Volatility of Inflation Rate in Mozambique: A Variance Decomposition

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January, 2015

Abstract

This paper investigates the volatility of inflation in Mozambique using a procedure to decompose inflation volatility according to the various CPI components. We show that monsoon weather has an intense effect on inflation and inflation volatility and that food, housing, water, electricity, gas and other fuels and transportation are the major contributors to the variability of inflation. Based on these findings we make some recommendations for economic policy.

Keywords: Mozambique; inflation; volatility decomposition.

Acknowledgements

This research was supported by the *Fundação para a Ciência e a Tecnologia* (Science and Technology Foundation).

1 Introduction

Inflation volatility in Mozambique is one of the highest in the world. For example, using data from the World Bank in the 1999-2012 period it can be shown that the variance of annual inflation is about 20 points, much higher than the Sub-Saharan Africa zone, 2.7, Middle East and North Africa zone, 6.6 and the group formed by all countries with "low and middle income", 2.4. High volatility means that it is very hard to anticipate inflation. This issue is particularly relevant as the main concerns with inflation are in general related to unexpected fluctuations, especially when it can reach high values, as is the case of Mozambique. When inflation can be anticipated, people and businesses can prepare and absorb much of its shock. For example, households may be able to switch savings into deposit accounts offering a higher nominal rate of interest or into other financial assets such as housing or equities where capital gains over a period of time might outstrip general price inflation, and companies can adjust prices and lenders can adjust interest rates. The costs of expected inflation are usually related to the so called "shoe-leather costs", menu costs, and increase in relative price volatility. The effects of inflation on poor people, for whom food is the most important component of their expenditure cannot also be neglected. However, the costs of unexpected inflation are generally higher and are related to arbitrary redistribution of wealth from lenders to borrowers and the costs to individuals of fixed nominal contracts. When it is difficult for people, businesses, and companies to make accurate predictions of inflation, they cannot take steps to protect themselves from its effects. This paper focuses mainly on this type of inflation, i.e. unexpected inflation that directly relates to inflation volatility. Specifically, we attempt to identify the components of the Consumer Price Index (CPI) that most contribute to inflation volatility. In principle, the methodology to study this issue could be based on the volatility of each price component and its contribution or weight in the CPI. This procedure would be adequate if the components were uncorrelated with each other. However, if the inflation components are correlated, as it is the case of Mozambique, and in general, of most countries, the volatility sum of the different components, (weighted by their contribution to the CPI) is not equal to the CPI inflation volatility. In this paper we propose a technique to decompose inflation volatility according to the various CPI components, based on the ideas of Bekaert et al. (2005) and Christiansen (2007), which allows us to find the components that most affect inflation volatility.

Although there is a lot of research on inflation, a wide array of models are used in the published papers. For example, Kapadia (1976) made one of the first contributions to the application of statistical methods in the study of inflation issues. De Bruin et al. (2011) adopted a survey-based measure of uncertainty about future inflation, asking consumers for density forecasts across inflation outcomes; Ilbas (2012) adopted a Bayesian model to estimate changes in US post-war monetary policy. More in line with our research, Clark (2006) used disaggregated inflation data spanning all consumption to examine the persistence of disaggregated inflation relative to aggregated inflation, the distribution of persistence across consumption sectors, and whether persistence has changed. Clark and McCracken (2010) adopted VAR models of output, inflation and interest rates, while Barros and Gil-Alana (2013) analyzed inflation forecasts in Angola using an ARFIMA model. Baharumshah et al. (2011) analyzed the relationship between inflation and growth using an ARCH model.

Our paper takes a new direction in relation to previous studies. We not only focus on the inflation volatility, but we also look at the contribution the various CPI components make to Mozambique's inflation volatility.

2 Inflation in Mozambique

The Mozambican inflation rate is split into twelve CPI components: 1) Food and non-alcoholic beverages; 2) Alcoholic beverages, tobacco and narcotics; 3) Clothing and shoes; 4) Housing, water, electricity, gas and other fuels; 5) Furniture and other home decorative equipment; 6) Health; 7) Transportation; 8) Communications; 9) Leisure, recreation and culture; 10) Education; 11) Restaurants, hotels, bars and similar and 12) Other goods and services. This paper analyzes the contribution of these components to the Mozambique inflation volatility. The procedure is described in section 3.

Figure 1 displays the graph of the Mozambican monthly inflation rate, and Table 1 displays its descriptive statistics.

Table 1 provides evidence of seasonality in mean inflation (mean inflation varies throughout the year), with systematically higher values of mean inflation in the months of December, January and February. These months are summer vacations in the southern hemisphere. In contrast, lower seasonality values are found in the months of April, May and June - the end of the monsoon period. However, the evidence of seasonal variation in inflation, also suggested by Table 1, displays an interesting view of Mozambique's inflation. Inflation volatility reaches its peak in February, during the monsoon period, whereas in the months between April and September, when the monsoon period ends, it shows a less volatile pattern. Notice that the standard deviation in February is almost three times higher than the standard deviation in September. As the monsoon period is a very important season in the southern hemisphere, it is remarkable to see the way it affects inflation volatility.

In the next section we explain the strategy we used to 1) uncover the determinants of the variability of the inflation rate, and 2) analyze the contribution of the variability of each CPI component to the conditional variance of inflation through volatility transmission ratios and conditional correlations between the CPI components and inflation.

3 Methodology

The inflation may be represented as

$$I_t = \sum_{i=1}^{k} \beta_i r_{it} + \varepsilon_t \tag{3.1}$$

where r_{it} is the inflation of component i, $\{\beta_1, ..., \beta_k\}$ are the components' weights in the CPI and ε_t represents a measurement error, which can be attributed to the variability of β_i $(i = 1, .., k)$ over time and to the transformation of the CPI index into inflation. k is the number of CPI components. In general, ε_t is a small quantity - the regression of I_t on the $(r_{1t}, ..., r_{kt})$ produces a R-squared close to .99. Our goal is to ascertain the contribution of each volatility component to the overall inflation volatility over time.

Let $\mathcal{F}_{t-1}$ be the information set generated by the available information until $t-1$. Given that

$$Var\left(I_t|\,\mathcal{F}_{t-1}\right) = \sum_{i=1}^{k} \beta_i^2 Var\left(r_{it}|\,\mathcal{F}_{t-1}\right) + 2\sum_{i=1}^{k-1}\sum_{j=i+1}^{k} Cov\left(r_{it}, r_{jt}|\,\mathcal{F}_{t-1}\right) \tag{3.2}$$

it is clear that the inflation's variance cannot be decomposed using solely the terms $Var\left(r_{it}|\,\mathcal{F}_{t-1}\right)$. To overcome this problem, we will develop a method based on Christiansen's methodology Christiansen (2007), which was used to extract the volatility spillover effects in bond markets. We assume that the inflation can be expressed as

$$I_t = \psi_t + \sum_{i=1}^{k} \eta_i e_{it} + v_t \tag{3.3}$$

where ψ_t is $\mathcal{F}_{t-1}$-measurable and may include a constant, seasonal dummies and possibly some dynamic terms (e.g. MA or AR), $\{e_{1t}, ..., e_{kt}\}$ are orthogonal shocks associated with each component and v_t is the error term of the equation. It turns out that the conditional variance of I_t can be decomposed into the sum of the individual shocks of each category plus the variance of the idiosyncratic term v_t,

$$Var\left(I_t|\,\mathcal{F}_{t-1}\right) = \sum_{i=1}^{k} \eta_i^2 Var\left(e_{it}|\,\mathcal{F}_{t-1}\right) + Var\left(v_t|\,\mathcal{F}_{t-1}\right) \tag{3.4}$$

This is the expression we were looking for to decompose the variance. It allows us to examine the relationship and volatility spillovers between inflation and its components, as the conditional variance of each component can be directly compared to the conditional variance of inflation. For example, the contribution of component 1 to the total variance $Var\left(I_t|\mathcal{F}_{t-1}\right)$ is simply $\eta_1^2 Var\left(e_{1t}|\mathcal{F}_{t-1}\right)/Var\left(I_t|\mathcal{F}_{t-1}\right)$. Hence, the larger this ratio the greater the contribution of the first component to the inflation volatility. It should be emphasized that this kind of comparison would be impossible to carry out using expression (3.2). However, this method to decompose the volatility involves two issues: 1) the shocks are unknown and need to be estimated and, 2) they must be orthogonal - only as such will the expression (3.4) make sense. We will address these issues below.

The shock e_{it} is unknown but can be consistently estimated from the residuals of the regression of r_i on μ_{it}, i.e.

$$r_{it} = \mu_{it} + e_{it} \tag{3.5}$$

The term μ_{it} is $\mathcal{F}_{t-1}$-mensurable and may include a constant, seasonal dummies, possibly some dynamic terms (e.g. MA or AR) and the other terms $r_{j,t-k}$ $(j \neq i, k \geq 1)$ that are correlated with r_{it}. This procedure aims to remove all the effects defined in μ_{it} from r_{it}, particularly the effects of other inflation components, and common underlying shocks (seasonal effects are the most important) so that e_{it} is uncorrelated with e_{jt} $(j \neq i)$. The difference between our approach and Christiansen (2007) is that we do not assume that causality goes in one direction only (as r_{it} may be correlated with the other components). However, we do assume that inflation is caused, in the Granger sense, by all the inflation components.

We assume $e_{it}|\mathcal{F}_{t-1} \sim N\left(0, \sigma_{it}^2\right)$ and $v_t|\mathcal{F}_{t-1} \sim N\left(0, \sigma_{vt}^2\right)$. A test based on the Lagrange multiplier principle confirms that most of the errors display GARCH type effects.

Let $\mathbf{y}_t = \left(r_{1t}, ..., r_{kt}, I_t\right)'$, $\mu_t = \left(\mu_{1t}, ..., \mu_{kt}, \omega_t\right)'$, $\mathbf{u}_t = \Psi \mathbf{e}_t$ where $\mathbf{\Psi}$ and $\mathbf{e}_t$ are defined as follows

$$\mathbf{u}_t = \underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ \eta_1 & \eta_2 & \cdots & \eta_k & 1 \end{bmatrix}}_{\Psi} \underbrace{\begin{bmatrix} e_{1t} \\ e_{2t} \\ \vdots \\ e_{kt} \\ v_t \end{bmatrix}}_{\mathbf{e}_t}. \tag{3.6}$$

The model can be succinctly written as $\mathbf{y}_t = \mu_t + \mathbf{u}_t$ where $\mathbf{u}_t = \Psi \mathbf{e}_t$ (note that Ψ is a triangular matrix). The conditional variance of $\mathbf{y}_t$ is

$$\begin{aligned}\mathbf{H}_t &= Var\left(\mathbf{y}_t | \mathcal{F}_{t-1}\right) = Var\left(\mathbf{u}_t | \mathcal{F}_{t-1}\right) = \mathbf{\Psi}\mathbf{\Sigma}_t\mathbf{\Psi}' \\ &= \begin{bmatrix} \sigma_{1t}^2 & 0 & \cdots & 0 & \eta_1\sigma_{1t}^2 \\ 0 & \sigma_{2t}^2 & \cdots & 0 & \eta_2\sigma_{2t}^2 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & \sigma_{kt}^2 & \eta_k\sigma_{kt}^2 \\ \eta_1\sigma_{1t}^2 & \eta_2\sigma_{2t}^2 & \cdots & \eta_k\sigma_{kt}^2 & \sum_{i=1}^k \eta_i^2\sigma_{it}^2 + \sigma_{vt}^2 \end{bmatrix} \end{aligned} \tag{3.7}$$

where $\Sigma_t := Var\left(\mathbf{e}_t | \mathcal{F}_{t-1}\right)$.

It follows that

$$Var\left(I_t | \mathcal{F}_{t-1}\right) = \sum_{i=1}^{k} \eta_i^2\sigma_{it}^2 + \sigma_{vt}^2 \text{ (as seen previously)} \tag{3.8}$$

$$Corr\left(r_{it}, I_t | \mathcal{F}_{t-1}\right) = \frac{\eta_i\sigma_{it}^2}{\sqrt{\sigma_{it}^2\left(\sum_{i=1}^k \eta_i^2\sigma_{it}^2 + \sigma_{vt}^2\right)}} = \frac{\eta_i\sigma_{it}}{\sqrt{\sum_{i=1}^k \eta_i^2\sigma_{it}^2 + \sigma_{vt}^2}} \tag{3.9}$$

In appendix 1 we provide a proof that the model can be estimated step by step i.e. equation by equation. This estimation procedure has been applied in similar models (see Bekaert et al. (2005) and Christiansen (2007)) but, as far as we know, it has not yet been proved that the estimators are consistent.

4 Results

Tables 2 and 3 present the estimation results of equations (3.5) and (3.3). As referred to previously, the model $\mathbf{y}_t = \mu_t + \mathbf{u}_t$ can be estimated equation by equation. Unlike what happens with the individual components, the term v_t (defined in the inflation equation) does not display any ARCH or GARCH type effects. That is, all sources of volatility come from the inflation components, as expected if the model is well defined.

It is of interest to test whether $E(\mathbf{e}_t\mathbf{e}_t')$ is a diagonal matrix. We consider Lagrange multiplier statistics, $\lambda_{LM} = n\sum_{i=2}^{M}\sum_{j=1}^{i-1}\hat{\rho}_{ij}^2$, where $\rho_{ij} := Corr(e_{it}, e_{jt})$ and M represents the number of contemporaneous errors included in the procedure (which in our case is 6). Under the null, $H_o : \rho_{ij} = 0$, λ_{LM} has a limiting chi-squared distribution with $M(M-1)/2$ degrees of freedom. We cannot reject the null hypothesis at the 5% level of significance, that is we have statistical evidence that the errors are contemporaneously orthogonal.

Based on the magnitude and statistical significance of the estimates, the components that most contribute to the volatility of the inflation rate are the following, in this order:

1. Food and non-alcoholic beverages (Component 1)

2. Housing, water, electricity, gas and other fuels (Component 4)

3. Transportation (Component 7)

4. Restaurants, hotels, bars and similar (Component 11)

5. Education (Component 10)

6. Communications (Component 8)

The other components were not statistically significant and were removed from the model, hence k in equation (3.3) was reduced to 6.

As we saw previously, we can calculate the fraction of inflation variability explained by the $i-th$ component of the CPI returns as follows:

$$VR_i = \frac{\eta_i^2 \sigma_{it}^2}{Var\left(I_t | \mathcal{F}_{t-1}\right)} \tag{4.1}$$

$$Var\left(I_t | \mathcal{F}_{t-1}\right) = \sum_{i=1}^{12} \eta_i^2 \sigma_{it}^2 + \sigma_{vt}^2. \tag{4.2}$$

The η_i coefficients are consistently estimated by $\hat{\eta}_i$ - the coefficients displayed in Table 3, and the series σ_{it}^2 are consistently estimated by $\hat{\sigma}_{it}^2$ - the estimated variance of $\hat{e}_{it}$.

Table 5 shows the variance ratios' means over time, Figure 2 plots the time series of the global conditional variances and the graphs in Figure 3 plot the time series of the variance ratios.

Figure 2 gives rise to a few comments. Firstly, the behavior of the global conditional variance has a slightly downward trend during the period, albeit with some peaks of huge volatility, particularly in the years 2000 and 2005, and high volatility in the years 2009, 2010 and 2011. It should be noted, moreover, that much more volatile behavior emerges from 2009 onwards. Although the peaks of high volatility are lower than those of 2000 and 2005, they occur more frequently. Secondly, the six major components identified in Table 4, on average, account for 88% of the inflation rate variability, which is a considerable proportion. The food component is the largest contributor to the variability of average inflation. More precisely, the variability of the food component explains about two thirds of the global variability of inflation, which is an expected result in land-based economies. The volatility of this component appears to be constant over time, which may be related to the fact that the demand for goods of this component is relatively inelastic.

Regarding the first comment, in the year 2000 Mozambique was hit by devastating floods, causing large fluctuations in agricultural production in the country, which contributed to an aggravation and a consequent increase in the inflation variability.

In 2005, there was a large currency devaluation against the dollar (in six months the Metical devalued about 45% according to the Bank of Portugal) and against the South African Rand. Devaluation resulted in the creation of a new currency in 2006 - The New Family's Metical. It is well-known that the Metical remains highly dependent on these two currencies, the US Dollar and the Rand, which themselves showed erratic behavior during the 2000s. While the Dollar has been depreciating, the Rand has shown very unstable behavior during the 2000s. Another relevant issue is related to the upward trend in oil prices. The price of oil began rising shortly after the occupation of Iraq in 2003, but it was in 2007 that the trend of rising Oil prices became more pronounced (in 2007 it amounted to $80 per barrel). If you add the instability effects deriving from the international financial crisis, then we have all the reasons to explain the variability peaks in the Mozambican inflation rate, which occurred in 2009, 2010 and 2011.

As for the second comment, it should be noted that the peaks of 2000, 2001 and 2005 in the conditional variance of inflation match with peaks in the variance of the transportation component. This may indicate that the shocks that occurred in the transportation component had a bearing on the volatility of inflation in this period, and it should not be forgotten that transportation is based on oil and priced in dollars.

To conclude this section, it is important to note that both the conditional variance of inflation and the variance ratios exhibit a strong seasonality pattern illustrated by Figure 4 and Figure 5 below.

The conditional variance of the inflation is systematically lower, on average, between April and September: period without monsoon weather. Moreover, the graph of the seasonal conditional variance of inflation takes a U-Shape form. It is noteworthy that the contribution of the volatility of Food and non-alcoholic beverages (a component that has a big bearing on the overall volatility) is 50% lower between the months of April and September.

As we have shown previously, the conditional correlation between the inflation rate and the i-th CPI component may be computed as:

$$Corr\left(r_{it}, I_t \middle| \mathcal{F}_{t-1}\right) = \frac{\eta_i \sigma_{it}^2}{\sqrt{\sigma_{it}^2 \left(\sum_{i=1}^{12} \eta_i^2 \sigma_{it}^2 + \sigma_{vt}^2\right)}} = \frac{\eta_i \sigma_{it}}{\sqrt{\sum_{i=1}^{12} \eta_i^2 \sigma_{it}^2 + \sigma_{vt}^2}} \tag{4.3}$$

Table 6 shows the means of the conditional correlations over time, and Figure 6 below plots the time series of conditional correlations over time.

On average, the highest correlations are as follows: 1) Food and non-alcoholic beverages (0.8084); 2) Housing, water, electricity, gas and other fuels (0.3497); 3) Transportation (0.1744); 4) Restaurants, hotels, bars and similar (0.0802); 5) Education (0.0506), 6) Communications (0.0409). An important fact is that all correlations are conditionally positive. This means that inflation varies in the same direction as the components of the CPI, given the unidirectional causal relationship: inflation is caused by the CPI components. In a similar manner to the variance ratios, there were noticeable peaks in the conditional correlation of the transportation component in the years 2000, 2001 and 2005. While on average the correlation between the transportation component and inflation in 2000 is 0.1; it reaches a peak of 0.44. In 2001 it reaches 0.31 and in 2005 it reaches 0.65.

An important issue is the seasonality in conditional correlations. Like the variance ratios, conditional correlations also exhibit strong seasonality, as can be seen from Figure 7.

We highlight the behavior of components 1 and 7. The correlation between inflation and component 1 is substantially lower in the months of April, July and September. In contrast, the correlation between component 7 and inflation has some peaks on April, May, August, September and December. These findings agree with the results of the seasonality in the variance ratios and help explain the pattern of seasonality in the inflation variance.

5 Conclusions

Based on these results, the following main conclusions can be drawn: 1) there is evidence of seasonality in Mozambique's inflation, both in mean and variance, reflecting the rise in economic activity. Furthermore, inflation volatility shows a more volatile pattern during the monsoon period, reaching a peak in February, whereas in the months between April and September it shows a less volatile pattern, when the monsoon period ends. Therefore, inflation in Mozambique reflects the seasonality of the monsoon weather, which has an intense effect on economic activity. 2) The variability of the food component is the largest contributor to the variability of inflation, which is an expected result in a land-based economy characterized by poverty. 3) Finally, inflation correlations are high with the component of Food and non-alcoholic beverages, revealing the importance of this activity in the country.

The food component is the major contributor to the inflation variability, reflecting the changes in demand that appears after the monsoon period. These changes induce poverty, since the poorest people are the most affected by this change. This is a worrying conclusion as 75% of the population lives below the poverty line (Castel-Branco and Ossemane (2012)). The government could increase the offer of food in order to match this increase in demand, but this has to be achieved through internal production, since importation also increases the country's dependence on foreign markets, putting pressure on the foreign exchange reserves. The policy implications of this research, based on the seasonality volatility found in Mozambique, is that the government should act to counter price increases during the monsoon period, granting credit to the food and beverage sectors to counteract the monsoon effects.

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6 Appendix

Given the Gaussian hypothesis, the conditional joint probability density function of $\mathbf{y}_t$ is

$$f\left(\mathbf{y}_t \middle| \mathcal{F}_{t-1}\right) = (2\pi)^{-m/2}\left|\mathbf{H}_t\right|^{-1/2}\exp\left\{-\frac{1}{2}\left(\mathbf{y}_t-\mu_t\right)'\mathbf{H}_t^{-1}\left(\mathbf{y}_t-\mu_t\right)\right\}.$$

and the log-likelihood function is

$$\log L_n\left(\theta\right) = c - \frac{1}{2}\sum_{t=1}^{n}\log\left|\mathbf{H}_t\right| - \frac{1}{2}\sum_{t=1}^{n}\left(\mathbf{y}_t-\mu_t\right)'\mathbf{H}_t^{-1}\left(\mathbf{y}_t-\mu_t\right).$$

where theta is the vector of all unknown parameters.

It can be easily proved that

$$\begin{aligned}
\log\left|\mathbf{H}_t\right| &= \log\left|\mathbf{\Psi}\mathbf{\Sigma}_t\mathbf{\Psi}'\right| = \log\left(\left|\mathbf{\Psi}\right|^2\left|\mathbf{\Sigma}_t\right|\right) \\
&= \log\left(\left|\mathbf{\Psi}\right|^2\right) + \log\left(\left|\mathbf{\Sigma}_t\right|\right) = \log\left(\left|\mathbf{\Sigma}_t\right|\right) = \\
&= \sum_{i=1}^{k}\log\left(\sigma_{it}^2\right) + \log\left(\sigma_{vt}^2\right)
\end{aligned}$$

$$\begin{aligned}
\left(\mathbf{y}_t-\mu_t\right)'\mathbf{H}_t^{-1}\left(\mathbf{y}_t-\mu_t\right) &= \mathbf{u}_t'\left(\mathbf{\Psi}'\right)^{-1}\mathbf{\Sigma}_t^{-1}\mathbf{\Psi}^{-1}\mathbf{u}_t \\
&= \mathbf{e}_t'\mathbf{\Sigma}_t^{-1}\mathbf{e}_t \\
&= \frac{1}{\sigma_{1t}^2}e_{1t}^2 + \ldots + \frac{1}{\sigma_{12,t}^2}e_{12,t}^2 + \\
&\quad \frac{1}{\sigma_{vt}^2}\left(I_t - \omega_t - \sum_{i=1}^{k}\eta_i e_{it}\right)^2
\end{aligned}$$

Therefore,

$$\begin{aligned}
\log L_n\left(\theta\right) &= c - \frac{1}{2}\sum_{t=1}^{n}\left(\sum_{i=1}^{k}\log\left(\sigma_{it}^2\right) + \log\left(\sigma_{vt}^2\right)\right) \\
&\quad -\frac{1}{2}\sum_{t=1}^{n}\left(\frac{1}{\sigma_{1t}^2}\left(r_{1t}-\mu_{1t}\right)^2 + \ldots + \frac{1}{\sigma_{k,t}^2}\left(r_{kt}-\mu_{12t}\right)^2 + \frac{1}{\sigma_{vt}^2}\left(I_t - \omega_t - \sum_{i=1}^{k}\eta_i e_{it}\right)^2\right) \\
&= \underbrace{c + \left(-\frac{1}{2}\sum_{t=1}^{n}\log\left(\sigma_{1t}^2\right) - \frac{1}{2}\sum_{t=1}^{n}\frac{1}{\sigma_{1t}^2}\left(r_{1t}-\mu_{1t}\right)^2\right)}_{\log L_{n,1}} \\
&\quad + \ldots + \underbrace{\left(-\frac{1}{2}\sum_{t=1}^{n}\log\left(\sigma_{kt}^2\right) - \frac{1}{2}\sum_{t=1}^{n}\frac{1}{\sigma_{kt}^2}\left(r_{kt}-\mu_{kt}\right)^2\right)}_{\log L_{n,k}} \\
&\quad \underbrace{\left(-\frac{1}{2}\sum_{t=1}^{n}\log\left(\sigma_{vt}^2\right) - \frac{1}{2}\sum_{t=1}^{n}\frac{1}{\sigma_{vt}^2}\left(I_t - \omega_t - \sum_{i=1}^{k}\eta_i e_{it}\right)^2\right)}_{\log L_{n,k+1}} \\
&= \log L_{n,1} + \log L_{n,2} + \ldots + \log L_{n,k}\log L_{n,k+1}.
\end{aligned}$$

Let us decompose θ as follows: $\theta = (\theta_1, ..., \theta_{13})$ where θ_i is the parameter vector in the term $\log L_{n,i}$. As such, $\log L_n(\theta)$ may be written as

$$\log L_n(\theta) = \log L_{n,1}(\theta_1) + \log L_{n,2}(\theta_2) + ... + \log L_{n,k}(\theta_k) + \log L_{n,k+1}(\theta_{k+1}).$$

By construction, the intersection $\theta_i \cap \theta_j$ $(i, j = 1, ..., k)$ is an empty set, meaning that the terms $\log L_{n,i}$ with $i = 1, ..., k$ do not involve common parameters. Consequently, θ_i can be consistently and efficiently estimated through the maximization of the $\log L_{n,i}$ term alone. The last term of the log-likelihood function, $\log L_{n,k+1}$, depends on the parameters η_i $(i = 1, ..., k)$ and σ^2_{vt}, those included in μ_t and the parameters defined in the other $\log L_{n,i}$ terms, through the errors $e_{it} = r_{it} - \mu_{it}$. Therefore, θ_{k+1} is the only vector included in $\theta = (\theta_1, ..., \theta_{k+1})$ that depends on the remaining parameters θ_i $(i = 1, ..., k)$. However, the substitution of e_{it} with $\hat{e}_{it} = r_{it} - \hat{\mu}_{it}$ (the estimate $\hat{\mu}_{it}$ is obtained through the maximization of $\log L_{n,i}$) does not affect the consistency of the estimators, as $\hat{e}_{it}$ is based on consistent and efficient estimators. Thus, we can maximize the $\log \hat{L}_{n,k+1}$function separately, based on the $\hat{e}_{it}$ residuals, without losing the consistency of the estimates. The conclusion is that the estimation model can be carried out step by step (i.e. equation by equation).

Tables

Table 1: Inflation of Mozambique: Descriptive Statistics

Month	Mean	Max	Min.	Std. Dev.	Obs.
Jan	1.5746	3.0402	-0.1562	1.0185	14
Feb	1.3878	4.4466	-0.5559	1.3012	14
Mar	0.4506	2.2652	-1.2655	0.9351	14
Apr	0.2068	1.1198	-0.8727	0.7068	14
May	-0.0412	1.3874	-1.6242	0.8735	14
Jun	0.0197	1.3545	-1.4633	0.8252	14
Jul	0.5452	2.4209	-0.6093	0.8109	14
Aug	0.3918	2.7661	-0.3444	0.7592	14
Sep	0.4864	1.5555	-0.0768	0.4728	14
Oct	0.7213	2.8074	-0.669	0.7722	14
Nov	1.329	3.8712	-0.1921	1.1522	13
Dec	2.4904	3.9025	0.8629	1.0089	13
All	0.7835	4.4466	-1.6242	1.1301	166

Table 2: Estimation Results I - Equation (3.5)

Only the components of CPI that most contribute to the volatility of the inflation rate are presented. The estimated residuals are the orthogonalized shocks that are statistically significant covariates in the model (3.3). The CPI components show evidence of seasonality in their mean and in their variance due to the significance of the seasonal dummy variables. All equations display ARCH type effects. The error terms were assumed to follow a standard normal distribution

Covariate	r_{1t}		r_{4t}		r_{7t}		r_{8t}		r_{10t}		r_{11t}	
	M.E.	V.E.	M.E.	V.E.	M.E.	V.E.	M.E.	V.E.	M.E.	V.E.	M.E.	V.E.
c	–	1.538 (0.409)	0.759 (0.141)	1.306 (0.371)	–	5.250 (0.436)	–	16.726 (0.718)	–	0.020 (0.002)	0.658 (0.115)	0.980 (0.120)
u_{t-1}^2	–	0.190 (0.081)	–	0.199 (0.106)	–	0.907 (0.136)	–	−0.0002 (0.0001)	–	3.325 (0.492)	–	0.368 (0.146)
σ_{t-1}^2	–	–	–	0.344 (0.154)	–	–	–	–	–	–	–	–
Jan	–	–	1.185 (0.524)	–	–	–	–	–	–	11.118 (3.883)	–	–
Mar	–	–	–	–	–	–	–	−16.428 (0.721)	–	–	–	–
Apr	–	−1.309 (0.414)	–	–	–	−5.205 (0.439)	–	–	–	−0.020 (0.002)	–	–
Mai	−0.686 (0.209)	–	–	–	0.866 (0.327)	−4.713 (0.547)	–	−16.659 (0.718)	–	–	−0.583 (0.097)	–
Jun	−0.967 (0.2446)	–	–	–	–	–	–	–	–	–	–	−0.968 (0.114)
Jul	−0.460 (0.196)	−1.341 (0.411)	–	–	–	–	–	–	–	–	–	–
Aug	–	–	–	–	–	−5.231 (0.436)	2.204 (0.798)	–	–	–	–	–
Sep	–	−1.272 (0.416)	–	–	–	−3.676 (0.527)	2.489 (0.803)	–	–	–	–	–
Oct	–	–	–	–	–	–	–	−16.566 (0.719)		–	–	–
Nov	–	–	−0.531 (0.135)	−2.045 (0.395)	0.227 (0.050)	–	–	−16.593 (0.720)	–	–	–	–
Dec	2.037 (0.369)	–	–	–	–	−5.257 (0.436)	–	−16.489 (0.720)	–	–	–	–
r_{1t-1}	0.621 (0.071)	–	–	–	–	–	–	–	–	–	–	–
r_{4t-1}	–	–	0.243 (0.060)	–	–	–	–	–	–	–	–	–
r_{5t-2}	0.246 (0.147)	–	–	–	–	–	–	–	–	–	–	–
r_{7t-1}	–	–	–	–	0.345 (0.069)	–	–	–	–	–	–	–
r_{10t-2}	–	–	–	–	–	–	–	–	0.101 (0.007)	–	–	–
r_{11t-2}	–	–	–	–	–	–	–	–	–	–	0.190 (0.090)	–
r_{12t-1}	0.242 (0.092)	–	–	–	–	–	–	–	–	–	–	–
$MA(1)$	–	–	–	–	–	–	–	–	−0.051 (0.006)	–	–	–

Coefficients are presented, standard errors between parentheses. All coefficients are statistically significant.

u_{t-1}^2 and σ_{t-1}^2 indicate the ARCH and GARCH terms respectively. M.E. and V.E. denote the mean and the variance equation.

Table 3: Estimation Results II - Equation (3.3)

All the coefficients shown are statistically significant. Moreover, the non-significant coefficients were removed from the estimation. This equation allows us to determine the six most important of the twelve CPI components.

e_{1t}	e_{4t}	e_{7t}	e_{8t}	e_{10t}	e_{11t}	$AR(12)$	$AR(1)$
0.58109	0.17725	0.06372	0.01063	0.03050	0.05635	0.68762	0.12563
(0.01868)	(0.01229)	(0.00729)	(0.00416)	(0.01128)	(0.01902)	(0.05918)	(0.05993)

Coefficients are presented, standard errors between parentheses. All coefficients are statistically significant.

Table 4: Means of the Variance Ratios

Table 5 exhibits the mean of the variance ratios over time - the contribution of each individual component to the global conditional variance of inflation. See model (4.1)

Component Number	1	4	7	8	10	11
Mean	0.6715	0.1326	0.0442	0.0024	0.0142	0.0073

Table 5: Means of the Conditional Correlations

Table 5 exhibits the mean of the conditional correlations over time between the inflation and the CPI components. See model (4.3)

Component Number	1	4	7	8	10	11
Mean	0.8105	0.3446	0.1707	0.0384	0.0477	0.0797

Figures

Figure 1: Inflation Rate of Mozambique

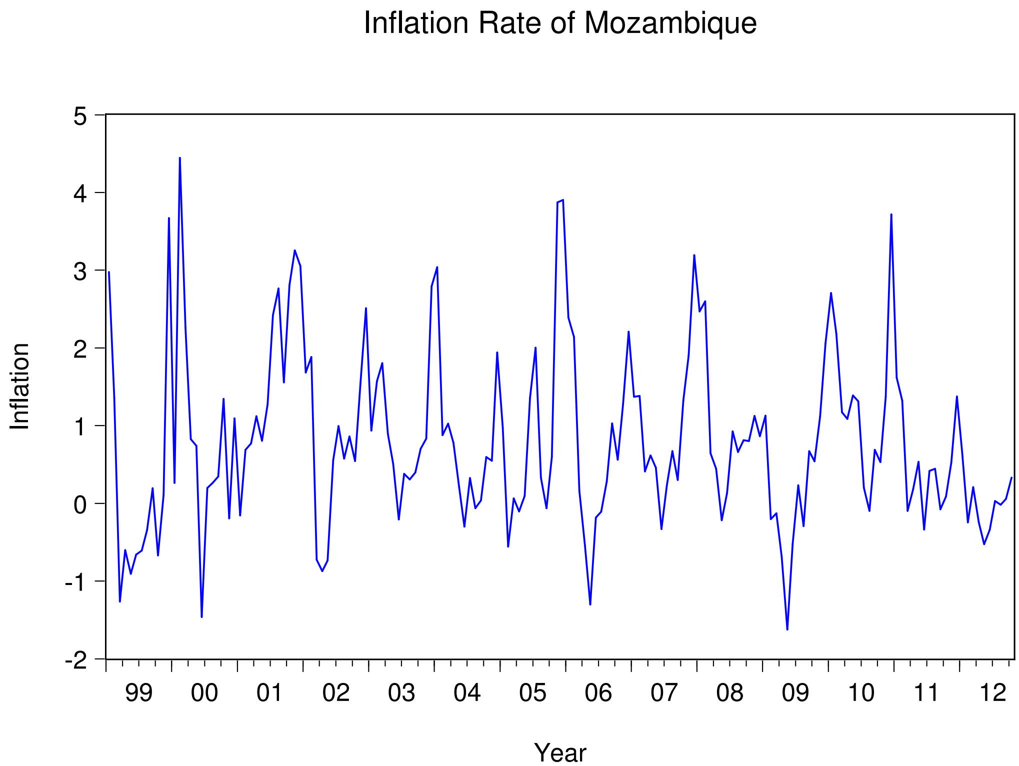

Figure 2: Conditional Variance of Inflation

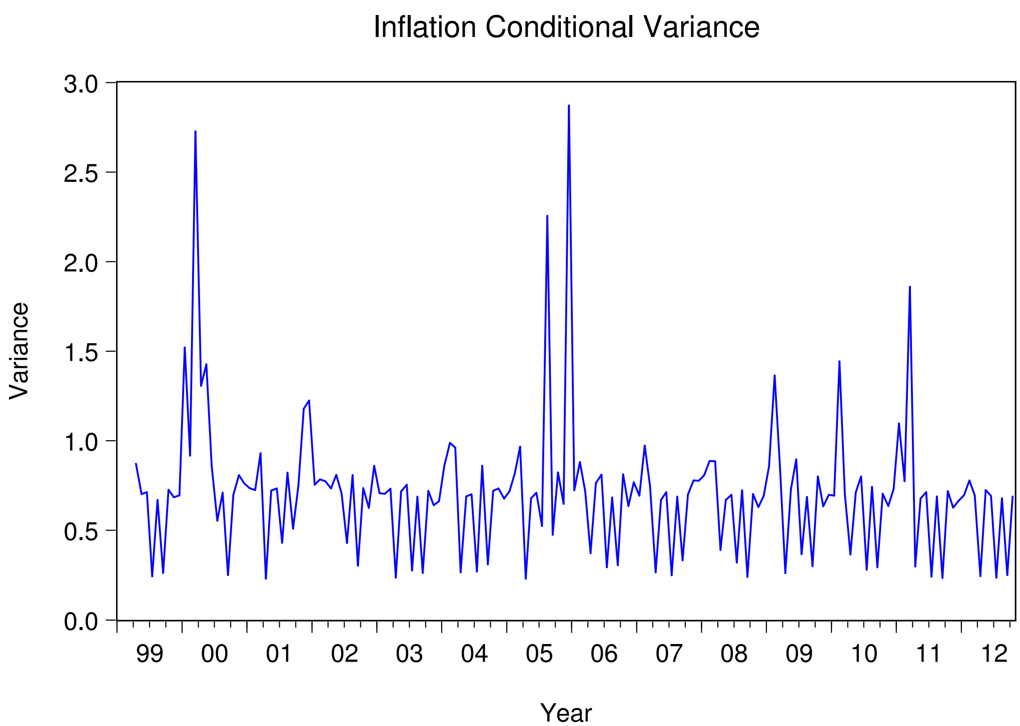

Figure 3: Variance Ratios

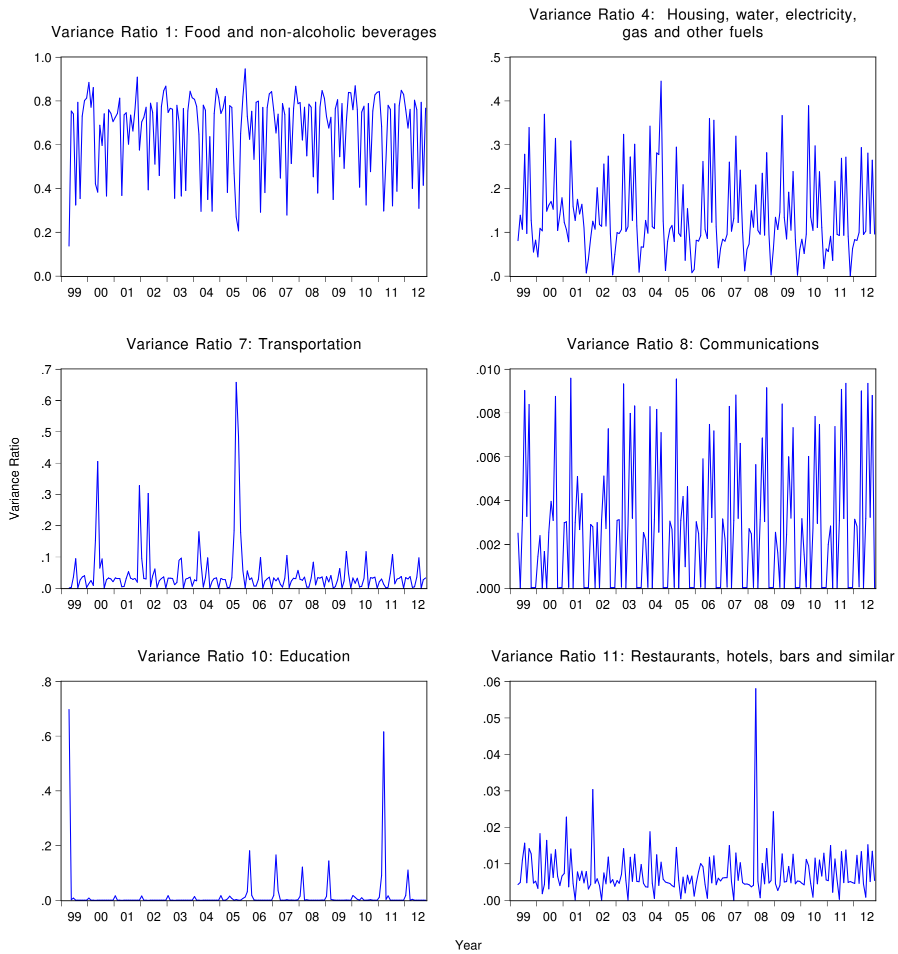

Figure 4: Seasonal Conditional Variance of Inflation

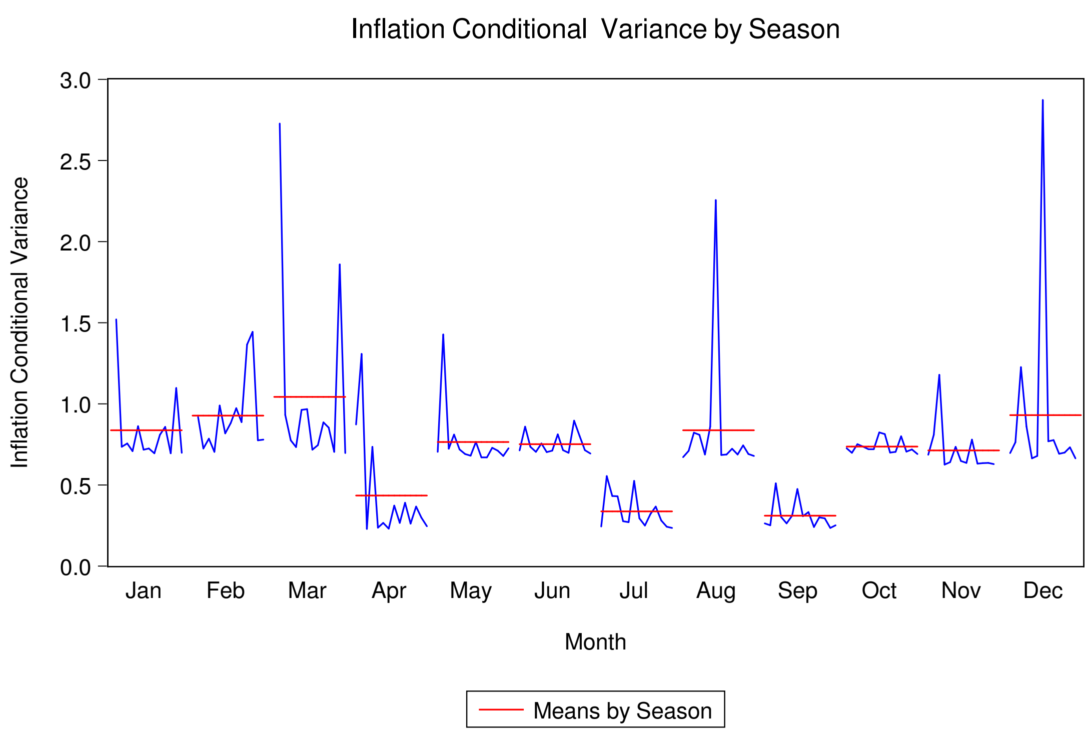

Figure 5: Seasonal Variance Ratios

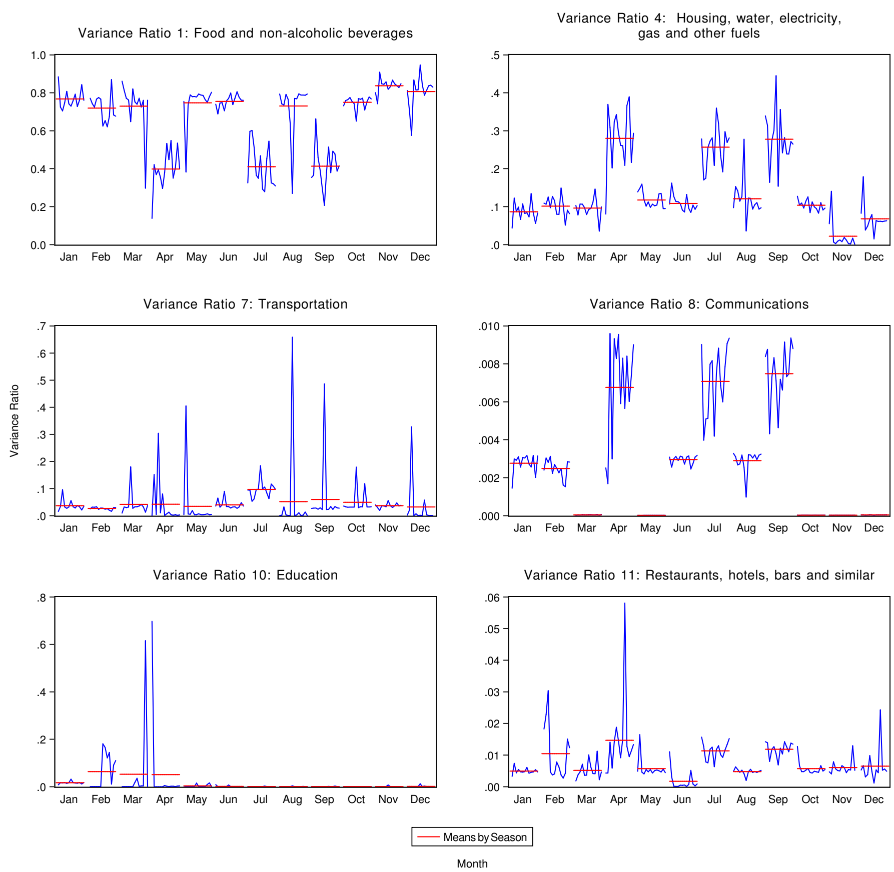

Figure 6: Conditional Correlations

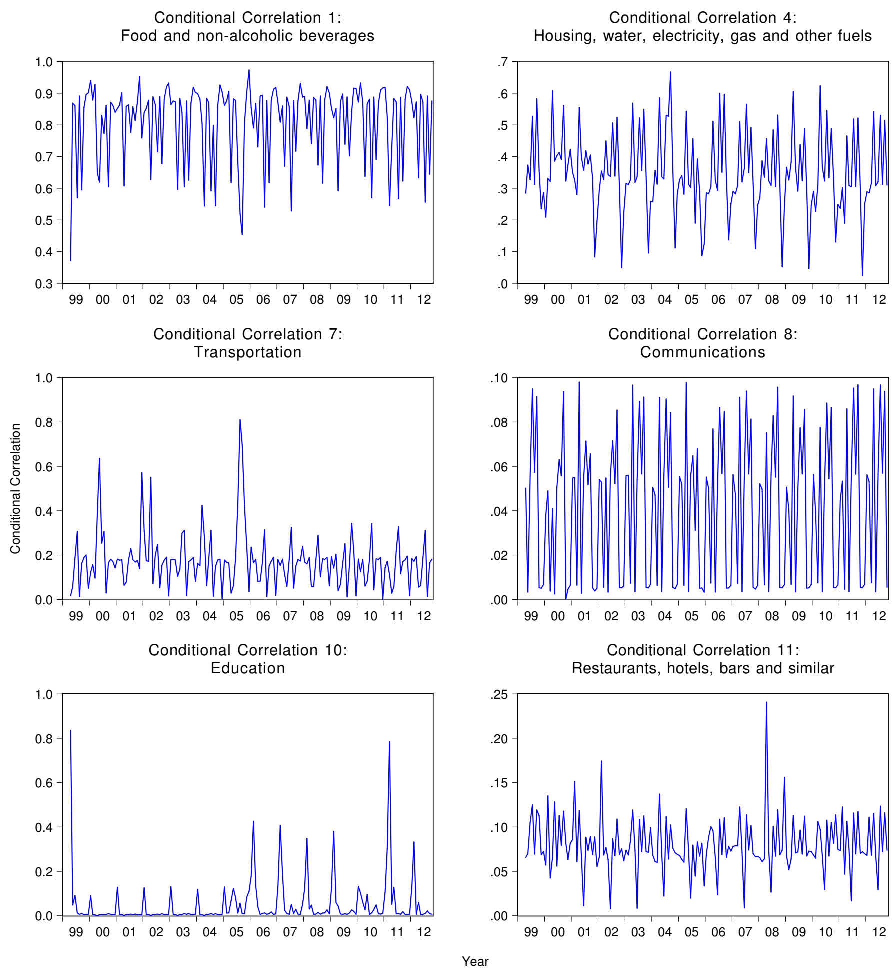

Figure 7: Seasonal Conditional Correlations