow \frac{(m)(m-1)}{2} = 28 \Leftrightarrow m^2 - m = 56 \Leftrightarrow m = 8 \vee \cancel{m = -7}$

Para $a^{m-3} b^3$ temos que $k = 3$ $\binom{8}{3} = \frac{8!}{(8-3)!\,3!} = \frac{8!}{5!\,3!} = 56$

17 $\left(x - \frac{1}{x}\right)^{100} = \sum_{k=0}^{100} \binom{100}{k} x^k \left(-\frac{1}{x}\right)^{100-k} = \sum_{k=0}^{100} \binom{100}{k} (-1)^{100-k} x^{2k-100}$

$2k - 100 = y \Leftrightarrow k = \frac{y}{2} + 50$, logo o coeficiente é

$\binom{100}{\frac{y}{2}+50} x^y (-1)^{50-\frac{k}{2}}$, para $y \in \{-100; 98, \dots, 98, 100\}$ e 0 para os restantes valores de k

Nota: O k do enunciado corresponde ao y usado em cima

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a) $\sum_{k=0}^{m} m\binom{m}{k} = m \times 2^m =$