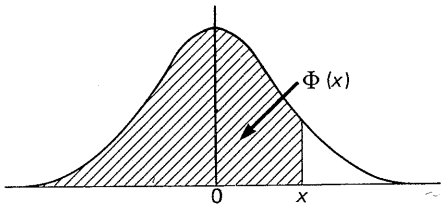
TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990	3·916	0·99995
3·105	0·9991	3·320	0·9995	3·759	0·99991	3·976	0·99996
3·138	0·9992	3·389	0·9996	3·791	0·99992	4·055	0·99997
3·174	0·9993	3·480	0·9997	3·826	0·99993	4·173	0·99998
3·215	0·9994	3·615	0·9998	3·867	0·99994	4·417	0·99999
			0·9999		0·99995		1·00000

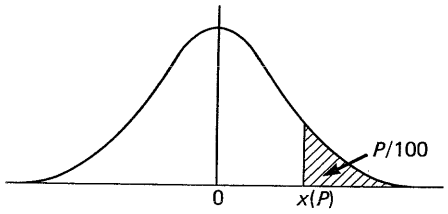
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

TABLE 7. THE χ^2-DISTRIBUTION FUNCTION

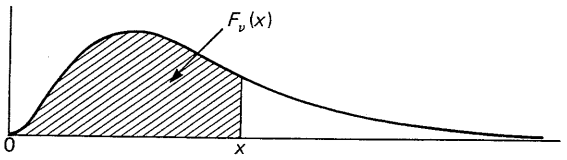
The function tabulated is

$$F_\nu(x) = \frac{1}{2^{\nu/2}\Gamma(\frac{\nu}{2})} \int_0^x t^{\frac{1}{2}\nu-1} e^{-\frac{1}{2}t}\, dt$$

for integer $\nu \leqslant 25$. $F_\nu(x)$ is the probability that a random variable X, distributed as χ^2 with ν degrees of freedom, will be less than or equal to x. Note that $F_1(x) = 2\Phi(x^{\frac{1}{2}}) - 1$ (cf. Table 4). For certain values of x and $\nu > 25$ use may be made of the following relation between the χ^2- and Poisson distributions:

$$F_\nu(x) = 1 - F(\tfrac{1}{2}\nu - 1 \,|\, \tfrac{1}{2}x)$$

where $F(r|\mu)$ is the Poisson distribution function (see Table 2). If $\nu > 25$, X is approximately normally distributed



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

with mean ν and variance 2ν. A better approximation is usually obtained by using the formula

$$F_\nu(x) \doteqdot \Phi(\sqrt{2x} - \sqrt{2\nu - 1})$$

where $\Phi(s)$ is the normal distribution function (see Table 4).

Omitted entries to the left and right of tabulated values are 1 and 0 respectively (to four decimal places).

$\nu =$	1	$\nu =$	1	$\nu =$	2	$\nu =$	2	$\nu =$	3	$\nu =$	3
$x = 0{\cdot}0$	0·0000	$x = 4{\cdot}0$	0·9545	$x = 0{\cdot}0$	0·0000	$x = 4{\cdot}0$	0·8647	$x = 0{\cdot}0$	0·0000	$x = 4{\cdot}0$	0·7385
·1	·2482	·1	·9571	·1	·0488	·1	·8713	·1	·0082	·2	·7593
·2	·3453	·2	·9596	·2	·0952	·2	·8775	·2	·0224	·4	·7786
·3	·4161	·3	·9619	·3	·1393	·3	·8835	·3	·0400	·6	·7965
·4	·4729	·4	·9641	·4	·1813	·4	·8892	·4	·0598	·8	·8130
0·5	0·5205	4·5	0·9661	0·5	0·2212	4·5	0·8946	0·5	0·0811	5·0	0·8282
·6	·5614	·6	·9680	·6	·2592	·6	·8997	·6	·1036	·2	·8423
·7	·5972	·7	·9698	·7	·2953	·7	·9046	·7	·1268	·4	·8553
·8	·6289	·8	·9715	·8	·3297	·8	·9093	·8	·1505	·6	·8672
·9	·6572	·9	·9731	·9	·3624	·9	·9137	·9	·1746	·8	·8782
1·0	0·6827	5·0	0·9747	1·0	0·3935	5·0	0·9179	1·0	0·1987	6·0	0·8884
·1	·7057	·1	·9761	·1	·4231	·1	·9219	·1	·2229	·2	·8977
·2	·7267	·2	·9774	·2	·4512	·2	·9257	·2	·2470	·4	·9063
·3	·7458	·3	·9787	·3	·4780	·3	·9293	·3	·2709	·6	·9142
·4	·7633	·4	·9799	·4	·5034	·4	·9328	·4	·2945	·8	·9214
1·5	0·7793	5·5	0·9810	1·5	0·5276	5·5	0·9361	1·5	0·3177	7·0	0·9281
·6	·7941	·6	·9820	·6	·5507	·6	·9392	·6	·3406	·2	·9342
·7	·8077	·7	·9830	·7	·5726	·7	·9422	·7	·3631	·4	·9398
·8	·8203	·8	·9840	·8	·5934	·8	·9450	·8	·3851	·6	·9450
·9	·8319	·9	·9849	·9	·6133	·9	·9477	·9	·4066	·8	·9497
2·0	0·8427	6·0	0·9857	2·0	0·6321	6·0	0·9502	2·0	0·4276	8·0	0·9540
·1	·8527	·1	·9865	·1	·6501	·2	·9550	·1	·4481	·2	·9579
·2	·8620	·2	·9872	·2	·6671	·4	·9592	·2	·4681	·4	·9616
·3	·8706	·3	·9879	·3	·6834	·6	·9631	·3	·4875	·6	·9649
·4	·8787	·4	·9886	·4	·6988	·8	·9666	·4	·5064	·8	·9679
2·5	0·8862	6·5	0·9892	2·5	0·7135	7·0	0·9698	2·5	0·5247	9·0	0·9707
·6	·8931	·6	·9898	·6	·7275	·2	·9727	·6	·5425	·2	·9733
·7	·8997	·7	·9904	·7	·7408	·4	·9753	·7	·5598	·4	·9756
·8	·9057	·8	·9909	·8	·7534	·6	·9776	·8	·5765	·6	·9777
·9	·9114	·9	·9914	·9	·7654	·8	·9798	·9	·5927	·8	·9797
3·0	0·9167	7·0	0·9918	3·0	0·7769	8·0	0·9817	3·0	0·6084	10·0	0·9814
·1	·9217	·1	·9923	·1	·7878	·2	·9834	·1	·6235	·2	·9831
·2	·9264	·2	·9927	·2	·7981	·4	·9850	·2	·6382	·4	·9845
·3	·9307	·3	·9931	·3	·8080	·6	·9864	·3	·6524	·6	·9859
·4	·9348	·4	·9935	·4	·8173	·8	·9877	·4	·6660	·8	·9871
3·5	0·9386	7·5	0·9938	3·5	0·8262	9·0	0·9889	3·5	0·6792	11·0	0·9883
·6	·9422	·6	·9942	·6	·8347	·2	·9899	·6	·6920	·2	·9893
·7	·9456	·7	·9945	·7	·8428	·4	·9909	·7	·7043	·4	·9903
·8	·9487	·8	·9948	·8	·8504	·6	·9918	·8	·7161	·6	·9911
·9	·9517	·9	·9951	·9	·8577	·8	·9926	·9	·7275	·8	·9919
4·0	0·9545	8·0	0·9953	4·0	0·8647	10·0	0·9933	4·0	0·7385	12·0	0·9926

TABLE 7. THE χ^2-DISTRIBUTION FUNCTION

$\nu =$	4	5	6	7	8	9	10	11	12	13	14
$x = 0{\cdot}5$	0·0265	0·0079	0·0022	0·0006	0·0001						
1·0	·0902	·0374	·0144	·0052	·0018	0·0006	0·0002	0·0001			
1·5	·1734	·0869	·0405	·0177	·0073	·0029	·0011	·0004	0·0001		
2·0	·2642	·1509	·0803	·0402	·0190	·0085	·0037	·0015	·0006	0·0002	0·0001
2·5	0·3554	0·2235	0·1315	0·0729	0·0383	0·0191	0·0091	0·0042	0·0018	0·0008	0·0003
3·0	·4422	·3000	·1912	·1150	·0656	·0357	·0186	·0093	·0045	·0021	·0009
3·5	·5221	·3766	·2560	·1648	·1008	·0589	·0329	·0177	·0091	·0046	·0022
4·0	·5940	·4506	·3233	·2202	·1429	·0886	·0527	·0301	·0166	·0088	·0045
4·5	·6575	·5201	·3907	·2793	·1906	·1245	·0780	·0471	·0274	·0154	·0084
5·0	0·7127	0·5841	0·4562	0·3400	0·2424	0·1657	0·1088	0·0688	0·0420	0·0248	0·0142
5·5	·7603	·6421	·5185	·4008	·2970	·2113	·1446	·0954	·0608	·0375	·0224
6·0	·8009	·6938	·5768	·4603	·3528	·2601	·1847	·1266	·0839	·0538	·0335
6·5	·8352	·7394	·6304	·5173	·4086	·3110	·2283	·1620	·1112	·0739	·0477
7·0	·8641	·7794	·6792	·5711	·4634	·3629	·2746	·2009	·1424	·0978	·0653
7·5	0·8883	0·8140	0·7229	0·6213	0·5162	0·4148	0·3225	0·2427	0·1771	0·1254	0·0863
8·0	·9084	·8438	·7619	·6674	·5665	·4659	·3712	·2867	·2149	·1564	·1107
8·5	·9251	·8693	·7963	·7094	·6138	·5154	·4199	·3321	·2551	·1904	·1383
9·0	·9389	·8909	·8264	·7473	·6577	·5627	·4679	·3781	·2971	·2271	·1689
9·5	·9503	·9093	·8527	·7813	·6981	·6075	·5146	·4242	·3403	·2658	·2022
10·0	0·9596	0·9248	0·8753	0·8114	0·7350	0·6495	0·5595	0·4696	0·3840	0·3061	0·2378
10·5	·9672	·9378	·8949	·8380	·7683	·6885	·6022	·5140	·4278	·3474	·2752
11·0	·9734	·9486	·9116	·8614	·7983	·7243	·6425	·5567	·4711	·3892	·3140
11·5	·9785	·9577	·9259	·8818	·8251	·7570	·6801	·5976	·5134	·4310	·3536
12·0	·9826	·9652	·9380	·8994	·8488	·7867	·7149	·6364	·5543	·4724	·3937
12·5	0·9860	0·9715	0·9483	0·9147	0·8697	0·8134	0·7470	0·6727	0·5936	0·5129	0·4338
13·0	·9887	·9766	·9570	·9279	·8882	·8374	·7763	·7067	·6310	·5522	·4735
13·5	·9909	·9809	·9643	·9392	·9042	·8587	·8030	·7381	·6662	·5900	·5124
14·0	·9927	·9844	·9704	·9488	·9182	·8777	·8270	·7670	·6993	·6262	·5503
14·5	·9941	·9873	·9755	·9570	·9304	·8944	·8486	·7935	·7301	·6604	·5868
15·0	0·9953	0·9896	0·9797	0·9640	0·9409	0·9091	0·8679	0·8175	0·7586	0·6926	0·6218
15·5	·9962	·9916	·9833	·9699	·9499	·9219	·8851	·8393	·7848	·7228	·6551
16·0	·9970	·9932	·9862	·9749	·9576	·9331	·9004	·8589	·8088	·7509	·6866
16·5	·9976	·9944	·9887	·9791	·9642	·9429	·9138	·8764	·8306	·7768	·7162
17·0	·9981	·9955	·9907	·9826	·9699	·9513	·9256	·8921	·8504	·8007	·7438
17·5	0·9985	0·9964	0·9924	0·9856	0·9747	0·9586	0·9360	0·9061	0·8683	0·8226	0·7695
18·0	·9988	·9971	·9938	·9880	·9788	·9648	·9450	·9184	·8843	·8425	·7932
18·5	·9990	·9976	·9949	·9901	·9822	·9702	·9529	·9293	·8987	·8606	·8151
19·0	·9992	·9981	·9958	·9918	·9851	·9748	·9597	·9389	·9115	·8769	·8351
19·5	·9994	·9984	·9966	·9932	·9876	·9787	·9656	·9473	·9228	·8916	·8533
20	0·9995	0·9988	0·9972	0·9944	0·9897	0·9821	0·9707	0·9547	0·9329	0·9048	0·8699
21	·9997	·9992	·9982	·9962	·9929	·9873	·9789	·9666	·9496	·9271	·8984
22	·9998	·9995	·9988	·9975	·9951	·9911	·9849	·9756	·9625	·9446	·9214
23	·9999	·9997	·9992	·9983	·9966	·9938	·9893	·9823	·9723	·9583	·9397
24	·9999	·9998	·9995	·9989	·9977	·9957	·9924	·9873	·9797	·9689	·9542
25	0·9999	0·9999	0·9997	0·9992	0·9984	0·9970	0·9947	0·9909	0·9852	0·9769	0·9654
26		·9999	·9998	·9995	·9989	·9980	·9963	·9935	·9893	·9830	·9741
27		·9999	·9999	·9997	·9993	·9986	·9974	·9954	·9923	·9876	·9807
28			·9999	·9998	·9995	·9990	·9982	·9968	·9945	·9910	·9858
29			·9999	·9999	·9997	·9994	·9988	·9977	·9961	·9935	·9895
30				0·9999	0·9998	0·9996	0·9991	0·9984	0·9972	0·9953	0·9924

TABLE 7. THE χ^2-DISTRIBUTION FUNCTION

$\nu =$	15	16	17	18	19	20	21	22	23	24	25
$x = 3$	0·0004	0·0002	0·0001								
4	·0023	·0011	·0005	0·0002	0·0001						
5	0·0079	0·0042	0·0022	0·0011	0·0006	0·0003	0·0001	0·0001			
6	·0203	·0119	·0068	·0038	·0021	·0011	·0006	·0003	0·0001	0·0001	
7	·0424	·0267	·0165	·0099	·0058	·0033	·0019	·0010	·0005	·0003	0·0001
8	·0762	·0511	·0335	·0214	·0133	·0081	·0049	·0028	·0016	·0009	·0005
9	·1225	·0866	·0597	·0403	·0265	·0171	·0108	·0067	·0040	·0024	·0014
10	0·1803	0·1334	0·0964	0·0681	0·0471	0·0318	0·0211	0·0137	0·0087	0·0055	0·0033
11	·2474	·1905	·1434	·1056	·0762	·0538	·0372	·0253	·0168	·0110	·0071
12	·3210	·2560	·1999	·1528	·1144	·0839	·0604	·0426	·0295	·0201	·0134
13	·3977	·3272	·2638	·2084	·1614	·1226	·0914	·0668	·0480	·0339	·0235
14	·4745	·4013	·3329	·2709	·2163	·1695	·1304	·0985	·0731	·0533	·0383
15	0·5486	0·4754	0·4045	0·3380	0·2774	0·2236	0·1770	0·1378	0·1054	0·0792	0·0586
16	·6179	·5470	·4762	·4075	·3427	·2834	·2303	·1841	·1447	·1119	·0852
17	·6811	·6144	·5456	·4769	·4101	·3470	·2889	·2366	·1907	·1513	·1182
18	·7373	·6761	·6112	·5443	·4776	·4126	·3510	·2940	·2425	·1970	·1576
19	·7863	·7313	·6715	·6082	·5432	·4782	·4149	·3547	·2988	·2480	·2029
20	0·8281	0·7798	0·7258	0·6672	0·6054	0·5421	0·4787	0·4170	0·3581	0·3032	0·2532
21	·8632	·8215	·7737	·7206	·6632	·6029	·5411	·4793	·4189	·3613	·3074
22	·8922	·8568	·8153	·7680	·7157	·6595	·6005	·5401	·4797	·4207	·3643
23	·9159	·8863	·8507	·8094	·7627	·7112	·6560	·5983	·5392	·4802	·4224
24	·9349	·9105	·8806	·8450	·8038	·7576	·7069	·6528	·5962	·5384	·4806
25	0·9501	0·9302	0·9053	0·8751	0·8395	0·7986	0·7528	0·7029	0·6497	0·5942	0·5376
26	·9620	·9460	·9255	·9002	·8698	·8342	·7936	·7483	·6991	·6468	·5924
27	·9713	·9585	·9419	·9210	·8953	·8647	·8291	·7888	·7440	·6955	·6441
28	·9784	·9684	·9551	·9379	·9166	·8906	·8598	·8243	·7842	·7400	·6921
29	·9839	·9761	·9655	·9516	·9340	·9122	·8860	·8551	·8197	·7799	·7361
30	0·9881	0·9820	0·9737	0·9626	0·9482	0·9301	0·9080	0·8815	0·8506	0·8152	0·7757
31	·9912	·9865	·9800	·9712	·9596	·9448	·9263	·9039	·8772	·8462	·8110
32	·9936	·9900	·9850	·9780	·9687	·9567	·9414	·9226	·8999	·8730	·8420
33	·9953	·9926	·9887	·9833	·9760	·9663	·9538	·9381	·9189	·8959	·8689
34	·9966	·9946	·9916	·9874	·9816	·9739	·9638	·9509	·9348	·9153	·8921
35	0·9975	0·9960	0·9938	0·9905	0·9860	0·9799	0·9718	0·9613	0·9480	0·9316	0·9118
36	·9982	·9971	·9954	·9929	·9894	·9846	·9781	·9696	·9587	·9451	·9284
37	·9987	·9979	·9966	·9948	·9921	·9883	·9832	·9763	·9675	·9562	·9423
38	·9991	·9985	·9975	·9961	·9941	·9911	·9871	·9817	·9745	·9653	·9537
39	·9994	·9989	·9982	·9972	·9956	·9933	·9902	·9859	·9802	·9727	·9632
40	0·9995	0·9992	0·9987	0·9979	0·9967	0·9950	0·9926	0·9892	0·9846	0·9786	0·9708
41	·9997	·9994	·9991	·9985	·9976	·9963	·9944	·9918	·9882	·9833	·9770
42	·9998	·9996	·9993	·9989	·9982	·9972	·9958	·9937	·9909	·9871	·9820
43	·9998	·9997	·9995	·9992	·9987	·9980	·9969	·9953	·9931	·9901	·9860
44	·9999	·9998	·9997	·9994	·9991	·9985	·9977	·9965	·9947	·9924	·9892
45	0·9999	0·9999	0·9998	0·9996	0·9993	0·9989	0·9983	0·9973	0·9960	0·9942	0·9916
46	·9999	·9999	·9998	·9997	·9995	·9992	·9987	·9980	·9970	·9956	·9936
47		·9999	·9999	·9998	·9996	·9994	·9991	·9985	·9978	·9967	·9951
48			·9999	·9998	·9997	·9996	·9993	·9989	·9983	·9975	·9963
49			·9999	·9999	·9998	·9997	·9995	·9992	·9988	·9981	·9972
50				0·9999	0·9999	0·9998	0·9996	0·9994	0·9991	0·9986	0·9979

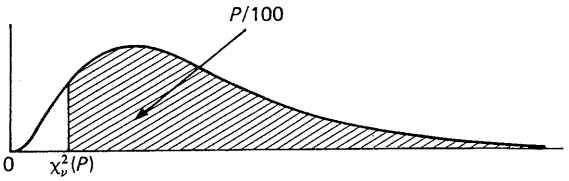
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} \int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}x}\, dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	99·95	99·9	99·5	99	97·5	95	90	80	70	60
$\nu = 1$	$0{\cdot}0^6 3927$	$0{\cdot}0^5 1571$	$0{\cdot}0^4 3927$	$0{\cdot}0^3 1571$	$0{\cdot}0^3 9821$	0·003932	0·01579	0·06418	0·1485	0·2750
2	0·001000	0·002001	0·01003	0·02010	0·05064	0·1026	0·2107	0·4463	0·7133	1·022
3	0·01528	0·02430	0·07172	0·1148	0·2158	0·3518	0·5844	1·005	1·424	1·869
4	0·06392	0·09080	0·2070	0·2971	0·4844	0·7107	1·064	1·649	2·195	2·753
5	0·1581	0·2102	0·4117	0·5543	0·8312	1·145	1·610	2·343	3·000	3·655
6	0·2994	0·3811	0·6757	0·8721	1·237	1·635	2·204	3·070	3·828	4·570
7	0·4849	0·5985	0·9893	1·239	1·690	2·167	2·833	3·822	4·671	5·493
8	0·7104	0·8571	1·344	1·646	2·180	2·733	3·490	4·594	5·527	6·423
9	0·9717	1·152	1·735	2·088	2·700	3·325	4·168	5·380	6·393	7·357
10	1·265	1·479	2·156	2·558	3·247	3·940	4·865	6·179	7·267	8·295
11	1·587	1·834	2·603	3·053	3·816	4·575	5·578	6·989	8·148	9·237
12	1·934	2·214	3·074	3·571	4·404	5·226	6·304	7·807	9·034	10·18
13	2·305	2·617	3·565	4·107	5·009	5·892	7·042	8·634	9·926	11·13
14	2·697	3·041	4·075	4·660	5·629	6·571	7·790	9·467	10·82	12·08
15	3·108	3·483	4·601	5·229	6·262	7·261	8·547	10·31	11·72	13·03
16	3·536	3·942	5·142	5·812	6·908	7·962	9·312	11·15	12·62	13·98
17	3·980	4·416	5·697	6·408	7·564	8·672	10·09	12·00	13·53	14·94
18	4·439	4·905	6·265	7·015	8·231	9·390	10·86	12·86	14·44	15·89
19	4·912	5·407	6·844	7·633	8·907	10·12	11·65	13·72	15·35	16·85
20	5·398	5·921	7·434	8·260	9·591	10·85	12·44	14·58	16·27	17·81
21	5·896	6·447	8·034	8·897	10·28	11·59	13·24	15·44	17·18	18·77
22	6·404	6·983	8·643	9·542	10·98	12·34	14·04	16·31	18·10	19·73
23	6·924	7·529	9·260	10·20	11·69	13·09	14·85	17·19	19·02	20·69
24	7·453	8·085	9·886	10·86	12·40	13·85	15·66	18·06	19·94	21·65
25	7·991	8·649	10·52	11·52	13·12	14·61	16·47	18·94	20·87	22·62
26	8·538	9·222	11·16	12·20	13·84	15·38	17·29	19·82	21·79	23·58
27	9·093	9·803	11·81	12·88	14·57	16·15	18·11	20·70	22·72	24·54
28	9·656	10·39	12·46	13·56	15·31	16·93	18·94	21·59	23·65	25·51
29	10·23	10·99	13·12	14·26	16·05	17·71	19·77	22·48	24·58	26·48
30	10·80	11·59	13·79	14·95	16·79	18·49	20·60	23·36	25·51	27·44
32	11·98	12·81	15·13	16·36	18·29	20·07	22·27	25·15	27·37	29·38
34	13·18	14·06	16·50	17·79	19·81	21·66	23·95	26·94	29·24	31·31
36	14·40	15·32	17·89	19·23	21·34	23·27	25·64	28·73	31·12	33·25
38	15·64	16·61	19·29	20·69	22·88	24·88	27·34	30·54	32·99	35·19
40	16·91	17·92	20·71	22·16	24·43	26·51	29·05	32·34	34·87	37·13
50	23·46	24·67	27·99	29·71	32·36	34·76	37·69	41·45	44·31	46·86
60	30·34	31·74	35·53	37·48	40·48	43·19	46·46	50·64	53·81	56·62
70	37·47	39·04	43·28	45·44	48·76	51·74	55·33	59·90	63·35	66·40
80	44·79	46·52	51·17	53·54	57·15	60·39	64·28	69·21	72·92	76·19
90	52·28	54·16	59·20	61·75	65·65	69·13	73·29	78·56	82·51	85·99
100	59·90	61·92	67·33	70·06	74·22	77·93	82·36	87·95	92·13	95·81

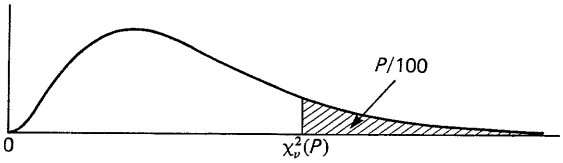
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} \int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1}\, e^{-\frac{1}{2}x}\, dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	50	40	30	20	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·4549	0·7083	1·074	1·642	2·706	3·841	5·024	6·635	7·879	10·83	12·12
2	1·386	1·833	2·408	3·219	4·605	5·991	7·378	9·210	10·60	13·82	15·20
3	2·366	2·946	3·665	4·642	6·251	7·815	9·348	11·34	12·84	16·27	17·73
4	3·357	4·045	4·878	5·989	7·779	9·488	11·14	13·28	14·86	18·47	20·00
5	4·351	5·132	6·064	7·289	9·236	11·07	12·83	15·09	16·75	20·52	22·11
6	5·348	6·211	7·231	8·558	10·64	12·59	14·45	16·81	18·55	22·46	24·10
7	6·346	7·283	8·383	9·803	12·02	14·07	16·01	18·48	20·28	24·32	26·02
8	7·344	8·351	9·524	11·03	13·36	15·51	17·53	20·09	21·95	26·12	27·87
9	8·343	9·414	10·66	12·24	14·68	16·92	19·02	21·67	23·59	27·88	29·67
10	9·342	10·47	11·78	13·44	15·99	18·31	20·48	23·21	25·19	29·59	31·42
11	10·34	11·53	12·90	14·63	17·28	19·68	21·92	24·72	26·76	31·26	33·14
12	11·34	12·58	14·01	15·81	18·55	21·03	23·34	26·22	28·30	32·91	34·82
13	12·34	13·64	15·12	16·98	19·81	22·36	24·74	27·69	29·82	34·53	36·48
14	13·34	14·69	16·22	18·15	21·06	23·68	26·12	29·14	31·32	36·12	38·11
15	14·34	15·73	17·32	19·31	22·31	25·00	27·49	30·58	32·80	37·70	39·72
16	15·34	16·78	18·42	20·47	23·54	26·30	28·85	32·00	34·27	39·25	41·31
17	16·34	17·82	19·51	21·61	24·77	27·59	30·19	33·41	35·72	40·79	42·88
18	17·34	18·87	20·60	22·76	25·99	28·87	31·53	34·81	37·16	42·31	44·43
19	18·34	19·91	21·69	23·90	27·20	30·14	32·85	36·19	38·58	43·82	45·97
20	19·34	20·95	22·77	25·04	28·41	31·41	34·17	37·57	40·00	45·31	47·50
21	20·34	21·99	23·86	26·17	29·62	32·67	35·48	38·93	41·40	46·80	49·01
22	21·34	23·03	24·94	27·30	30·81	33·92	36·78	40·29	42·80	48·27	50·51
23	22·34	24·07	26·02	28·43	32·01	35·17	38·08	41·64	44·18	49·73	52·00
24	23·34	25·11	27·10	29·55	33·20	36·42	39·36	42·98	45·56	51·18	53·48
25	24·34	26·14	28·17	30·68	34·38	37·65	40·65	44·31	46·93	52·62	54·95
26	25·34	27·18	29·25	31·79	35·56	38·89	41·92	45·64	48·29	54·05	56·41
27	26·34	28·21	30·32	32·91	36·74	40·11	43·19	46·96	49·64	55·48	57·86
28	27·34	29·25	31·39	34·03	37·92	41·34	44·46	48·28	50·99	56·89	59·30
29	28·34	30·28	32·46	35·14	39·09	42·56	45·72	49·59	52·34	58·30	60·73
30	29·34	31·32	33·53	36·25	40·26	43·77	46·98	50·89	53·67	59·70	62·16
32	31·34	33·38	35·66	38·47	42·58	46·19	49·48	53·49	56·33	62·49	65·00
34	33·34	35·44	37·80	40·68	44·90	48·60	51·97	56·06	58·96	65·25	67·80
36	35·34	37·50	39·92	42·88	47·21	51·00	54·44	58·62	61·58	67·99	70·59
38	37·34	39·56	42·05	45·08	49·51	53·38	56·90	61·16	64·18	70·70	73·35
40	39·34	41·62	44·16	47·27	51·81	55·76	59·34	63·69	66·77	73·40	76·09
50	49·33	51·89	54·72	58·16	63·17	67·50	71·42	76·15	79·49	86·66	89·56
60	59·33	62·13	65·23	68·97	74·40	79·08	83·30	88·38	91·95	99·61	102·7
70	69·33	72·36	75·69	79·71	85·53	90·53	95·02	100·4	104·2	112·3	115·6
80	79·33	82·57	86·12	90·41	96·58	101·9	106·6	112·3	116·3	124·8	128·3
90	89·33	92·76	96·52	101·1	107·6	113·1	118·1	124·1	128·3	137·2	140·8
100	99·33	102·9	106·9	111·7	118·5	124·3	129·6	135·8	140·2	149·4	153·2

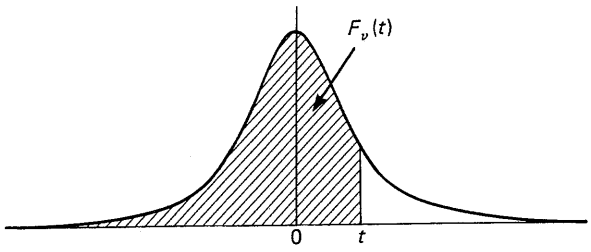
TABLE 9. THE t-DISTRIBUTION FUNCTION

The function tabulated is

$$F_\nu(t) = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{-\infty}^{t} \frac{ds}{(1+s^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

$F_\nu(t)$ is the probability that a random variable, distributed as t with ν degrees of freedom, will be less than or equal to t. When $t < 0$ use $F_\nu(t) = 1 - F_\nu(-t)$, the t distribution being symmetric about zero.

The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance (see Table 4). When ν is large interpolation in ν should be harmonic.



Omitted entries to the right of tabulated values are 1 (to four decimal places).

ν =	1	ν =	1	ν =	2	ν =	2	ν =	3	ν =	3
t = 0·0	0·5000	t = 4·0	0·9220	t = 0·0	0·5000	t = 4·0	0·9714	t = 0·0	0·5000	t = 4·0	0·9860
·1	·5317	4·2	·9256	·1	·5353	·1	·9727	·1	·5367	·1	·9869
·2	·5628	4·4	·9289	·2	·5700	·2	·9739	·2	·5729	·2	·9877
·3	·5928	4·6	·9319	·3	·6038	·3	·9750	·3	·6081	·3	·9884
·4	·6211	4·8	·9346	·4	·6361	·4	·9760	·4	·6420	·4	·9891
0·5	0·6476	5·0	0·9372	0·5	0·6667	4·5	0·9770	0·5	0·6743	4·5	0·9898
·6	·6720	5·5	·9428	·6	·6953	·6	·9779	·6	·7046	·6	·9903
·7	·6944	6·0	·9474	·7	·7218	·7	·9788	·7	·7328	·7	·9909
·8	·7148	6·5	·9514	·8	·7462	·8	·9796	·8	·7589	·8	·9914
·9	·7333	7·0	·9548	·9	·7684	·9	·9804	·9	·7828	·9	·9919
1·0	0·7500	7·5	0·9578	1·0	0·7887	5·0	0·9811	1·0	0·8045	5·0	0·9923
·1	·7651	8·0	·9604	·1	·8070	·1	·9818	·1	·8242	·1	·9927
·2	·7789	8·5	·9627	·2	·8235	·2	·9825	·2	·8419	·2	·9931
·3	·7913	9·0	·9648	·3	·8384	·3	·9831	·3	·8578	·3	·9934
·4	·8026	9·5	·9666	·4	·8518	·4	·9837	·4	·8720	·4	·9938
1·5	0·8128	10·0	0·9683	1·5	0·8638	5·5	0·9842	1·5	0·8847	5·5	0·9941
·6	·8222	10·5	·9698	·6	·8746	·6	·9848	·6	·8960	·6	·9944
·7	·8307	11·0	·9711	·7	·8844	·7	·9853	·7	·9062	·7	·9946
·8	·8386	11·5	·9724	·8	·8932	·8	·9858	·8	·9152	·8	·9949
·9	·8458	12·0	·9735	·9	·9011	·9	·9862	·9	·9232	·9	·9951
2·0	0·8524	12·5	0·9746	2·0	0·9082	6·0	0·9867	2·0	0·9303	6·0	0·9954
·1	·8585	13·0	·9756	·1	·9147	·1	·9871	·1	·9367	·1	·9956
·2	·8642	13·5	·9765	·2	·9206	·2	·9875	·2	·9424	·2	·9958
·3	·8695	14·0	·9773	·3	·9259	·3	·9879	·3	·9475	·3	·9960
·4	·8743	14·5	·9781	·4	·9308	·4	·9882	·4	·9521	·4	·9961
2·5	0·8789	15	0·9788	2·5	0·9352	6·5	0·9886	2·5	0·9561	6·5	0·9963
·6	·8831	16	·9801	·6	·9392	·6	·9889	·6	·9598	·6	·9965
·7	·8871	17	·9813	·7	·9429	·7	·9892	·7	·9631	·7	·9966
·8	·8908	18	·9823	·8	·9463	·8	·9895	·8	·9661	·8	·9967
·9	·8943	19	·9833	·9	·9494	·9	·9898	·9	·9687	·9	·9969
3·0	0·8976	20	0·9841	3·0	0·9523	7·0	0·9901	3·0	0·9712	7·0	0·9970
·1	·9007	21	·9849	·1	·9549	·1	·9904	·1	·9734	·1	·9971
·2	·9036	22	·9855	·2	·9573	·2	·9906	·2	·9753	·2	·9972
·3	·9063	23	·9862	·3	·9596	·3	·9909	·3	·9771	·3	·9973
·4	·9089	24	·9867	·4	·9617	·4	·9911	·4	·9788	·4	·9974
3·5	0·9114	25	0·9873	3·5	0·9636	7·5	0·9913	3·5	0·9803	7·5	0·9975
·6	·9138	30	·9894	·6	·9654	·6	·9916	·6	·9816	·6	·9976
·7	·9160	35	·9909	·7	·9670	·7	·9918	·7	·9829	·7	·9977
·8	·9181	40	·9920	·8	·9686	·8	·9920	·8	·9840	·8	·9978
·9	·9201	45	·9929	·9	·9701	·9	·9922	·9	·9850	·9	·9979
4·0	0·9220	50	0·9936	4·0	0·9714	8·0	0·9924	4·0	0·9860	8·0	0·9980

TABLE 9. THE t-DISTRIBUTION FUNCTION

$\nu =$	4	5	6	7	8	9	10	11	12	13	14
$t = 0.0$	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000
·1	·5374	·5379	·5382	·5384	·5386	·5387	·5388	·5389	·5390	·5391	·5391
·2	·5744	·5753	·5760	·5764	·5768	·5770	·5773	·5774	·5776	·5777	·5778
·3	·6104	·6119	·6129	·6136	·6141	·6145	·6148	·6151	·6153	·6155	·6157
·4	·6452	·6472	·6485	·6495	·6502	·6508	·6512	·6516	·6519	·6522	·6524
0·5	0·6783	0·6809	0·6826	0·6838	0·6847	0·6855	0·6861	0·6865	0·6869	0·6873	0·6876
·6	·7096	·7127	·7148	·7163	·7174	·7183	·7191	·7197	·7202	·7206	·7210
·7	·7387	·7424	·7449	·7467	·7481	·7492	·7501	·7508	·7514	·7519	·7523
·8	·7657	·7700	·7729	·7750	·7766	·7778	·7788	·7797	·7804	·7810	·7815
·9	·7905	·7953	·7986	·8010	·8028	·8042	·8054	·8063	·8071	·8078	·8083
1·0	0·8130	0·8184	0·8220	0·8247	0·8267	0·8283	0·8296	0·8306	0·8315	0·8322	0·8329
·1	·8335	·8393	·8433	·8461	·8483	·8501	·8514	·8526	·8535	·8544	·8551
·2	·8518	·8581	·8623	·8654	·8678	·8696	·8711	·8723	·8734	·8742	·8750
·3	·8683	·8748	·8793	·8826	·8851	·8870	·8886	·8899	·8910	·8919	·8927
·4	·8829	·8898	·8945	·8979	·9005	·9025	·9041	·9055	·9066	·9075	·9084
1·5	0·8960	0·9030	0·9079	0·9114	0·9140	0·9161	0·9177	0·9191	0·9203	0·9212	0·9221
·6	·9076	·9148	·9196	·9232	·9259	·9280	·9297	·9310	·9322	·9332	·9340
·7	·9178	·9251	·9300	·9335	·9362	·9383	·9400	·9414	·9426	·9435	·9444
·8	·9269	·9341	·9390	·9426	·9452	·9473	·9490	·9503	·9515	·9525	·9533
·9	·9349	·9421	·9469	·9504	·9530	·9551	·9567	·9580	·9591	·9601	·9609
2·0	0·9419	0·9490	0·9538	0·9572	0·9597	0·9617	0·9633	0·9646	0·9657	0·9666	0·9674
·1	·9482	·9551	·9598	·9631	·9655	·9674	·9690	·9702	·9712	·9721	·9728
·2	·9537	·9605	·9649	·9681	·9705	·9723	·9738	·9750	·9759	·9768	·9774
·3	·9585	·9651	·9694	·9725	·9748	·9765	·9779	·9790	·9799	·9807	·9813
·4	·9628	·9692	·9734	·9763	·9784	·9801	·9813	·9824	·9832	·9840	·9846
2·5	0·9666	0·9728	0·9767	0·9795	0·9815	0·9831	0·9843	0·9852	0·9860	0·9867	0·9873
·6	·9700	·9759	·9797	·9823	·9842	·9856	·9868	·9877	·9884	·9890	·9895
·7	·9730	·9786	·9822	·9847	·9865	·9878	·9888	·9897	·9903	·9909	·9914
·8	·9756	·9810	·9844	·9867	·9884	·9896	·9906	·9914	·9920	·9925	·9929
·9	·9779	·9831	·9863	·9885	·9901	·9912	·9921	·9928	·9933	·9938	·9942
3·0	0·9800	0·9850	0·9880	0·9900	0·9915	0·9925	0·9933	0·9940	0·9945	0·9949	0·9952
·1	·9819	·9866	·9894	·9913	·9927	·9936	·9944	·9949	·9954	·9958	·9961
·2	·9835	·9880	·9907	·9925	·9937	·9946	·9953	·9958	·9962	·9965	·9968
·3	·9850	·9893	·9918	·9934	·9946	·9954	·9960	·9965	·9968	·9971	·9974
·4	·9864	·9904	·9928	·9943	·9953	·9961	·9966	·9970	·9974	·9976	·9978
3·5	0·9876	0·9914	0·9936	0·9950	0·9960	0·9966	0·9971	0·9975	0·9978	0·9980	0·9982
·6	·9886	·9922	·9943	·9956	·9965	·9971	·9976	·9979	·9982	·9984	·9986
·7	·9896	·9930	·9950	·9962	·9970	·9975	·9979	·9982	·9985	·9987	·9988
·8	·9904	·9937	·9955	·9966	·9974	·9979	·9983	·9985	·9987	·9989	·9990
·9	·9912	·9943	·9960	·9971	·9977	·9982	·9985	·9988	·9989	·9991	·9992
4·0	0·9919	0·9948	0·9964	0·9974	0·9980	0·9984	0·9987	0·9990	0·9991	0·9992	0·9993
·1	·9926	·9953	·9968	·9977	·9983	·9987	·9989	·9991	·9993	·9994	·9995
·2	·9932	·9958	·9972	·9980	·9985	·9988	·9991	·9993	·9994	·9995	·9996
·3	·9937	·9961	·9975	·9982	·9987	·9990	·9992	·9994	·9995	·9996	·9996
·4	·9942	·9965	·9977	·9984	·9989	·9991	·9993	·9995	·9996	·9996	·9997
4·5	0·9946	0·9968	0·9979	0·9986	0·9990	0·9993	0·9994	0·9995	0·9996	0·9997	0·9998
·6	·9950	·9971	·9982	·9988	·9991	·9994	·9995	·9996	·9997	·9998	·9998
·7	·9953	·9973	·9983	·9989	·9992	·9994	·9996	·9997	·9997	·9998	·9998
·8	·9957	·9976	·9985	·9990	·9993	·9995	·9996	·9997	·9998	·9998	·9999
·9	·9960	·9978	·9986	·9991	·9994	·9996	·9997	·9998	·9998	·9999	·9999
5·0	0·9963	0·9979	0·9988	0·9992	0·9995	0·9996	0·9997	0·9998	0·9998	0·9999	0·9999

TABLE 9. THE *t*-DISTRIBUTION FUNCTION

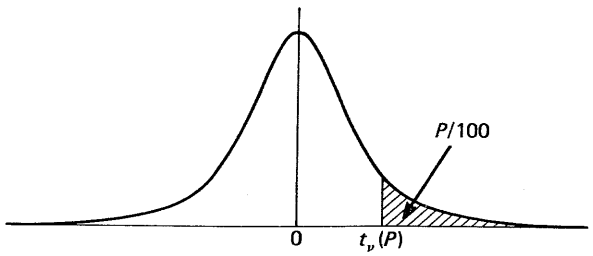
$\nu =$	15	16	17	18	19	20	24	30	40	60	∞
$t = 0.0$	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000	0·5000
·1	·5392	·5392	·5392	·5393	·5393	·5393	·5394	·5395	·5396	·5397	·5398
·2	·5779	·5780	·5781	·5781	·5782	·5782	·5784	·5786	·5788	·5789	·5793
·3	·6159	·6160	·6161	·6162	·6163	·6164	·6166	·6169	·6171	·6174	·6179
·4	·6526	·6528	·6529	·6531	·6532	·6533	·6537	·6540	·6544	·6547	·6554
0·5	0·6878	0·6881	0·6883	0·6884	0·6886	0·6887	0·6892	0·6896	0·6901	0·6905	0·6915
·6	·7213	·7215	·7218	·7220	·7222	·7224	·7229	·7235	·7241	·7246	·7257
·7	·7527	·7530	·7533	·7536	·7538	·7540	·7547	·7553	·7560	·7567	·7580
·8	·7819	·7823	·7826	·7829	·7832	·7834	·7842	·7850	·7858	·7866	·7881
·9	·8088	·8093	·8097	·8100	·8103	·8106	·8115	·8124	·8132	·8141	·8159
1·0	0·8334	0·8339	0·8343	0·8347	0·8351	0·8354	0·8364	0·8373	0·8383	0·8393	0·8413
·1	·8557	·8562	·8567	·8571	·8575	·8578	·8589	·8600	·8610	·8621	·8643
·2	·8756	·8762	·8767	·8772	·8776	·8779	·8791	·8802	·8814	·8826	·8849
·3	·8934	·8940	·8945	·8950	·8954	·8958	·8970	·8982	·8995	·9007	·9032
·4	·9091	·9097	·9103	·9107	·9112	·9116	·9128	·9141	·9154	·9167	·9192
1·5	0·9228	0·9235	0·9240	0·9245	0·9250	0·9254	0·9267	0·9280	0·9293	0·9306	0·9332
·6	·9348	·9354	·9360	·9365	·9370	·9374	·9387	·9400	·9413	·9426	·9452
·7	·9451	·9458	·9463	·9468	·9473	·9477	·9490	·9503	·9516	·9528	·9554
·8	·9540	·9546	·9552	·9557	·9561	·9565	·9578	·9590	·9603	·9616	·9641
·9	·9616	·9622	·9627	·9632	·9636	·9640	·9652	·9665	·9677	·9689	·9713
2·0	0·9680	0·9686	0·9691	0·9696	0·9700	0·9704	0·9715	0·9727	0·9738	0·9750	0·9772
·1	·9735	·9740	·9745	·9750	·9753	·9757	·9768	·9779	·9790	·9800	·9821
·2	·9781	·9786	·9790	·9794	·9798	·9801	·9812	·9822	·9832	·9842	·9861
·3	·9819	·9824	·9828	·9832	·9835	·9838	·9848	·9857	·9866	·9875	·9893
·4	·9851	·9855	·9859	·9863	·9866	·9869	·9877	·9886	·9894	·9902	·9918
2·5	0·9877	0·9882	0·9885	0·9888	0·9891	0·9894	0·9902	0·9909	0·9917	0·9924	0·9938
·6	·9900	·9903	·9907	·9910	·9912	·9914	·9921	·9928	·9935	·9941	·9953
·7	·9918	·9921	·9924	·9927	·9929	·9931	·9937	·9944	·9949	·9955	·9965
·8	·9933	·9936	·9938	·9941	·9943	·9945	·9950	·9956	·9961	·9966	·9974
·9	·9945	·9948	·9950	·9952	·9954	·9956	·9961	·9965	·9970	·9974	·9981
3·0	0·9955	0·9958	0·9960	0·9962	0·9963	0·9965	0·9969	0·9973	0·9977	0·9980	0·9987
·1	·9963	·9966	·9967	·9969	·9971	·9972	·9976	·9979	·9982	·9985	·9990
·2	·9970	·9972	·9974	·9975	·9976	·9978	·9981	·9984	·9987	·9989	·9993
·3	·9976	·9977	·9979	·9980	·9981	·9982	·9985	·9988	·9990	·9992	·9995
·4	·9980	·9982	·9983	·9984	·9985	·9986	·9988	·9990	·9992	·9994	·9997
3·5	0·9984	0·9985	0·9986	0·9987	0·9988	0·9989	0·9991	0·9993	0·9994	0·9996	0·9998
·6	·9987	·9988	·9989	·9990	·9990	·9991	·9993	·9994	·9996	·9997	·9998
·7	·9989	·9990	·9991	·9992	·9992	·9993	·9994	·9996	·9997	·9998	·9999
·8	·9991	·9992	·9993	·9993	·9994	·9994	·9996	·9997	·9998	·9998	·9999
·9	·9993	·9994	·9994	·9995	·9995	·9996	·9997	·9997	·9998	·9999	
4·0	0·9994	0·9995	0·9995	0·9996	0·9996	0·9996	0·9997	0·9998	0·9999	0·9999	
·1	·9995	·9996	·9996	·9997	·9997	·9997	·9998	·9999	·9999	·9999	
·2	·9996	·9997	·9997	·9997	·9998	·9998	·9998	·9999	·9999		
·3	·9997	·9997	·9998	·9998	·9998	·9998	·9999	·9999	·9999		
·4	·9997	·9998	·9998	·9998	·9998	·9999	·9999	·9999			
4·5	0·9998	0·9998	0·9998	0·9999	0·9999	0·9999	0·9999				

TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

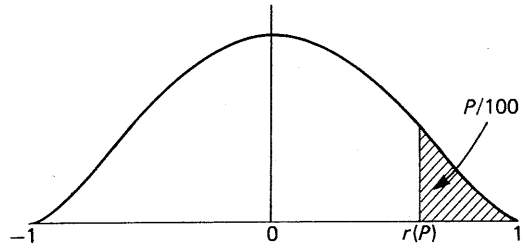
P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291

TABLE 13. PERCENTAGE POINTS OF THE CORRELATION COEFFICIENT r WHEN $\rho = 0$

The function tabulated is $r(P) = r(P|\nu)$ defined by the equation

$$\frac{\Gamma\left(\frac{\nu-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{\nu-2}{2}\right)} \int_{r(P)}^{1} (1-r^2)^{\frac{\nu-4}{2}} dr = P/100.$$

Let r be a partial correlation coefficient, after s variables have been eliminated, in a sample of size n from a multivariate normal population with corresponding true partial correlation coefficient $\rho = 0$, and let $\nu = n-s$. This table gives upper P per cent points of r; the corresponding lower P per cent points are given by $-r(P)$, and the tabulated values are also upper $2P$ per cent points of $|r|$. For $s = 0$ we have $\nu = n$ and r is the ordinary correlation coefficient. When $\nu > 130$ use the results that r is approximately normally distributed with zero mean and variance $\frac{1}{\nu-1}$, or (more accurately) that $z = \tanh^{-1} r$ is approximately normally distributed with zero mean and variance $\frac{1}{\nu-3}$ (cf. Tables 16 and 17).



(This shape applies for $\nu \geqslant 5$ only. When $\nu = 4$ the distribution is uniform and when $\nu = 3$ the probability density function is U-shaped.)

Tables of the distribution of r for various values of ρ are given by, for example, F. N. David, *Tables of the Ordinates and Probability Integral of the Distribution of the Correlation Coefficient in Small Samples*, Cambridge University Press (1954), and R. E. Odeh, 'Critical values of the sample product-moment correlation coefficient in the bivariate normal distribution', *Commun. Statist. – Simula Computa.* 11 (1) (1982), pp. 1–26. The z-transformation may also be used (cf. Tables 16 and 17).

P	5	2·5	1	0·5	0·1
$\nu = 3$	0·9877	0·9969	0·9995	0·9999	0·999995
4	·9000	·9500	·9800	·9900	·9980
5	0·8054	0·8783	0·9343	0·9587	0·9859
6	·7293	·8114	·8822	·9172	·9633
7	·6694	·7545	·8329	·8745	·9350
8	·6215	·7067	·7887	·8343	·9049
9	·5822	·6664	·7498	·7977	·8751
10	0·5494	0·6319	0·7155	0·7646	0·8467
11	·5214	·6021	·6851	·7348	·8199
12	·4973	·5760	·6581	·7079	·7950
13	·4762	·5529	·6339	·6835	·7717
14	·4575	·5324	·6120	·6614	·7501
15	0·4409	0·5140	0·5923	0·6411	0·7301
16	·4259	·4973	·5742	·6226	·7114
17	·4124	·4821	·5577	·6055	·6940
18	·4000	·4683	·5425	·5897	·6777
19	·3887	·4555	·5285	·5751	·6624
20	0·3783	0·4438	0·5155	0·5614	0·6481
21	·3687	·4329	·5034	·5487	·6346
22	·3598	·4227	·4921	·5368	·6219
23	·3515	·4132	·4815	·5256	·6099
24	·3438	·4044	·4716	·5151	·5986
25	0·3365	0·3961	0·4622	0·5052	0·5879
26	·3297	·3882	·4534	·4958	·5776
27	·3233	·3809	·4451	·4869	·5679
28	·3172	·3739	·4372	·4785	·5587
29	·3115	·3673	·4297	·4705	·5499
30	0·3061	0·3610	0·4226	0·4629	0·5415
31	·3009	·3550	·4158	·4556	·5334
32	·2960	·3494	·4093	·4487	·5257
33	·2913	·3440	·4032	·4421	·5184
34	·2869	·3388	·3972	·4357	·5113
35	0·2826	0·3338	0·3916	0·4296	0·5045
36	·2785	·3291	·3862	·4238	·4979
37	·2746	·3246	·3810	·4182	·4916
38	·2709	·3202	·3760	·4128	·4856
39	·2673	·3160	·3712	·4076	·4797

P	5	2·5	1	0·5	0·1
$\nu = 40$	0·2638	0·3120	0·3665	0·4026	0·4741
42	·2573	·3044	·3578	·3932	·4633
44	·2512	·2973	·3496	·3843	·4533
46	·2455	·2907	·3420	·3761	·4439
48	·2403	·2845	·3348	·3683	·4351
50	0·2353	0·2787	0·3281	0·3610	0·4267
52	·2306	·2732	·3218	·3542	·4188
54	·2262	·2681	·3158	·3477	·4114
56	·2221	·2632	·3102	·3415	·4043
58	·2181	·2586	·3048	·3357	·3976
60	0·2144	0·2542	0·2997	0·3301	0·3912
62	·2108	·2500	·2948	·3248	·3850
64	·2075	·2461	·2902	·3198	·3792
66	·2042	·2423	·2858	·3150	·3736
68	·2012	·2387	·2816	·3104	·3683
70	0·1982	0·2352	0·2776	0·3060	0·3632
72	·1954	·2319	·2737	·3017	·3583
74	·1927	·2287	·2700	·2977	·3536
76	·1901	·2257	·2664	·2938	·3490
78	·1876	·2227	·2630	·2900	·3447
80	0·1852	0·2199	0·2597	0·2864	0·3405
82	·1829	·2172	·2565	·2830	·3364
84	·1807	·2146	·2535	·2796	·3325
86	·1786	·2120	·2505	·2764	·3287
88	·1765	·2096	·2477	·2732	·3251
90	0·1745	0·2072	0·2449	0·2702	0·3215
92	·1726	·2050	·2422	·2673	·3181
94	·1707	·2028	·2396	·2645	·3148
96	·1689	·2006	·2371	·2617	·3116
98	·1671	·1986	·2347	·2591	·3085
100	0·1654	0·1966	0·2324	0·2565	0·3054
105	·1614	·1918	·2268	·2504	·2983
110	·1576	·1874	·2216	·2446	·2915
115	·1541	·1832	·2167	·2393	·2853
120	·1509	·1793	·2122	·2343	·2794
125	0·1478	0·1757	0·2079	0·2296	0·2738
130	·1449	·1723	·2039	·2252	·2686

TABLE 14. PERCENTAGE POINTS OF SPEARMAN'S S
TABLE 15. PERCENTAGE POINTS OF KENDALL'S K

Spearman's S and Kendall's K are both used to measure the degree of association between two rankings of n objects. Let d_i ($1 \leqslant i \leqslant n$) be the difference in the ranks of the ith object; Spearman's S is defined as $\sum_{i=1}^{n} d_i^2$. To define Kendall's K, reorder the pairs of ranks so that the first set is in natural order from left to right, and let m_i ($1 \leqslant i \leqslant n$) be the number of ranks greater than i in the second ranking which are to the right of rank i. Kendall's K is defined as $\sum_{i=1}^{n} m_i$.

For Table 14 the tabulated value $x(P)$ is the lower percentage point, i.e. the largest value x such that, in independent rankings, $\Pr(S \leqslant x) \leqslant P/100$; in Table 15, K replaces S and the upper percentage point is given. A dash indicates that there is no value with the required property. The distributions are symmetric about means $\frac{1}{6}(n^3-n)$ for S and $\frac{1}{4}n(n-1)$ for K, with maxima equal to twice the means; hence the upper percentage points of S are $\frac{1}{3}(n^3-n)-x(P)$ and the lower percentage points of K are $\frac{1}{2}n(n-1)-x(P)$. The variances are $\frac{1}{36}n^2(n+1)^2(n-1)$ for S and $\frac{1}{72}n(n-1)(2n+5)$ for K, and when $n > 40$ both statistics are approximately normally distributed; more accurately, the distribution function of $X = [S-\frac{1}{6}(n^3-n)]/[\frac{1}{6}n(n+1)\sqrt{n-1}]$ is approximately equal to $\Phi(x) - \dfrac{\gamma}{24\sqrt{2\pi}} e^{-\frac{1}{2}x^2}(x^3-3x)$, where $\gamma = \dfrac{-0{\cdot}04(19n^2+5n-36)}{\frac{1}{6}(n^3-n)}$ and $\Phi(x)$ is the normal distribution function (see Table 4).

A test of the null hypothesis of independent rankings is provided by rejecting at the P per cent level if $S \leqslant x(P)$, or $K \geqslant x(P)$, when the alternative is contrary rankings. The other points are similarly used when the alternative is similar rankings. To cover both alternatives reject at the $2P$ per cent level if S, or K, lies in either tail. Spearman's rank correlation coefficient r_S is defined as $1-6S/(n^3-n)$, and has upper and lower P per cent points $1-6x(P)/(n^3-n)$ and $-[1-6x(P)/(n^3-n)]$ respectively. Kendall's rank correlation coefficient r_K is defined as $4K/[n(n-1)]-1$, and has upper and lower P per cent points $4x(P)/[n(n-1)]-1$ and $-\{4x(P)/[n(n-1)]-1\}$ respectively.

SPEARMAN'S S

P	5	2·5	1	0·5	0·1	$\frac{1}{6}(n^3-n)$
$n=4$	0	—	—	—	—	10
5	2	0	0	—	—	20
6	6	4	2	0	—	35
7	16	12	6	4	0	56
8	30	22	14	10	4	84
9	48	36	26	20	10	120
10	72	58	42	34	20	165
11	102	84	64	54	34	220
12	142	118	92	78	52	286
13	188	160	128	108	76	364
14	244	210	170	146	104	455
15	310	268	222	194	140	560
16	388	338	284	248	184	680
17	478	418	354	312	236	816
18	580	512	436	388	298	969
19	694	616	530	474	370	1140
20	824	736	636	572	452	1330
21	970	868	756	684	544	1540
22	1132	1018	890	808	650	1771
23	1310	1182	1040	948	768	2024
24	1508	1364	1206	1102	900	2300
25	1724	1566	1388	1272	1048	2600
26	1958	1784	1588	1460	1210	2925
27	2214	2022	1806	1664	1388	3276
28	2492	2282	2044	1888	1584	3654
29	2794	2564	2304	2132	1796	4060
30	3118	2866	2584	2396	2028	4495
31	3466	3194	2884	2682	2280	4960
32	3840	3544	3210	2988	2552	5456
33	4240	3920	3558	3318	2844	5984
34	4666	4322	3930	3672	3160	6545
35	5120	4750	4330	4050	3498	7140
36	5604	5206	4754	4454	3858	7770
37	6118	5692	5206	4884	4244	8436
38	6662	6206	5686	5342	4656	9139
39	7238	6750	6196	5826	5092	9880
40	7846	7326	6736	6342	5556	10660

KENDALL'S K

P	5	2·5	1	0·5	0·1	$\frac{1}{4}n(n-1)$
$n=4$	6	—	—	—	—	3
5	9	10	10	—	—	5
6	13	14	14	15	—	7·5
7	17	18	19	20	21	10·5
8	22	23	24	25	26	14
9	27	28	30	31	33	18
10	33	34	36	37	40	22·5
11	39	41	43	44	47	27·5
12	46	48	51	52	55	33
13	53	56	59	61	64	39
14	62	64	67	69	73	45·5
15	70	73	77	79	83	52·5
16	79	83	86	89	94	60
17	89	93	97	100	105	68
18	99	103	108	111	117	76·5
19	110	114	119	123	129	85·5
20	121	126	131	135	142	95
21	133	138	144	148	156	105
22	146	151	157	161	170	115·5
23	159	164	171	176	184	126·5
24	172	178	185	190	200	138
25	186	193	200	205	216	150
26	201	208	216	221	232	162·5
27	216	223	232	238	249	175·5
28	232	239	248	254	267	189
29	248	256	266	272	285	203
30	265	273	283	290	303	217·5
31	282	291	301	308	323	232·5
32	300	309	320	328	342	248
33	318	328	340	347	363	264
34	337	347	359	368	384	280·5
35	356	367	380	388	405	297·5
36	376	388	401	410	428	315
37	397	409	422	432	450	333
38	418	430	444	454	473	351·5
39	440	452	467	477	497	370·5
40	462	475	490	501	522	390