# MathOptInterface

The JuMP core developers and contributors

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# Part I Introduction

# Introduction

Welcome to the documentation for MathOptInterface.

#### Note

This documentation is also available in PDF format: MathOptInterface.pdf.

#### 1.1 What is MathOptInterface?

MathOptInterface.jl (MOI) is an abstraction layer designed to provide a unified interface to mathematical optimization solvers so that users do not need to understand multiple solver-specific APIs.

#### Tip

This documentation is aimed at developers writing software interfaces to solvers and modeling languages using the MathOptInterface API. If you are a user interested in solving optimization problems, we encourage you instead to use MOI through a higher-level modeling interface like JuMP or Convex.jl.

#### 1.2 How the documentation is structured

Having a high-level overview of how this documentation is structured will help you know where to look for certain things.

- The **Tutorials** section contains articles on how to use and implement the MathOptInteraface API. Look here if you want to write a model in MOI, or write an interface to a new solver.
- The Manual contains short code-snippets that explain how to use the MOI API. Look here for more details on particular areas of MOI.
- The **Background** section contains articles on the theory behind MathOptInterface. Look here if you want to understand why, rather than how.
- The **API Reference** contains a complete list of functions and types that comprise the MOI API. Look here is you want to know how to use (or implement) a particular function.
- The **Submodules** section contains stand-alone documentation for each of the submodules within MOI.
   These submodules are not required to interface a solver with MOI, but they make the job much easier.

## 1.3 Citing MathOptInterface

A paper describing the design and features of MathOptInterface is available on arXiv.

If you find MathOptInterface useful in your work, we kindly request that you cite the following paper:

```
@article{legat2021mathoptinterface,
    title={{MathOptInterface}: a data structure for mathematical optimization problems},
    author={Legat, Beno{\^\i}t and Dowson, Oscar and Garcia, Joaquim Dias and Lubin, Miles},
    journal={INFORMS Journal on Computing},
    year={2021},
    doi={10.1287/ijoc.2021.1067},
    publisher={INFORMS}
}
```

## **Motivation**

MathOptInterface (MOI) is a replacement for MathProgBase, the first-generation abstraction layer for mathematical optimization previously used by JuMP and Convex.jl.

To address a number of limitations of MathProgBase, MOI is designed to:

- · Be simple and extensible
  - unifying linear, quadratic, and conic optimization,
  - seamlessly facilitating extensions to essentially arbitrary constraints and functions (e.g., indicator constraints, complementarity constraints, and piecewise-linear functions)
- Be fast
  - by allowing access to a solver's in-memory representation of a problem without writing intermediate files (when possible)
  - by using multiple dispatch and avoiding requiring containers of nonconcrete types
- Allow a solver to return multiple results (e.g., a pool of solutions)
- Allow a solver to return extra arbitrary information via attributes (e.g., variable- and constraint-wise membership in an irreducible inconsistent subset for infeasibility analysis)
- Provide a greatly expanded set of status codes explaining what happened during the optimization procedure
- Enable a solver to more precisely specify which problem classes it supports
- Enable both primal and dual warm starts
- Enable adding and removing both variables and constraints by indices that are not required to be consecutive
- Enable any modification that the solver supports to an existing model
- · Avoid requiring the solver wrapper to store an additional copy of the problem data

Part II

**Tutorials** 

# Solving a problem using MathOptInterface

In this tutorial we demonstrate how to use MathOptInterface to solve the binary-constrained knapsack problem:

$$\max c^{\top} x$$

$$s.t. \ w^{\top} x \le C$$

$$x_i \in \{0, 1\}, \quad \forall i = 1, \dots, n$$

#### 3.1 Required packages

Load the MathOptInterface module and define the shorthand MOI:

```
using MathOptInterface
const MOI = MathOptInterface
```

As an optimizer, we choose GLPK:

```
using GLPK
optimizer = GLPK.Optimizer()
```

#### 3.2 Define the data

We first define the constants of the problem:

```
julia> c = [1.0, 2.0, 3.0]
3-element Vector{Float64}:
1.0
2.0
3.0

julia> w = [0.3, 0.5, 1.0]
3-element Vector{Float64}:
0.3
0.5
1.0

julia> C = 3.2
3.2
```

#### 3.3 Add the variables

```
| julia> x = MOI.add_variables(optimizer, length(c));
```

#### 3.4 Set the objective

MOI.ScalarAffineTerm.(c, x) is a shortcut for [MOI.ScalarAffineTerm(c[i], x[i]) for i = 1:3]. This is Julia's broadcast syntax in action, and is used quite often throughout MOI.

#### 3.5 Add the constraints

We add the knapsack constraint and integrality constraints:

Add integrality constraints:

#### 3.6 Optimize the model

```
julia> MOI.optimize!(optimizer)
```

#### 3.7 Understand why the solver stopped

The first thing to check after optimization is why the solver stopped, e.g., did it stop because of a time limit or did it stop because it found the optimal solution?

```
julia> MOI.get(optimizer, MOI.TerminationStatus())
OPTIMAL::TerminationStatusCode = 1
```

Looks like we found an optimal solution!

#### 3.8 Understand what solution was returned

```
julia> MOI.get(optimizer, MOI.ResultCount())
1

julia> MOI.get(optimizer, MOI.PrimalStatus())
FEASIBLE_POINT::ResultStatusCode = 1

julia> MOI.get(optimizer, MOI.DualStatus())
NO_SOLUTION::ResultStatusCode = 0
```

#### 3.9 Query the objective

What is its objective value?

```
julia> MOI.get(optimizer, MOI.ObjectiveValue())
6.0
```

#### 3.10 Query the primal solution

And what is the value of the variables x?

```
julia> MOI.get(optimizer, MOI.VariablePrimal(), x)
3-element Vector{Float64}:
1.0
1.0
1.0
```

# Implementing a solver interface

This guide outlines the basic steps to implement an interface to MathOptInterface for a new solver.

#### **Danger**

Implementing an interface to MathOptInterface for a new solver is a lot of work. Before starting, we recommend that you join the Developer chatroom and explain a little bit about the solver you are wrapping. If you have questions that are not answered by this guide, please ask them in the Developer chatroom so we can improve this guide!

#### 4.1 A note on the API

The API of MathOptInterface is large and varied. In order to support the diversity of solvers and use-cases, we make heavy use of duck-typing. That is, solvers are not expected to implement the full API, nor is there a well-defined minimal subset of what must be implemented. Instead, you should implement the API as necessary in order to make the solver function as you require.

The main reason for using duck-typing is that solvers work in different ways and target different use-cases.

For example:

- Some solvers support incremental problem construction, support modification after a solve, and have native support for things like variable names.
- Other solvers are "one-shot" solvers that require all of the problem data to construct and solve the problem in a single function call. They do not support modification or things like variable names.
- Other "solvers" are not solvers at all, but things like file readers. These may only support functions like read\_from\_file, and may not even support the ability to add variables or constraints directly!
- Finally, some "solvers" are layers which take a problem as input, transform it according to some rules, and pass the transformed problem to an inner solver.

#### 4.2 Preliminaries

Before starting on your wrapper, you should do some background research and make the solver accessible via Julia.

#### Decide if MathOptInterface is right for you

The first step in writing a wrapper is to decide whether implementing an interface is the right thing to do.

MathOptInterface is an abstraction layer for unifying constrained mathematical optimization solvers. If your solver doesn't fit in the category, i.e., it implements a derivative-free algorithm for unconstrained objective functions, MathOptInterface may not be the right tool for the job.

#### Tip

If you're not sure whether you should write an interface, ask in the Developer chatroom.

#### Find a similar solver already wrapped

The next step is to find (if possible) a similar solver that is already wrapped. Although not strictly necessary, this will be a good place to look for inspiration when implementing your wrapper.

The JuMP documentation has a good list of solvers, along with the problem classes they support.

#### Tip

If you're not sure which solver is most similar, ask in the Developer chatroom.

#### Create a low-level interface

Before writing a MathOptInterface wrapper, you first need to be able to call the solver from Julia.

#### Wrapping solvers written in Julia

If your solver is written in Julia, there's nothing to do here! Go to the next section.

#### Wrapping solvers written in C

Julia is well suited to wrapping solvers written in C.

#### Info

This is not true for C++. If you have a solver written in C++, first write a C interface, then wrap the C interface.

Before writing a MathOptInterface wrapper, there are a few extra steps.

**Create a JLL** If the C code is publicly available under an open-source license, create a JLL package via Yggdrasil. The easiest way to do this is to copy an existing solver. Good examples to follow are the COIN-OR solvers.

#### Warning

Building the solver via Yggdrasil is non-trivial. Please ask the Developer chatroom for help.

If the code is commercial or not publicly available, the user will need to manually install the solver. See Gurobi.jl or CPLEX.jl for examples of how to structure this.

**Use Clang.jl to wrap the C API** The next step is to use Clang.jl to automatically wrap the C API. The easiest way to do this is to follow an example. Good examples to follow are Cbc.jl and HiGHS.jl.

Sometimes, you will need to make manual modifications to the resulting files.

#### Solvers written in other languages

Ask the Developer chatroom for advice. You may be able to use one of the JuliaInterop packages to call out to the solver.

For example, SeDuMi.jl uses MATLAB.jl to call the SeDuMi solver written in MATLAB.

#### 4.3 Structuring the package

Structure your wrapper as a Julia package. Consult the Julia documentation if you haven't done this before.

MOI solver interfaces may be in the same package as the solver itself (either the C wrapper if the solver is accessible through C, or the Julia code if the solver is written in Julia, for example), or in a separate package which depends on the solver package.

#### Note

The JuMP core contributors request that you do not use "JuMP" in the name of your package without prior consent.

Your package should have the following structure:

```
/.github
    /workflows
        ci.yml
        format_check.yml
        TagBot.yml
/gen
    gen.jl # Code to wrap the C API
/src
    NewSolver.jl
    /gen
        libnewsolver_api.jl
        libnewsolver_common.jl
    /MOI wrapper
        MOI wrapper.jl
        other files.jl
/test
    runtests.jl
    /MOI_wrapper
        MOI_wrapper.jl
.gitignore
.JuliaFormatter.toml
README.md
LICENSE.md
Project.toml
```

- The /.github folder contains the scripts for GitHub actions. The easiest way to write these is to copy the ones from an existing solver.
- The /gen and /src/gen folders are only needed if you are wrapping a solver written in C.

- The /src/MOI\_wrapper folder contains the Julia code for the MOI wrapper.
- The /test folder contains code for testing your package. See Setup tests for more information.
- The .JuliaFormatter.toml and .github/workflows/format\_check.yml enforce code formatting using JuliaFormatter.jl. Check existing solvers or JuMP.jl for details.

#### **Documentation**

Your package must include documentation explaining how to use the package. The easiest approach is to include documentation in your README.md. A more involved option is to use Documenter.jl.

Examples of packages with README-based documentation include:

- Cbc.jl
- HiGHS.jl
- SCS.jl

Examples of packages with Documenter-based documentation include:

- Alpine.jl
- COSMO.il
- Juniper.jl

#### **Setup tests**

The best way to implement an interface to MathOptInterface is via test-driven development.

The MOI. Test submodule contains a large test suite to help check that you have implemented things correctly. Follow the guide How to test a solver to set up the tests for your package.

#### Tip

Run the tests frequently when developing. However, at the start there is going to be a lot of errors! Start by excluding large classes of tests (e.g., exclude = ["test\_basic\_", "test\_model\_"], implement any missing methods until the tests pass, then remove an exclusion and repeat.

#### 4.4 Initial code

By this point, you should have a package setup with tests, formatting, and access to the underlying solver. Now it's time to start writing the wrapper.

#### The Optimizer object

The first object to create is a subtype of AbstractOptimizer. This type is going to store everything related to the problem.

By convention, these optimizers should not be exported and should be named PackageName.Optimizer.

```
import MathOptInterface
const MOI = MathOptInterface
struct Optimizer <: MOI.AbstractOptimizer
    # Fields go here
end</pre>
```

#### Optimizer objects for C solvers

#### Warning

This section is important if you wrap a solver written in C.

Wrapping a solver written in C will require the use of pointers, and for you to manually free the solver's memory when the Optimizer is garbage collected by Julia.

#### Never pass a pointer directly to a Julia ccall function.

Instead, store the pointer as a field in your Optimizer, and implement Base.cconvert and Base.unsafe\_convert. Then you can pass Optimizer to any ccall function that expects the pointer.

In addition, make sure you implement a finalizer for each model you create.

If newsolver\_createProblem() is the low-level function that creates the problem pointer in C, and newsolver\_freeProblem(::Pt is the low-level function that frees memory associated with the pointer, your Optimizer() function should look like this:

```
struct Optimizer <: MOI.AbstractOptimizer
   ptr::Ptr{Cvoid}

function Optimizer()
   ptr = newsolver_createProblem()
   model = Optimizer(ptr)
   finalizer(model) do m
        newsolver_freeProblem(m)
        return
   end
   return model
end

Base.cconvert(::Type{Ptr{Cvoid}}, model::Optimizer) = model
Base.unsafe_convert(::Type{Ptr{Cvoid}}, model::Optimizer) = model.ptr</pre>
```

#### Implement methods for Optimizer

All Optimizers must implement the following methods:

- empty!
- is\_empty
- optimize!

Other methods, detailed below, are optional or depend on how you implement the interface.

#### Tip

For this and all future methods, read the docstrings to understand what each method does, what it expects as input, and what it produces as output. If it isn't clear, let us know and we will improve the docstrings! It is also very helpful to look at an existing wrapper for a similar solver.

You should also implement Base.show(::I0, ::Optimizer) to print a nice string when someone prints your model. For example

```
function Base.show(io::IO, model::Optimizer)
    return print(io, "NewSolver with the pointer $(model.ptr)")
end
```

#### Implement attributes

MathOptInterface uses attributes to manage different aspects of the problem.

For each attribute

- get gets the current value of the attribute
- set sets a new value of the attribute. Not all attributes can be set. For example, the user can't modify the SolverName.
- supports returns a Bool indicating whether the solver supports the attribute.

#### Info

Use attribute\_value\_type to check the value expected by a given attribute. You should make sure that your get function correctly infers to this type (or a subtype of it).

Each column in the table indicates whether you need to implement the particular method for each attribute.

Attribute	get	set	supports
SolverName	Yes	No	No
SolverVersion	Yes	No	No
RawSolver	Yes	No	No
Name	Yes	Yes	Yes
Silent	Yes	Yes	Yes
TimeLimitSec	Yes	Yes	Yes
RawOptimizerAttribute	Yes	Yes	Yes
NumberOfThreads	Yes	Yes	Yes
Humber of Threads	103	.03	103

For example:

```
function MOI.get(model::Optimizer, ::MOI.Silent)
    return # true if MOI.Silent is set
end

function MOI.set(model::Optimizer, ::MOI.Silent, v::Bool)
    if v
        # Set a parameter to turn off printing
    else
```

```
# Restore the default printing
end
return
end

MOI.supports(::Optimizer, ::MOI.Silent) = true
```

#### Define supports\_constraint

The next step is to define which constraints and objective functions you plan to support.

For each function-set constraint pair, define supports constraint:

To make this easier, you may want to use Unions:

```
function MOI.supports_constraint(
     ::Optimizer,
     ::Type{MOI.VariableIndex},
     ::Type<<:Union{MOI.LessThan,MOI.GreaterThan,MOI.EqualTo}},
)
    return true
end</pre>
```

#### Tip

Only support a constraint if your solver has native support for it.

#### 4.5 The big decision: copy-to or incremental modifications?

Now you need to decide whether to support incremental modification or not.

Incremental modification means that the user can add variables and constraints one-by-one without needing to rebuild the entire problem, and they can modify the problem data after an optimize! call. Supporting incremental modification means implementing functions like add variable and add constraint.

The alternative is to accept the problem data in a single copy\_to function call, afterwhich it cannot be modified. Because copy\_to sees all of the data at once, it can typically call a more efficient function to load data into the underlying solver.

Good examples of solvers supporting incremental modification are MILP solvers like GLPK.jl and Gurobi.jl. Examples of copy to solvers are AmplNLWriter.jl and SCS.jl

It is possible to implement both approaches, but you should probably start with one for simplicity.

#### Tip

Only support incremental modification if your solver has native support for it.

In general, supporting incremental modification is more work, and it usually requires some extra book-keeping. However, it provides a more efficient interface to the solver if the problem is going to be resolved multiple times with small modifications. Moreover, once you've implemented incremental modification, it's usually not much extra work to add a copy\_to interface. The converse is not true.

#### Tip

If this is your first time writing an interface, start with copy to.

#### The copy\_to interface

To implement the copy\_to interface, implement the following function:

• copy\_to

#### The incremental interface

#### Warning

Writing this interface is a lot of work. The easiest way is to consult the source code of a similar solver!

To implement the incremental interface, implement the following functions:

- add variable
- add\_variables
- add\_constraint
- add\_constraints
- is\_valid
- delete

#### Info

Solvers do not have to support AbstractScalarFunction in GreaterThan, LessThan, EqualTo, or Interval with a nonzero constant in the function. Throw ScalarFunctionConstantNotZero if the function constant is not zero.

In addition, you should implement the following model attributes:

Attribute	get	set	supports
ListOfModelAttributesSet	Yes	No	No
ObjectiveFunctionType	Yes	No	No
ObjectiveFunction	Yes	Yes	Yes
ObjectiveSense	Yes	Yes	Yes
Name	Yes	Yes	Yes

Variable-related attributes:

Constraint-related attributes:

Attribute	get	set	supports
ListOfVariableAttributesSet	Yes	No	No
NumberOfVariables	Yes	No	No
ListOfVariableIndices	Yes	No	No

Attribute	get	set	supports
ListOfConstraintAttributesSet	Yes	No	No
NumberOfConstraints	Yes	No	No
ListOfConstraintTypesPresent	Yes	No	No
ConstraintFunction	Yes	Yes	No
ConstraintSet	Yes	Yes	No

#### **Modifications**

If your solver supports modifying data in-place, implement modify for the following AbstractModifications:

- ScalarConstantChange
- ScalarCoefficientChange
- VectorConstantChange
- MultirowChange

#### Variables constrained on creation

Some solvers require variables be associated with a set when they are created. This conflicts with the incremental modification approach, since you cannot first add a free variable and then constrain it to the set.

If this is the case, implement:

- add constrained variable
- add\_constrained\_variables
- supports\_add\_constrained\_variables

By default, MathOptInterface assumes solvers support free variables. If your solver does not support free variables, define:

```
MOI.supports_add_constrained_variables(::Optimizer, ::Type{Reals}) = false
```

#### Incremental and copy\_to

If you implement the incremental interface, you have the option of also implementing copy to.

If you don't want to implement copy\_to, e.g., because the solver has no API for building the problem in a single function call, define the following fallback:

```
MOI.supports_incremental_interface(::Optimizer) = true
function MOI.copy_to(dest::Optimizer, src::MOI.ModelLike)
    return MOI.Utilities.default_copy_to(dest, src)
end
```

#### 4.6 Names

Regardless of which interface you implement, you have the option of implementing the Name attribute for variables and constraints:

Attribute	get	set	supports
VariableName	Yes	Yes	Yes
ConstraintName	Yes	Yes	Yes

If you implement names, you must also implement the following three methods:

```
function MOI.get(model::Optimizer, ::Type{MOI.VariableIndex}, name::String)
    return # The variable named `name`.
end

function MOI.get(model::Optimizer, ::Type{MOI.ConstraintIndex}, name::String)
    return # The constraint any type named `name`.
end

function MOI.get(
    model::Optimizer,
    ::Type{MOI.ConstraintIndex{F,S}},
    name::String,
) where {F,S}
    return # The constraint of type F-in-S named `name`.
end
```

These methods have the following rules:

- If there is no variable or constraint with the name, return nothing
- If there is a single variable or constraint with that name, return the variable or constraint
- If there are multiple variables or constraints with the name, throw an error.

#### Warning

You should not implement ConstraintName for VariableIndex constraints. If you implement ConstraintName for other constraints, you can add the following two methods to disable ConstraintName for VariableIndex constraints.

```
function MOI.supports(
    ::Optimizer,
    ::MOI.ConstraintName,
    ::Type{<:MOI.ConstraintIndex{MOI.VariableIndex,<:MOI.AbstractScalarSet}},
)
    return throw(MOI.VariableIndexConstraintNameError())
end
function MOI.set(
    ::Optimizer,
    ::MOI.ConstraintName,
    ::MOI.ConstraintIndex{MOI.VariableIndex,<:MOI.AbstractScalarSet},
    ::String,
)
    return throw(MOI.VariableIndexConstraintNameError())
end</pre>
```

#### 4.7 Solutions

Implement optimize! to solve the model:

• optimize!

All Optimizers must implement the following attributes:

- DualStatus
- PrimalStatus
- RawStatusString
- ResultCount
- TerminationStatus

#### Info

You only need to implement get for solution attributes. Don't implement set or supports.

#### Note

Solver wrappers should document how the low-level statuses map to the MOI statuses. Statuses like NEARLY\_FEASIBLE\_POINT and INFEASIBLE\_POINT, are designed to be used when the solver explicitly indicates that relaxed tolerances are satisfied or the returned point is infeasible, respectively.

You should also implement the following attributes:

- ObjectiveValue
- SolveTimeSec
- VariablePrimal

#### Tip

Attributes like VariablePrimal and ObjectiveValue are indexed by the result count. Use MOI.check\_result\_index\_bound attr) to throw an error if the attribute is not available.

If your solver returns dual solutions, implement:

- ConstraintDual
- DualObjectiveValue

For integer solvers, implement:

- ObjectiveBound
- RelativeGap

If applicable, implement:

- SimplexIterations
- BarrierIterations
- NodeCount

If your solver uses the Simplex method, implement:

• ConstraintBasisStatus

If your solver accepts primal or dual warm-starts, implement:

- VariablePrimalStart
- ConstraintDualStart

#### 4.8 Other tips

Here are some other points to be aware of when writing your wrapper.

#### Unsupported constraints at runtime

In some cases, your solver may support a particular type of constraint (e.g., quadratic constraints), but only if the data meets some condition (e.g., it is convex).

In this case, declare that you support the constraint, and throw AddConstraintNotAllowed.

#### Dealing with multiple variable bounds

MathOptInterface uses VariableIndex constraints to represent variable bounds. Defining multiple variable bounds on a single variable is not allowed.

Throw LowerBoundAlreadySet or UpperBoundAlreadySet if the user adds a constraint that results in multiple bounds.

Only throw if the constraints conflict. It is okay to add VariableIndex-in-GreaterThan and then VariableIndex-in-LessThan, but not VariableIndex-in-Interval and then VariableIndex-in-LessThan,

#### **Expect duplicate coefficients**

Solvers must expect that functions such as ScalarAffineFunction and VectorQuadraticFunction may contain duplicate coefficents.

For example, ScalarAffineFunction([ScalarAffineTerm(x, 1), ScalarAffineTerm(x, 1)], 0.0).

Use Utilities.canonical to return a new function with the duplicate coefficients aggregated together.

#### Don't modify user-data

All data passed to the solver must be copied immediately to internal data structures. Solvers may not modify any input vectors and must assume that input vectors will not be modified by users in the future.

This applies, for example, to the terms vector in ScalarAffineFunction. Vectors returned to the user, e.g., via ObjectiveFunction or ConstraintFunction attributes, must not be modified by the solver afterwards. The in-place version of get! can be used by users to avoid extra copies in this case.

#### **Column Generation**

There is no special interface for column generation. If the solver has a special API for setting coefficients in existing constraints when adding a new variable, it is possible to queue modifications and new variables and then call the solver's API once all of the new coefficients are known.

#### Solver-specific attributes

You don't need to restrict yourself to the attributes defined in the MathOptInterface.jl package.

Solver-specific attributes should be specified by creating an appropriate subtype of AbstractModelAttribute, AbstractOptimizerAttribute, AbstractVariableAttribute, or AbstractConstraintAttribute.

For example, Gurobi.jl adds attributes for multiobjective optimization by defining:

```
struct NumberOfObjectives <: MOI.AbstractModelAttribute end

function MOI.set(model::Optimizer, ::NumberOfObjectives, n::Integer)
    # Code to set NumberOfObjectives
    return
end

function MOI.get(model::Optimizer, ::NumberOfObjectives)
    n = # Code to get NumberOfObjectives
    return n
end</pre>
```

Then, the user can write:

```
model = Gurobi.Optimizer()
MOI.set(model, Gurobi.NumberofObjectives(), 3)
```

# Transitioning from MathProgBase

MathOptInterface is a replacement for MathProgBase.jl. However, it is not a direct replacement.

#### 5.1 Transitioning a solver interface

MathOptInterface is more extensive than MathProgBase which may make its implementation seem daunting at first. There are however numerous utilities in MathOptInterface that the simplify implementation process.

For more information, read Implementing a solver interface.

#### 5.2 Transitioning the high-level functions

MathOptInterface doesn't provide replacements for the high-level interfaces in MathProgBase. We recommend you use |uMP as a modeling interface instead.

#### Tip

If you haven't used JuMP before, start with the tutorial Getting started with JuMP

#### linprog

Here is one way of transitioning from linprog:

```
using JuMP

function linprog(c, A, sense, b, l, u, solver)
    N = length(c)
    model = Model(solver)
    @variable(model, l[i] <= x[i=1:N] <= u[i])
    @objective(model, Min, c' * x)
    eq_rows, ge_rows, le_rows = sense .== '=', sense .== '>', sense .== '<'
    @constraint(model, A[eq_rows, :] * x .== b[eq_rows])
    @constraint(model, A[ge_rows, :] * x .>= b[ge_rows])
    @constraint(model, A[le_rows, :] * x .<= b[le_rows])
    optimize!(model)
    return (
        status = termination_status(model),
        objval = objective_value(model),
        sol = value.(x)
    )
end</pre>
```

#### mixintprog

Here is one way of transitioning from mixintprog:

```
using JuMP
function mixintprog(c, A, rowlb, rowub, vartypes, lb, ub, solver)
    N = length(c)
    model = Model(solver)
    @variable(model, lb[i] <= x[i=1:N] <= ub[i])</pre>
    for i in 1:N
        if vartypes[i] == :Bin
            set_binary(x[i])
        elseif vartypes[i] == :Int
            set_integer(x[i])
        end
    end
    @objective(model, Min, c' * x)
    @constraint(model, rowlb .<= A * x .<= rowub)</pre>
    optimize!(model)
    return (
        status = termination_status(model),
        objval = objective_value(model),
        sol = value.(x)
end
```

#### quadprog

Here is one way of transitioning from quadprog:

```
function quadprog(c, Q, A, rowlb, rowub, lb, ub, solver)
  N = length(c)
  model = Model(solver)
  @variable(model, lb[i] <= x[i=1:N] <= ub[i])
  @objective(model, Min, c' * x + 0.5 * x' * Q * x)
  @constraint(model, rowlb .<= A * x .<= rowub)
  optimize!(model)
  return (
        status = termination_status(model),
        objval = objective_value(model),
        sol = value.(x)
  )
end</pre>
```

# Implementing a constraint bridge

This guide outlines the basic steps to create a new bridge from a constraint expressed in the formalism Function-in-Set.

#### 6.1 Preliminaries

First, decide on the set you want to bridge. Then, study its properties: the most important one is whether the set is scalar or vector, which impacts the dimensionality of the functions that can be used with the set.

- A scalar function only has one dimension. MOI defines three types of scalar functions: a variable (VariableIndex), an affine function (ScalarAffineFunction), or a quadratic function (ScalarQuadraticFunction).
- A vector function has several dimensions (at least one). MOI defines three types of vector functions: several variables (VectorOfVariables), an affine function (VectorAffineFunction), or a quadratic function (VectorQuadraticFunction). The main difference with scalar functions is that the order of dimensions can be very important: for instance, in an indicator constraint (Indicator), the first dimension indicates whether the constraint about the second dimension is active.

To explain how to implement a bridge, we present the example of Bridges.Constraint.FlipSignBridge. This bridge maps <= (LessThan) constraints to >= (GreaterThan) constraints. This corresponds to reversing the sign of the inequality. We focus on scalar affine functions (we disregard the cases of a single variable or of quadratic functions). This example is a simplified version of the code included in MOI.

#### 6.2 Four mandatory parts in a constraint bridge

The first part of a constraint bridge is a new concrete subtype of Bridges. Constraint. AbstractBridge. This type must have fields to store all the new variables and constraints that the bridge will add. Typically, these types are parametrized by the type of the coefficients in the model.

Then, three sets of functions must be defined:

- 1. Bridges.Constraint.bridge\_constraint: this function implements the bridge and creates the required variables and constraints.
- 2. supports\_constraint: these functions must return true when the combination of function and set is supported by the bridge. By default, the base implementation always returns false and the bridge does not have to provide this implementation.

3. Bridges.added\_constrained\_variable\_types and Bridges.added\_constraint\_types: these functions return the types of variables and constraints that this bridge adds. They are used to compute the set of other bridges that are required to use the one you are defining, if need be.

More functions can be implemented, for instance to retrieve properties from the bridge or deleting a bridged constraint.

#### 1. Structure for the bridge

A typical struct behind a bridge depends on the type of the coefficients that are used for the model (typically Float64, but coefficients might also be integers or complex numbers).

This structure must hold a reference to all the variables and the constraints that are created as part of the bridge.

The type of this structure is used throughout MOI as an identifier for the bridge. It is passed as argument to most functions related to bridges.

The best practice is to have the name of this type end with Bridge.

In our example, the bridge maps any ScalarAffineFunction{T}-in-LessThan{T} constraint to a single ScalarAffineFunction{T in-GreaterThan{T} constraint. The affine function has coefficients of type T. The bridge is parametrized with T, so that the constraint that the bridge creates also has coefficients of type T.

```
struct SignBridge{T<:Number} <: Bridges.Constraint.AbstractBridge
  constraint::ConstraintIndex{ScalarAffineFunction{T}, GreaterThan{T}}
end</pre>
```

#### 2. Bridge creation

The function <code>Bridges.Constraint.bridge\_constraint</code> is called whenever the bridge is instantiated for a specific model, with the given function and set. The arguments to <code>bridge\_constraint</code> are similar to <code>add\_constraint</code>, with the exception of the first argument: it is the Type of the struct defined in the first step (for our example, <code>Type{SignBridge{T}})</code>.

bridge\_constraint returns an instance of the struct defined in the first step. the first step.

In our example, the bridge constraint could be defined as:

```
function Bridges.Constraint.bridge_constraint(
    ::Type{SignBridge{T}}, # Bridge to use.
    model::ModelLike, # Model to which the constraint is being added.
    f::ScalarAffineFunction{T}, # Function to rewrite.
    s::LessThan{T}, # Set to rewrite.
) where {T}
    # Create the variables and constraints required for the bridge.
    con = add_constraint(model, -f, GreaterThan(-s.upper))

# Return an instance of the bridge type with a reference to all the
    # variables and constraints that were created in this function.
    return SignBridge(con)
end
```

#### 3. Supported constraint types

The function supports\_constraint determines whether the bridge type supports a given combination of function and set.

This function must closely match bridge\_constraint, because it will not be called if supports\_constraint returns false.

```
function supports_constraint(
    ::Type{SignBridge{T}}, # Bridge to use.
    ::Type{ScalarAffineFunction{T}}, # Function to rewrite.
    ::Type{LessThan{T}}, # Set to rewrite.
) where {T}
    # Do some computation to ensure that the constraint is supported.
    # Typically, you can directly return true.
    return true
end
```

#### 4. Metadata about the bridge

To determine whether a bridge can be used, MOI uses a shortest-path algorithm that uses the variable types and the constraints that the bridge can create. This information is communicated from the bridge to MOI using the functions <code>Bridges.added\_constrained\_variable\_types</code> and <code>Bridges.added\_constraint\_types</code>. Both return lists of tuples: either a list of 1-tuples containing the variable types (typically, Zero0ne or Integer) or a list of 2-tuples contained the functions and sets (like ScalarAffineFunction{T}-GreaterThan).

For our example, the bridge does not create any constrained variables, and only  $ScalarAffineFunction\{T\}-in-GreaterThan\{T\}$  constraints:

```
function Bridges.added_constrained_variable_types(::Type{SignBridge{T}}) where {T}
    # The bridge does not create variables, return an empty list of tuples:
    return Tuple{Type}[]
end

function Bridges.added_constraint_types(::Type{SignBridge{T}}) where {T}
    return Tuple{Type, Type}[
        # One element per F-in-S the bridge creates.
        (ScalarAffineFunction{T}, GreaterThan{T}),
    ]
end
```

A bridge that creates binary variables would rather have this definition of added\_constrained\_variable\_types:

```
function Bridges.added_constrained_variable_types(::Type{SomeBridge{T}}) where {T}
    # The bridge only creates binary variables:
    return Tuple{Type}[(ZeroOne,)]
end
```

#### Warning

If you declare the creation of constrained variables in added\_constrained\_variable\_types, the corresponding constraint type VariableIndex must not be indicated in added\_constraint\_types. This would restrict the use of the bridge to solvers that can add such a constraint after the variable is created.

More concretely, if you declare in added\_constrained\_variable\_types that your bridge creates binary variables (ZeroOne), and if you never add such a constraint afterward (you do not call add\_constraint(model, var, ZeroOne())), then you must not list (VariableIndex, ZeroOne) in added\_constraint\_types.

Typically, the function Bridges.Constraint.concrete\_bridge\_type does not have to be defined for most bridges.

#### 6.3 Bridge registration

For a bridge to be used by MOI, it must be known by MOI.

#### SingleBridgeOptimizer

The first way to do so is to create a single-bridge optimizer. This type of optimizer wraps another optimizer and adds the possibility to use only one bridge. It is especially useful when unit testing bridges.

It is common practice to use the same name as the type defined for the bridge (SignBridge, in our example) without the suffix Bridge.

```
const Sign{T,0T<: ModelLike} =
    SingleBridge0ptimizer{SignBridge{T}, 0T}</pre>
```

In the context of unit tests, this bridge is used in conjunction with a Utilities.MockOptimizer:

```
mock = Utilities.MockOptimizer(
    Utilities.UniversalFallback(Utilities.Model{Float64}()),
)
bridged_mock = Sign{Float64}(mock)
```

#### New bridge for a LazyBridgeOptimizer

Typical user-facing models for MOI are based on Bridges.LazyBridgeOptimizer. For instance, this type of model is returned by Bridges.full\_bridge\_optimizer. These models can be added more bridges by using Bridges.add\_bridge:

```
inner_optimizer = Utilities.Model{Float64}()
optimizer = Bridges.full_bridge_optimizer(inner_optimizer, Float64)
Bridges.add_bridge(optimizer, SignBridge{Float64})
```

#### 6.4 Bridge improvements

#### **Attribute retrieval**

Like models, bridges have attributes that can be retrieved using get and set. The most important ones are the number of variables and constraints, but also the lists of variables and constraints.

In our example, we only have one constraint and only have to implement the NumberOfConstraints and ListOfConstraintIndices attributes:

```
function get(
    ::SignBridge{T},
    ::NumberOfConstraints{
        ScalarAffineFunction{T},
        GreaterThan{T},
   },
) where {T}
    return 1
end
function get(
    bridge::SignBridge{T},
    ::ListOfConstraintIndices{
        ScalarAffineFunction{T},
        GreaterThan{T},
   },
) where {T}
    return [bridge.constraint]
end
```

You must implement one such pair of functions for each type of constraint the bridge adds to the model.

#### Warning

Avoid returning a list from the bridge object without copying it. Users must be able to change the contents of the returned list without altering the bridge object.

For variables, the situation is simpler. If your bridge creates new variables, you must implement the NumberOfVariables and ListOfVariableIndices attributes. However, these attributes do not have parameters, unlike their constraint counterparts. Only two functions suffice:

```
function get(
    ::SignBridge{T},
    ::NumberOfVariables,
) where {T}
    return 0
end

function get(
    ::SignBridge{T},
    ::ListOfVariableIndices,
) where {T}
    return VariableIndex[]
end
```

#### **Model modifications**

To avoid copying the model when the user request to change a constraint, MOI provides modify. Bridges can also implement this API to allow certain changes, such as coefficient changes.

In our case, a modification of a coefficient in the original constraint (i.e. replacing the value of the coefficient of a variable in the affine function) must be transmitted to the constraint created by the bridge, but with a sign change.

```
function modify(
    model::ModelLike,
    bridge::SignBridge,
    change::ScalarCoefficientChange,
)
    modify(
        model,
        bridge.constraint,
        ScalarCoefficientChange(change.variable, -change.new_coefficient),
    )
    return
end
```

## **Bridge deletion**

When a bridge is deleted, the constraints it added must be deleted too.

```
function delete(model::ModelLike, bridge::SignBridge)
  delete(model, bridge.constraint)
  return
end
```

# **Manipulating expressions**

This guide highlights a syntactically appealing way to build expressions at the MOI level, but also to look at their contents. It may be especially useful when writing models or bridge code.

#### 7.1 Creating functions

This section details the ways to create functions with MathOptInterface.

#### Creating scalar affine functions

The simplest scalar function is simply a variable:

```
julia> x = MOI.add_variable(model) # Create the variable x
MathOptInterface.VariableIndex(1)
```

This type of function is extremely simple; to express more complex functions, other types must be used. For instance, a ScalarAffineFunction is a sum of linear terms (a factor times a variable) and a constant. Such an object can be built using the standard constructor:

```
julia> f = MOI.ScalarAffineFunction([MOI.ScalarAffineTerm(1, x)], 2) # x + 2
MathOptInterface.ScalarAffineFunction{Int64}(MathOptInterface.ScalarAffineTerm{Int64}[MathOptInterface.ScalarAffineTerm
→ MathOptInterface.VariableIndex(1))], 2)
```

However, you can also use operators to build the same scalar function:

#### Creating scalar quadratic functions

Scalar quadratic functions are stored in ScalarQuadraticFunction objects, in a way that is highly similar to scalar affine functions. You can obtain a quadratic function as a product of affine functions:

#### **Creating vector functions**

A vector function is a function with several values, irrespective of the number of input variables. Similarly to scalar functions, there are three main types of vector functions: VectorOfVariables, VectorAffineFunction, and VectorQuadraticFunction.

The easiest way to create a vector function is to stack several scalar functions using Utilities.vectorize. It takes a vector as input, and the generated vector function (of the most appropriate type) has each dimension corresponding to a dimension of the vector.

```
julia> g = MOI.Utilities.vectorize([f, 2 * f])
MathOptInterface.VectorAffineFunction{Int64}(MathOptInterface.VectorAffineTerm{Int64}[MathOptInterface.VectorAffineTerm
→ MathOptInterface.ScalarAffineTerm{Int64}(1, MathOptInterface.VariableIndex(1))),
→ MathOptInterface.VectorAffineTerm{Int64}(2, MathOptInterface.ScalarAffineTerm{Int64}(2,
→ MathOptInterface.VariableIndex(1)))], [2, 4])
```

#### Warning

Utilities.vectorize only takes a vector of similar scalar functions: you cannot mix VariableIndex and ScalarAffineFunction, for instance. In practice, it means that Utilities.vectorize([x, f]) does not work; you should rather use Utilities.vectorize([1 \* x, f]) instead to only have ScalarAffineFunction objects.

#### 7.2 Canonicalizing functions

In more advanced use cases, you might need to ensure that a function is "canonical". Functions are stored as an array of terms, but there is no check that these terms are redundant: a ScalarAffineFunction object might have two terms with the same variable, like x + x + 1. These terms could be merged without changing the semantics of the function: 2x + 1.

Working with these objects might be cumbersome. Canonicalization helps maintain redundancy to zero.

Utilities.is\_canonical checks whether a function is already in its canonical form:

```
julia> MOI.Utilities.is\_canonical(f + f) # (x + 2) + (x + 2) is stored as x + x + 4 false
```

Utilities.canonical returns the equivalent canonical version of the function:

```
julia> MOI.Utilities.canonical(f + f) # Returns 2x + 4
MathOptInterface.ScalarAffineFunction{Int64}(MathOptInterface.ScalarAffineTerm{Int64}[MathOptInterface.ScalarAffineTerm
→ MathOptInterface.VariableIndex(1))], 4)
```

# 7.3 Exploring functions

At some point, you might need to dig into a function, for instance to map it into solver constructs.

#### **Vector functions**

Utilities.scalarize returns a vector of scalar functions from a vector function:

#### Note

Utilities.eachscalar returns an iterator on the dimensions, which serves the same purpose as Utilities.scalarize.

output dimension returns the number of dimensions of the output of a function:

```
julia> MOI.output_dimension(g)
```

# Latency

MathOptInterface suffers the "time-to-first-solve" problem of start-up latency.

This hurts both the user- and developer-experience of MathOptInterface. In the first case, because simple models have a multi-second delay before solving, and in the latter, because our tests take so long to run!

This page contains some advice on profiling and fixing latency-related problems in the MathOptInterface.jl repository.

## 8.1 Background

Before reading this part of the documentation, you should familiarize yourself with the reasons for latency in Julia and how to fix them.

- Read the blogposts on julialang.org on precompilation and SnoopCompile
- Read the SnoopCompile documentation.
- Watch Tim Holy's talk at JuliaCon 2021
- Watch the package development workshop at JuliaCon 2021

#### 8.2 Causes

There are three main causes of latency in MathOptInterface:

- 1. A large number of types
- 2. Lack of method ownership
- 3. Type-instability in the bridge layer

#### A large number of types

Julia is very good at specializing method calls based on the input type. Each specialization has a compilation cost, but the benefit of faster run-time performance.

The best-case scenario is for a method to be called a large number of times with a single set of argument types. The worst-case scenario is for a method to be called a single time for a large set of argument types.

Because of MathOptInterface's function-in-set formulation, we fall into the worst-case situation.

This is a fundamental limitation of Julia, so there isn't much we can do about it. However, if we can precompile MathOptInterface, much of the cost can be shifted from start-up latency to the time it takes to precompile a package on installation.

However, there are two things which make MathOptInterface hard to precompile...

#### Lack of method ownership

Lack of method ownership happens when a call is made using a mix of structs and methods from different modules. Because of this, no single module "owns" the method that is being dispatched, and so it cannot be precompiled.

#### Tip

This is a slightly simplified explanation. Read the precompilation tutorial for a more in-depth discussion on back-edges.

Unfortunately, the design of MOI means that this is a frequent occurrence! We have a bunch of types in MOI.Utilities that wrap types defined in external packages (i.e., the Optimizers), which implement methods of functions defined in MOI (e.g., add\_variable, add\_constraint).

Here's a simple example of method-ownership in practice:

```
module MyMOI
struct Wrapper{T}
                    inner::T
end
optimize!(x::Wrapper) = optimize!(x.inner)
end # MyMOI
module MyOptimizer
using ..MyMOI
struct Optimizer end
MyMOI.optimize!(x::Optimizer) = 1
end # MyOptimizer
using SnoopCompile
model = MyMOI.Wrapper(MyOptimizer.Optimizer())
julia> tinf = @snoopi_deep MyMOI.optimize!(model)
Inference Timing Node: 0.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.008543 \ on \ Inference Frame Info \ for \ Core. Compiler. Timings. ROOT() \ with \ 10.008256/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00856/0.00
\,\hookrightarrow\,\,\text{direct children}
```

The result is that there was one method that required type inference. If we visualize tinf:

```
using ProfileView
ProfileView.view(flamegraph(tinf))
```

we see a flamegraph with a large red-bar indicating that the method MyMOI.optimize(MyMOI.Wrapper{MyOptimizer.Optimizer} cannot be precompiled.

To fix this, we need to designate a module to "own" that method (i.e., create a back-edge). The easiest way to do this is for MyOptimizer to call MyMOI.optimize(MyMOI.Wrapper{MyOptimizer.Optimizer}) during using MyOptimizer. Let's see that in practice:

```
module MyMOI
struct Wrapper{T}
    inner::T
optimize(x::Wrapper) = optimize(x.inner)
end # MyMOI
module MyOptimizer
using ..MyMOI
struct Optimizer end
MyMOI.optimize(x::Optimizer) = 1
# The syntax of this let-while loop is very particular:
\# * `let ... end` keeps everything local to avoid polluting the MyOptimizer
    namespace
\# * `while true ... break end` runs the code once, and forces Julia to compile
    the inner loop, rather than interpret it.
   while true
        model = MyMOI.Wrapper(Optimizer())
        MyMOI.optimize(model)
        break
    end
end
end # MyOptimizer
using SnoopCompile
model = MyMOI.Wrapper(MyOptimizer.Optimizer())
julia> tinf = @snoopi deep MyMOI.optimize(model)
InferenceTimingNode: 0.006822/0.006822 on InferenceFrameInfo for Core.Compiler.Timings.ROOT() with 0
\hookrightarrow direct children
```

There are now 0 direct children that required type inference because the method was already stored in MyOptimizer!

Unfortunately, this trick only works if the call-chain is fully inferrable. If there are breaks (due to type instability), then the benefit of doing this is reduced. And unfortunately for us, the design of MathOptInterface has a lot of type instabilities...

## Type instability in the bridge layer

Most of MathOptInterface is pretty good at ensuring type-stability. However, a key component is not type stable, and that is the bridging layer.

In particular, the bridging layer defines Bridges.LazyBridgeOptimizer, which has fields like:

```
struct LazyBridgeOptimizer
    constraint_bridge_types::Vector{Any}
    constraint_node::Dict{Tuple{Type,Type},ConstraintNode}
    constraint_types::Vector{Tuple{Type,Type}}
```

This is because the LazyBridgeOptimizer needs to be able to deal with any function-in-set type passed to it, and we also allow users to pass additional bridges that they defined in external packages.

So to recap, MathOptInterface suffers package latency because:

- 1. there are a large number of types and functions...
- 2. and these are split between multiple modules, including external packages...
- 3. and there are type-instabilities like those in the bridging layer.

#### 8.3 Resolutions

There are no magic solutions to reduce latency. Issue #1313 tracks progress on reducing latency in MathOpt-Interface.

A useful script is the following (replace GLPK as needed):

```
using MathOptInterface, GLPK
const MOI = MathOptInterface
function example_diet(optimizer, bridge)
    category_data = [
       1800.0 2200.0;
         91.0 Inf;
          0.0 65.0;
           0.0 1779.0
    1
    cost = [2.49, 2.89, 1.50, 1.89, 2.09, 1.99, 2.49, 0.89, 1.59]
    food_data = [
        410 24 26 730;
        420 32 10 1190;
        560 20 32 1800;
       380 4 19 270;
       320 12 10 930;
        320 15 12 820;
        320 31 12 1230;
       100 8 2.5 125;
        330 8 10 180
    ]
    bridge_model = if bridge
        {\tt MOI.instantiate(optimizer; with\_bridge\_type=} {\tt Float64})
    else
        MOI.instantiate(optimizer)
    end
    model = MOI.Utilities.CachingOptimizer(
        MOI. Utilities. UniversalFallback(MOI. Utilities. Model(Float64)()),
        MOI.Utilities.AUTOMATIC,
    )
   MOI.Utilities.reset_optimizer(model, bridge_model)
   MOI.set(model, MOI.Silent(), true)
    nutrition = MOI.add variables(model, size(category data, 1))
    for (i, v) in enumerate(nutrition)
        MOI.add_constraint(model, v, MOI.GreaterThan(category_data[i, 1]))
        MOI.add_constraint(model, v, MOI.LessThan(category_data[i, 2]))
    buy = MOI.add_variables(model, size(food_data, 1))
   MOI.add_constraint.(model, buy, MOI.GreaterThan(0.0))
   MOI.set(model, MOI.ObjectiveSense(), MOI.MIN SENSE)
    f = MOI.ScalarAffineFunction(MOI.ScalarAffineTerm.(cost, buy), 0.0)
   MOI.set(model, MOI.ObjectiveFunction{typeof(f)}(), f)
```

```
for (j, n) in enumerate(nutrition)
         f = MOI.ScalarAffineFunction(
             MOI.ScalarAffineTerm.(food_data[:, j], buy),
         push!(f.terms, MOI.ScalarAffineTerm(-1.0, n))
         \texttt{MOI.add\_constraint}(\texttt{model}, \texttt{ f, MOI.EqualTo}(\theta.\theta))
     end
    MOI.optimize!(model)
     term_status = MOI.get(model, MOI.TerminationStatus())
    @assert term_status == MOI.OPTIMAL
    {\tt MOI.add\_constraint}(
         model,
         MOI.ScalarAffineFunction(
             MOI.ScalarAffineTerm.(1.0, [buy[end-1], buy[end]]),
             0.0,
         ),
         MOI.LessThan(6.0),
     )
    MOI.optimize!(model)
    @assert MOI.get(model, MOI.TerminationStatus()) == MOI.INFEASIBLE
     return
end
if length(ARGS) > 0
    bridge = get(ARGS, 2, "") != "--no-bridge"
     println("Running: $(ARGS[1]) $(get(ARGS, 2, ""))")
    @time example_diet(GLPK.Optimizer, bridge)
    @time example_diet(GLPK.Optimizer, bridge)
     exit(0)
end
You can create a flame-graph via
```

```
using SnoopComile
tinf = @snoopi_deep example_diet(GLPK.Optimizer, true)
using ProfileView
ProfileView.view(flamegraph(tinf))
```

Here's how things looked in mid-August 2021:

There are a few opportunities for improvement (non-red flames, particularly on the right). But the main problem is a large red (non-precompilable due to method ownership) flame.



Figure 8.1: flamegraph

Part III

Manual

# Standard form problem

MathOptInterface represents optimization problems in the standard form:

$$\min_{x \in \mathbb{R}^n} \qquad \qquad f_0(x) \tag{9.1}$$

s.t. 
$$f_i(x) \in \mathcal{S}_i$$
  $i=1\dots m$  (9.2)

where:

- ullet the functions  $f_0, f_1, \dots, f_m$  are specified by <code>AbstractFunction</code> objects
- ullet the sets  $\mathcal{S}_1,\dots,\mathcal{S}_m$  are specified by <code>AbstractSet</code> objects

#### Tip

For more information on this standard form, read our paper.

MOI defines some commonly used functions and sets, but the interface is extensible to other sets recognized by the solver.

#### 9.1 Functions

The function types implemented in MathOptInterface.jl are:

- VariableIndex:  $x_j$ , i.e., projection onto a single coordinate defined by a variable index j.
- VectorOfVariables: projection onto multiple coordinates (i.e., extracting a subvector).
- ScalarAffineFunction:  $a^Tx + b$ , where a is a vector and b scalar.
- VectorAffineFunction: Ax + b, where A is a matrix and b is a vector.
- ScalarQuadraticFunction:  $\frac{1}{2}x^TQx + a^Tx + b$ , where Q is a symmetric matrix, a is a vector, and b is a constant.
- VectorQuadraticFunction: a vector of scalar-valued quadratic functions.

Extensions for nonlinear programming are present but not yet well documented.

## 9.2 One-dimensional sets

The one-dimensional set types implemented in MathOptInterface.jl are:

```
• LessThan(upper): \{x \in \mathbb{R} : x \leq \text{upper}\}
• GreaterThan(lower): \{x \in \mathbb{R} : x \geq \text{lower}\}
• EqualTo(value): \{x \in \mathbb{R} : x = \text{value}\}
• Interval(lower, upper): \{x \in \mathbb{R} : x \in [\text{lower}, \text{upper}]\}
• Integer(): \mathbb{Z}
• ZeroOne(): \{0,1\}
• Semicontinuous(lower, upper): \{0\} \cup [\text{lower}, \text{upper}]
• Semiinteger(lower, upper): \{0\} \cup \{\text{lower}, \text{lower} + 1, \dots, \text{upper} - 1, \text{upper}\}
```

#### 9.3 Vector cones

The vector-valued set types implemented in MathOptInterface.jl are:

```
• Reals (dimension): \mathbb{R} dimension • Zeros (dimension): 0 dimension • Nonnegatives (dimension): \{x \in \mathbb{R}^{\text{dimension}} : x \geq 0\} • Nonpositives (dimension): \{x \in \mathbb{R}^{\text{dimension}} : x \leq 0\} • SecondOrderCone (dimension): \{(t, x) \in \mathbb{R}^{\text{dimension}} : t \geq \|x\|_2\} • RotatedSecondOrderCone (dimension): \{(t, u, x) \in \mathbb{R}^{\text{dimension}} : 2tu \geq \|x\|_2^2, t, u \geq 0\} • ExponentialCone(): \{(x, y, z) \in \mathbb{R}^3 : y \exp(x/y) \leq z, y > 0\} • DualExponentialCone(): \{(u, v, w) \in \mathbb{R}^3 : -u \exp(v/u) \leq \exp(1)w, u < 0\} • GeometricMeanCone (dimension): \{(t, x) \in \mathbb{R}^{n+1} : x \geq 0, t \leq \sqrt[n]{x_1x_2 \cdots x_n}\} where n is dimension—1
• PowerCone (exponent): \{(x, y, z) \in \mathbb{R}^3 : x \text{exponent} y^1 - \text{exponent} \geq |z|, x, y \geq 0\}
• DualPowerCone (exponent): \{(u, v, w) \in \mathbb{R}^3 : \frac{u}{\text{exponent}} \text{exponent} \frac{v}{1 - \text{exponent}} \frac{1 - \text{exponent}}{1 - \text{exponent}} \geq |w|, u, v \geq 0\}
• NormOneCone (dimension): \{(t, x) \in \mathbb{R}^{\text{dimension}} : t \geq ||x||_1\} where ||x||_1 = \sum_i |x_i|
• NormInfinityCone (dimension): \{(t, x) \in \mathbb{R}^{\text{dimension}} : t \geq ||x||_1\} where ||x||_1 = \sum_i |x_i|
```

• RelativeEntropyCone(dimension):  $\{(u,v,w) \in \mathbb{R}^{\text{dimension}} : u \geq \sum_i w_i \log(\frac{w_i}{v_i}), v_i \geq 0, w_i \geq 0\}$ 

### 9.4 Matrix cones

The matrix-valued set types implemented in MathOptInterface.jl are:

- RootDetConeTriangle(dimension):  $\{(t,X) \in \mathbb{R}^{1+\operatorname{dimension}(1+\operatorname{dimension})/2}: t \leq \det(X)^{1/\operatorname{dimension}}, X \text{ is the upper superior} \}$
- RootDetConeSquare(dimension):  $\{(t,X) \in \mathbb{R}^{1+\text{dimension}^2} : t \leq \det(X)^{1/\text{dimension}}, X \text{ is a PSD matrix} \}$
- PositiveSemidefiniteConeTriangle(dimension):  $\{X \in \mathbb{R}^{\mathsf{dimension}(\mathsf{dimension}+1)/2} : X \text{ is the upper triangle of a PSI of the context of the property of the context of the con$
- PositiveSemidefiniteConeSquare(dimension):  $\{X \in \mathbb{R}^{\text{dimension}^2} : X \text{ is a PSD matrix}\}$
- LogDetConeTriangle(dimension):  $\{(t,u,X) \in \mathbb{R}^{2+\text{dimension}(1+\text{dimension})/2}: t \leq u \log(\det(X/u)), X \text{ is the upper } 0\}$
- LogDetConeSquare(dimension):  $\{(t,u,X) \in \mathbb{R}^{2+\text{dimension}^2} : t \leq u \log(\det(X/u)), X \text{ is a PSD matrix}, u > 0\}$
- NormSpectralCone(row\_dim, column\_dim):  $\{(t,X) \in \mathbb{R}^{1+\text{row\_dim} \times \text{column\_dim}} : t \geq \sigma_1(X), X \text{ is a matrix with row\_dim} \times \sigma_1(X) \}$
- NormNuclearCone(row\_dim, column\_dim):  $\{(t,X) \in \mathbb{R}^{1+\text{row\_dim} \times \text{column\_dim}} : t \geq \sum_i \sigma_i(X), X \text{ is a matrix with row property of the property of$

Some of these cones can take two forms: XXXConeTriangle and XXXConeSquare.

In XXXConeTriangle sets, the matrix is assumed to be symmetric, and the elements are provided by a vector, in which the entries of the upper-right triangular part of the matrix are given column by column (or equivalently, the entries of the lower-left triangular part are given row by row).

In XXXConeSquare sets, the entries of the matrix are given column by column (or equivalently, row by row), and the matrix is constrained to be symmetric. As an example, given a 2-by-2 matrix of variables X and a one-dimensional variable t, we can specify a root-det constraint as [t, X11, X12, X22] ∈ RootDetConeTriangle or [t, X11, X12, X21, X22] ∈ RootDetConeSquare.

We provide both forms to enable flexibility for solvers who may natively support one or the other. Transformations between XXXConeTriangle and XXXConeSquare are handled by bridges, which removes the chance of conversion mistakes by users or solver developers.

#### 9.5 Multi-dimensional sets with combinatorial structure

- SOS1(weights): A special ordered set of Type I.
- SOS2(weights): A special ordered set of Type II.
- Indicator(set): A set to specify indicator constraints.
- Complements (dimension): A set for mixed complementarity constraints.

# **Models**

The most significant part of MOI is the definition of the **model API** that is used to specify an instance of an optimization problem (e.g., by adding variables and constraints). Objects that implement the model API must inherit from the ModelLike abstract type.

Notably missing from the model API is the method to solve an optimization problem. ModelLike objects may store an instance (e.g., in memory or backed by a file format) without being linked to a particular solver. In addition to the model API, MOI defines AbstractOptimizer and provides methods to solve the model and interact with solutions. See the Solutions section for more details.

#### Info

Throughout the rest of the manual, model is used as a generic ModelLike, and optimizer is used as a generic AbstractOptimizer.

#### Tip

MOI does not export functions, but for brevity we often omit qualifying names with the MOI module. Best practice is to have

```
using MathOptInterface
const MOI = MathOptInterface
```

and prefix all MOI methods with MOI. in user code. If a name is also available in base Julia, we always explicitly use the module prefix, for example, with MOI.get.

## 10.1 Attributes

Attributes are properties of the model that can be queried and modified. These include constants such as the number of variables in a model NumberOfVariables), and properties of variables and constraints such as the name of a variable (VariableName).

There are four types of attributes:

- Model attributes (subtypes of AbstractModelAttribute) refer to properties of a model.
- Optimizer attributes (subtypes of AbstractOptimizerAttribute) refer to properties of an optimizer.
- Constraint attributes (subtypes of AbstractConstraintAttribute) refer to properties of an individual constraint.

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Variable attributes (subtypes of AbstractVariableAttribute) refer to properties of an individual variable.

Some attributes are values that can be queried by the user but not modified, while other attributes can be modified by the user.

All interactions with attributes occur through the get and set functions.

Consult the docstsrings of each attribute for information on what it represents.

#### 10.2 ModelLike API

The following attributes are available:

- ListOfConstraintAttributesSet
- ListOfConstraintIndices
- ListOfConstraintTypesPresent
- ListOfModelAttributesSet
- ListOfVariableAttributesSet
- ListOfVariableIndices
- NumberOfConstraints
- NumberOfVariables
- Name
- ObjectiveFunction
- ObjectiveFunctionType
- ObjectiveSense

## 10.3 AbstractOptimizer API

The following attributes are available:

- DualStatus
- PrimalStatus
- RawStatusString
- ResultCount
- TerminationStatus
- BarrierIterations
- DualObjectiveValue
- NodeCount

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- NumberOfThreads
- ObjectiveBound
- ObjectiveValue
- RelativeGap
- RawOptimizerAttribute
- RawSolver
- Silent
- SimplexIterations
- SolverName
- SolverVersion
- SolveTimeSec
- TimeLimitSec

# **Variables**

#### 11.1 Add a variable

Use add\_variable to add a single variable.

```
julia> x = MOI.add_variable(model)
MathOptInterface.VariableIndex(1)
```

add\_variable returns a VariableIndex type, which is used to refer to the added variable in other calls.

Check if a VariableIndex is valid using is\_valid.

```
julia> MOI.is_valid(model, x)
true
```

Use add variables to add a number of variables.

```
julia> y = MOI.add_variables(model, 2)
2-element Vector{MathOptInterface.VariableIndex}:
MathOptInterface.VariableIndex(2)
MathOptInterface.VariableIndex(3)
```

#### Warning

The integer does not necessarily corresond to the column inside an optimizer!

#### 11.2 Delete a variable

Delete a variable using delete.

```
julia> MOI.delete(model, x)
julia> MOI.is_valid(model, x)
false
```

#### Warning

Not all ModelLike models support deleting variables. A DeleteNotAllowed error is thrown if this is not supported.

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## 11.3 Variable attributes

The following attributes are available for variables:

- VariableName
- VariablePrimalStart
- VariablePrimal

Get and set these attributes using get and set.

```
julia> MOI.set(model, MOI.VariableName(), x, "var_x")
julia> MOI.get(model, MOI.VariableName(), x)
"var_x"
```

# **Constraints**

#### 12.1 Add a constraint

Use add\_constraint to add a single constraint.

add constraint returns a ConstraintIndex type, which is used to refer to the added constraint in other calls.

Check if a ConstraintIndex is valid using is valid.

```
julia> MOI.is_valid(model, c)
true
```

Use add\_constraints to add a number of constraints of the same type.

This time, a vector of ConstraintIndex are returned.

Use supports\_constraint to check if the model supports adding a constraint type.

#### 12.2 Delete a constraint

Use delete to delete a constraint.

```
julia> MOI.delete(model, c)
julia> MOI.is_valid(model, c)
false
```

#### 12.3 Constraint attributes

The following attributes are available for constraints:

- ConstraintName
- ConstraintPrimalStart
- ConstraintDualStart
- ConstraintPrimal
- ConstraintDual
- ConstraintBasisStatus
- ConstraintFunction
- CanonicalConstraintFunction
- ConstraintSet

Get and set these attributes using get and set.

```
julia> MOI.set(model, MOI.ConstraintName(), c, "con_c")
julia> MOI.get(model, MOI.ConstraintName(), c)
"con_c"
```

#### 12.4 Constraints by function-set pairs

Below is a list of common constraint types and how they are represented as function-set pairs in MOI. In the notation below, x is a vector of decision variables,  $x_i$  is a scalar decision variable,  $\alpha, \beta$  are scalar constants, a, b are constant vectors, A is a constant matrix and  $\mathbb{R}_+$  (resp.  $\mathbb{R}_-$ ) is the set of nonnegative (resp. nonpositive) real numbers.

#### **Linear constraints**

By convention, solvers are not expected to support nonzero constant terms in the ScalarAffineFunctions the first four rows above, because they are redundant with the parameters of the sets. For example, encode  $2x+1\leq 2$  as  $2x\leq 1$ .

Constraints with VariableIndex in LessThan, GreaterThan, EqualTo, or Interval sets have a natural interpretation as variable bounds. As such, it is typically not natural to impose multiple lower- or upper-bounds on the same variable, and the solver interfaces will throw respectively LowerBoundAlreadySet or UpperBoundAlreadySet.

Mathematical Constraint	MOI Function	MOI Set
$a^T x \le \beta$	ScalarAffineFunction	LessThan
$a^T x \ge \alpha$	ScalarAffineFunction	GreaterThan
$a^T x = \beta$	ScalarAffineFunction	EqualTo
$\alpha \le a^T x \le \beta$	ScalarAffineFunction	Interval
$x_i \leq \beta$	VariableIndex	LessThan
$x_i \ge \alpha$	VariableIndex	GreaterThan
$x_i = \beta$	VariableIndex	EqualTo
$\alpha \le x_i \le \beta$	VariableIndex	Interval
$Ax + b \in \mathbb{R}^n_+$	VectorAffineFunction	Nonnegatives
$Ax + b \in \mathbb{R}^n$	VectorAffineFunction	Nonpositives
Ax + b = 0	VectorAffineFunction	Zeros

Moreover, adding two VariableIndex constraints on the same variable with the same set is impossible because they share the same index as it is the index of the variable, see ConstraintIndex.

It is natural, however, to impose upper- and lower-bounds separately as two different constraints on a single variable. The difference between imposing bounds by using a single Interval constraint and by using separate LessThan and GreaterThan constraints is that the latter will allow the solver to return separate dual multipliers for the two bounds, while the former will allow the solver to return only a single dual for the interval constraint.

#### **Conic constraints**

Mathematical Constraint	MOI Function	MOI Set
		MOI Set
$  Ax + b  _2 \le c^T x + d$	VectorAffineFunction	SecondOrderCone
$y \ge   x  _2$	VectorOfVariables	SecondOrderCone
$2yz \ge   x  _2^2, y, z \ge 0$	VectorOfVariables	RotatedSecondOrderCone
$(a_1^T x + b_1, a_2^T x + b_2, a_3^T x + b_3) \in \mathcal{E}$	VectorAffineFunction	ExponentialCone
$A(x) \in \mathcal{S}_+$	VectorAffineFunction	PositiveSemidefiniteConeTriangle
$B(x) \in \mathcal{S}_+$	VectorAffineFunction	PositiveSemidefiniteConeSquare
$x \in \mathcal{S}_+$	VectorOfVariables	PositiveSemidefiniteConeTriangle
$x \in \mathcal{S}_+$	VectorOfVariables	PositiveSemidefiniteConeSquare

where  $\mathcal{E}$  is the exponential cone (see ExponentialCone),  $\mathcal{S}_+$  is the set of positive semidefinite symmetric matrices, A is an affine map that outputs symmetric matrices and B is an affine map that outputs square matrices.

## **Quadratic constraints**

Mathematical Constraint	MOI Function	MOI Set
$x^T Q x + a^T x + b \ge 0$	ScalarQuadraticFunction	GreaterThan
	ScalarQuadraticFunction	LessThan
$x^T Q x + a^T x + b = 0$	ScalarQuadraticFunction	EqualTo
Bilinear matrix inequality	VectorQuadraticFunction	PositiveSemidefiniteCone

## Discrete and logical constraints

## 12.5 JuMP mapping

The following bullet points show examples of how JuMP constraints are translated into MOI function-set pairs:

Mathematical Constraint	MOI Function	MOI Set
$x_i \in \mathbb{Z}$	VariableIndex	Integer
$x_i \in \{0, 1\}$	VariableIndex	Zero0ne
$x_i \in \{0\} \cup [l, u]$	VariableIndex	Semicontinuous
$x_i \in \{0\} \cup \{l, l+1, \dots, u-1, u\}$	VariableIndex	Semiinteger
At most one component of $\boldsymbol{x}$ can be nonzero	VectorOfVariables	SOS1
At most two components of $\boldsymbol{x}$ can be nonzero, and if so they must be	VectorOfVariables	S S0S2
adjacent components		
$y = 1 \implies a^T x \in S$	VectorAffineFunct	ionIndicator

- @constraint(m,  $2x + y \le 10$ ) becomes ScalarAffineFunction-in-LessThan
- @constraint(m, 2x + y >= 10) becomes ScalarAffineFunction-in-GreaterThan
- @constraint(m, 2x + y == 10) becomes ScalarAffineFunction-in-EqualTo
- @constraint(m, 0 <= 2x + y <= 10) becomes ScalarAffineFunction-in-Interval
- @constraint(m, 2x + y in ArbitrarySet()) becomes ScalarAffineFunction-in-ArbitrarySet.

Variable bounds are handled in a similar fashion:

- @variable(m, x <= 1) becomes VariableIndex-in-LessThan
- @variable(m, x >= 1) becomes VariableIndex-in-GreaterThan

One notable difference is that a variable with an upper and lower bound is translated into two constraints, rather than an interval. i.e.:

• @variable(m,  $0 \le x \le 1$ ) becomes VariableIndex-in-LessThan and VariableIndex-in-GreaterThan.

# **Solutions**

## 13.1 Solving and retrieving the results

Once an optimizer is loaded with the objective function and all of the constraints, we can ask the solver to solve the model by calling optimize!.

```
MOI.optimize!(optimizer)
```

#### 13.2 Why did the solver stop?

The optimization procedure may terminate for a number of reasons. The TerminationStatus attribute of the optimizer returns a TerminationStatusCode object which explains why the solver stopped.

The termination statuses distinguish between proofs of optimality, infeasibility, local convergence, limits, and termination because of something unexpected like invalid problem data or failure to converge.

A typical usage of the TerminationStatus attribute is as follows:

```
status = MOI.get(optimizer, TerminationStatus())
if status == MOI.OPTIMAL
    # Ok, we solved the problem!
else
    # Handle other cases.
end
```

After checking the TerminationStatus, check ResultCount. This attribute returns the number of results that the solver has available to return. A result is defined as a primal-dual pair, but either the primal or the dual may be missing from the result. While the OPTIMAL termination status normally implies that at least one result is available, other statuses do not. For example, in the case of infeasibility, a solver may return no result or a proof of infeasibility. The ResultCount attribute distinguishes between these two cases.

#### 13.3 Primal solutions

Use the PrimalStatus optimizer attribute to return a ResultStatusCode describing the status of the primal solution.

Common returns are described below in the Common status situations section.

Query the primal solution using the VariablePrimal and ConstraintPrimal attributes.

Query the objective function value using the ObjectiveValue attribute.

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# 13.4 Dual solutions

#### Warning

See Duality for a discussion of the MOI conventions for primal-dual pairs and certificates.

Use the DualStatus optimizer attribute to return a ResultStatusCode describing the status of the dual solution.

Query the dual solution using the ConstraintDual attribute.

Query the dual objective function value using the DualObjectiveValue attribute.

#### 13.5 Common status situations

The sections below describe how to interpret typical or interesting status cases for three common classes of solvers. The example cases are illustrative, not comprehensive. Solver wrappers may provide additional information on how the solver's statuses map to MOI statuses.

#### Info

\* in the tables indicate that multiple different values are possible.

#### Primal-dual convex solver

Linear programming and conic optimization solvers fall into this category.

What happened?	TerminationSt	a <b>f</b> lessultCou	nt PrimalStatus	DualStatus	
Proved optimality	OPTIMAL	1	FEASIBLE_POINT	FEASIBLE_POINT	
Proved infeasible	INFEASIBLE	1	NO_SOLUTION	INFEASIBILITY_CERTI	FICATE
Optimal within relaxed	ALMOST_OPTIMA	L 1	FEASIBLE_POINT	FEASIBLE_POINT	
tolerances					
Optimal within relaxed	ALMOST_OPTIMA	L 1	ALMOST_FEASIBLE_PO	NATLMOST_FEASIBLE_P01	:NT
tolerances					
Detected an unbounded ray	DUAL_INFEASIB	LE 1	INFEASIBILITY_CERT	FICATE NO_SOLUTION	
of the primal					
Stall	SLOW_PROGRESS	1	*	*	

#### Global branch-and-bound solvers

Mixed-integer programming solvers fall into this category.

What happened?	TerminationStatus	ResultCour	t PrimalStatus	DualStatus
Proved optimality	OPTIMAL	1	FEASIBLE_POIN	NO_SOLUTION
Presolve detected infeasibility or	INFEASIBLE_OR_UNBOU	NDED 0	NO_SOLUTION	NO_SOLUTION
unboundedness				
Proved infeasibility	INFEASIBLE	0	NO_SOLUTION	NO_SOLUTION
Timed out (no solution)	TIME_LIMIT	0	NO_SOLUTION	NO_SOLUTION
Timed out (with a solution)	TIME_LIMIT	1	FEASIBLE_POIN	NO_SOLUTION
CPXMIP_OPTIMAL_INFEAS	ALMOST_OPTIMAL	1	INFEASIBLE_PO	NNTO_SOLUTION

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#### Info

CPXMIP\_OPTIMAL\_INFEAS is a CPLEX status that indicates that a preprocessed problem was solved to optimality, but the solver was unable to recover a feasible solution to the original problem. Handling this status was one of the motivating drivers behind the design of MOI.

#### Local search solvers

Nonlinear programming solvers fall into this category. It also includes non-global tree search solvers like Juniper.

What happened?	TerminationStatus	ResultCou	n₱rimalStatus	DualStatus
Converged to a stationary point	LOCALLY_SOLVED	1	FEASIBLE_P0I	NTFEASIBLE_POIN
Completed a non-global tree search	LOCALLY_SOLVED	1	FEASIBLE_P0I	NTFEASIBLE_POIN
(with a solution)				
Converged to an infeasible point	LOCALLY_INFEASIBLE	1	INFEASIBLE_P	OINT *
Completed a non-global tree search	LOCALLY_INFEASIBLE	0	NO_SOLUTION	NO_SOLUTION
(no solution found)				
Iteration limit	ITERATION_LIMIT	1	*	*
Diverging iterates	NORM_LIMIT or	1	*	*
	OBJECTIVE_LIMIT			

## 13.6 Querying solution attributes

Some solvers will not implement every solution attribute. Therefore, a call like MOI.get(model, MOI.SolveTimeSec()) may throw an UnsupportedAttribute error.

If you need to write code that is agnostic to the solver (for example, you are writing a library that an end-user passes their choice of solver to), you can work-around this problem using a try-catch:

```
function get_solve_time(model)
    try
        return MOI.get(model, MOI.SolveTimeSec())
    catch err
        if err isa MOI.UnsupportedAttribute
            return NaN # Solver doesn't support. Return a placeholder value.
        end
        rethrow(err) # Something else went wrong. Rethrow the error
    end
end
```

If, after careful profiling, you find that the try-catch is taking a significant portion of your runtime, you can improve performance by caching the result of the try-catch:

```
mutable struct CachedSolveTime{M}
   model::M
   supports_solve_time::Bool
   CachedSolveTime(model::M) where {M} = new(model, true)
end

function get_solve_time(model::CachedSolveTime)
   if !model.supports_solve_time
       return NaN
   end
```

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```
try
    return MOI.get(model, MOI.SolveTimeSec())
catch err
    if err isa MOI.UnsupportedAttribute
        model.supports_solve_time = false
        return NaN
    end
    rethrow(err) # Something else went wrong. Rethrow the error
end
end
```

# **Problem modification**

In addition to adding and deleting constraints and variables, MathOptInterface supports modifying, in-place, coefficients in the constraints and the objective function of a model.

These modifications can be grouped into two categories:

- · modifications which replace the set of function of a constraint with a new set or function
- modifications which change, in-place, a component of a function

#### Warning

Solve ModelLike objects do not support problem modification.

## 14.1 Modify the set of a constraint

Use set and ConstraintSet to modify the set of a constraint by replacing it with a new instance of the same type.

However, the following will fail as the new set is of a different type to the original set:

```
julia> MOI.set(model, MOI.ConstraintSet(), c, MOI.GreaterThan(2.0)) ERROR: [...]
```

#### Special cases: set transforms

If our constraint is an affine inequality, then this corresponds to modifying the right-hand side of a constraint in linear programming.

In some special cases, solvers may support efficiently changing the set of a constraint (for example, from LessThan to GreaterThan). For these cases, MathOptInterface provides the transform method.

The transform function returns a new constraint index, and the old constraint index (i.e., c) is no longer valid.

#### Note

transform cannot be called with a set of the same type. Use set instead.

## 14.2 Modify the function of a constraint

Use set and ConstraintFunction to modify the function of a constraint by replacing it with a new instance of the same type.

However, the following will fail as the new function is of a different type to the original function:

```
julia> MOI.set(model, MOI.ConstraintFunction(), c, x)
ERROR: [...]
```

## 14.3 Modify constant term in a scalar function

 $Use \ modify\ and\ Scalar Constant Change\ to\ modify\ the\ constant\ term\ in\ a\ Scalar Affine Function\ or\ Scalar Quadratic Function.$ 

#### Tip

ScalarConstantChange can also be used to modify the objective function by passing an instance of ObjectiveFunction instead of the constraint index c as we saw above.

## 14.4 Modify constant terms in a vector function

Use modify and VectorConstantChange to modify the constant vector in a VectorAffineFunction or VectorQuadraticFunction

### 14.5 Modify affine coefficients in a scalar function

Use modify and ScalarCoefficientChange to modify the affine coefficient of a ScalarAffineFunction or ScalarQuadraticFunction.

#### Tip

ScalarCoefficientChange can also be used to modify the objective function by passing an instance of ObjectiveFunction instead of the constraint index c as we saw above.

#### 14.6 Modify affine coefficients in a vector function

Use modify and MultirowChange to modify a vector of affine coefficients in a VectorAffineFunction or a VectorQuadraticFunction.

# Part IV

# **Background**

# **Duality**

Conic duality is the starting point for MOI's duality conventions. When all functions are affine (or coordinate projections), and all constraint sets are closed convex cones, the model may be called a conic optimization problem.

For a minimization problem in geometric conic form, the primal is:

$$\min_{a_0^T x + b_0 \tag{15.1}$$

s.t. 
$$A_i x + b_i \in \mathcal{C}_i$$
  $i = 1 \dots m$  (15.2)

and the dual is a maximization problem in standard conic form:

$$\max_{y_1, \dots, y_m} -\sum_{i=1}^m b_i^T y_i + b_0 \tag{15.3}$$

s.t. 
$$a_0 - \sum_{i=1}^m A_i^T y_i = 0$$
 (15.4)

$$y_i \in \mathcal{C}_i^* \qquad \qquad i = 1 \dots m \tag{15.5}$$

where each  $\mathcal{C}_i$  is a closed convex cone and  $\mathcal{C}_i^*$  is its dual cone.

For a maximization problem in geometric conic form, the primal is:

$$\max_{a_0^T x + b_0} \qquad (15.6)$$

s.t. 
$$A_i x + b_i \in \mathcal{C}_i$$
  $i = 1 \dots m$  (15.7)

and the dual is a minimization problem in standard conic form:

$$\min_{y_1, \dots, y_m} \sum_{i=1}^m b_i^T y_i + b_0 \tag{15.8}$$

s.t. 
$$a_0 + \sum_{i=1}^m A_i^T y_i = 0 ag{15.9}$$

$$y_i \in \mathcal{C}_i^* \qquad \qquad i = 1 \dots m \tag{15.10}$$

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A linear inequality constraint  $a^Tx+b\geq c$  is equivalent to  $a^Tx+b-c\in\mathbb{R}_+$ , and  $a^Tx+b\leq c$  is equivalent to  $a^Tx+b-c\in\mathbb{R}_+$ . Variable-wise constraints are affine constraints with the appropriate identity mapping in place of  $A_i$ .

For the special case of minimization LPs, the MOI primal form can be stated as:

$$\min_{x \in \mathbb{P}^n} \qquad \qquad a_0^T x + b_0 \tag{15.11}$$

s.t. 
$$A_1 x \ge b_1$$
 (15.12)

$$A_2 x \le b_2$$
 (15.13)

$$A_3 x = b_3 {(15.14)}$$

By applying the stated transformations to conic form, taking the dual, and transforming back into linear inequality form, one obtains the following dual:

$$\max_{y_1, y_2, y_3} b_1^T y_1 + b_2^T y_2 + b_3^T y_3 + b_0$$
 (15.15)

s.t. 
$$A_1^T y_1 + A_2^T y_2 + A_3^T y_3 = a_0$$
 (15.16)

$$y_1 \ge 0$$
 (15.17)

$$y_2 \le 0$$
 (15.18)

For maximization LPs, the MOI primal form can be stated as:

$$\max_{x \in \mathbb{D}^n} \qquad a_0^T x + b_0 \tag{15.19}$$

s.t. 
$$A_1 x \ge b_1$$
 (15.20)

$$A_2 x \le b_2 \tag{15.21}$$

$$A_3 x = b_3 (15.22)$$

and similarly, the dual is:

s.t. 
$$A_1^T y_1 + A_2^T y_2 + A_3^T y_3 = -a_0$$
 (15.24)

$$y_1 \ge 0$$
 (15.25)

$$y_2 \le 0$$
 (15.26)

# Warning

For the LP case, the signs of the feasible dual variables depend only on the sense of the corresponding primal inequality and not on the objective sense.

#### 15.1 Duality and scalar product

The scalar product is different from the canonical one for the sets PositiveSemidefiniteConeTriangle, LogDetConeTriangle, RootDetConeTriangle.

If the set  $C_i$  of the section Duality is one of these three cones, then the rows of the matrix  $A_i$  corresponding to off-diagonal entries are twice the value of the coefficients field in the VectorAffineFunction for the corresponding rows. See PositiveSemidefiniteConeTriangle for details.

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# 15.2 Dual for problems with quadratic functions

## **Quadratic Programs (QPs)**

For quadratic programs with only affine conic constraints,

$$\min_{x\in\mathbb{R}^n} \qquad \qquad \frac{1}{2}x^TQ_0x + a_0^Tx + b_0$$
 s.t. 
$$A_ix + b_i \in \mathcal{C}_i \qquad \qquad i=1\dots m.$$

with cones  $\mathcal{C}_i \subseteq \mathbb{R}^{m_i}$  for  $i=1,\ldots,m$ , consider the Lagrangian function

$$L(x,y) = \frac{1}{2}x^{T}Q_{0}x + a_{0}^{T}x + b_{0} - \sum_{i=1}^{m} y_{i}^{T}(A_{i}x + b_{i}).$$

Let z(y) denote  $\sum_{i=1}^m A_i^T y_i - a_0$ , the Lagrangian can be rewritten as

$$L(x,y) = \frac{1}{2}x^{T}Q_{0}x - z(y)^{T}x + b_{0} - \sum_{i=1}^{m} y_{i}^{T}b_{i}.$$

The condition  $\nabla_x L(x,y) = 0$  gives

$$0 = \nabla_x L(x, y) = Q_0 x + a_0 - \sum_{i=1}^m y_i^T b_i$$

which gives  $Q_0x=z(y)$ . This allows to obtain that

$$\min_{x \in \mathbb{R}^n} L(x, y) = -\frac{1}{2} x^T Q_0 x + b_0 - \sum_{i=1}^m y_i^T b_i$$

so the dual problem is

$$\max_{y_i \in \mathcal{C}_i^*} \min_{x \in \mathbb{R}^n} -\frac{1}{2} x^T Q_0 x + b_0 - \sum_{i=1}^m y_i^T b_i.$$

If  $Q_0$  is invertible, we have  $x=Q_0^{-1}z(y)$  hence

$$\min_{x \in \mathbb{R}^n} L(x, y) = -\frac{1}{2} z(y)^T Q_0^{-1} z(y) + b_0 - \sum_{i=1}^m y_i^T b_i$$

so the dual problem is

$$\max_{y_i \in \mathcal{C}_i^*} -\frac{1}{2} z(y)^T Q_0^{-1} z(y) + b_0 - \sum_{i=1}^m y_i^T b_i.$$

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## **Quadratically Constrained Quadratic Programs (QCQPs)**

Given a problem with both quadratic function and quadratic objectives:

$$\begin{aligned} \min_{x \in \mathbb{R}^n} & \frac{1}{2} x^T Q_0 x + a_0^T x + b_0 \\ \text{s.t.} & \frac{1}{2} x^T Q_i x + a_i^T x + b_i \in \mathcal{C}_i \end{aligned} \qquad i = 1 \dots m.$$

with cones  $\mathcal{C}_i \subseteq \mathbb{R}$  for  $i=1\dots m$ , consider the Lagrangian function

$$L(x,y) = \frac{1}{2}x^{T}Q_{0}x + a_{0}^{T}x + b_{0} - \sum_{i=1}^{m} y_{i}(\frac{1}{2}x^{T}Q_{i}x + a_{i}^{T}x + b_{i})$$

A pair of primal-dual variables  $(x^\star, y^\star)$  is optimal if

•  $x^{\star}$  is a minimizer of

$$\min_{x \in \mathbb{R}^n} L(x, y^*).$$

That is,

$$0 = \nabla_x L(x, y^*) = Q_0 x + a_0 - \sum_{i=1}^m y_i^* (Q_i x + a_i).$$

• and  $y^*$  is a maximizer of

$$\max_{y_i \in \mathcal{C}_i^*} L(x^*, y).$$

That is, for all  $i=1,\ldots,m$ ,  $\frac{1}{2}x^TQ_ix+a_i^Tx+b_i$  is either zero or in the normal cone of  $\mathcal{C}_i^*$  at  $y^\star$ . For instance, if  $\mathcal{C}_i$  is  $\{z\in\mathbb{R}:z\leq 0\}$ , this means that if  $\frac{1}{2}x^TQ_ix+a_i^Tx+b_i$  is nonzero at  $x^\star$  then  $y_i^\star=0$ . This is the classical complementary slackness condition.

If  $C_i$  is a vector set, the discussion remains valid with  $y_i(\frac{1}{2}x^TQ_ix + a_i^Tx + b_i)$  replaced with the scalar product between  $y_i$  and the vector of scalar-valued quadratic functions.

# Infeasibility certificates

When given a conic problem that is infeasible or unbounded, some solvers can produce a certificate of infeasibility. This page explains what a certificate of infeasibility is, and the related conventions that MathOptInterface adopts.

## 16.1 Conic duality

MathOptInterface uses conic duality to define infeasibility certificates. A full explanation is given in the section Duality, but here is a brief overview.

## **Minimization problems**

For a minimization problem in geometric conic form, the primal is:

$$\min_{x \in \mathbb{D}^n} \qquad \qquad a_0^\top x + b_0 \tag{16.1}$$

s.t. 
$$A_i x + b_i \in \mathcal{C}_i$$
  $i = 1 \dots m,$  (16.2)

and the dual is a maximization problem in standard conic form:

$$\max_{y_1, \dots, y_m} \qquad -\sum_{i=1}^m b_i^\top y_i + b_0 \tag{16.3}$$

s.t. 
$$a_0 - \sum_{i=1}^m A_i^\top y_i = 0 \tag{16.4}$$

$$y_i \in \mathcal{C}_i^* \qquad \qquad i = 1 \dots m, \tag{16.5}$$

where each  $\mathcal{C}_i$  is a closed convex cone and  $\mathcal{C}_i^*$  is its dual cone.

#### **Maximization problems**

For a maximization problem in geometric conic form, the primal is:

$$\max_{x \in \mathbb{R}^n} \qquad \qquad a_0^\top x + b_0 \tag{16.6}$$

s.t. 
$$A_i x + b_i \in \mathcal{C}_i$$
  $i = 1 \dots m,$  (16.7)

and the dual is a minimization problem in standard conic form:

$$\min_{y_1, \dots, y_m} \sum_{i=1}^m b_i^\top y_i + b_0$$
(16.8)

s.t. 
$$a_0 + \sum_{i=1}^m A_i^\top y_i = 0 \tag{16.9}$$

$$y_i \in \mathcal{C}_i^* \qquad \qquad i = 1 \dots m. \tag{16.10}$$

#### 16.2 Unbounded problems

A problem is unbounded if and only if:

- 1. there exists a feasible primal solution
- 2. the dual is infeasible.

A feasible primal solution—if one exists—can be obtained by setting <code>ObjectiveSense</code> to <code>FEASIBILITY\_SENSE</code> before optimizing. Therefore, most solvers terminate after they prove the dual is infeasible via a certificate of dual infeasibility, but before they have found a feasible primal solution. This is also the reason that <code>MathOptInterface</code> defines the <code>DUAL\_INFEASIBLE</code> status instead of <code>UNBOUNDED</code>.

A certificate of dual infeasibility is an improving ray of the primal problem. That is, there exists some vector d such that for all  $\eta > 0$ :

$$A_i(x + \eta d) + b_i \in \mathcal{C}_i, i = 1 \dots m,$$

and (for minimization problems):

$$a_0^{\top}(x + \eta d) + b_0 < a_0^{\top}x + b_0,$$

for any feasible point x. The latter simplifies to  $a_0^\top d < 0$ . For maximization problems, the inequality is reversed, so that  $a_0^\top d > 0$ .

If the solver has found a certificate of dual infeasibility:

- TerminationStatus must be DUAL\_INFEASIBLE
- PrimalStatus must be INFEASIBILITY\_CERTIFICATE
- ullet VariablePrimal must be the corresponding value of d
- ullet ConstraintPrimal must be the corresponding value of  $A_id$
- ObjectiveValue must be the value  $a_0^\top d$ . Note that this is the value of the objective function at d, ignoring the constant b\_0.

#### Note

The choice of whether to scale the ray d to have magnitude 1 is left to the solver.

# 16.3 Infeasible problems

A certificate of primal infeasibility is an improving ray of the dual problem. However, because infeasibility is independent of the objective function, we first homogenize the primal problem by removing its objective.

For a minimization problem, a dual improving ray is some vector d such that for all  $\eta > 0$ :

$$-\sum_{i=1}^{m} A_i^{\top}(y_i + \eta d_i) = 0$$
 (16.11)

$$(y_i + \eta d_i) \in \mathcal{C}_i^* \qquad \qquad i = 1 \dots m, \tag{16.12}$$

and:

$$-\sum_{i=1}^{m} b_{i}^{\top}(y_{i} + \eta d_{i}) > -\sum_{i=1}^{m} b_{i}^{\top} y_{i},$$

for any feasible dual solution y. The latter simplifies to  $-\sum_{i=1}^m b_i^\top d_i > 0$ . For a maximization problem, the inequality is  $\sum_{i=1}^m b_i^\top d_i < 0$ . (Note that these are the same inequality, modulo a - sign.)

If the solver has found a certificate of primal infeasibility:

- TerminationStatus must be INFEASIBLE
- DualStatus must be INFEASIBILITY CERTIFICATE
- ullet ConstraintDual must be the corresponding value of d
- DualObjectiveValue must be the value  $-\sum_{i=1}^m b_i^\top d_i$  for minimization problems and  $\sum_{i=1}^m b_i^\top d_i$  for maximization problems.

# Note

The choice of whether to scale the ray d to have magnitude 1 is left to the solver.

# Infeasibility certificates of variable bounds

Many linear solvers (e.g., Gurobi) do not provide explicit access to the primal infeasibility certificate of a variable bound. However, given a set of linear constraints:

$$l_A \le Ax \le u_A \tag{16.13}$$

$$l_x \le x \le u_x,\tag{16.14}$$

the primal certificate of the variable bounds can be computed using the primal certificate associated with the affine constraints, d. (Note that d will have one element for each row of the A matrix, and that some or all of the elements in the vectors  $l_A$  and  $u_A$  may be  $\pm\infty$ . If both  $l_A$  and  $u_A$  are finite for some row, the corresponding element in 'd must be 0.)

Given d, compute  $\bar{d} = d^{\top}A$ . If the bound is finite, a certificate for the lower variable bound of  $x_i$  is  $\max\{\bar{d}_i,0\}$ , and a certificate for the upper variable bound is  $\min\{\bar{d}_i,0\}$ .

# **Chapter 17**

# **Naming conventions**

MOI follows several conventions for naming functions and structures. These should also be followed by packages extending MOI.

#### 17.1 Sets

Sets encode the structure of constraints. Their names should follow the following conventions:

- Abstract types in the set hierarchy should begin with Abstract and end in Set, e.g., AbstractScalarSet, AbstractVectorSet.
- Vector-valued conic sets should end with Cone, e.g., NormInfinityCone, SecondOrderCone.
- Vector-valued Cartesian products should be plural and not end in Cone, e.g., Nonnegatives, not NonnegativeCone.
- Matrix-valued conic sets should provide two representations: ConeSquare and ConeTriangle, e.g., RootDetConeTriangle and RootDetConeSquare. See Matrix cones for more details.
- Scalar sets should be singular, not plural, e.g., Integer, not Integers.
- As much as possible, the names should follow established conventions in the domain where this set is used: for instance, convex sets should have names close to those of CVX, and constraint-programming sets should follow MiniZinc's constraints.

# Part V API Reference

# **Chapter 18**

# Standard form

# 18.1 Functions

MathOptInterface.AbstractFunction - Type.

AbstractFunction

Abstract supertype for function objects.

source

MathOptInterface.AbstractScalarFunction - Type.

AbstractScalarFunction

Abstract supertype for scalar-valued function objects.

source

MathOptInterface.AbstractVectorFunction - Type.

AbstractVectorFunction

Abstract supertype for vector-valued function objects.

source

MathOptInterface.VariableIndex - Type.

VariableIndex

A type-safe wrapper for Int64 for use in referencing variables in a model. To allow for deletion, indices need not be consecutive.

source

MathOptInterface.VectorOfVariables - Type.

| VectorOfVariables(variables)

The function that extracts the vector of variables referenced by variables, a Vector{VariableIndex}. This function is naturally be used for constraints that apply to groups of variables, such as an "all different" constraint, an indicator constraint, or a complementarity constraint.

MathOptInterface.ScalarAffineTerm - Type.

```
struct ScalarAffineTerm{T}
    coefficient::T
    variable::VariableIndex
end
```

Represents  $cx_i$  where c is coefficient and  $x_i$  is the variable identified by variable.

source

MathOptInterface.ScalarAffineFunction - Type.

```
| ScalarAffineFunction{T}(terms, constant)
```

The scalar-valued affine function  $a^Tx + b$ , where:

- a is a sparse vector specified by a list of ScalarAffineTerm structs.
- b is a scalar specified by constant::T

Duplicate variable indices in terms are accepted, and the corresponding coefficients are summed together.

source

MathOptInterface.VectorAffineTerm - Type.

```
struct VectorAffineTerm{T}
  output_index::Int64
  scalar_term::ScalarAffineTerm{T}
end
```

A ScalarAffineTerm plus its index of the output component of a VectorAffineFunction or VectorQuadraticFunction. output\_index can also be interpreted as a row index into a sparse matrix, where the scalar\_term contains the column index and coefficient.

source

 ${\tt MathOptInterface.VectorAffineFunction-Type.}$ 

```
VectorAffineFunction{T}(terms, constants)
```

The vector-valued affine function Ax + b, where:

- ullet A is a sparse matrix specified by a list of VectorAffineTerm objects.
- ullet b is a vector specified by constants

Duplicate indices in the A are accepted, and the corresponding coefficients are summed together.

source

MathOptInterface.ScalarQuadraticTerm - Type.

```
struct ScalarQuadraticTerm{T}
    coefficient::T
    variable_1::VariableIndex
    variable_2::VariableIndex
end
```

Represents  $cx_ix_j$  where c is coefficient,  $x_i$  is the variable identified by variable\_1 and  $x_j$  is the variable identified by variable\_2.

source

MathOptInterface.ScalarQuadraticFunction - Type.

| ScalarQuadraticFunction{T}(quadratic\_terms, affine\_terms, constant)

The scalar-valued quadratic function  $\frac{1}{2}x^TQx + a^Tx + b$ , where:

- ullet a is a sparse vector specified by a list of ScalarAffineTerm structs.
- b is a scalar specified by constant.
- ullet Q is a symmetric matrix specified by a list of ScalarQuadraticTerm structs.

Duplicate indices in a or Q are accepted, and the corresponding coefficients are summed together. "Mirrored" indices (q,r) and (r,q) (where r and q are VariableIndexes) are considered duplicates; only one need be specified.

For example, for two scalar variables y,z, the quadratic expression  $yz+y^2$  is represented by the terms ScalarQuadraticTerm.([1.0, 2.0], [y, y], [z, y]).

source

MathOptInterface.VectorQuadraticTerm - Type.

```
struct VectorQuadraticTerm{T}
  output_index::Int64
  scalar_term::ScalarQuadraticTerm{T}
end
```

A ScalarQuadraticTerm plus its index of the output component of a VectorQuadraticFunction. Each output component corresponds to a distinct sparse matrix  $Q_i$ .

source

MathOptInterface.VectorQuadraticFunction - Type.

```
| VectorQuadraticFunction{T}(quadratic_terms, affine_terms, constants)
```

The vector-valued quadratic function with ith component ("output index") defined as  $\frac{1}{2}x^TQ_ix + a_i^Tx + b_i$ , where:

- $a_i$  is a sparse vector specified by the VectorAffineTerms with output\_index == i.
- $b_i$  is a scalar specified by constants[i]
- $Q_i$  is a symmetric matrix specified by the VectorQuadraticTerm with output index == i.

Duplicate indices in  $a_i$  or  $Q_i$  are accepted, and the corresponding coefficients are summed together. "Mirrored" indices (q,r) and (r,q) (where r and q are VariableIndexes) are considered duplicates; only one need be specified.

#### **Utilities**

```
MathOptInterface.output_dimension - Function.
   output_dimension(f::AbstractFunction)
   Return 1 if f has a scalar output and the number of output components if f has a vector output.
    source
MathOptInterface.constant - Method.
   constant(f::Union{ScalarAffineFunction, ScalarQuadraticFunction})
   Returns the constant term of the scalar function
    source
MathOptInterface.constant - Method.
   constant(f::Union{VectorAffineFunction, VectorQuadraticFunction})
   Returns the vector of constant terms of the vector function
    source
MathOptInterface.constant - Method.
   constant(f::VariableIndex, ::Type{T}) where {T}
   The constant term of a VariableIndex function is the zero value of the specified type T.
    source
MathOptInterface.constant - Method.
   constant(f::VectorOfVariables, ::Type{T}) where {T}
   The constant term of a VectorOfVariables function is a vector of zero values of the specified type T.
    source
18.2 Sets
MathOptInterface.AbstractSet - Type.
   AbstractSet
   Abstract supertype for set objects used to encode constraints. A set object should not contain any VariableIndex
   or ConstraintIndex as the set is passed unmodifed during copy_to.
    source
MathOptInterface.AbstractScalarSet - Type.
   AbstractScalarSet
   Abstract supertype for subsets of \mathbb{R}.
```

MathOptInterface.AbstractVectorSet - Type.

```
AbstractVectorSet
```

Abstract supertype for subsets of  $\mathbb{R}^n$  for some n.

source

# **Utilities**

MathOptInterface.dimension - Function.

```
dimension(s::AbstractSet)
```

Return the output\_dimension that an AbstractFunction should have to be used with the set s.

#### **Examples**

source

```
julia> dimension(Reals(4))
4

julia> dimension(LessThan(3.0))
1

julia> dimension(PositiveSemidefiniteConeTriangle(2))
3
```

MathOptInterface.dual\_set - Function.

```
dual_set(s::AbstractSet)
```

Return the dual set of s, that is the dual cone of the set. This follows the definition of duality discussed in Duality.

See Dual cone for more information.

If the dual cone is not defined it returns an error.

## **Examples**

```
julia> dual_set(Reals(4))
Zeros(4)

julia> dual_set(SecondOrderCone(5))
SecondOrderCone(5)

julia> dual_set(ExponentialCone())
DualExponentialCone()

source

MathOptInterface.dual_set_type - Function.
```

dual\_set\_type(S::Type{<:AbstractSet})</pre>

```
Return the type of dual set of sets of type S, as returned by dual_set. If the dual cone is not defined it
```

#### **Examples**

returns an error.

```
julia> dual_set_type(Reals)
Zeros

julia> dual_set_type(SecondOrderCone)
SecondOrderCone

julia> dual_set_type(ExponentialCone)
DualExponentialCone

source

MathOptInterface.constant - Method.

| constant(s::Union{EqualTo, GreaterThan, LessThan})

Returns the constant of the set.
source

MathOptInterface.supports_dimension_update - Function.

| supports_dimension_update(S::Type{<:MOI.AbstractVectorSet})</pre>
```

Return a Bool indicating whether the elimination of any dimension of n-dimensional sets of type S give an n-1-dimensional set S. By default, this function returns false so it should only be implemented for sets that supports dimension update.

For instance, supports\_dimension\_update(MOI.Nonnegatives) is true because the elimination of any dimension of the n-dimensional nonnegative orthant gives the n-1-dimensional nonnegative orthant. However supports\_dimension\_update(MOI.ExponentialCone) is false.

```
source
```

```
MathOptInterface.update dimension - Function.
```

```
update_dimension(s::AbstractVectorSet, new_dim)
```

Returns a set with the dimension modified to new\_dim.

source

## 18.3 Scalar sets

List of recognized scalar sets.

```
\label{eq:mathoptInterface.GreaterThan-Type.} $$ | GreaterThan \{T <: Real\} (lower::T) $$ The set $[lower, \infty) \subseteq \mathbb{R}$. $$ source $$ $$ MathOptInterface.LessThan - Type. $$ | LessThan \{T <: Real\} (upper::T) $$ The set $(-\infty, upper] \subseteq \mathbb{R}$. $$ source $$ $$
```

```
MathOptInterface.EqualTo - Type.
    EqualTo{T <: Number}(value::T)</pre>
    The set containing the single point x \in \mathbb{R} where x is given by value.
    source
MathOptInterface.Interval - Type.
    Interval{T <: Real}(lower::T,upper::T)</pre>
    The interval [lower, upper] \subseteq \mathbb{R}. If lower or upper is -Inf or Inf, respectively, the set is interpreted as
    a one-sided interval.
    Interval(s::GreaterThan{<:AbstractFloat})</pre>
    Construct a (right-unbounded) Interval equivalent to the given GreaterThan set.
    Interval(s::LessThan{<:AbstractFloat})</pre>
    Construct a (left-unbounded) Interval equivalent to the given LessThan set.
    Interval(s::EqualTo{<:Real})</pre>
    Construct a (degenerate) Interval equivalent to the given EqualTo set.
    source
MathOptInterface.Integer - Type.
    Integer()
    The set of integers \mathbb{Z}.
    source
MathOptInterface.ZeroOne - Type.
    ZeroOne()
    The set \{0, 1\}.
    source
MathOptInterface.Semicontinuous - Type.
    | Semicontinuous{T <: Real}(lower::T,upper::T)
    The set \{0\} \cup [lower, upper].
    source
MathOptInterface.Semiinteger - Type.
    | Semiinteger{T <: Real}(lower::T,upper::T)
    The set \{0\} \cup \{lower, lower + 1, \dots, upper - 1, upper\}.
    source
```

# 18.4 Vector sets

```
List of recognized vector sets.
MathOptInterface.Reals - Type.
    Reals(dimension)
    The set \mathbb{R}^{dimension} (containing all points) of dimension dimension.
    source
MathOptInterface.Zeros - Type.
    Zeros(dimension)
    The set \{0\}^{dimension} (containing only the origin) of dimension dimension.
    source
MathOptInterface.Nonnegatives - Type.
    Nonnegatives(dimension)
    The nonnegative orthant \{x \in \mathbb{R}^{dimension} : x \geq 0\} of dimension dimension.
    source
MathOptInterface.Nonpositives - Type.
    | Nonpositives(dimension)
    The nonpositive orthant \{x \in \mathbb{R}^{dimension} : x \leq 0\} of dimension dimension.
    source
MathOptInterface.NormInfinityCone - Type.
    | NormInfinityCone(dimension)
    The \ell_\infty-norm cone \{(t,x)\in\mathbb{R}^{dimension}:t\geq \|x\|_\infty=\max_i|x_i|\} of dimension dimension.
MathOptInterface.NormOneCone - Type.
    NormOneCone(dimension)
   The \ell_1-norm cone \{(t,x)\in\mathbb{R}^{dimension}:t\geq \|x\|_1=\sum_i |x_i|\} of dimension dimension.
MathOptInterface.SecondOrderCone - Type.
    | SecondOrderCone(dimension)
    The second-order cone (or Lorenz cone or \ell_2-norm cone) \{(t,x)\in\mathbb{R}^{dimension}:t\geq \|x\|_2\} of dimension
    dimension.
    source
```

MathOptInterface.RotatedSecondOrderCone - Type. RotatedSecondOrderCone(dimension) The rotated second-order cone  $\{(t,u,x)\in\mathbb{R}^{dimension}:2tu\geq\|x\|_2^2,t,u\geq0\}$  of dimension dimension. source  ${\tt MathOptInterface.GeometricMeanCone-Type.}$ GeometricMeanCone(dimension) The geometric mean cone  $\{(t,x)\in\mathbb{R}^{n+1}:x\geq 0,t\leq \sqrt[n]{x_1x_2\cdots x_n}\}$ , where dimension = n + 1 >= **Duality note** The dual of the geometric mean cone is  $\{(u,v)\in\mathbb{R}^{n+1}:u\leq 0,v\geq 0,-u\leq n\sqrt[n]{\prod_i v_i}\}$ , where dimension = n + 1 >= 2. source MathOptInterface.ExponentialCone - Type. ExponentialCone() The 3-dimensional exponential cone  $\{(x,y,z) \in \mathbb{R}^3 : y \exp(x/y) \le z, y > 0\}.$ source MathOptInterface.DualExponentialCone - Type. DualExponentialCone() The 3-dimensional dual exponential cone  $\{(u,v,w)\in\mathbb{R}^3: -u\exp(v/u)\leq \exp(1)w, u<0\}.$ source MathOptInterface.PowerCone - Type. | PowerCone{T <: Real}(exponent::T) The 3-dimensional power cone  $\{(x,y,z)\in\mathbb{R}^3:x^{exponent}y^{1-exponent}\geq |z|,x\geq 0,y\geq 0\}$  with parameter exponent. source MathOptInterface.DualPowerCone - Type. | DualPowerCone{T <: Real}(exponent::T) The 3-dimensional power cone  $\{(u,v,w)\in\mathbb{R}^3: (\frac{u}{exponent})^{exponent}(\frac{v}{1-exponent})^{1-exponent}\geq |w|, u\geq 1$  $0,v\geq 0\}$  with parameter exponent. source

MathOptInterface.RelativeEntropyCone - Type.

RelativeEntropyCone(dimension)

The relative entropy cone  $\{(u,v,w)\in\mathbb{R}^{1+2n}:u\geq\sum_{i=1}^nw_i\log(\frac{w_i}{v_i}),v_i\geq0,w_i\geq0\}$ , where dimension = 2n + 1 >= 1.

#### **Duality note**

The dual of the relative entropy cone is  $\{(u,v,w)\in\mathbb{R}^{1+2n}: \forall i,w_i\geq u(\log(\frac{u}{v_i})-1),v_i\geq 0,u>0\}$  of dimension =2n+1.

source

MathOptInterface.NormSpectralCone - Type.

```
NormSpectralCone(row_dim, column_dim)
```

The epigraph of the matrix spectral norm (maximum singular value function)  $\{(t,X)\in\mathbb{R}^{1+row_dim\times column_dim}:t\geq\sigma_1(X)\}$ , where  $\sigma_i$  is the ith singular value of the matrix X of row dimension row\_dim and column dimension column\_dim.

The matrix X is vectorized by stacking the columns, matching the behavior of Julia's vec function.

source

MathOptInterface.NormNuclearCone - Type.

```
| NormNuclearCone(row_dim, column_dim)
```

The epigraph of the matrix nuclear norm (sum of singular values function)  $\{(t,X)\in\mathbb{R}^{1+row_dim imes column_dim}:t\geq\sum_i\sigma_i(X)\}$ , where  $\sigma_i$  is the ith singular value of the matrix X of row dimension row\_dim and column dimension column dim.

The matrix X is vectorized by stacking the columns, matching the behavior of Julia's vec function.

source

MathOptInterface.SOS1 - Type.

```
| SOS1{T <: Real}(weights::Vector{T})
```

The set corresponding to the special ordered set (SOS) constraint of type 1. Of the variables in the set, at most one can be nonzero. The weights induce an ordering of the variables; as such, they should be unique values. The kth element in the set corresponds to the kth weight in weights. See here for a description of SOS constraints and their potential uses.

source

MathOptInterface.SOS2 - Type.

```
SOS2{T <: Real}(weights::Vector{T})
```

The set corresponding to the special ordered set (SOS) constraint of type 2. Of the variables in the set, at most two can be nonzero, and if two are nonzero, they must be adjacent in the ordering of the set. The weights induce an ordering of the variables; as such, they should be unique values. The kth element in the set corresponds to the kth weight in weights. See here for a description of SOS constraints and their potential uses.

source

MathOptInterface.Indicator - Type.

```
Indicator{A<:ActivationCondition,S<:AbstractScalarSet}(set::S)</pre>
```

The set corresponding to an indicator constraint.

```
When A is ACTIVATE_ON_ZERO, this means: \{(y,x)\in\{0,1\}\times\mathbb{R}^n:y=0\implies x\in set\} When A is ACTIVATE_ON_ONE, this means: \{(y,x)\in\{0,1\}\times\mathbb{R}^n:y=1\implies x\in set\}
```

#### Notes

Most solvers expect that the first row of the function is interpretable as a variable index  $x_i$  (e.g., 1.0 \* x + 0.0). An error will be thrown if this is not the case.

#### **Example**

The constraint  $\{(y,x)\in\{0,1\}\times\mathbb{R}^2:y=1\implies x_1+x_2\leq 9\}$  is defined as

source

MathOptInterface.Complements - Type.

```
| Complements(dimension::Base.Integer)
```

The set corresponding to a mixed complementarity constraint.

Complementarity constraints should be specified with an AbstractVectorFunction-in-Complements (dimension) constraint.

The dimension of the vector-valued function F must be dimension. This defines a complementarity constraint between the scalar function F[i] and the variable in F[i + dimension/2]. Thus, F[i + dimension/2] must be interpretable as a single variable  $x_i$  (e.g., 1.0 \*  $x_i$  + 0.0), and dimension must be even.

The mixed complementarity problem consists of finding  $x_i$  in the interval [lb, ub] (i.e., in the set Interval(lb, ub)), such that the following holds:

```
    F_i(x) == 0 if lb_i < x_i < ub_i</li>
    F_i(x) >= 0 if lb_i == x_i
    F i(x) <= 0 if x i == ub i</li>
```

Classically, the bounding set for x\_i is Interval(0, Inf), which recovers:  $0 \le F_i(x) \perp x_i \ge 0$ , where the  $\bot$  operator implies  $F_i(x) * x_i = 0$ .

# **Examples**

The problem:

```
| x -in- Interval(-1, 1)
| [-4 * x - 3, x] -in- Complements(2)
```

defines the mixed complementarity problem where the following holds:

```
1. -4 * x - 3 == 0 \text{ if } -1 < x < 1
```

2. 
$$-4 * x - 3 >= 0 \text{ if } x == -1$$

3. 
$$-4 * x - 3 \le 0 \text{ if } x == 1$$

There are three solutions:

```
1. x = -3/4 with F(x) = 0
```

2. 
$$x = -1$$
 with  $F(x) = 1$ 

3. 
$$x = 1$$
 with  $F(x) = -7$ 

The function F can also be defined in terms of single variables. For example, the problem:

```
[x_3, x_4] -in- Nonnegatives(2)
[x_1, x_2, x_3, x_4] -in- Complements(4)
```

defines the complementarity problem where  $0 \le x_1 \perp x_3 \ge 0$  and  $0 \le x_2 \perp x_4 \ge 0$ .

source

#### 18.5 Matrix sets

Matrix sets are vectorized in order to be subtypes of AbstractVectorSet.

For sets of symmetric matrices, storing both the (i, j) and (j, i) elements is redundant. Use the AbstractSymmetricMatrixSe set to represent only the vectorization of the upper triangular part of the matrix.

When the matrix of expressions constrained to be in the set is not symmetric, and hence additional constraints are needed to force the equality of the (i, j) and (j, i) elements, use the AbstractSymmetricMatrixSetSquare set.

The Bridges.Constraint.SquareBridge can transform a set from the square form to the triangular\_form by adding appropriate constraints if the (i, j) and (j, i) expressions are different.

MathOptInterface.AbstractSymmetricMatrixSetTriangle - Type.

Abstract supertype for subsets of the (vectorized) cone of symmetric matrices, with side\_dimension rows and columns. The entries of the upper-right triangular part of the matrix are given column by column (or equivalently, the entries of the lower-left triangular part are given row by row). A vectorized cone of dimension n corresponds to a square matrix with side dimension  $\sqrt{1/4+2n}-1/2$ . (Because a  $d\times d$  matrix has d(d+1)/2 elements in the upper or lower triangle.)

#### **Examples**

The matrix

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

has side\_dimension 3 and vectorization (1, 2, 3, 4, 5, 6).

#### Note

Two packed storage formats exist for symmetric matrices, the respective orders of the entries are:

- upper triangular column by column (or lower triangular row by row);
- lower triangular column by column (or upper triangular row by row).

The advantage of the first format is the mapping between the (i, j) matrix indices and the k index of the vectorized form. It is simpler and does not depend on the side dimension of the matrix. Indeed,

- the entry of matrix indices (i, j) has vectorized index k = div((j 1) \* j, 2) + i if  $i \le j$  and k = div((i 1) \* i, 2) + j if  $j \le i$ ;
- and the entry with vectorized index k has matrix indices i = div(1 + isqrt(8k 7), 2) and j = k div((i 1) \* i, 2) or j = div(1 + isqrt(8k 7), 2) and i = k div((j 1) \* j, 2).

### **Duality note**

The scalar product for the symmetric matrix in its vectorized form is the sum of the pairwise product of the diagonal entries plus twice the sum of the pairwise product of the upper diagonal entries; see [p. 634, 1]. This has important consequence for duality.

Consider for example the following problem (Positive Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Triangle is a subtype of Abstract Symmetric Matrix and Semidefinite Cone Tr

$$\max_{x \in \mathbb{R}} \qquad \qquad x$$
 s.t. 
$$(1,-x,1) \in \mathsf{PositiveSemidefiniteConeTriangle}(2).$$

The dual is the following problem

$$\min_{x \in \mathbb{R}^3} \qquad y_1 + y_3$$
 s.t. 
$$2y_2 = 1$$
 
$$y \in \mathsf{PositiveSemidefiniteConeTriangle}(2).$$

Why do we use  $2y_2$  in the dual constraint instead of  $y_2$ ? The reason is that  $2y_2$  is the scalar product between y and the symmetric matrix whose vectorized form is (0,1,0). Indeed, with our modified scalar products we have

$$\langle (0,1,0), (y_1,y_2,y_3) \rangle = \operatorname{trace} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} y_1 & y_2 \\ y_2 & y_3 \end{pmatrix} = 2y_2.$$

#### References

[1] Boyd, S. and Vandenberghe, L.. Convex optimization. Cambridge university press, 2004. source

MathOptInterface.AbstractSymmetricMatrixSetSquare - Type.

```
| abstract type AbstractSymmetricMatrixSetSquare <: AbstractVectorSet end
```

Abstract supertype for subsets of the (vectorized) cone of symmetric matrices, with side\_dimension rows and columns. The entries of the matrix are given column by column (or equivalently, row by row). The matrix is both constrained to be symmetric and to have its triangular\_form belong to the corresponding

set. That is, if the functions in entries (i,j) and (j,i) are different, then a constraint will be added to make sure that the entries are equal.

#### **Examples**

 $Positive Semidefinite Cone Square \ is \ a \ subtype \ of \ Abstract Symmetric Matrix Set Square \ and \ constraining \ the \ matrix$ 

$$\begin{bmatrix} 1 & -y \\ -z & 0 \end{bmatrix}$$

to be symmetric positive semidefinite can be achieved by constraining the vector (1,-z,-y,0) (or (1,-y,-z,0)) to belong to the PositiveSemidefiniteConeSquare(2). It both constrains y=z and (1,-y,0) (or (1,-z,0)) to be in PositiveSemidefiniteConeTriangle(2), since triangular\_form(PositiveSemidefiniteConeSquare) is PositiveSemidefiniteConeTriangle.

source

MathOptInterface.side\_dimension - Function.

Side dimension of the matrices in set. By convention, it should be stored in the side\_dimension field but if it is not the case for a subtype of AbstractSymmetricMatrixSetTriangle, the method should be implemented for this subtype.

source

MathOptInterface.triangular form - Function.

```
triangular_form(S::Type{<:AbstractSymmetricMatrixSetSquare})
triangular_form(set::AbstractSymmetricMatrixSetSquare)</pre>
```

Return the AbstractSymmetricMatrixSetTriangle corresponding to the vectorization of the upper triangular part of matrices in the AbstractSymmetricMatrixSetSquare set.

source

List of recognized matrix sets.

MathOptInterface.PositiveSemidefiniteConeTriangle - Type.

```
| PositiveSemidefiniteConeTriangle(side_dimension) <: AbstractSymmetricMatrixSetTriangle
```

The (vectorized) cone of symmetric positive semidefinite matrices, with side\_dimension rows and columns.

See AbstractSymmetricMatrixSetTriangle for more details on the vectorized form.

source

MathOptInterface.PositiveSemidefiniteConeSquare - Type.

```
| PositiveSemidefiniteConeSquare(side_dimension) <: AbstractSymmetricMatrixSetSquare
```

The cone of symmetric positive semidefinite matrices, with side length side\_dimension.

See AbstractSymmetricMatrixSetSquare for more details on the vectorized form.

The entries of the matrix are given column by column (or equivalently, row by row).

The matrix is both constrained to be symmetric and to be positive semidefinite. That is, if the functions in entries (i,j) and (j,i) are different, then a constraint will be added to make sure that the entries are equal.

# **Examples**

Constraining the matrix

$$\begin{bmatrix} 1 & -y \\ -z & 0 \end{bmatrix}$$

to be symmetric positive semidefinite can be achieved by constraining the vector (1, -z, -y, 0) (or (1, -y, -z, 0)) to belong to the PositiveSemidefiniteConeSquare(2).

It both constrains y=z and (1,-y,0) (or (1,-z,0)) to be in PositiveSemidefiniteConeTriangle(2).

source

MathOptInterface.LogDetConeTriangle - Type.

LogDetConeTriangle(side dimension)

The log-determinant cone  $\{(t,u,X)\in\mathbb{R}^{2+d(d+1)/2}:t\leq u\log(\det(X/u)),u>0\}$ , where the matrix X is represented in the same symmetric packed format as in the PositiveSemidefiniteConeTriangle.

The argument side\_dimension is the side dimension of the matrix X, i.e., its number of rows or columns.

source

MathOptInterface.LogDetConeSquare - Type.

LogDetConeSquare(side dimension)

The log-determinant cone  $\{(t,u,X)\in\mathbb{R}^{2+d^2}:t\leq u\log(\det(X/u)),X \text{ symmetric},u>0\}$ , where the matrix X is represented in the same format as in the PositiveSemidefiniteConeSquare.

 $Similarly\ to\ {\tt PositiveSemidefiniteConeSquare},\ constraints\ are\ added\ to\ ensure\ that\ X\ is\ symmetric.$ 

The argument side\_dimension is the side dimension of the matrix X, i.e., its number of rows or columns.

source

 ${\tt MathOptInterface.RootDetConeTriangle-Type.}$ 

RootDetConeTriangle(side\_dimension)

The root-determinant cone  $\{(t,X)\in\mathbb{R}^{1+d(d+1)/2}:t\leq \det(X)^{1/d}\}$ , where the matrix X is represented in the same symmetric packed format as in the PositiveSemidefiniteConeTriangle.

The argument side\_dimension is the side dimension of the matrix X, i.e., its number of rows or columns.

source

MathOptInterface.RootDetConeSquare - Type.

| RootDetConeSquare(side\_dimension)

The root-determinant cone  $\{(t,X)\in\mathbb{R}^{1+d^2}:t\leq \det(X)^{1/d},X \text{ symmetric}\}$ , where the matrix X is represented in the same format as PositiveSemidefiniteConeSquare.

 $Similarly\ to\ {\tt PositiveSemidefiniteConeSquare},\ constraints\ are\ added\ to\ ensure\ that\ X\ is\ symmetric.$ 

The argument side\_dimension is the side dimension of the matrix X, i.e., its number of rows or columns.

# **Chapter 19**

# **Models**

# 19.1 Attribute interface

MathOptInterface.is\_set\_by\_optimize - Function.

```
is_set_by_optimize(::AnyAttribute)
```

Return a Bool indicating whether the value of the attribute is modified during an optimize! call, that is, the attribute is used to query the result of the optimization.

#### Important note when defining new attributes

This function returns false by default so it should be implemented for attributes that are modified by optimize!.

source

MathOptInterface.is\_copyable - Function.

```
| is_copyable(::AnyAttribute)
```

Return a Bool indicating whether the value of the attribute may be copied during copy\_to using set.

### Important note when defining new attributes

By default is\_copyable(attr) returns !is\_set\_by\_optimize(attr). A specific method should be defined for attributes which are copied indirectly during copy\_to. For instance, both is\_copyable and is\_set\_by\_optimize return false for the following attributes:

- ListOfOptimizerAttributesSet, ListOfModelAttributesSet, ListOfConstraintAttributesSet and ListOfVariableAttributesSet.
- SolverName and RawSolver: these attributes cannot be set.
- NumberOfVariables and ListOfVariableIndices: these attributes are set indirectly by add\_variable and add variables.
- ObjectiveFunctionType: this attribute is set indirectly when setting the ObjectiveFunction attribute.
- NumberOfConstraints, ListOfConstraintIndices, ListOfConstraintTypesPresent, CanonicalConstraintFunction ConstraintFunction and ConstraintSet: these attributes are set indirectly by add\_constraint and add\_constraints.

MathOptInterface.get - Function.

```
get(optimizer::AbstractOptimizer, attr::AbstractOptimizerAttribute)
```

Return an attribute attr of the optimizer optimizer.

```
get(model::ModelLike, attr::AbstractModelAttribute)
```

Return an attribute attr of the model model.

```
get(model::ModelLike, attr::AbstractVariableAttribute, v::VariableIndex)
```

If the attribute attr is set for the variable v in the model model, return its value, return nothing otherwise. If the attribute attr is not supported by model then an error should be thrown instead of returning nothing.

```
get(model::ModelLike, attr::AbstractVariableAttribute, v::Vector{VariableIndex})
```

Return a vector of attributes corresponding to each variable in the collection v in the model model.

```
get(model::ModelLike, attr::AbstractConstraintAttribute, c::ConstraintIndex)
```

If the attribute attr is set for the constraint c in the model model, return its value, return nothing otherwise. If the attribute attr is not supported by model then an error should be thrown instead of returning nothing.

```
get(model::ModelLike, attr::AbstractConstraintAttribute, c::Vector{ConstraintIndex{F,S}})
```

Return a vector of attributes corresponding to each constraint in the collection c in the model model.

```
| get(model::ModelLike, ::Type{VariableIndex}, name::String)
```

If a variable with name name exists in the model model, return the corresponding index, otherwise return nothing. Errors if two variables have the same name.

```
get(model::ModelLike, ::Type{ConstraintIndex{F,S}}, name::String) where {F<:AbstractFunction,S<:
    AbstractSet}</pre>
```

If an F-in-S constraint with name name exists in the model model, return the corresponding index, otherwise return nothing. Errors if two constraints have the same name.

```
| get(model::ModelLike, ::Type{ConstraintIndex}, name::String)
```

If any constraint with name name exists in the model model, return the corresponding index, otherwise return nothing. This version is available for convenience but may incur a performance penalty because it is not type stable. Errors if two constraints have the same name.

# **Examples**

```
get(model, ObjectiveValue())
get(model, VariablePrimal(), ref)
get(model, VariablePrimal(5), [ref1, ref2])
get(model, OtherAttribute("something specific to cplex"))
get(model, VariableIndex, "var1")
get(model, ConstraintIndex{ScalarAffineFunction{Float64}, LessThan{Float64}}, "con1")
get(model, ConstraintIndex, "con1")
```

```
get!(output, model::ModelLike, args...)
```

An in-place version of get.

The signature matches that of get except that the tresult is placed in the vector output.

source

MathOptInterface.set - Function.

```
| set(optimizer::AbstractOptimizer, attr::AbstractOptimizerAttribute, value)
```

Assign value to the attribute attr of the optimizer optimizer.

```
set(model::ModelLike, attr::AbstractModelAttribute, value)
```

Assign value to the attribute attr of the model model.

```
set(model::ModelLike, attr::AbstractVariableAttribute, v::VariableIndex, value)
```

Assign value to the attribute attr of variable v in model model.

```
set(model::ModelLike, attr::AbstractVariableAttribute, v::Vector{VariableIndex}, vector_of_values
)
```

Assign a value respectively to the attribute attr of each variable in the collection v in model model.

```
set(model::ModelLike, attr::AbstractConstraintAttribute, c::ConstraintIndex, value)
```

Assign a value to the attribute attr of constraint c in model model.

```
set(model::ModelLike, attr::AbstractConstraintAttribute, c::Vector{ConstraintIndex{F,S}},
    vector_of_values)
```

Assign a value respectively to the attribute attr of each constraint in the collection c in model model.

An UnsupportedAttribute error is thrown if model does not support the attribute attr (see supports) and a SetAttributeNotAllowed error is thrown if it supports the attribute attr but it cannot be set.

# Replace set in a constraint

```
| set(model::ModelLike, ::ConstraintSet, c::ConstraintIndex{F,S}, set::S)
```

Change the set of constraint c to the new set set which should be of the same type as the original set.

#### **Examples**

If c is a ConstraintIndex{F,Interval}

```
set(model, ConstraintSet(), c, Interval(0, 5))
set(model, ConstraintSet(), c, GreaterThan(0.0)) # Error
```

# Replace function in a constraint

```
set(model::ModelLike, ::ConstraintFunction, c::ConstraintIndex{F,S}, func::F)
```

Replace the function in constraint c with func. F must match the original function type used to define the constraint.

#### Note

Setting the constraint function is not allowed if F is VariableIndex, it throws a SettingVariableIndexNotAllowed error. Indeed, it would require changing the index c as the index of VariableIndex constraints should be the same as the index of the variable.

# **Examples**

If c is a ConstraintIndex{ScalarAffineFunction, S} and v1 and v2 are VariableIndex objects,

```
set(model, ConstraintFunction(), c,
    ScalarAffineFunction(ScalarAffineTerm.([1.0, 2.0], [v1, v2]), 5.0))
set(model, ConstraintFunction(), c, v1) # Error
```

source

MathOptInterface.supports - Function.

```
| supports(model::ModelLike, sub::AbstractSubmittable)::Bool
```

Return a Bool indicating whether model supports the submittable sub.

```
| supports(model::ModelLike, attr::AbstractOptimizerAttribute)::Bool
```

Return a Bool indicating whether model supports the optimizer attribute attr. That is, it returns false if copy\_to(model, src) shows a warning in case attr is in the ListOfOptimizerAttributesSet of src; see copy\_to for more details on how unsupported optimizer attributes are handled in copy.

```
| supports(model::ModelLike, attr::AbstractModelAttribute)::Bool
```

Return a Bool indicating whether model supports the model attribute attr. That is, it returns false if copy\_to(model, src) cannot be performed in case attr is in the ListOfModelAttributesSet of src.

```
| supports(model::ModelLike, attr::AbstractVariableAttribute, ::Type{VariableIndex})::Bool
```

Return a Bool indicating whether model supports the variable attribute attr. That is, it returns false if copy\_to(model, src) cannot be performed in case attr is in the ListOfVariableAttributesSet of src.

```
supports(model::ModelLike, attr::AbstractConstraintAttribute, ::Type{ConstraintIndex{F,S}})::Bool
    where {F,S}
```

Return a Bool indicating whether model supports the constraint attribute attr applied to an F-in-S constraint. That is, it returns false if copy\_to(model, src) cannot be performed in case attr is in the ListOfConstraintAttributesSet of src.

For all five methods, if the attribute is only not supported in specific circumstances, it should still return true.

Note that supports is only defined for attributes for which is\_copyable returns true as other attributes do not appear in the list of attributes set obtained by ListOf...AttributesSet.

source

MathOptInterface.attribute\_value\_type - Function.

```
| attribute_value_type(attr::AnyAttribute)
```

Given an attribute attr, return the type of value expected by get, or returned by set.

#### **Notes**

• Only implement this if it make sense to do so. If un-implemented, the default is Any.

# 19.2 Model interface

```
MathOptInterface.ModelLike - Type.
```

```
ModelLike
```

Abstract supertype for objects that implement the "Model" interface for defining an optimization problem.

source

MathOptInterface.is\_empty - Function.

```
is_empty(model::ModelLike)
```

Returns false if the model has any model attribute set or has any variables or constraints.

Note that an empty model can have optimizer attributes set.

source

MathOptInterface.empty! - Function.

```
empty!(model::ModelLike)
```

Empty the model, that is, remove all variables, constraints and model attributes but not optimizer attributes.

source

MathOptInterface.write\_to\_file - Function.

```
write_to_file(model::ModelLike, filename::String)
```

Writes the current model data to the given file. Supported file types depend on the model type.

source

MathOptInterface.read\_from\_file - Function.

```
read_from_file(model::ModelLike, filename::String)
```

Read the file filename into the model model. If model is non-empty, this may throw an error.

Supported file types depend on the model type.

# Note

Once the contents of the file are loaded into the model, users can query the variables via get(model, ListOfVariableIndices()). However, some filetypes, such as LP files, do not maintain an explicit ordering of the variables. Therefore, the returned list may be in an arbitrary order. To avoid depending on the order of the indices, users should look up each variable index by name: get(model, VariableIndex, "name").

source

MathOptInterface.supports incremental interface - Function.

```
| supports_incremental_interface(model::ModelLike)
```

Return a Bool indicating whether model supports building incrementally via add\_variable and add\_constraint.

The main purpose of this function is to determine whether a model can be loaded into model incrementally or whether it should be cached and copied at once instead.

source

MathOptInterface.copy\_to - Function.

```
copy_to(dest::ModelLike, src::ModelLike)::IndexMap
```

Copy the model from src into dest.

The target dest is emptied, and all previous indices to variables and constraints in dest are invalidated.

Returns an IndexMap object that translates variable and constraint indices from the src model to the corresponding indices in the dest model.

#### Notes

- If a constraint that in src is not supported by dest, then an UnsupportedConstraint error is thrown.
- If an AbstractModelAttribute, AbstractVariableAttribute, or AbstractConstraintAttribute is set in src but not supported by dest, then an UnsupportedAttribute error is thrown.

AbstractOptimizerAttributes are not copied to the dest model.

#### IndexMap

Implementations of copy\_to must return an IndexMap. For technical reasons, this type is defined in the Utilities submodule as MOI.Utilities.IndexMap. However, since it is an integral part of the MOI API, we provide MOI.IndexMap as an alias.

# Example

```
# Given empty `ModelLike` objects `src` and `dest`.

x = add_variable(src)

is_valid(src, x)  # true
is_valid(dest, x)  # false (`dest` has no variables)

index_map = copy_to(dest, src)
is_valid(dest, x)  # false (unless index_map[x] == x)
is_valid(dest, index_map[x])  # true
```

MathOptInterface.IndexMap - Type.

```
IndexMap()
```

The dictionary-like object returned by copy\_to.

## IndexMap

Implementations of copy\_to must return an IndexMap. For technical reasons, the IndexMap type is defined in the Utilities submodule as MOI.Utilities.IndexMap. However, since it is an integral part of the MOI API, we provide this MOI.IndexMap as an alias.

# 19.3 Model attributes

MathOptInterface.AbstractModelAttribute - Type.

AbstractModelAttribute

Abstract supertype for attribute objects that can be used to set or get attributes (properties) of the model.

source

MathOptInterface.Name - Type.

Name()

A model attribute for the string identifying the model. It has a default value of "" if not set'.

source

MathOptInterface.ObjectiveFunction - Type.

```
| ObjectiveFunction{F<:AbstractScalarFunction}()
```

A model attribute for the objective function which has a type F<:AbstractScalarFunction. F should be guaranteed to be equivalent but not necessarily identical to the function type provided by the user. Throws an InexactError if the objective function cannot be converted to F, e.g. the objective function is quadratic and F is ScalarAffineFunction{Float64} or it has non-integer coefficient and F is ScalarAffineFunction{Int}.

source

MathOptInterface.ObjectiveFunctionType - Type.

```
ObjectiveFunctionType()
```

A model attribute for the type F of the objective function set using the ObjectiveFunction{F} attribute.

## **Examples**

In the following code, attr should be equal to MOI. VariableIndex:

MathOptInterface.ObjectiveSense - Type.

```
ObjectiveSense()
```

A model attribute for the objective sense of the objective function, which must be an OptimizationSense: MIN SENSE, MAX SENSE, or FEASIBILITY SENSE. The default is FEASIBILITY SENSE.

source

MathOptInterface.NumberOfVariables - Type.

```
NumberOfVariables()
```

A model attribute for the number of variables in the model.

source

MathOptInterface.ListOfVariableIndices - Type.

```
ListOfVariableIndices()
```

A model attribute for the Vector{VariableIndex} of all variable indices present in the model (i.e., of length equal to the value of NumberOfVariables()) in the order in which they were added.

source

MathOptInterface.ListOfConstraintTypesPresent - Type.

```
ListOfConstraintTypesPresent()
```

A model attribute for the list of tuples of the form (F,S), where F is a function type and S is a set type indicating that the attribute NumberOfConstraints $\{F,S\}$ () has value greater than zero.

source

MathOptInterface.NumberOfConstraints - Type.

```
NumberOfConstraints{F,S}()
```

A model attribute for the number of constraints of the type F-in-S present in the model.

source

 ${\tt MathOptInterface.ListOfConstraintIndices-Type.}\\$ 

```
ListOfConstraintIndices{F,S}()
```

A model attribute for the  $Vector\{ConstraintIndex\{F,S\}\}\)$  of all constraint indices of type F-in-S in the model (i.e., of length equal to the value of  $NumberOfConstraints\{F,S\}$ ()) in the order in which they were added.

source

MathOptInterface.ListOfOptimizerAttributesSet - Type.

```
ListOfOptimizerAttributesSet()
```

An optimizer attribute for the  $Vector\{Abstract0ptimizerAttribute\}$  of all optimizer attributes that were set.

source

MathOptInterface.ListOfModelAttributesSet - Type.

```
ListOfModelAttributesSet()
```

A model attribute for the Vector{AbstractModelAttribute} of all model attributes attr such that 1) is\_copyable(attr) returns true and 2) the attribute was set to the model.

source

 ${\tt MathOptInterface.ListOfVariableAttributesSet-Type.}$ 

```
ListOfVariableAttributesSet()
```

A model attribute for the Vector{AbstractVariableAttribute} of all variable attributes attr such that 1) is\_copyable(attr) returns true and 2) the attribute was set to variables.

source

MathOptInterface.ListOfConstraintAttributesSet - Type.

```
ListOfConstraintAttributesSet{F, S}()
```

A model attribute for the Vector{AbstractConstraintAttribute} of all constraint attributes attr such that 1) is\_copyable(attr) returns true and

2. the attribute was set to F-in-S constraints.

#### Note

The attributes ConstraintFunction and ConstraintSet should not be included in the list even if then have been set with set.

source

# 19.4 Optimizer interface

MathOptInterface.AbstractOptimizer - Type.

```
| AbstractOptimizer <: ModelLike
```

Abstract supertype for objects representing an instance of an optimization problem tied to a particular solver. This is typically a solver's in-memory representation. In addition to ModelLike, AbstractOptimizer objects let you solve the model and query the solution.

source

MathOptInterface.OptimizerWithAttributes - Type.

```
struct OptimizerWithAttributes
    optimizer_constructor
    params::Vector{Pair{AbstractOptimizerAttribute,<:Any}}
end</pre>
```

Object grouping an optimizer constructor and a list of optimizer attributes. Instances are created with instantiate.

source

MathOptInterface.optimize! - Function.

```
optimize!(optimizer::AbstractOptimizer)
```

Optimize the problem contained in optimizer.

Before calling optimize!, the problem should first be constructed using the incremental interface (see supports\_incremental\_interface) or copy\_to.

MathOptInterface.instantiate - Function.

```
instantiate(
    optimizer_constructor,
    with_bridge_type::Union{Nothing, Type} = nothing,
)
```

Creates an instance of optimizer by either:

- calling optimizer\_constructor.optimizer\_constructor() and setting the parameters in optimizer\_constructor.p if optimizer constructor is a OptimizerWithAttributes
- calling optimizer\_constructor() if optimizer\_constructor is callable.

If with\_bridge\_type is not nothing, it enables all the bridges defined in the MathOptInterface.Bridges submodule with coefficient type with\_bridge\_type.

If the optimizer created by optimizer\_constructor does not support loading the problem incrementally (see supports\_incremental\_interface), then a Utilities.CachingOptimizer is added to store a cache of the bridged model.

source

MathOptInterface.default\_cache - Function.

```
default_cache(optimizer::ModelLike, ::Type{T}) where {T}
```

Return a new instance of the default model type to be used as cache for optimizer in a Utilities. CachingOptimizer for holding constraints of coefficient type T. By default, this returns Utilities. UniversalFallback(Utilities.Model{T}()) If copying from a instance of a given model type is faster for optimizer then a new method returning an instance of this model type should be defined.

source

# 19.5 Optimizer attributes

MathOptInterface.AbstractOptimizerAttribute - Type.

```
AbstractOptimizerAttribute
```

Abstract supertype for attribute objects that can be used to set or get attributes (properties) of the optimizer.

#### Note

The difference between AbstractOptimizerAttribute and AbstractModelAttribute lies in the behavior of is\_empty, empty! and copy\_to. Typically optimizer attributes only affect how the model is solved.

source

MathOptInterface.SolverName - Type.

```
| SolverName()
```

An optimizer attribute for the string identifying the solver/optimizer.

source

MathOptInterface.SolverVersion - Type.

```
|SolverVersion()
```

An optimizer attribute for the string identifying the version of the solver.

#### Note

For solvers supporting semantic versioning, the SolverVersion should be a string of the form "vMAJOR.MINOR.PATCH", so that it can be converted to a Julia VersionNumber (e.g., 'Version-Number("v1.2.3")).

We do not require Semantic Versioning because some solvers use alternate versioning systems. For example, CPLEX uses Calendar Versioning, so SolverVersion will return a string like "202001".

source

MathOptInterface.Silent - Type.

```
|Silent()
```

An optimizer attribute for silencing the output of an optimizer. When set to true, it takes precedence over any other attribute controlling verbosity and requires the solver to produce no output. The default value is false which has no effect. In this case the verbosity is controlled by other attributes.

#### Note

Every optimizer should have verbosity on by default. For instance, if a solver has a solver-specific log level attribute, the MOI implementation should set it to 1 by default. If the user sets Silent to true, then the log level should be set to 0, even if the user specifically sets a value of log level. If the value of Silent is false then the log level set to the solver is the value given by the user for this solver-specific parameter or 1 if none is given.

source

MathOptInterface.TimeLimitSec - Type.

```
|TimeLimitSec()
```

An optimizer attribute for setting a time limit for an optimization. When set to nothing, it deactivates the solver time limit. The default value is nothing. The time limit is in seconds.

source

MathOptInterface.RawOptimizerAttribute - Type.

```
RawOptimizerAttribute(name::String)
```

An optimizer attribute for the solver-specific parameter identified by name.

source

MathOptInterface.NumberOfThreads - Type.

```
NumberOfThreads()
```

An optimizer attribute for setting the number of threads used for an optimization. When set to nothing uses solver default. Values are positive integers. The default value is nothing.

MathOptInterface.RawSolver - Type.

```
RawSolver()
```

A model attribute for the object that may be used to access a solver-specific API for this optimizer.

source

List of attributes useful for optimizers

MathOptInterface.TerminationStatus - Type.

```
TerminationStatus()
```

A model attribute for the TerminationStatusCode explaining why the optimizer stopped.

source

MathOptInterface.TerminationStatusCode - Type.

TerminationStatusCode

An Enum of possible values for the TerminationStatus attribute. This attribute is meant to explain the reason why the optimizer stopped executing in the most recent call to optimize!.

If no call has been made to optimize!, then the TerminationStatus is:

• OPTIMIZE\_NOT\_CALLED: The algorithm has not started.

### OK

These are generally OK statuses, i.e., the algorithm ran to completion normally.

- OPTIMAL: The algorithm found a globally optimal solution.
- INFEASIBLE: The algorithm concluded that no feasible solution exists.
- DUAL\_INFEASIBLE: The algorithm concluded that no dual bound exists for the problem. If, additionally, a feasible (primal) solution is known to exist, this status typically implies that the problem is unbounded, with some technical exceptions.
- LOCALLY\_SOLVED: The algorithm converged to a stationary point, local optimal solution, could not find directions for improvement, or otherwise completed its search without global guarantees.
- LOCALLY\_INFEASIBLE: The algorithm converged to an infeasible point or otherwise completed its search without finding a feasible solution, without guarantees that no feasible solution exists.
- INFEASIBLE\_OR\_UNBOUNDED: The algorithm stopped because it decided that the problem is infeasible or unbounded; this occasionally happens during MIP presolve.

# Solved to relaxed tolerances

- ALMOST OPTIMAL: The algorithm found a globally optimal solution to relaxed tolerances.
- ALMOST\_INFEASIBLE: The algorithm concluded that no feasible solution exists within relaxed tolerances.
- ALMOST\_DUAL\_INFEASIBLE: The algorithm concluded that no dual bound exists for the problem within relaxed tolerances.
- ALMOST\_LOCALLY\_SOLVED: The algorithm converged to a stationary point, local optimal solution, or could not find directions for improvement within relaxed tolerances.

#### Limits

The optimizer stopped because of some user-defined limit.

ITERATION\_LIMIT: An iterative algorithm stopped after conducting the maximum number of iterations.

- TIME\_LIMIT: The algorithm stopped after a user-specified computation time.
- NODE\_LIMIT: A branch-and-bound algorithm stopped because it explored a maximum number of nodes in the branch-and-bound tree.
- SOLUTION\_LIMIT: The algorithm stopped because it found the required number of solutions. This is often used in MIPs to get the solver to return the first feasible solution it encounters.
- MEMORY\_LIMIT: The algorithm stopped because it ran out of memory.
- OBJECTIVE\_LIMIT: The algorithm stopped because it found a solution better than a minimum limit set by the user.
- NORM\_LIMIT: The algorithm stopped because the norm of an iterate became too large.
- OTHER LIMIT: The algorithm stopped due to a limit not covered by one of the above.

#### **Problematic**

This group of statuses means that something unexpected or problematic happened.

- SLOW\_PROGRESS: The algorithm stopped because it was unable to continue making progress towards the solution.
- NUMERICAL\_ERROR: The algorithm stopped because it encountered unrecoverable numerical error.
- INVALID\_MODEL: The algorithm stopped because the model is invalid.
- INVALID\_OPTION: The algorithm stopped because it was provided an invalid option.
- INTERRUPTED: The algorithm stopped because of an interrupt signal.
- OTHER\_ERROR: The algorithm stopped because of an error not covered by one of the statuses defined above.

source

MathOptInterface.PrimalStatus - Type.

```
PrimalStatus(result_index::Int = 1)
```

A model attribute for the ResultStatusCode of the primal result result\_index. If result\_index is omitted, it defaults to 1.

See ResultCount for information on how the results are ordered.

If result\_index is larger than the value of ResultCount then NO\_SOLUTION is returned.

source

MathOptInterface.DualStatus - Type.

```
DualStatus(result_index::Int = 1)
```

A model attribute for the ResultStatusCode of the dual result result\_index. If result\_index is omitted, it defaults to 1.

See ResultCount for information on how the results are ordered.

If result\_index is larger than the value of ResultCount then NO\_SOLUTION is returned.

# MathOptInterface.ResultStatusCode - Type.

```
ResultStatusCode
```

An Enum of possible values for the PrimalStatus and DualStatus attributes. The values indicate how to interpret the result vector.

- NO\_SOLUTION: the result vector is empty.
- FEASIBLE\_POINT: the result vector is a feasible point.
- NEARLY FEASIBLE POINT: the result vector is feasible if some constraint tolerances are relaxed.
- INFEASIBLE POINT: the result vector is an infeasible point.
- INFEASIBILITY\_CERTIFICATE: the result vector is an infeasibility certificate. If the PrimalStatus is INFEASIBILITY\_CERTIFICATE, then the primal result vector is a certificate of dual infeasibility. If the DualStatus is INFEASIBILITY\_CERTIFICATE, then the dual result vector is a proof of primal infeasibility.
- NEARLY\_INFEASIBILITY\_CERTIFICATE: the result satisfies a relaxed criterion for a certificate of infeasibility.
- REDUCTION\_CERTIFICATE: the result vector is an ill-posed certificate; see this article for details. If the PrimalStatus is REDUCTION\_CERTIFICATE, then the primal result vector is a proof that the dual problem is ill-posed. If the DualStatus is REDUCTION\_CERTIFICATE, then the dual result vector is a proof that the primal is ill-posed.
- NEARLY\_REDUCTION\_CERTIFICATE: the result satisfies a relaxed criterion for an ill-posed certificate.
- UNKNOWN RESULT STATUS: the result vector contains a solution with an unknown interpretation.
- OTHER\_RESULT\_STATUS: the result vector contains a solution with an interpretation not covered by one of the statuses defined above.

source

MathOptInterface.RawStatusString - Type.

```
RawStatusString()
```

A model attribute for a solver specific string explaining why the optimizer stopped.

source

MathOptInterface.ResultCount - Type.

```
ResultCount()
```

A model attribute for the number of results available.

### Order of solutions

A number of attributes contain an index, result\_index, which is used to refer to one of the available results. Thus, result\_index must be an integer between 1 and the number of available results.

As a general rule, the first result (result\_index=1) is the most important result (e.g., an optimal solution or an infeasibility certificate). Other results will typically be alternate solutions that the solver found during the search for the first result.

If a (local) optimal solution is available, i.e., TerminationStatus is OPTIMAL or LOCALLY\_SOLVED, the first result must correspond to the (locally) optimal solution. Other results may be alternative optimal solutions, or they may be other suboptimal solutions; use ObjectiveValue to distingiush between them.

If a primal or dual infeasibility certificate is available, i.e., TerminationStatus is INFEASIBLE or DUAL\_INFEASIBLE and the corresponding PrimalStatus or DualStatus is INFEASIBILITY\_CERTIFICATE, then the first result must be a certificate. Other results may be alternate certificates, or infeasible points.

source

MathOptInterface.ObjectiveValue - Type.

```
ObjectiveValue(result_index::Int = 1)
```

A model attribute for the objective value of the primal solution result\_index.

If the solver does not have a primal value for the objective because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError. Otherwise, if the result is unavailable for another reason (for instance, only a dual solution is available), the result is undefined. Users should first check PrimalStatus before accessing the ObjectiveValue attribute.

See ResultCount for information on how the results are ordered.

source

MathOptInterface.DualObjectiveValue - Type.

```
DualObjectiveValue(result index::Int = 1)
```

A model attribute for the value of the objective function of the dual problem for the result\_indexth dual result.

If the solver does not have a dual value for the objective because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError. Otherwise, if the result is unavailable for another reason (for instance, only a primal solution is available), the result is undefined. Users should first check DualStatus before accessing the DualObjectiveValue attribute.

See ResultCount for information on how the results are ordered.

source

MathOptInterface.ObjectiveBound - Type.

```
ObjectiveBound()
```

A model attribute for the best known bound on the optimal objective value.

source

MathOptInterface.RelativeGap - Type.

```
RelativeGap()
```

A model attribute for the final relative optimality gap.

#### Warning

The definition of this gap is solver-dependent. However, most solvers implementing this attribute define the relative gap as some variation of  $\frac{|b-f|}{|f|}$ , where b is the best bound and f is the best feasible objective value.

```
MathOptInterface.SolveTimeSec - Type.
```

```
|SolveTimeSec()
```

A model attribute for the total elapsed solution time (in seconds) as reported by the optimizer.

source

MathOptInterface.SimplexIterations - Type.

```
SimplexIterations()
```

A model attribute for the cumulative number of simplex iterations during the optimization process. In particular, for a mixed-integer program (MIP), the total simplex iterations for all nodes.

source

MathOptInterface.BarrierIterations - Type.

```
|BarrierIterations()
```

A model attribute for the cumulative number of barrier iterations while solving a problem.

source

MathOptInterface.NodeCount - Type.

```
NodeCount()
```

A model attribute for the total number of branch-and-bound nodes explored while solving a mixed-integer program (MIP).

source

# **Conflict Status**

MathOptInterface.compute\_conflict! - Function.

```
compute_conflict!(optimizer::AbstractOptimizer)
```

Computes a minimal subset of constraints such that the model with the other constraint removed is still infeasible.

Some solvers call a set of conflicting constraints an Irreducible Inconsistent Subsystem (IIS).

See also ConflictStatus and ConstraintConflictStatus.

#### Note

If the model is modified after a call to compute\_conflict!, the implementor is not obliged to purge the conflict. Any calls to the above attributes may return values for the original conflict without a warning. Similarly, when modifying the model, the conflict can be discarded.

source

MathOptInterface.ConflictStatus - Type.

```
ConflictStatus()
```

A model attribute for the ConflictStatusCode explaining why the conflict refiner stopped when computing the conflict.

source

MathOptInterface.ConflictStatusCode - Type.

ConflictStatusCode

An Enum of possible values for the ConflictStatus attribute. This attribute is meant to explain the reason why the conflict finder stopped executing in the most recent call to compute conflict!.

Possible values are:

- COMPUTE\_CONFLICT\_NOT\_CALLED: the function compute\_conflict! has not yet been called
- NO\_CONFLICT\_EXISTS: there is no conflict because the problem is feasible
- NO\_CONFLICT\_FOUND: the solver could not find a conflict
- · CONFLICT FOUND: at least one conflict could be found

source

MathOptInterface.ConstraintConflictStatus - Type.

ConstraintConflictStatus()

 $A constraint attribute indicating whether the constraint participates in the conflict. Its type is {\tt ConflictParticipationStatus} (a) the conflict of {\tt ConflictParticipationStatus} (b) the {\tt ConflictParticipationStatus} (c) the {\tt ConflictPa$ 

source

 ${\tt MathOptInterface.ConflictParticipationStatusCode-Type.}$ 

ConflictParticipationStatusCode

An Enum of possible values for the ConstraintConflictStatus attribute. This attribute is meant to indicate whether a given constraint participates or not in the last computed conflict.

Possible values are:

- NOT IN CONFLICT: the constraint does not participate in the conflict
- IN\_CONFLICT: the constraint participates in the conflict
- MAYBE\_IN\_CONFLICT: the constraint may participate in the conflict, the solver was not able to prove that the constraint can be excluded from the conflict

# **Variables**

#### 20.1 Functions

MathOptInterface.add\_variable - Function.

```
| add_variable(model::ModelLike)::VariableIndex
```

Add a scalar variable to the model, returning a variable index.

A AddVariableNotAllowed error is thrown if adding variables cannot be done in the current state of the model model.

source

MathOptInterface.add\_variables - Function.

```
| add_variables(model::ModelLike, n::Int)::Vector{VariableIndex}
```

Add n scalar variables to the model, returning a vector of variable indices.

A AddVariableNotAllowed error is thrown if adding variables cannot be done in the current state of the model model.

source

MathOptInterface.add\_constrained\_variable - Function.

Add to model a scalar variable constrained to belong to set, returning the index of the variable created and the index of the constraint constraining the variable to belong to set.

By default, this function falls back to creating a free variable with add\_variable and then constraining it to belong to set with add\_constraint.

source

MathOptInterface.add\_constrained\_variables - Function.

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```
add_constrained_variables(
    model::ModelLike,
    sets::AbstractVector{<:AbstractScalarSet}
)::Tuple{
    Vector{MOI.VariableIndex},
    Vector{MOI.ConstraintIndex{MOI.VariableIndex,eltype(sets)}},
}</pre>
```

Add to model scalar variables constrained to belong to sets, returning the indices of the variables created and the indices of the constraints constraining the variables to belong to each set in sets. That is, if it returns variables and constraints, constraints[i] is the index of the constraint constraining variable[i] to belong to sets[i].

By default, this function falls back to calling add constrained variable on each set.

source

```
add_constrained_variables(
    model::ModelLike,
    set::AbstractVectorSet,
)::Tuple{
    Vector{MOI.VariableIndex},
    MOI.ConstraintIndex{MOI.VectorOfVariables,typeof(set)},
}
```

Add to model a vector of variables constrained to belong to set, returning the indices of the variables created and the index of the constraint constraining the vector of variables to belong to set.

By default, this function falls back to creating free variables with add\_variables and then constraining it to belong to set with add\_constraint.

source

MathOptInterface.supports\_add\_constrained\_variable - Function.

```
supports_add_constrained_variable(
    model::ModelLike,
    S::Type{<:AbstractScalarSet}
)::Bool</pre>
```

Return a Bool indicating whether model supports constraining a variable to belong to a set of type S either on creation of the variable with add\_constrained\_variable or after the variable is created with add\_constraint.

By default, this function falls back to supports\_add\_constrained\_variables(model, Reals) && supports\_constraint(model. VariableIndex, S) which is the correct definition for most models.

#### **Example**

Suppose that a solver supports only two kind of variables: binary variables and continuous variables with a lower bound. If the solver decides not to support VariableIndex-in-Binary and VariableIndex-in-GreaterThan constraints, it only has to implement add\_constrained\_variable for these two sets which prevents the user to add both a binary constraint and a lower bound on the same variable. Moreover, if the user adds a VariableIndex-in-GreaterThan constraint, implementing this interface (i.e., supports\_add\_constrained\_varia enables the constraint to be transparently bridged into a supported constraint.

```
supports_add_constrained_variables(
    model::ModelLike,
    S::Type{<:AbstractVectorSet}
)::Bool</pre>
```

Return a Bool indicating whether model supports constraining a vector of variables to belong to a set of type S either on creation of the vector of variables with add\_constrained\_variables or after the variable is created with add\_constraint.

By default, if S is Reals then this function returns true and otherwise, it falls back to supports\_add\_constrained\_variables (Reals) && supports\_constraint(model, MOI.VectorOfVariables, S) which is the correct definition for most models.

#### **Example**

In the standard conic form (see Duality), the variables are grouped into several cones and the constraints are affine equality constraints. If Reals is not one of the cones supported by the solvers then it needs to implement supports\_add\_constrained\_variables(::0ptimizer, ::Type{Reals}) = false as free variables are not supported. The solvers should then implement supports\_add\_constrained\_variables(::0ptimizer, ::Type{<:SupportedCones}) = true where SupportedCones is the union of all cone types that are supported; it does not have to implement the method supports\_constraint(::Type{VectorOfVariables}, Type{<:SupportedCones}) as it should return false and it's the default. This prevents the user to constrain the same variable in two different cones. When a VectorOfVariables-in-S is added, the variables of the vector have already been created so they already belong to given cones. If bridges are enabled, the constraint will therefore be bridged by adding slack variables in S and equality constraints ensuring that the slack variables are equal to the corresponding variables of the given constraint function.

Note that there may also be sets for which !supports\_add\_constrained\_variables(model, S) and supports\_constraint(model, MOI.VectorOfVariables, S). For instance, suppose a solver supports positive semidefinite variable constraints and two types of variables: binary variables and nonnegative variables. Then the solver should support adding VectorOfVariables-in-PositiveSemidefiniteConeTriangle constraints, but it should not support creating variables constrained to belong to the PositiveSemidefiniteConeTriangle because the variables in PositiveSemidefiniteConeTriangle should first be created as either binary or non-negative.

source

MathOptInterface.is\_valid - Method.

```
is_valid(model::ModelLike, index::Index)::Bool
```

Return a Bool indicating whether this index refers to a valid object in the model model.

source

MathOptInterface.delete - Method.

```
delete(model::ModelLike, index::Index)
```

Delete the referenced object from the model. Throw DeleteNotAllowed if if index cannot be deleted.

The following modifications also take effect if Index is VariableIndex:

- If index used in the objective function, it is removed from the function, i.e., it is substituted for zero.
- For each func-in-set constraint of the model:
  - If func isa VariableIndex and func == index then the constraint is deleted.

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- If func isa VectorOfVariables and index in func.variables then
  - \* if length(func.variables) == 1 is one, the constraint is deleted;
  - \* iflength(func.variables) > 1 and supports\_dimension\_update(set) then then the variable is removed from func and set is replaced by update\_dimension(set, MOI.dimension(set) - 1).
  - \* Otherwise, a DeleteNotAllowed error is thrown.
- Otherwise, the variable is removed from func, i.e., it is substituted for zero.

source

MathOptInterface.delete - Method.

```
| delete(model::ModelLike, indices::Vector{R<:Index}) where {R}
```

Delete the referenced objects in the vector indices from the model. It may be assumed that R is a concrete type. The default fallback sequentially deletes the individual items in indices, although specialized implementations may be more efficient.

source

#### 20.2 Attributes

MathOptInterface.AbstractVariableAttribute - Type.

AbstractVariableAttribute

Abstract supertype for attribute objects that can be used to set or get attributes (properties) of variables in the model.

source

MathOptInterface.VariableName - Type.

```
VariableName()
```

A variable attribute for a string identifying the variable. It is valid for two variables to have the same name; however, variables with duplicate names cannot be looked up using get. It has a default value of "" if not set'.

source

MathOptInterface.VariablePrimalStart - Type.

```
VariablePrimalStart()
```

A variable attribute for the initial assignment to some primal variable's value that the optimizer may use to warm-start the solve. May be a number or nothing (unset).

source

MathOptInterface.VariablePrimal - Type.

```
VariablePrimal(result_index::Int = 1)
```

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A variable attribute for the assignment to some primal variable's value in result result\_index. If result\_index is omitted, it is 1 by default.

If the solver does not have a primal value for the variable because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError. Otherwise, if the result is unavailable for another reason (for instance, only a dual solution is available), the result is undefined. Users should first check PrimalStatus before accessing the VariablePrimal attribute.

See ResultCount for information on how the results are ordered.

source

MathOptInterface.VariableBasisStatus - Type.

VariableBasisStatus(result\_index::Int = 1)

A variable attribute for the BasisStatusCode of a variable in result result\_index, with respect to an available optimal solution basis.

If the solver does not have a basis statue for the variable because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError. Otherwise, if the result is unavailable for another reason (for instance, only a dual solution is available), the result is undefined. Users should first check PrimalStatus before accessing the VariableBasisStatus attribute.

See ResultCount for information on how the results are ordered.

# **Constraints**

### **21.1 Types**

MathOptInterface.ConstraintIndex - Type.

```
ConstraintIndex{F, S}
```

A type-safe wrapper for Int64 for use in referencing F-in-S constraints in a model. The parameter F is the type of the function in the constraint, and the parameter S is the type of set in the constraint. To allow for deletion, indices need not be consecutive. Indices within a constraint type (i.e. F-in-S) must be unique, but non-unique indices across different constraint types are allowed. If F is VariableIndex then the index is equal to the index of the variable. That is for an index::ConstraintIndex{VariableIndex}, we always have

```
index.value == MOI.get(model, MOI.ConstraintFunction(), index).value
source
```

#### 21.2 Functions

```
{\tt MathOptInterface.is\_valid-Method}.
```

```
is_valid(model::ModelLike, index::Index)::Bool
```

Return a Bool indicating whether this index refers to a valid object in the model model.

source

MathOptInterface.add\_constraint - Function.

```
| add_constraint(model::ModelLike, func::F, set::S)::ConstraintIndex{F,S} where {F,S}
```

Add the constraint  $f(x) \in \mathcal{S}$  where f is defined by func, and  $\mathcal{S}$  is defined by set.

```
add_constraint(model::ModelLike, v::VariableIndex, set::S)::ConstraintIndex{VariableIndex,S}
    where {S}
add_constraint(model::ModelLike, vec::Vector{VariableIndex}, set::S)::ConstraintIndex{
    VectorOfVariables,S} where {S}
```

Add the constraint  $v \in \mathcal{S}$  where v is the variable (or vector of variables) referenced by v and  $\mathcal{S}$  is defined by set.

- An UnsupportedConstraint error is thrown if model does not support F-in-S constraints,
- a AddConstraintNotAllowed error is thrown if it supports F-in-S constraints but it cannot add the constraint(s) in its current state and
- a ScalarFunctionConstantNotZero error may be thrown if func is an AbstractScalarFunction with nonzero constant and set is EqualTo, GreaterThan, LessThan or Interval.
- a LowerBoundAlreadySet error is thrown if F is a VariableIndex and a constraint was already added to this variable that sets a lower bound.
- a UpperBoundAlreadySet error is thrown if F is a VariableIndex and a constraint was already added to this variable that sets an upper bound.

source

MathOptInterface.add constraints - Function.

Add the set of constraints specified by each function-set pair in funcs and sets. F and S should be concrete types. This call is equivalent to add constraint. (model, funcs, sets) but may be more efficient.

source

MathOptInterface.transform - Function.

#### **Transform Constraint Set**

```
transform(model::ModelLike, c::ConstraintIndex{F,S1}, newset::S2)::ConstraintIndex{F,S2}
```

Replace the set in constraint c with newset. The constraint index c will no longer be valid, and the function returns a new constraint index with the correct type.

Solvers may only support a subset of constraint transforms that they perform efficiently (for example, changing from a LessThan to GreaterThan set). In addition, set modification (where S1 = S2) should be performed via the modify function.

Typically, the user should delete the constraint and add a new one.

#### **Examples**

If c is a ConstraintIndex{ScalarAffineFunction{Float64}, LessThan{Float64}},

```
c2 = transform(model, c, GreaterThan(0.0))
transform(model, c, LessThan(0.0)) # errors
```

MathOptInterface.supports\_constraint - Function.

```
MOI.supports_constraint(
   BT::Type{<:AbstractBridge},
   F::Type{<:MOI.AbstractFunction},
   S::Type{<:MOI.AbstractSet},
)::Bool</pre>
```

Return a Bool indicating whether the bridges of type BT support bridging F-in-S constraints.

```
supports_constraint(
   model::ModelLike,
   ::Type{F},
   ::Type{S},
)::Bool where {F<:AbstractFunction,S<:AbstractSet}</pre>
```

Return a Bool indicating whether model supports F-in-S constraints, that is, copy\_to(model, src) does not throw UnsupportedConstraint when src contains F-in-S constraints. If F-in-S constraints are only not supported in specific circumstances, e.g. F-in-S constraints cannot be combined with another type of constraint, it should still return true.

source

#### 21.3 Attributes

MathOptInterface.AbstractConstraintAttribute - Type.

AbstractConstraintAttribute

Abstract supertype for attribute objects that can be used to set or get attributes (properties) of constraints in the model.

source

MathOptInterface.ConstraintName - Type.

```
ConstraintName()
```

A constraint attribute for a string identifying the constraint.

It is valid for constraints variables to have the same name; however, constraints with duplicate names cannot be looked up using get, regardless of whether they have the same F-in-S type.

ConstraintName has a default value of "" if not set.

#### Notes

You should not implement ConstraintName for VariableIndex constraints.

source

MathOptInterface.ConstraintPrimalStart - Type.

```
ConstraintPrimalStart()
```

A constraint attribute for the initial assignment to some constraint's ConstraintPrimal that the optimizer may use to warm-start the solve.

May be nothing (unset), a number for AbstractScalarFunction, or a vector for AbstractVectorFunction.

source

MathOptInterface.ConstraintDualStart - Type.

```
ConstraintDualStart()
```

A constraint attribute for the initial assignment to some constraint's ConstraintDual that the optimizer may use to warm-start the solve.

 $May \ be \ nothing \ (unset), a \ number for \ Abstract Scalar Function, or a \ vector for \ Abstract Vector Function.$ 

MathOptInterface.ConstraintPrimal - Type.

```
ConstraintPrimal(result_index::Int = 1)
```

A constraint attribute for the assignment to some constraint's primal value(s) in result result index.

If the constraint is f(x) in S, then in most cases the ConstraintPrimal is the value of f, evaluated at the corresponding VariablePrimal solution.

However, some conic solvers reformulate b - Ax in S to s = b - Ax, s in S. These solvers may return the value of s for ConstraintPrimal, rather than b - Ax. (Although these are constrained by an equality constraint, due to numerical tolerances they may not be identical.)

If the solver does not have a primal value for the constraint because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError. Otherwise, if the result is unavailable for another reason (for instance, only a dual solution is available), the result is undefined. Users should first check PrimalStatus before accessing the ConstraintPrimal attribute.

If result\_index is omitted, it is 1 by default. See ResultCount for information on how the results are ordered.

source

MathOptInterface.ConstraintDual - Type.

```
| ConstraintDual(result_index::Int = 1)
```

A constraint attribute for the assignment to some constraint's dual value(s) in result result\_index. If result\_index is omitted, it is 1 by default.

If the solver does not have a dual value for the variable because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError. Otherwise, if the result is unavailable for another reason (for instance, only a primal solution is available), the result is undefined. Users should first check DualStatus before accessing the ConstraintDual attribute.

See ResultCount for information on how the results are ordered.

source

MathOptInterface.ConstraintBasisStatus - Type.

```
ConstraintBasisStatus(result_index::Int = 1)
```

A constraint attribute for the BasisStatusCode of some constraint in result result\_index, with respect to an available optimal solution basis. If result\_index is omitted, it is 1 by default.

If the solver does not have a basis statue for the constraint because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError. Otherwise, if the result is unavailable for another reason (for instance, only a dual solution is available), the result is undefined. Users should first check PrimalStatus before accessing the ConstraintBasisStatus attribute.

See ResultCount for information on how the results are ordered.

#### **Notes**

For the basis status of a variable, query VariableBasisStatus.

ConstraintBasisStatus does not apply to VariableIndex constraints. You can infer the basis status of a VariableIndex constraint by looking at the result of VariableBasisStatus.

source

MathOptInterface.BasisStatusCode - Type.

BasisStatusCode

An Enum of possible values for the ConstraintBasisStatus and VariableBasisStatus attributes, explaining the status of a given element with respect to an optimal solution basis.

Possible values are:

- BASIC: element is in the basis
- NONBASIC: element is not in the basis
- NONBASIC AT LOWER: element is not in the basis and is at its lower bound
- · NONBASIC AT UPPER: element is not in the basis and is at its upper bound
- SUPER BASIC: element is not in the basis but is also not at one of its bounds

#### Notes

- NONBASIC\_AT\_LOWER and NONBASIC\_AT\_UPPER should be used only for constraints with the Interval
  set. In this case, they are necessary to distinguish which side of the constraint is active. One-sided
  constraints (e.g., LessThan and GreaterThan) should use NONBASIC instead of the NONBASIC\_AT\_\*
  values. This restriction does not apply to VariableBasisStatus, which should return NONBASIC\_AT\_\*
  regardless of whether the alternative bound exists.
- In linear programs, SUPER BASIC occurs when a variable with no bounds is not in the basis.

source

MathOptInterface.ConstraintFunction - Type.

ConstraintFunction()

A constraint attribute for the AbstractFunction object used to define the constraint. It is guaranteed to be equivalent but not necessarily identical to the function provided by the user.

source

MathOptInterface.CanonicalConstraintFunction - Type.

CanonicalConstraintFunction()

A constraint attribute for a canonical representation of the AbstractFunction object used to define the constraint. Getting this attribute is guaranteed to return a function that is equivalent but not necessarily identical to the function provided by the user.

By default, MOI.get(model, MOI.CanonicalConstraintFunction(), ci) fallbacks to MOI.Utilities.canonical(MOI.get MOI.ConstraintFunction(), ci)). However, if model knows that the constraint function is canonical then it can implement a specialized method that directly return the function without calling Utilities.canonical. Therefore, the value returned cannot be assumed to be a copy of the function stored in model. Moreover, Utilities.Model checks with Utilities.is\_canonical whether the function stored internally is already canonical and if it's the case, then it returns the function stored internally instead of a copy.

MathOptInterface.ConstraintSet - Type.

ConstraintSet()

 $\ensuremath{\mathsf{A}}$  constraint attribute for the  $\ensuremath{\mathsf{AbstractSet}}$  object used to define the constraint.

# **Modifications**

MathOptInterface.modify - Function.

#### **Constraint Function**

```
| modify(model::ModelLike, ci::ConstraintIndex, change::AbstractFunctionModification)
```

Apply the modification specified by change to the function of constraint ci.

An ModifyConstraintNotAllowed error is thrown if modifying constraints is not supported by the model model.

#### **Examples**

```
modify(model, ci, ScalarConstantChange(10.0))
```

### **Objective Function**

```
| modify(model::ModelLike, ::ObjectiveFunction, change::AbstractFunctionModification)
```

Apply the modification specified by change to the objective function of model. To change the function completely, call set instead.

An ModifyObjectiveNotAllowed error is thrown if modifying objectives is not supported by the model model.

#### **Examples**

```
| modify(model, ObjectiveFunction{ScalarAffineFunction{Float64}}(), ScalarConstantChange(10.0)) source
```

MathOptInterface.AbstractFunctionModification - Type.

```
AbstractFunctionModification
```

An abstract supertype for structs which specify partial modifications to functions, to be used for making small modifications instead of replacing the functions entirely.

source

MathOptInterface.ScalarConstantChange - Type.

```
| ScalarConstantChange{T}(new_constant::T)
```

A struct used to request a change in the constant term of a scalar-valued function. Applicable to ScalarAffineFunction and ScalarQuadraticFunction.

source

MathOptInterface.VectorConstantChange - Type.

```
VectorConstantChange{T}(new_constant::Vector{T})
```

A struct used to request a change in the constant vector of a vector-valued function. Applicable to VectorAffineFunction and VectorQuadraticFunction.

source

MathOptInterface.ScalarCoefficientChange - Type.

```
| ScalarCoefficientChange{T}(variable::VariableIndex, new_coefficient::T)
```

A struct used to request a change in the linear coefficient of a single variable in a scalar-valued function. Applicable to ScalarAffineFunction and ScalarQuadraticFunction.

source

MathOptInterface.MultirowChange - Type.

```
| MultirowChange{T}(variable::VariableIndex, new_coefficients::Vector{Tuple{Int64, T}})
```

A struct used to request a change in the linear coefficients of a single variable in a vector-valued function. New coefficients are specified by (output\_index, coefficient) tuples. Applicable to VectorAffineFunction and VectorQuadraticFunction.

# Nonlinear programming

### **23.1 Types**

MathOptInterface.AbstractNLPEvaluator - Type.

```
AbstractNLPEvaluator
```

Abstract supertype for the callback object that is used to query function values, derivatives, and expression graphs. It is used in NLPBlock.

source

MathOptInterface.NLPBoundsPair - Type.

```
NLPBoundsPair(lower,upper)
```

A struct holding a pair of lower and upper bounds. -Inf and Inf can be used to indicate no lower or upper bound, respectively.

source

MathOptInterface.NLPBlockData - Type.

```
struct NLPBlockData
    constraint_bounds::Vector{NLPBoundsPair}
    evaluator::AbstractNLPEvaluator
    has_objective::Bool
end
```

A struct encoding a set of nonlinear constraints of the form  $lb \leq g(x) \leq ub$  and, if has\_objective == true, a nonlinear objective function f(x). constraint\_bounds holds the pairs of lb and ub elements. Nonlinear objectives override any objective set by using the ObjectiveFunction attribute. The evaluator is a callback object that is used to query function values, derivatives, and expression graphs. If has\_objective == false, then it is an error to query properties of the objective function, and in Hessian-of-the-Lagrangian queries,  $\sigma$  must be set to zero.

#### Note

Throughout the evaluator, all variables are ordered according to ListOfVariableIndices. Hence, MOI copies of nonlinear problems should be done with attention.

#### 23.2 Attributes

```
MathOptInterface.NLPBlock - Type.
```

```
NLPBlock()
```

Holds the NLPBlockData that represents a set of nonlinear constraints, and optionally a nonlinear objective.

source

MathOptInterface.NLPBlockDual - Type.

```
NLPBlockDual(result_index::Int)
NLPBlockDual()
```

The Lagrange multipliers on the constraints from the NLPBlock in result result\_index. If result\_index is omitted, it is 1 by default.

source

MathOptInterface.NLPBlockDualStart - Type.

```
NLPBlockDualStart()
```

An initial assignment of the Lagrange multipliers on the constraints from the NLPBlock that the solver may use to warm-start the solve.

source

#### 23.3 Functions

MathOptInterface.initialize - Function.

```
initialize(d::AbstractNLPEvaluator, requested_features::Vector{Symbol})
```

Must be called before any other methods. The vector requested\_features lists features requested by the solver. These may include : Grad for gradients of the obejctive, f, : Jac for explicit Jacobians of constraints, g, : JacVec for Jacobian-vector products, :HessVec for Hessian-vector and Hessian-of-Lagrangian-vector products, :Hess for explicit Hessians and Hessian-of-Lagrangians, and :ExprGraph for expression graphs.

source

MathOptInterface.features\_available - Function.

```
features_available(d::AbstractNLPEvaluator)
```

Returns the subset of features available for this problem instance, as a vector of symbols in the same format as in initialize.

source

 ${\tt MathOptInterface.eval\_objective-Function}.$ 

```
eval_objective(d::AbstractNLPEvaluator, x)
```

Evaluate the objective f(x), returning a scalar value.

MathOptInterface.eval\_constraint - Function.

```
eval_constraint(d::AbstractNLPEvaluator, g, x)
```

Evaluate the constraint function g(x), storing the result in the vector  ${\bf g}$  which must be of the appropriate size.

source

MathOptInterface.eval objective gradient - Function.

```
eval_objective_gradient(d::AbstractNLPEvaluator, df, x)
```

Evaluate  $\nabla f(x)$  as a dense vector, storing the result in the vector df which must be of the appropriate size.

source

MathOptInterface.jacobian\_structure - Function.

jacobian\_structure(d::AbstractNLPEvaluator)::Vector{Tuple{Int64,Int64}}

Returns the sparsity structure of the Jacobian matrix  $J_g(x) = \begin{bmatrix} \nabla g_1(x) \\ \nabla g_2(x) \\ \vdots \\ \nabla g_m(x) \end{bmatrix}$  where  $g_i$  is the ith component

of g. The sparsity structure is assumed to be independent of the point x. Returns a vector of tuples, (row, column), where each indicates the position of a structurally nonzero element. These indices are not required to be sorted and can contain duplicates, in which case the solver should combine the corresponding elements by adding them together.

source

MathOptInterface.hessian\_lagrangian\_structure - Function.

```
| hessian_lagrangian_structure(d::AbstractNLPEvaluator)::Vector{Tuple{Int64,Int64}}
```

Returns the sparsity structure of the Hessian-of-the-Lagrangian matrix  $\nabla^2 f + \sum_{i=1}^m \nabla^2 g_i$  as a vector of tuples, where each indicates the position of a structurally nonzero element. These indices are not required to be sorted and can contain duplicates, in which case the solver should combine the corresponding elements by adding them together. Any mix of lower and upper-triangular indices is valid. Elements (i,j) and (j,i), if both present, should be treated as duplicates.

source

MathOptInterface.eval\_constraint\_jacobian - Function.

```
eval_constraint_jacobian(d::AbstractNLPEvaluator, J, x)
```

Evaluates the sparse Jacobian matrix  $J_g(x)=\begin{bmatrix} \nabla g_1(x) \\ \nabla g_2(x) \\ \vdots \\ \nabla g_m(x) \end{bmatrix}$  . The result is stored in the vector J in the

same order as the indices returned by jacobian structure.

```
MathOptInterface.eval_constraint_jacobian_product - Function.
```

```
| eval_constraint_jacobian_product(d::AbstractNLPEvaluator, y, x, w)
```

Computes the Jacobian-vector product  $J_q(x)w$ , storing the result in the vector y.

source

 ${\tt MathOptInterface.eval\_constraint\_jacobian\_transpose\_product-Function}.$ 

```
| eval_constraint_jacobian_transpose_product(d::AbstractNLPEvaluator, y, x, w)
```

Computes the Jacobian-transpose-vector product  $J_q(x)^T w$ , storing the result in the vector y.

source

MathOptInterface.eval\_hessian\_lagrangian - Function.

```
| eval_hessian_lagrangian(d::AbstractNLPEvaluator, Η, x, σ, μ)
```

Given scalar weight  $\sigma$  and vector of constraint weights  $\mu$ , computes the sparse Hessian-of-the-Lagrangian matrix  $\sigma \nabla^2 f(x) + \sum_{i=1}^m \mu_i \nabla^2 g_i(x)$ , storing the result in the vector H in the same order as the indices returned by hessian lagrangian structure.

source

MathOptInterface.eval\_hessian\_lagrangian\_product - Function.

```
| eval hessian lagrangian product(d::AbstractNLPEvaluator, h, x, v, σ, μ)
```

Given scalar weight  $\sigma$  and vector of constraint weights  $\mu$ , computes the Hessian-of-the-Lagrangian-vector product  $\left(\sigma\nabla^2 f(x) + \sum_{i=1}^m \mu_i \nabla^2 g_i(x)\right) v$ , storing the result in the vector h.

source

MathOptInterface.objective\_expr - Function.

```
objective_expr(d::AbstractNLPEvaluator)
```

Returns an expression graph for the objective function as a standard Julia Expr object. All sums and products are flattened out as simple  $\operatorname{Expr}(:+,\dots)$  and  $\operatorname{Expr}(:*,\dots)$  objects. The symbol x is used as a placeholder for the vector of decision variables. No other undefined symbols are permitted; coefficients are embedded as explicit values. For example, the expression  $x_1+\sin(x_2/\exp(x_3))$  would be represented as the Julia object : (x[1] +  $\sin(x[2]/\exp(x[3]))$ ). Each integer index is wrapped in a VariableIndex. See the Julia manual for more information on the structure of Expr objects. There are currently no restrictions on recognized functions; typically these will be built-in Julia functions like ^, exp, log, cos, tan, sqrt, etc., but modeling interfaces may choose to extend these basic functions.

source

MathOptInterface.constraint\_expr - Function.

```
constraint_expr(d::AbstractNLPEvaluator, i)
```

Returns an expression graph for the ith constraint in the same format as described above, with an additional comparison operator indicating the sense of and bounds on the constraint. The right-hand side of the comparison must be a constant; that is, :(x[1]^3 <= 1) is allowed, while :(1 <= x[1]^3) is not valid. Double-sided constraints are allowed, in which case both the lower bound and upper bounds should be constants; for example, :(-1 <=  $\cos(x[1]) + \sin(x[2]) <= 1$ ) is valid.

# **Callbacks**

MathOptInterface.AbstractCallback - Type.

```
| abstract type AbstractCallback <: AbstractModelAttribute end
```

Abstract type for a model attribute representing a callback function. The value set to subtypes of AbstractCallback is a function that may be called during optimize!. As optimize! is in progress, the result attributes (i.e, the attributes attr such that is\_set\_by\_optimize(attr)) may not be accessible from the callback, hence trying to get result attributes might throw a OptimizeInProgress error.

At most one callback of each type can be registered. If an optimizer already has a function for a callback type, and the user registers a new function, then the old one is replaced.

The value of the attribute should be a function taking only one argument, commonly called callback\_data, that can be used for instance in LazyConstraintCallback, HeuristicCallback and UserCutCallback.

source

MathOptInterface.AbstractSubmittable - Type.

AbstractSubmittable

Abstract supertype for objects that can be submitted to the model.

source

MathOptInterface.submit - Function.

```
| submit(optimizer::AbstractOptimizer, sub::AbstractSubmittable, values...)::Nothing
```

Submit values to the submittable sub of the optimizer optimizer.

An UnsupportedSubmittable error is thrown if model does not support the attribute attr (see supports) and a SubmitNotAllowed error is thrown if it supports the submittable sub but it cannot be submitted.

source

### 24.1 Attributes

MathOptInterface.CallbackNodeStatus - Type.

```
CallbackNodeStatus(callback_data)
```

An optimizer attribute describing the (in)feasibility of the primal solution available from CallbackVariablePrimal during a callback identified by callback\_data.

Returns a CallbackNodeStatusCode Enum.

source

MathOptInterface.CallbackNodeStatusCode - Type.

CallbackNodeStatusCode

An Enum of possible return values from calling get with CallbackNodeStatus.

Possible values are:

- CALLBACK\_NODE\_STATUS\_INTEGER: the primal solution available from CallbackVariablePrimal is integer feasible.
- CALLBACK\_NODE\_STATUS\_FRACTIONAL: the primal solution available from CallbackVariablePrimal is integer infeasible.
- CALLBACK\_NODE\_STATUS\_UNKNOWN: the primal solution available from CallbackVariablePrimal might be integer feasible or infeasible.

source

MathOptInterface.CallbackVariablePrimal - Type.

CallbackVariablePrimal(callback\_data)

A variable attribute for the assignment to some primal variable's value during the callback identified by callback\_data.

source

### 24.2 Lazy constraints

MathOptInterface.LazyConstraintCallback - Type.

```
| LazyConstraintCallback() <: AbstractCallback
```

The callback can be used to reduce the feasible set given the current primal solution by submitting a LazyConstraint. For instance, it may be called at an incumbent of a mixed-integer problem. Note that there is no guarantee that the callback is called at every feasible primal solution.

The current primal solution is accessed through CallbackVariablePrimal. Trying to access other result attributes will throw OptimizeInProgress as discussed in AbstractCallback.

### **Examples**

```
x = MOI.add_variables(optimizer, 8)
MOI.set(optimizer, MOI.LazyConstraintCallback(), callback_data -> begin
    sol = MOI.get(optimizer, MOI.CallbackVariablePrimal(callback_data), x)
    if # should add a lazy constraint
        func = # computes function
        set = # computes set
        MOI.submit(optimizer, MOI.LazyConstraint(callback_data), func, set)
    end
end)
```

source

MathOptInterface.LazyConstraint - Type.

```
LazyConstraint(callback_data)
```

Lazy constraint func-in-set submitted as func, set. The optimal solution returned by VariablePrimal will satisfy all lazy constraints that have been submitted.

This can be submitted only from the LazyConstraintCallback. The field callback\_data is a solver-specific callback type that is passed as the argument to the feasible solution callback.

#### **Examples**

Suppose x and y are VariableIndexs of optimizer. To add a LazyConstraint for  $2x + 3y \le 1$ , write

```
func = 2.0x + 3.0y
set = MOI.LessThan(1.0)
MOI.submit(optimizer, MOI.LazyConstraint(callback_data), func, set)
```

inside a LazyConstraintCallback of data callback\_data.

source

#### 24.3 User cuts

MathOptInterface.UserCutCallback - Type.

```
UserCutCallback() <: AbstractCallback</pre>
```

The callback can be used to submit UserCut given the current primal solution. For instance, it may be called at fractional (i.e., non-integer) nodes in the branch and bound tree of a mixed-integer problem. Note that there is not guarantee that the callback is called everytime the solver has an infeasible solution.

The infeasible solution is accessed through CallbackVariablePrimal. Trying to access other result attributes will throw OptimizeInProgress as discussed in AbstractCallback.

#### **Examples**

```
x = MOI.add_variables(optimizer, 8)
MOI.set(optimizer, MOI.UserCutCallback(), callback_data -> begin
    sol = MOI.get(optimizer, MOI.CallbackVariablePrimal(callback_data), x)
    if # can find a user cut
        func = # computes function
        set = # computes set
        MOI.submit(optimizer, MOI.UserCut(callback_data), func, set)
    end
end
```

 ${\tt MathOptInterface.UserCut-Type.}$ 

```
UserCut(callback_data)
```

Constraint func-to-set suggested to help the solver detect the solution given by CallbackVariablePrimal as infeasible. The cut is submitted as func, set. Typically CallbackVariablePrimal will violate integrality constraints, and a cut would be of the form ScalarAffineFunction-in-LessThan or ScalarAffineFunction-in-GreaterThan. Note that, as opposed to LazyConstraint, the provided constraint cannot modify the feasible set, the constraint should be redundant, e.g., it may be a consequence of affine and integrality constraints.

This can be submitted only from the UserCutCallback. The field callback\_data is a solver-specific callback type that is passed as the argument to the infeasible solution callback.

Note that the solver may silently ignore the provided constraint.

source

#### 24.4 Heuristic solutions

MathOptInterface.HeuristicCallback - Type.

```
| HeuristicCallback() <: AbstractCallback
```

The callback can be used to submit HeuristicSolution given the current primal solution. For instance, it may be called at fractional (i.e., non-integer) nodes in the branch and bound tree of a mixed-integer problem. Note that there is not guarantee that the callback is called everytime the solver has an infeasible solution.

The current primal solution is accessed through CallbackVariablePrimal. Trying to access other result attributes will throw OptimizeInProgress as discussed in AbstractCallback.

#### **Examples**

MathOptInterface.HeuristicSolutionStatus - Type.

```
HeuristicSolutionStatus
```

An Enum of possible return values for submit with HeuristicSolution. This informs whether the heuristic solution was accepted or rejected. Possible values are:

- HEURISTIC SOLUTION ACCEPTED: The heuristic solution was accepted.
- HEURISTIC\_SOLUTION\_REJECTED: The heuristic solution was rejected.
- HEURISTIC SOLUTION UNKNOWN: No information available on the acceptance.

source

### HeuristicSolution(callback\_data)

Heuristically obtained feasible solution. The solution is submitted as variables, values where values[i] gives the value of variables[i], similarly to set. The submit call returns a HeuristicSolutionStatus indicating whether the provided solution was accepted or rejected.

This can be submitted only from the HeuristicCallback. The field callback\_data is a solver-specific callback type that is passed as the argument to the heuristic callback.

Some solvers require a complete solution, others only partial solutions.

### **Errors**

When an MOI call fails on a model, precise errors should be thrown when possible instead of simply calling error with a message. The docstrings for the respective methods describe the errors that the implementation should throw in certain situations. This error-reporting system allows code to distinguish between internal errors (that should be shown to the user) and unsupported operations which may have automatic workarounds.

When an invalid index is used in an MOI call, an InvalidIndex is thrown:

MathOptInterface.InvalidIndex - Type.

```
struct InvalidIndex{IndexType<:Index} <: Exception
  index::IndexType
end</pre>
```

An error indicating that the index index is invalid.

source

When an invalid result index is used to retrieve an attribute, a ResultIndexBoundsError is thrown:

MathOptInterface.ResultIndexBoundsError - Type.

```
struct ResultIndexBoundsError{AttrType} <: Exception
   attr::AttrType
   result_count::Int
end</pre>
```

An error indicating that the requested attribute attr could not be retrieved, because the solver returned too few results compared to what was requested. For instance, the user tries to retrieve VariablePrimal(2) when only one solution is available, or when the model is infeasible and has no solution.

```
See also: check_result_index_bounds.
source

MathOptInterface.check_result_index_bounds - Function.
| check_result_index_bounds(model::ModelLike, attr)
```

This function checks whether enough results are available in the model for the requested attr, using its result\_index field. If the model does not have sufficient results to answer the query, it throws a ResultIndexBoundsError.

As discussed in JuMP mapping, for scalar constraint with a nonzero function constant, a ScalarFunctionConstantNotZero exception may be thrown:

MathOptInterface.ScalarFunctionConstantNotZero - Type.

```
struct ScalarFunctionConstantNotZero{T, F, S} <: Exception
    constant::T
end</pre>
```

An error indicating that the constant part of the function in the constraint F-in-S is nonzero.

source

Some VariableIndex constraints cannot be combined on the same variable:

MathOptInterface.LowerBoundAlreadySet - Type.

```
LowerBoundAlreadySet{S1, S2}
```

Error thrown when setting a VariableIndex-in-S2 when a VariableIndex-in-S1 has already been added and the sets S1, S2 both set a lower bound, i.e. they are EqualTo, GreaterThan, Interval, Semicontinuous or Semiinteger.

source

MathOptInterface.UpperBoundAlreadySet - Type.

```
UpperBoundAlreadySet{S1, S2}
```

Error thrown when setting a VariableIndex-in-S2 when a VariableIndex-in-S1 has already been added and the sets S1, S2 both set an upper bound, i.e. they are EqualTo, LessThan, Interval, Semicontinuous or Semiinteger.

source

As discussed in AbstractCallback, trying to get attributes inside a callback may throw:

MathOptInterface.OptimizeInProgress - Type.

```
struct OptimizeInProgress{AttrType<:AnyAttribute} <: Exception
  attr::AttrType
end</pre>
```

Error thrown from optimizer when MOI.get(optimizer, attr) is called inside an AbstractCallback while it is only defined once optimize! has completed. This can only happen when is\_set\_by\_optimize(attr) is true.

source

Trying to submit the wrong type of AbstractSubmittable inside an AbstractCallback (e.g., a UserCut inside a LazyConstraintCallback) will throw:

MathOptInterface.InvalidCallbackUsage - Type.

```
struct InvalidCallbackUsage{C, S} <: Exception
    callback::C
    submittable::S
end</pre>
```

An error indicating that submittable cannot be submitted inside callback.

For example, UserCut cannot be submitted inside LazyConstraintCallback.

source

The rest of the errors defined in MOI fall in two categories represented by the following two abstract types:

MathOptInterface.UnsupportedError - Type.

```
UnsupportedError <: Exception
```

Abstract type for error thrown when an element is not supported by the model.

source

MathOptInterface.NotAllowedError - Type.

```
NotAllowedError <: Exception
```

Abstract type for error thrown when an operation is supported but cannot be applied in the current state of the model.

source

The different UnsupportedError and NotAllowedError are the following errors:

MathOptInterface.UnsupportedAttribute - Type.

```
struct UnsupportedAttribute{AttrType} <: UnsupportedError
  attr::AttrType
  message::String
end</pre>
```

An error indicating that the attribute attr is not supported by the model, i.e. that supports returns false.

source

MathOptInterface.SetAttributeNotAllowed - Type.

```
struct SetAttributeNotAllowed{AttrType} <: NotAllowedError
   attr::AttrType
   message::String # Human-friendly explanation why the attribute cannot be set
end</pre>
```

An error indicating that the attribute attr is supported (see supports) but cannot be set for some reason (see the error string).

source

MathOptInterface.AddVariableNotAllowed - Type.

```
struct AddVariableNotAllowed <: NotAllowedError
  message::String # Human-friendly explanation why the attribute cannot be set
end</pre>
```

An error indicating that variables cannot be added to the model.

source

MathOptInterface.UnsupportedConstraint - Type.

```
struct UnsupportedConstraint{F<:AbstractFunction, S<:AbstractSet} <: UnsupportedError
   message::String # Human-friendly explanation why the attribute cannot be set
end</pre>
```

An error indicating that constraints of type F-in-S are not supported by the model, i.e. that supports\_constraint returns false.

source

MathOptInterface.AddConstraintNotAllowed - Type.

```
struct AddConstraintNotAllowed{F<:AbstractFunction, S<:AbstractSet} <: NotAllowedError
    message::String # Human-friendly explanation why the attribute cannot be set
end</pre>
```

An error indicating that constraints of type F-in-S are supported (see supports\_constraint) but cannot be added.

source

MathOptInterface.ModifyConstraintNotAllowed - Type.

An error indicating that the constraint modification change cannot be applied to the constraint of index ci.

source

MathOptInterface.ModifyObjectiveNotAllowed - Type.

```
struct ModifyObjectiveNotAllowed{C<:AbstractFunctionModification} <: NotAllowedError
    change::C
    message::String
end</pre>
```

An error indicating that the objective modification change cannot be applied to the objective.

source

MathOptInterface.DeleteNotAllowed - Type.

```
struct DeleteNotAllowed{IndexType <: Index} <: NotAllowedError
  index::IndexType
  message::String
end</pre>
```

An error indicating that the index index cannot be deleted.

source

MathOptInterface.UnsupportedSubmittable - Type.

```
struct UnsupportedSubmittable{SubmitType} <: UnsupportedError
    sub::SubmitType
    message::String
end</pre>
```

An error indicating that the submittable sub is not supported by the model, i.e. that supports returns false.

source

MathOptInterface.SubmitNotAllowed - Type.

```
struct SubmitNotAllowed{SubmitTyp<:AbstractSubmittable} <: NotAllowedError
    sub::SubmitType
    message::String # Human-friendly explanation why the attribute cannot be set
end</pre>
```

An error indicating that the submittable sub is supported (see supports) but cannot be added for some reason (see the error string).

source

Note that setting the  ${\tt ConstraintFunction}$  of a  ${\tt VariableIndex}$  constraint is not allowed:

MathOptInterface.SettingVariableIndexNotAllowed - Type.

```
| SettingVariableIndexNotAllowed()
```

Error type that should be thrown when the user calls set to change the ConstraintFunction of a VariableIndex constraint.

# Part VI

# **Submodules**

# **Benchmarks**

#### 26.1 Overview

#### The Benchmarks submodule

To aid the development of efficient solver wrappers, MathOptInterface provides benchmarking functionality. Benchmarking a wrapper follows a two-step process.

First, prior to making changes, run and save the benchmark results on a given benchmark suite as follows:

```
using SolverPackage # Replace with your choice of solver.
using MathOptInterface
const MOI = MathOptInterface
suite = MOI.Benchmarks.suite() do
    SolverPackage.Optimizer()
end

MOI.Benchmarks.create_baseline(
    suite, "current"; directory = "/tmp", verbose = true
)
```

Use the exclude argument to Benchmarks.suite to exclude benchmarks that the solver doesn't support.

Second, after making changes to the package, re-run the benchmark suite and compare to the prior saved results:

```
using SolverPackage, MathOptInterface

const MOI = MathOptInterface

suite = MOI.Benchmarks.suite() do
    SolverPackage.Optimizer()
end

MOI.Benchmarks.compare_against_baseline(
    suite, "current"; directory = "/tmp", verbose = true
)
```

This comparison will create a report detailing improvements and regressions.

#### 26.2 API Reference

#### **Benchmarks**

Functions to help benchmark the performance of solver wrappers. See The Benchmarks submodule for more details

MathOptInterface.Benchmarks.suite - Function.

```
suite(
    new_model::Function;
    exclude::Vector{Regex} = Regex[]
)
```

Create a suite of benchmarks. new\_model should be a function that takes no arguments, and returns a new instance of the optimizer you wish to benchmark.

Use exclude to exclude a subset of benchmarks.

#### **Examples**

```
suite() do
   GLPK.Optimizer()
end
suite(exclude = [r"delete"]) do
   Gurobi.Optimizer(OutputFlag=0)
end
```

source

 ${\tt MathOptInterface.Benchmarks.create\_baseline-Function}.$ 

```
create_baseline(suite, name::String; directory::String = ""; kwargs...)
```

Run all benchmarks in suite and save to files called name in directory.

Extra kwargs are based to BenchmarkTools.run.

### **Examples**

```
my_suite = suite(() -> GLPK.Optimizer())
create_baseline(my_suite, "glpk_master"; directory = "/tmp", verbose = true)
```

MathOptInterface.Benchmarks.compare\_against\_baseline - Function.

```
compare_against_baseline(
   suite, name::String; directory::String = "",
   report_filename::String = "report.txt"
)
```

Run all benchmarks in suite and compare against files called name in directory that were created by a call to create\_baseline.

A report summarizing the comparison is written to report\_filename in directory.

Extra kwargs are based to BenchmarkTools.run.

#### **Examples**

```
my_suite = suite(() -> GLPK.Optimizer())
compare_against_baseline(
   my_suite, "glpk_master"; directory = "/tmp", verbose = true
)
source
```

# **Bridges**

#### 27.1 Overview

### The Bridges submodule

The Bridges module simplifies the process of converting models between equivalent formulations.

#### Tip

Read our paper for more details on how bridges are implemented.

### Why bridges?

A constraint can often be written in a number of equivalent formulations. For example, the constraint  $l \leq a^\top x \leq u$  (ScalarAffineFunction-in-Interval) could be re-formulated as two constraints:  $a^\top x \geq l$  (ScalarAffineFunction-in-GreaterThan) and  $a^\top x \leq u$  (ScalarAffineFunction-in-LessThan). An alternative re-formulation is to add a dummy variable y with the constraints  $l \leq y \leq u$  (VariableIndex-in-Interval) and  $a^\top x - y = 0$  (ScalarAffineFunction-in-EqualTo).

To avoid each solver having to code these transformations manually, MathOptInterface provides bridges.

A bridge is a small transformation from one constraint type to another (potentially collection of) constraint type.

Because these bridges are included in MathOptInterface, they can be re-used by any optimizer. Some bridges also implement constraint modifications and constraint primal and dual translations.

Several bridges can be used in combination to transform a single constraint into a form that the solver may understand. Choosing the bridges to use takes the form of finding a shortest path in the hypergraph of bridges. The methodology is detailed in the MOI paper.

#### The three types of bridges

There are three types of bridges in MathOptInterface:

- 1. Constraint bridges
- 2. Variable bridges
- 3. Objective bridges

**Constraint bridges** Constraint bridges convert constraints formulated by the user into an equivalent form supported by the solver. Constraint bridges are subtypes of Bridges.Constraint.AbstractBridge.

The equivalent formulation may add constraints (and possibly also variables) in the underlying model.

In particular, constraint bridges can focus on rewriting the function of a constraint, and do not change the set. Function bridges are subtypes of Bridges.Constraint.AbstractFunctionConversionBridge.

Read the list of implemented constraint bridges for more details on the types of transformations that are available. Function bridges are Bridges. Constraint. Scalar Functionize Bridge and Bridges. Constraint. Vector Functionize Bridges.

**Variable bridges** Variable bridges convert variables added by the user, either free with add\_variable/add\_variables, or constrained with add\_constrained\_variable/add\_constrained\_variables, into an equivalent form supported by the solver. Variable bridges are subtypes of Bridges.Variable.AbstractBridge.

The equivalent formulation may add constraints (and possibly also variables) in the underlying model.

Read the list of implemented variable bridges for more details on the types of transformations that are available.

**Objective bridges** Objective bridges convert the ObjectiveFunction set by the user into an equivalent form supported by the solver. Objective bridges are subtypes of Bridges.Objective.AbstractBridge.

The equivalent formulation may add constraints (and possibly also variables) in the underlying model.

Read the list of implemented objective bridges for more details on the types of transformations that are available.

#### Bridges.full\_bridge\_optimizer

#### Tip

Unless you have an advanced use-case, this is probably the only function you need to care about.

To enable the full power of MathOptInterface's bridges, wrap an optimizer in a Bridges.full bridge optimizer.

```
julia> inner_optimizer = MOI.Utilities.Model{Float64}()
MOIU.Model{Float64}

julia> optimizer = MOI.Bridges.full_bridge_optimizer(inner_optimizer, Float64)
MOIB.LazyBridgeOptimizer{MOIU.Model{Float64}}
with 0 variable bridges
with 0 constraint bridges
with 0 objective bridges
with inner model MOIU.Model{Float64}
```

That's all you have to do! Use optimizer as normal, and bridging will happen lazily behind the scenes. By lazily, we mean that bridging will only happen if the constraint is not supported by the inner optimizer.

#### Info

Most bridges are added by default in Bridges.full\_bridge\_optimizer. However, for technical reasons, some bridges are not added by default. Three examples include Bridges.Constraint.SOCtoPSDBridge, Bridges.Constraint.SOCtoNonConvexQuadBridge and Bridges.Constraint.RSOCtoNonConvexQuadBridge. See the docs of those bridges for more information.

#### Add a single bridge

If you don't want to use Bridges.full bridge optimizer, you can wrap an optimizer in a single bridge.

However, this will force the constraint to be bridged, even if the inner optimizer supports it.

```
julia> inner_optimizer = MOI.Utilities.Model{Float64}()
MOIU.Model{Float64}
julia> optimizer = MOI.Bridges.Constraint.SplitInterval{Float64}(inner_optimizer)
{\tt MOIB.Constraint.SplitIntervalBridge} \{Float 64, \ F, \ S, \ LS, \ US\}
\quad \  \  \, \hookrightarrow \  \  \, \text{where } \{ \text{F<:MOI.AbstractFunction, S<:MOI.AbstractSet, LS<:MOI.AbstractSet, US<:MOI.AbstractSet} \}, \\
→ MOIU.Model{Float64}}
with 0 constraint bridges
with inner model MOIU.Model{Float64}
julia> x = MOI.add_variable(optimizer)
MOI.VariableIndex(1)
julia> MOI.add_constraint(optimizer, x, MOI.Interval(0.0, 1.0))
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,
→ MathOptInterface.Interval{Float64}}(1)
julia> MOI.get(optimizer, MOI.ListOfConstraintTypesPresent())
1-element Vector{Tuple{Type, Type}}:
 (MathOptInterface.VariableIndex, MathOptInterface.Interval{Float64})
julia> MOI.get(inner_optimizer, MOI.ListOfConstraintTypesPresent())
2-element Vector{Tuple{Type, Type}}:
 (MathOptInterface.VariableIndex, MathOptInterface.GreaterThan{Float64})
(MathOptInterface.VariableIndex, MathOptInterface.LessThan{Float64})
```

#### Bridges.LazyBridgeOptimizer

If you don't want to use Bridges.full\_bridge\_optimizer, but you need more than a single bridge (or you want the bridging to happen lazily), you can manually construct a Bridges.LazyBridgeOptimizer.

First, wrap an inner optimizer:

```
julia> inner_optimizer = MOI.Utilities.Model{Float64}()
MOIU.Model{Float64}

julia> optimizer = MOI.Bridges.LazyBridgeOptimizer(inner_optimizer)
MOIB.LazyBridgeOptimizer{MOIU.Model{Float64}}
with 0 variable bridges
with 0 constraint bridges
with 0 objective bridges
with inner model MOIU.Model{Float64}
```

Then use Bridges.add bridge to add individual bridges:

```
julia> MOI.Bridges.add_bridge(optimizer, MOI.Bridges.Constraint.SplitIntervalBridge{Float64})
julia> MOI.Bridges.add_bridge(optimizer, MOI.Bridges.Objective.FunctionizeBridge{Float64})
```

Now the constraints will be bridged only if needed:

### 27.2 Implementation

### **Bridge interface**

To be usable by a bridge optimizer, a bridge must implement the following functions:

MathOptInterface.Bridges.added\_constrained\_variable\_types - Function.

```
added_constrained_variable_types(
   BT::Type{<:Variable.AbstractBridge},
)::Vector{Tuple{Type}}</pre>
```

Return a list of the types of constrained variables that bridges of concrete type BT add. This is used by the LazyBridgeOptimizer.

source

MathOptInterface.Bridges.added\_constraint\_types - Function.

```
added_constraint_types(
    BT::Type{<:Constraint.AbstractBridge},
)::Vector{Tuple{Type, Type}}</pre>
```

Return a list of the types of constraints that bridges of concrete type BT add. This is used by the LazyBridgeOptimizer.

source

Additionally, variable bridges must implement:

MathOptInterface.Bridges.Variable.supports\_constrained\_variable - Function.

```
supports_constrained_variable(
    ::Type{<:AbstractBridge},
    ::Type{<:MOI.AbstractSet},
)::Bool</pre>
```

Return a Bool indicating whether the bridges of type BT support bridging constrained variables in S.

source

MathOptInterface.Bridges.Variable.concrete\_bridge\_type - Function.

```
concrete_bridge_type(
   BT::Type{<:AbstractBridge},
   S::Type{<:MOI.AbstractSet},
)::Type</pre>
```

Return the concrete type of the bridge supporting variables in S constraints. This function can only be called if MOI.supports constrained variable(BT, S) is true.

#### **Examples**

As a variable in MathOptInterface. GreaterThan is bridged into variables in MathOptInterface. Nonnegatives by the VectorizeBridge:

```
MOI.Bridges.Variable.concrete_bridge_type(
    MOI.Bridges.Variable.VectorizeBridge{Float64},
    MOI.GreaterThan{Float64},
)

# output

MathOptInterface.Bridges.Variable.VectorizeBridge{Float64, MathOptInterface.Nonnegatives}
```

source

MathOptInterface.Bridges.Variable.bridge\_constrained\_variable - Function.

```
bridge_constrained_variable(
   BT::Type{<:AbstractBridge},
   model::MOI.ModelLike,
   set::MOI.AbstractSet,
)</pre>
```

Bridge the constrained variable in set using bridge BT to model and returns a bridge object of type BT. The bridge type BT should be a concrete type, that is, all the type parameters of the bridge should be set. Use concrete\_bridge\_type to obtain a concrete type for given set types.

source

constraint bridges must implement:

MathOptInterface.supports\_constraint - Method.

```
MOI.supports_constraint(
   BT::Type{<:AbstractBridge},
   F::Type{<:MOI.AbstractFunction},
   S::Type{<:MOI.AbstractSet},
)::Bool</pre>
```

Return a Bool indicating whether the bridges of type BT support bridging F-in-S constraints.

source

MathOptInterface.Bridges.Constraint.concrete bridge type - Function.

```
concrete_bridge_type(
   BT::Type{<:AbstractBridge},
   F::Type{<:MOI.AbstractFunction},
   S::Type{<:MOI.AbstractSet}
)::Type</pre>
```

Return the concrete type of the bridge supporting F-in-S constraints. This function can only be called if MOI.supports\_constraint(BT, F, S) is true.

### **Examples**

As a MathOptInterface.VariableIndex-in-MathOptInterface.Interval constraint is bridged into a MathOptInterface.VariableIndex-in-MathOptInterface.LessThan by the SplitIntervalBridge:

Bridge the constraint func-in-set using bridge BT to model and returns a bridge object of type BT. The bridge type BT should be a concrete type, that is, all the type parameters of the bridge should be set. Use concrete\_bridge\_type to obtain a concrete type for given function and set types.

source

)

and objective bridges must implement:

set::MOI.AbstractSet,

MathOptInterface.Bridges.set\_objective\_function\_type - Function.

```
set_objective_function_type(
    BT::Type{<:Objective.AbstractBridge},
)::Type{<:MOI.AbstractScalarFunction}</pre>
```

Return the type of objective function that bridges of concrete type BT set. This is used by the LazyBridgeOptimizer.

source

MathOptInterface.Bridges.Objective.concrete bridge type - Function.

```
concrete_bridge_type(
   BT::Type{<:MOI.Bridges.Objective.AbstractBridge},
   F::Type{<:MOI.AbstractScalarFunction},
)::Type</pre>
```

Return the concrete type of the bridge supporting objective functions of type F. This function can only be called if MOI.supports\_objective\_function(BT, F) is true.

MathOptInterface.Bridges.Objective.bridge\_objective - Function.

```
bridge_objective(
   BT::Type{<:MOI.Bridges.Objective.AbstractBridge},
   model::MOI.ModelLike,
   func::MOI.AbstractScalarFunction,
)</pre>
```

Bridge the objective function func using bridge BT to model and returns a bridge object of type BT. The bridge type BT should be a concrete type, that is, all the type parameters of the bridge should be set. Use concrete\_bridge\_type to obtain a concrete type for a given function type.

source

When querying the NumberOfVariables, NumberOfConstraints ListOfVariableIndices, and ListOfConstraintIndices, the variables and constraints created by the bridges in the underlying model are hidden by the bridge optimizer. For this purpose, the bridge must provide access to the variables and constraints it has created by implementing the following methods of get:

```
MathOptInterface.get - Method.
```

```
MOI.get(b::AbstractBridge, ::MOI.NumberOfVariables)
```

The number of variables created by the bridge b in the model.

source

MathOptInterface.get - Method.

```
|MOI.get(b::AbstractBridge, ::MOI.ListOfVariableIndices)
```

The list of variables created by the bridge b in the model.

source

MathOptInterface.get - Method.

```
| MOI.get(b::AbstractBridge, ::MOI.NumberOfConstraints{F, S}) where {F, S}
```

The number of constraints of the type F-in-S created by the bridge b in the model.

source

MathOptInterface.get - Method.

```
| \begin{tabular}{ll} MOI.get(b::AbstractBridge, ::MOI.ListOfConstraintIndices{F, S}) & where {F, S} \\ \hline \end{tabular}
```

A  $Vector{ConstraintIndex{F,S}}$  with indices of all constraints of type F-inS created by the bride b in the model (i.e., of length equal to the value of  $NumberOfConstraints{F,S}()$ ).

source

## SetMap bridges

Implementing a constraint bridge relying on linear transformation between two sets is easier thanks to the SetMap interface. The bridge simply needs to be a subtype of [Bridges.Variable.SetMapBridge] for a variable bridge and [Bridges.Constraint.SetMapBridge] for a constraint bridge and the linear transformation is represented with Bridges.map\_set, Bridges.map\_function, Bridges.inverse\_map\_set, Bridges.inverse\_map\_function,

Bridges.adjoint\_map\_function and Bridges.inverse\_adjoint\_map\_function. Note that the implementing last 4 methods is optional in the sense that if they are not implemented, bridging constraint would still work but some features would be missing as described in the docstrings. See [L20, Section 2.1.2] for more details including [L20, Example 2.1.1] that illustrates the idea for Bridges.Variable.SOCtoRSOCBridge, Bridges.Variable.RSOCtoSOCBridge and Bridges.Constraint.RSOCtoSOCBridge.

[L20] Legat, Benoît. Set Programming: Theory and Computation. PhD thesis. 2020.

#### 27.3 API Reference

### **Bridges**

MathOptInterface.Bridges.AbstractBridge - Type.

AbstractBridge

Represents a bridged constraint or variable in a MathOptInterface.Bridges.AbstractBridgeOptimizer. It contains the indices of the variables and constraints that it has created in the model. These can be obtained using MathOptInterface.NumberOfVariables, MathOptInterface.ListOfVariableIndices, MathOptInterface.Nu and MathOptInterface.ListOfConstraintIndices using MathOptInterface.get with the bridge in place of the MathOptInterface.ModelLike. Attributes of the bridged model such as MathOptInterface.ConstraintDual and MathOptInterface.ConstraintPrimal, can be obtained using MathOptInterface.get with the bridge in place of the constraint index. These calls are used by the MathOptInterface.Bridges.AbstractBridgeOptimizer to communicate with the bridge so they should be implemented by the bridge.

source

MathOptInterface.Bridges.AbstractBridgeOptimizer - Type.

AbstractBridgeOptimizer

A bridge optimizer applies given constraint bridges to a given optimizer thus extending the types of supported constraints. The attributes of the inner optimizer are automatically transformed to make the bridges transparent, e.g. the variables and constraints created by the bridges are hidden.

By convention, the inner optimizer should be stored in a model field and the dictionary mapping constraint indices to bridges should be stored in a bridges field. If a bridge optimizer deviates from these conventions, it should implement the functions MOI.optimize! and bridge respectively.

source

MathOptInterface.Bridges.LazyBridgeOptimizer - Type.

| LazyBridgeOptimizer{OT<:MOI.ModelLike} <: AbstractBridgeOptimizer

The LazyBridgeOptimizer combines several bridges, which are added using the add\_bridge function.

Whenever a constraint is added, it only attempts to bridge it if it is not supported by the internal model (hence its name Lazy).

When bridging a constraint, it selects the minimal number of bridges needed.

For example, if a constraint F-in-S can be bridged into a constraint F1-in-S1 (supported by the internal model) using bridge 1 or bridged into a constraint F2-in-S2 (unsupported by the internal model) using bridge 2 which can then be bridged into a constraint F3-in-S3 (supported by the internal model) using bridge 3, it will choose bridge 1 as it allows to bridge F-in-'S using only one bridge instead of two if it uses bridge 2 and 3.

```
MathOptInterface.Bridges.add_bridge - Function.
   add bridge(b::LazyBridgeOptimizer, BT::Type{<:AbstractBridge})
   Enable the use of the bridges of type BT by b.
    source
MathOptInterface.Bridges.remove_bridge - Function.
   remove bridge(b::LazyBridgeOptimizer, BT::Type{<:AbstractBridge})</pre>
   Disable the use of the bridges of type BT by b.
    source
MathOptInterface.Bridges.has bridge - Function.
   has bridge(b::LazyBridgeOptimizer, BT::Type{<:AbstractBridge})</pre>
   Return a Bool indicating whether the bridges of type BT are used by b.
   source
MathOptInterface.Bridges.full_bridge_optimizer - Function.
   full_bridge_optimizer(model::MOI.ModelLike, ::Type{T}) where {T}
   Returns a LazyBridgeOptimizer bridging model for every bridge defined in this package (see below
   for the few exceptions) and for the coefficient type T in addition to the bridges in the list returned by
   MOI.get(model, MOI.Bridges.ListOfNonstandardBridges{T}()).
```

 $See\ also\ List Of Nonstandard Bridges.$ 

#### Note

The following bridges are not added by full\_bridge\_optimizer except if they are in the list returned by MOI.get(model, MOI.Bridges.ListOfNonstandardBridges{T}()) (see the docstrings of the corresponding bridge for the reason they are not added):

- Constraint.SOCtoNonConvexQuadBridge, Constraint.RSOCtoNonConvexQuadBridge and Constraint.SOCtoPSDBridge.
- The subtypes of Constraint. AbstractToIntervalBridge (i.e. Constraint. GreaterToIntervalBridge and Constraint. LessToIntervalBridge) if T is not a subtype of AbstractFloat.

source

 ${\tt MathOptInterface.Bridges.ListOfNonstandardBridges-Type.}$ 

```
| ListOfNonstandardBridges{T}() <: MOI.AbstractOptimizerAttribute
```

Any optimizer can be wrapped in a LazyBridgeOptimizer using full\_bridge\_optimizer. However, by default LazyBridgeOptimizer uses a limited set of bridges that are:

- 1. implemented in MOI.Bridges
- 2. generally applicable for all optimizers.

For some optimizers however, it is useful to add additional bridges, such as those that are implemented in external packages (e.g., within the solver package itself) or only apply in certain circumstances (e.g., Constraint.SOCtoNonConvexQuadBridge).

Such optimizers should implement the ListOfNonstandardBridges attribute to return a vector of bridge types that are added by full\_bridge\_optimizer in addition to the list of default bridges.

Note that optimizers implementing ListOfNonstandardBridges may require package-specific functions or sets to be used if the non-standard bridges are not added. Therefore, you are recommended to use model = MOI.instantiate(Package.Optimizer; with\_bridge\_type = T) instead of model = MOI.instantiate(Package.Optimizer) See MathOptInterface.instantiate.

### **Examples**

### An optimizer using a non-default bridge in MOI.Bridges

Solvers supporting MOI. ScalarQuadraticFunction can support MOI. SecondOrderCone and MOI. RotatedSecondOrderCone by defining:

# An optimizer defining an internal bridge

Suppose an optimizer can exploit specific structure of a constraint, e.g., it can exploit the structure of the matrix A in the linear system of equations A \* x = b.

The optimizer can define the function:

```
struct MatrixAffineFunction{T} <: MOI.AbstractVectorFunction
    A::SomeStructuredMatrixType{T}
    b::Vector{T}
end

and then a bridge

struct MatrixAffineFunctionBridge{T} <: MOI.Constraint.AbstractBridge
    # ...
end
# ...</pre>
```

```
from\ Vector Affine Function \{T\}\ to\ the\ Matrix Affine Function.\ Finally,\ it\ defines:
```

```
function MOI.get(::Optimizer{T}, ::ListOfNonstandardBridges{T}) where {T}
    return Type[MatrixAffineFunctionBridge{T}]
end

source
```

MathOptInterface.Bridges.debug\_supports\_constraint - Function.

```
debug_supports_constraint(
    b::LazyBridgeOptimizer,
    F::Type{<:MOI.AbstractFunction},
    S::Type{<:MOI.AbstractSet};
    io::IO = Base.stdout,
)</pre>
```

Prints to io explanations for the value of MOI.supports\_constraint with the same arguments.

source

MathOptInterface.Bridges.debug supports - Function.

```
debug_supports(
    b::LazyBridgeOptimizer,
    ::MOI.ObjectiveFunction{F};
    io::IO = Base.stdout,
) where F
```

Prints to io explanations for the value of MOI. supports with the same arguments.

source

MathOptInterface.Bridges.bridged\_variable\_function - Function.

```
bridged_variable_function(
    b::AbstractBridgeOptimizer,
    vi::MOI.VariableIndex,
)
```

Return a MOI.AbstractScalarFunction of variables of b.model that equals vi. That is, if the variable vi is bridged, it returns its expression in terms of the variables of b.model. Otherwise, it returns vi.

source

MathOptInterface.Bridges.unbridged\_variable\_function - Function.

```
unbridged_variable_function(
    b::AbstractBridgeOptimizer,
    vi::MOI.VariableIndex,
)
```

Return a MOI.AbstractScalarFunction of variables of b that equals vi. That is, if the variable vi is an internal variable of b.model created by a bridge but not visible to the user, it returns its expression in terms of the variables of bridged variables. Otherwise, it returns vi.

source

 ${\tt MathOptInterface.Bridges.bridged\_function-Function}.$ 

```
| bridged_function(b::AbstractBridgeOptimizer, value)::typeof(value)
```

Substitute any bridged MOI. VariableIndex in value by an equivalent expression in terms of variables of b.model.

source

MathOptInterface.Bridges.Variable.unbridged\_map - Function.

unbridged\_map( bridge::MOI.Bridges.Variable.AbstractBridge, vi::MOI.VariableIndex, )

For a bridged variable in a scalar set, return a tuple of pairs mapping the variables created by the bridge to an affine expression in terms of the bridged variable vi.

```
unbridged_map(
    bridge::MOI.Bridges.Variable.AbstractBridge,
    vis::Vector{MOI.VariableIndex},
)
```

For a bridged variable in a vector set, return a tuple of pairs mapping the variables created by the bridge to an affine expression in terms of the bridged variable vis. If this method is not implemented, it falls back to calling the following method for every variable of vis.

```
unbridged_map(
    bridge::MOI.Bridges.Variable.AbstractBridge,
    vi::MOI.VariableIndex,
    i::MOIB.IndexInVector,
)
```

For a bridged variable in a vector set, return a tuple of pairs mapping the variables created by the bridge to an affine expression in terms of the bridged variable vi corresponding to the ith variable of the vector.

If there is no way to recover the expression in terms of the bridged variable(s) vi(s), return nothing. See ZerosBridge for an example of bridge returning nothing.

source

## **Constraint bridges**

MathOptInterface.Bridges.Constraint.AbstractBridge - Type.

```
AbstractBridge
```

Subtype of MathOptInterface.Bridges.AbstractBridge for constraint bridges.

source

 ${\tt MathOptInterface.Bridges.Constraint.AbstractFunctionConversionBridge-Type.}$ 

```
abstract type AbstractFunctionConversionBridge{F, S} <: AbstractBridge end
```

Bridge a constraint G-in-S into a constraint F-in-S where F and G are equivalent representations of the same function. By convention, the transformed function is stored in the constraint field.

source

MathOptInterface.Bridges.Constraint.SingleBridgeOptimizer - Type.

```
\label{thm:condition} Single Bridge Optimizer \{BT<:AbstractBridge,\ OT<:MOI.ModelLike\} <: AbstractBridge Optimizer
```

The SingleBridgeOptimizer bridges any constraint supported by the bridge BT. This is in contrast with the MathOptInterface.Bridges.LazyBridgeOptimizer which only bridges the constraints that are unsupported by the internal model, even if they are supported by one of its bridges.

source

MathOptInterface.Bridges.Constraint.add\_all\_bridges - Function.

```
add_all_bridges(bridged_model, ::Type{T}) where {T}
```

Add all bridges defined in the Bridges.Constraint submodule to bridged\_model. The coefficient type used is T.

source

**SetMap bridges** MathOptInterface.Bridges.Variable.SetMapBridge - Type.

```
abstract type SetMapBridge{T,S1,S2} <: AbstractBridge end</pre>
```

Consider two type of sets S1, S2 and a linear mapping A that the image of a set of type S1 under A is a set of type S2. A SetMapBridge{T,S1,S2} is a bridge that substitutes constrained variables in S2 into the image through A of constrained variables in S1.

The linear map A is described by MathOptInterface.Bridges.map\_set, MathOptInterface.Bridges.map\_function. Implementing a method for these two functions is sufficient to bridge constrained variables. In order for the getters and setters of dual solutions, starting values, etc... to work as well a method for the following functions should be implemented as well: MathOptInterface.Bridges.inverse\_map\_set, MathOptInterface.Bridges.inverse\_MathOptInterface.Bridges.inverse\_adjoint\_map\_function. See the docstrings of the function to see which feature would be missing it it was not implemented for a given bridge.

source

MathOptInterface.Bridges.Constraint.SetMapBridge - Type.

```
| abstract type SetMapBridge{T,S2,S1,F,G} <: AbstractBridge end
```

Consider two type of sets S1, S2 and a linear mapping A that the image of a set of type S1 under A is a set of type S2. A SetMapBridge{T,S2,S1,F,G} is a bridge that maps G-in-S2 constraints into F-in-S1 by mapping the function through A.

The linear map A is described by MathOptInterface.Bridges.map\_set, MathOptInterface.Bridges.map\_function.

Implementing a method for these two functions is sufficient to bridge constraints. In order for the getters and setters of dual solutions, starting values, etc... to work as well a method for the following functions should be implemented as well: MathOptInterface.Bridges.inverse\_map\_set, MathOptInterface.Bridges.inverse\_map MathOptInterface.Bridges.adjoint\_map\_function and MathOptInterface.Bridges.inverse\_adjoint\_map\_function.

See the docstrings of the function to see which feature would be missing it it was not implemented for a

source

given bridge.

MathOptInterface.Bridges.map set - Function.

```
map_set(::Type{BT}, set) where {BT}
```

Return the image of set through the linear map A defined in Variable. SetMapBridge and Constraint. SetMapBridge. This is used for bridging the constraint and setting the MathOptInterface. ConstraintSet.

source

MathOptInterface.Bridges.inverse\_map\_set - Function.

```
inverse_map_set(::Type{BT}, set) where {BT}
```

Return the preimage of set through the linear map A defined in Variable. SetMapBridge and Constraint. SetMapBridge. This is used for getting the MathOptInterface. ConstraintSet.

SOURCE

MathOptInterface.Bridges.map\_function - Function.

```
map_function(::Type{BT}, func) where {BT}
```

Return the image of func through the linear map A defined in Variable. SetMapBridge and Constraint. SetMapBridge. This is used for getting the MathOptInterface. ConstraintPrimal of variable bridges. For constraint bridges, this is used for bridging the constraint, setting the MathOptInterface. ConstraintFunction and MathOptInterface. ConstraintPrimalStart and modifying the function with MathOptInterface.modify.

```
map_function(::Type{BT}, func, i::IndexInVector) where {BT}
```

Return the scalar function at the ith index of the vector function that would be returned by map\_function (BT, func) except that it may compute the ith element. This is used by bridged\_function and for getting the MathOptInterface.VariablePrimal and MathOptInterface.VariablePrimalStart of variable bridges.

source

MathOptInterface.Bridges.inverse\_map\_function - Function.

```
inverse_map_function(::Type{BT}, func) where {BT}
```

Return the image of func through the inverse of the linear map A defined in Variable.SetMapBridge and Constraint.SetMapBridge. This is used by Variable.unbridged\_map and for setting the MathOptInterface.VariablePrim of variable bridges and for getting the MathOptInterface.ConstraintFunction, the MathOptInterface.ConstraintPrimal and the MathOptInterface.ConstraintPrimalStart of constraint bridges.

source

 ${\tt MathOptInterface.Bridges.adjoint\_map\_function-Function}.$ 

```
adjoint_map_function(::Type{BT}, func) where {BT}
```

Return the image of func through the adjoint of the linear map A defined in Variable.SetMapBridge and Constraint.SetMapBridge. This is used for getting the MathOptInterface.ConstraintDual and MathOptInterface.ConstraintDualStart of constraint bridges.

source

 ${\tt MathOptInterface.Bridges.inverse\_adjoint\_map\_function-Function}.$ 

```
inverse_adjoint_map_function(::Type{BT}, func) where {BT}
```

Return the image of func through the inverse of the adjoint of the linear map A defined in Variable. SetMapBridge and Constraint. SetMapBridge. This is used for getting the MathOptInterface. ConstraintDual of variable bridges and setting the MathOptInterface. ConstraintDualStart of constraint bridges.

source

**Bridges implemented** MathOptInterface.Bridges.Constraint.FlipSignBridge - Type.

```
FlipSignBridge{T, S1, S2, F, G}
```

Bridge a G-in-S1 constraint into an F-in-S2 constraint by multiplying the function by -1 and taking the point reflection of the set across the origin. The flipped F-in-S constraint is stored in the constraint field by convention.

source

MathOptInterface.Bridges.Constraint.AbstractToIntervalBridge - Type.

```
AbstractToIntervalBridge{T, S1, F}
```

Bridge a F-in-Interval constraint into an F-in-Interval {T} constraint where we have either:

- S1 = MOI.GreaterThan{T}
- S1 = MOI.LessThan{T}

The F-in-Interval{T} constraint is stored in the constraint field by convention.

#### Warning

It is required that T be a AbstractFloat type because otherwise typemin and typemax would either be not implemented (e.g. BigInt) or would not give infinite value (e.g. Int). For this reason, this bridge is only added to MathOptInterface.Bridges.full\_bridge\_optimizer. when T is a subtype of AbstractFloat.

source

MathOptInterface.Bridges.Constraint.GreaterToIntervalBridge - Type.

```
GreaterToIntervalBridge{T, F<:MOI.AbstractScalarFunction} <:
    AbstractToIntervalBridge{T, MOI.GreaterThan{T}, F}</pre>
```

Transforms a F-in-GreaterThan $\{T\}$  constraint into an F-in-Interval $\{T\}$  constraint.

source

MathOptInterface.Bridges.Constraint.LessToIntervalBridge - Type.

```
LessToIntervalBridge{T, F<:MOI.AbstractScalarFunction} <:
    AbstractToIntervalBridge{T, MOI.LessThan{T}, F}</pre>
```

Transforms a F-in-LessThan{T} constraint into an F-in-Interval{T} constraint.

source

MathOptInterface.Bridges.Constraint.GreaterToLessBridge - Type.

```
GreaterToLessBridge{
    T,
    F<:MOI.AbstractScalarFunction,
    G<:MOI.AbstractScalarFunction
} <: FlipSignBridge{T, MOI.GreaterThan{T}, MOI.LessThan{T}, F, G}</pre>
```

Transforms a G-in-GreaterThan{T} constraint into an F-in-LessThan{T} constraint.

source

MathOptInterface.Bridges.Constraint.LessToGreaterBridge - Type.

```
LessToGreaterBridge{
        F<:MOI.AbstractScalarFunction,
        G<:MOI.AbstractScalarFunction
    } <: FlipSignBridge{T, MOI.LessThan{T}, MOI.GreaterThan{T}, F, G}</pre>
   Transforms a G-in-LessThan\{T\} constraint into an F-in-GreaterThan\{T\} constraint.
   source
MathOptInterface.Bridges.Constraint.NonnegToNonposBridge - Type.
    NonnegToNonposBridge{
        Τ.
        F<:MOI.AbstractVectorFunction,
        {\sf G}{<}:{\sf MOI.AbstractVectorFunction}
    } <: FlipSignBridge{T, MOI.Nonnegatives, MOI.Nonpositives, F, G}</pre>
   Transforms a G-in-Nonnegatives constraint into a F-in-Nonpositives constraint.
   source
{\tt MathOptInterface.Bridges.Constraint.NonposToNonnegBridge-Type.}
    NonposToNonnegBridge{
        Τ,
        F<:MOI.AbstractVectorFunction.
        G<:MOI.AbstractVectorFunction,</pre>
    } <: FlipSignBridge{T, MOI.Nonpositives, MOI.Nonnegatives, F, G}</pre>
   Transforms a G-in-Nonpositives constraint into a F-in-Nonnegatives constraint.
   source
{\tt MathOptInterface.Bridges.Constraint.VectorizeBridge-Type.}
   VectorizeBridge{T,F,S,G}
   Transforms a constraint G-in-scalar_set_type(S, T) where S <: VectorLinearSet to F-in-S.
   Examples
   The constraint VariableIndex-in-LessThan{Float64} becomes VectorAffineFunction{Float64}-in-Nonpositives,
   where T = Float64, F = VectorAffineFunction\{Float64\}, S = Nonpositives, and G = VariableIndex.
   source
MathOptInterface.Bridges.Constraint.ScalarizeBridge - Type.
   ScalarizeBridge{T, F, S}
   Transforms a constraint AbstractVectorFunction-in-vector set type(S) where S <: LPCone{T} to F-
   in-S.
   source
MathOptInterface.Bridges.Constraint.ScalarSlackBridge - Type.
   ScalarSlackBridge{T, F, S}
```

The ScalarSlackBridge converts a constraint G-in-S where G is a function different from VariableIndex into the constraints F-in-EqualTo{T} and VariableIndex-in-S.

F is the result of subtracting a VariableIndex from G. Typically G is the same as F, but that is not mandatory.

source

MathOptInterface.Bridges.Constraint.VectorSlackBridge - Type.

```
VectorSlackBridge{T, F, S}
```

The VectorSlackBridge converts a constraint G-in-S where G is a function different from VectorOfVariables into the constraints Fin-Zeros and VectorOfVariables-in-S.

F is the result of subtracting a VectorOfVariables from G. Typically G is the same as F, but that is not mandatory.

source

MathOptInterface.Bridges.Constraint.ScalarFunctionizeBridge - Type.

```
ScalarFunctionizeBridge{T, S}
```

 $The Scalar Functionize Bridge \ converts \ a \ constraint \ Variable Index-in-S \ into \ the \ constraint \ Scalar Affine Function \ \{T\}-in-S.$ 

source

MathOptInterface.Bridges.Constraint.VectorFunctionizeBridge - Type.

```
VectorFunctionizeBridge{T, S}
```

The VectorFunctionizeBridge converts a constraint VectorOfVariables-in-S into the constraint  $VectorAffineFunction\{T\}$  in-S.

source

MathOptInterface.Bridges.Constraint.SplitIntervalBridge - Type.

```
SplitIntervalBridge{T, F, S, LS, US}
```

The SplitIntervalBridge splits a F-in-S constraint into a F-in-LS and a F-in-US constraint where we have either:

- S = MOI.Interval{T}, LS = MOI.GreaterThan{T} and US = MOI.LessThan{T},
- S = MOI.EqualTo{T}, LS = MOI.GreaterThan{T} and US = MOI.LessThan{T}, or
- S = MOI.Zeros, LS = MOI.Nonnegatives and US = MOI.Nonpositives.

For instance, if F is MOI. Scalar Affine Function and S is MOI. Interval, it transforms the constraint la, x+u into the constraints a, x+l and a, x+u.

## Note

If T<:AbstractFloat and S is MOI.Interval{T} then no lower (resp. upper) bound constraint is created if the lower (resp. upper) bound is typemin(T) (resp. typemax(T)). Similarly, when MathOptInterface.ConstraintSet is set, a lower or upper bound constraint may be deleted or created accordingly.

source

MathOptInterface.Bridges.Constraint.SOCtoRSOCBridge - Type.

SOCtoRSOCBridge{T, F, G}

We simply do the inverse transformation of RSOCtoSOCBridge. In fact, as the transformation is an involution, we do the same transformation.

source

MathOptInterface.Bridges.Constraint.RSOCtoSOCBridge - Type.

RSOCtoSOCBridge{T, F, G}

The RotatedSecondOrderCone is SecondOrderCone representable; see [BN01, p. 104]. Indeed, we have  $2tu=(t/\sqrt{2}+u/\sqrt{2})^2-(t/\sqrt{2}-u/\sqrt{2})^2$  hence

$$2tu > ||x||_2^2$$

is equivalent to

$$(t/\sqrt{2} + u/\sqrt{2})^2 \ge ||x||_2^2 + (t/\sqrt{2} - u/\sqrt{2})^2.$$

We can therefore use the transformation  $(t,u,x)\mapsto (t/\sqrt{2}+u/\sqrt{2},t/\sqrt{2}-u/\sqrt{2},x)$ . Note that the linear transformation is a symmetric involution (i.e. it is its own transpose and its own inverse). That means in particular that the norm of constraint primal and dual values are preserved by the transformation.

[BN01] Ben-Tal, Aharon, and Nemirovski, Arkadi. Lectures on modern convex optimization: analysis, algorithms, and engineering applications. Society for Industrial and Applied Mathematics, 2001.

source

MathOptInterface.Bridges.Constraint.SOCtoNonConvexQuadBridge - Type.

SOCtoNonConvexQuadBridge{T}

Constraints of the form VectorOfVariables-in-SecondOrderCone can be transformed into a ScalarQuadraticFunction-in-LessThan and a ScalarAffineFunction-in-GreaterThan. Indeed, the definition of the second-order cone

$$t \ge ||x||_2 (1)$$

is equivalent to

$$\sum x_i^2 \le t^2(2)$$

with  $t \geq 0$ . (3)

### Warning

This transformation starts from a convex constraint (1) and creates a non-convex constraint (2), because the Q matrix associated with the constraint (2) has one negative eigenvalue. This might be wrongly interpreted by a solver. Some solvers can look at (2) and understand that it is a second order cone, but this is not a general rule. For these reasons this bridge is not automatically added by MOI.Bridges.full\_bridge\_optimizer. Care is recommended when adding this bridge to a optimizer.

source

MathOptInterface.Bridges.Constraint.RSOCtoNonConvexQuadBridge - Type.

RSOCtoNonConvexQuadBridge{T}

Constraints of the form VectorOfVariables-in-SecondOrderCone can be transformed into a ScalarQuadraticFunction-in-LessThan and a ScalarAffineFunction-in-GreaterThan. Indeed, the definition of the second-order cone

$$2tu \ge ||x||_2^2, t, u \ge 0(1)$$

is equivalent to

$$\sum x_i^2 \le 2tu(2)$$

with  $t,u\geq 0$ . (3)

WARNING This transformation starts from a convex constraint (1) and creates a non-convex constraint (2), because the Q matrix associated with the constraint 2 has two negative eigenvalues. This might be wrongly interpreted by a solver. Some solvers can look at (2) and understand that it is a rotated second order cone, but this is not a general rule. For these reasons, this bridge is not automatically added by MOI.Bridges.full\_bridge\_optimizer. Care is recommended when adding this bridge to an optimizer.

source

MathOptInterface.Bridges.Constraint.QuadtoSOCBridge - Type.

| QuadtoSOCBridge{T}

The set of points x satisfying the constraint

$$\frac{1}{2}x^TQx + a^Tx + b \le 0$$

is a convex set if Q is positive semidefinite and is the union of two convex cones if a and b are zero (i.e. homogeneous case) and Q has only one negative eigenvalue. Currently, only the non-homogeneous transformation is implemented, see the Note section below for more details.

## Non-homogeneous case

If Q is positive semidefinite, there exists U such that  $Q = U^T U$ , the inequality can then be rewritten as

$$||Ux||_2^2 \le 2(-a^Tx - b)$$

which is equivalent to the membership of  $(1, -a^T x - b, Ux)$  to the rotated second-order cone.

### Homogeneous case

If Q has only one negative eigenvalue, the set of x such that  $x^TQx \leq 0$  is the union of a convex cone and its opposite. We can choose which one to model by checking the existence of bounds on variables as shown below.

#### Second-order cone

If Q is diagonal and has eigenvalues (1, 1, -1), the inequality  $x^2+x^2 \le z^2$  combined with  $z \ge 0$  defines the Lorenz cone (i.e. the second-order cone) but when combined with  $z \le 0$ , it gives the opposite of the second order cone. Therefore, we need to check if the variable z has a lower bound 0 or an upper bound 0 in order to determine which cone is

#### Rotated second-order cone

The matrix Q corresponding to the inequality  $x^2 \leq 2yz$  has one eigenvalue 1 with eigenvectors  $(1, \ 0, \ 0)$  and  $(0, \ 1, \ -1)$  and one eigenvalue -1 corresponding to the eigenvector  $(0, \ 1, \ 1)$ . Hence if we intersect this union of two convex cone with the halfspace  $x+y \geq 0$ , we get the rotated second-order cone and if we intersect it with the halfspace  $x+y \leq 0$  we get the opposite of the rotated second-order cone. Note that y and z have the same sign since yz is nonnegative hence  $x+y \geq 0$  is equivalent to  $x \geq 0$  and  $y \geq 0$ .

#### Note

The check for existence of bound can be implemented (but inefficiently) with the current interface but if bound is removed or transformed (e.g.  $\leq 0$  transformed into  $\geq 0$ ) then the bridge is no longer valid. For this reason the homogeneous version of the bridge is not implemented yet.

source

MathOptInterface.Bridges.Constraint.SOCtoPSDBridge - Type.

The SOCtoPSDBridge transforms the second order cone constraint  $\|x\| \leq t$  into the semidefinite cone constraints

$$\begin{pmatrix} t & x^{\top} \\ x & tI \end{pmatrix} \succeq 0$$

Indeed by the Schur Complement, it is positive definite iff

$$tI \succ 0$$
$$t - x^{\top}(tI)^{-1}x \succ 0$$

which is equivalent to

$$t > 0$$
$$t^2 > x^{\top} x$$

## Warning

This bridge is not added by default by MOI.Bridges.full\_bridge\_optimizer as bridging second order cone constraints to semidefinite constraints can be achieved by the SOCtoRSOCBridge followed by the RSOCtoPSDBridge while creating a smaller semidefinite constraint.

source

MathOptInterface.Bridges.Constraint.RSOCtoPSDBridge - Type.

The RSOCtoPSDBridge transforms the second order cone constraint  $\|x\| \leq 2tu$  with  $u \geq 0$  into the semidefinite cone constraints

$$\begin{pmatrix} t & x^\top \\ x & 2uI \end{pmatrix} \succeq 0$$

Indeed by the Schur Complement, it is positive definite iff

$$uI \succ 0$$
$$t - x^{\top} (2uI)^{-1} x \succ 0$$

which is equivalent to

$$u > 0$$
$$2tu > x^{\top}x$$

source

MathOptInterface.Bridges.Constraint.NormInfinityBridge - Type.

NormInfinityBridge{T}

The NormInfinityCone is representable with LP constraints, since  $t \ge \max_i |x_i|$  if and only if  $t \ge x_i$  and  $t \ge -x_i$  for all i.

source

MathOptInterface.Bridges.Constraint.NormOneBridge - Type.

|NormOneBridge{T}

The NormOneCone is representable with LP constraints, since  $t \geq \sum_i |x_i|$  if and only if there exists a vector y such that  $t \geq \sum_i y_i$  and  $y_i \geq x_i$ ,  $y_i \geq -x_i$  for all i.

source

MathOptInterface.Bridges.Constraint.GeoMeantoRelEntrBridge - Type.

GeoMeantoRelEntrBridge{T}

The geometric mean cone is representable with a relative entropy constraint and a nonnegative auxiliary variable.

This is because  $u \leq \prod_{i=1}^n w_i^{1/n}$  is equivalent to  $y \geq 0$  and  $0 \leq u+y \leq \prod_{i=1}^n w_i^{1/n}$ , and the latter inequality is equivalent to  $1 \leq \prod_{i=1}^n (\frac{w_i}{u+y})^{1/n}$ , which is equivalent to  $0 \leq \sum_{i=1}^n \log(\frac{w_i}{u+y})^{1/n}$ , which is equivalent to  $0 \geq \sum_{i=1}^n (u+y) \log(\frac{u+y}{w_i})$ .

Thus  $(u, w) \in GeometricMeanCone(1+n)$  is representable as  $y \ge 0$ ,  $(0, w, (u+y)e) \in RelativeEntropyCone(1+2n)$ , where e is a vector of ones.

MathOptInterface.Bridges.Constraint.GeoMeanBridge - Type.

GeoMeanBridge{T, F, G, H}

The GeometricMeanCone is SecondOrderCone representable; see [1, p. 105].

The reformulation is best described in an example.

Consider the cone of dimension 4:

$$t \le \sqrt[3]{x_1 x_2 x_3}$$

This can be rewritten as  $\exists x_{21} \geq 0$  such that:

$$t \le x_{21},$$
  
$$x_{21}^4 \le x_1 x_2 x_3 x_{21}.$$

Note that we need to create  $x_{21}$  and not use  $t^4$  directly as t is allowed to be negative. Now, this is equivalent to:

$$t \le x_{21}/\sqrt{4}$$
,  
 $x_{21}^2 \le 2x_{11}x_{12}$ ,  
 $x_{11}^2 \le 2x_1x_2$ ,  $x_{12}^2 \le 2x_3(x_{21}/\sqrt{4})$ .

[1] Ben-Tal, Aharon, and Arkadi Nemirovski. Lectures on modern convex optimization: analysis, algorithms, and engineering applications. Society for Industrial and Applied Mathematics, 2001.

source

MathOptInterface.Bridges.Constraint.RelativeEntropyBridge - Type.

RelativeEntropyBridge{T}

The RelativeEntropyCone is representable with exponential cone and LP constraints, since  $u \geq \sum_{i=1}^n w_i \log(\frac{w_i}{v_i})$  if and only if there exists a vector y such that  $u \geq \sum_i y_i$  and  $y_i \geq w_i \log(\frac{w_i}{v_i})$  or equivalently  $v_i \geq w_i \exp(\frac{-y_i}{w_i})$  or equivalently  $(-y_i, w_i, v_i) \in ExponentialCone$ , for all i.

source

MathOptInterface.Bridges.Constraint.NormSpectralBridge - Type.

NormSpectralBridge{T}

The NormSpect ralCone is representable with a PSD constraint, since  $t \geq \sigma_1(X)$  if and only if  $[tIX^\top; XtI] \succ 0$ .

source

MathOptInterface.Bridges.Constraint.NormNuclearBridge - Type.

|NormNuclearBridge{T}

The NormNuclearCone is representable with an SDP constraint and extra variables, since  $t \geq \sum_i \sigma_i(X)$  if and only if there exists symmetric matrices U, V such that  $[UX^\top; XV] \succ 0$  and  $t \geq (tr(U) + tr(V))/2$ .

source

MathOptInterface.Bridges.Constraint.SquareBridge - Type.

The SquareBridge reformulates the constraint of a square matrix to be in ST to a list of equality constraints for pair or off-diagonal entries with different expressions and a TT constraint the upper triangular part of the matrix.

For instance, the constraint for the matrix

$$\begin{pmatrix} 1 & 1+x & 2-3x \\ 1+x & 2+x & 3-x \\ 2-3x & 2+x & 2x \end{pmatrix}$$

to be PSD can be broken down to the constraint of the symmetric matrix

$$\begin{pmatrix} 1 & 1+x & 2-3x \\ \cdot & 2+x & 3-x \\ \cdot & \cdot & 2x \end{pmatrix}$$

and the equality constraint between the off-diagonal entries (2, 3) and (3, 2) 2x == 1. Note that now symmetrization constraint need to be added between the off-diagonal entries (1, 2) and (2, 1) or between (1, 3) and (3, 1) since the expressions are the same.

source

MathOptInterface.Bridges.Constraint.RootDetBridge - Type.

```
RootDetBridge{T,F,G,H}
```

The RootDetConeTriangle is representable by a PositiveSemidefiniteConeTriangle and an GeometricMeanCone constraints; see [1, p. 149].

Indeed,  $t \leq \det(X)^{1/n}$  if and only if there exists a lower triangular matrix such that:

$$\begin{pmatrix} X \\ \top & \text{Diag}() \end{pmatrix} \succeq 0$$
$$t \le ({}_{1122} \cdots {}_{nn})^{1/n}$$

[1] Ben-Tal, Aharon, and Arkadi Nemirovski. Lectures on modern convex optimization: analysis, algorithms, and engineering applications. Society for Industrial and Applied Mathematics, 2001.

LogDetBridge{T,F,G,H,I}

 $The \ LogDet Cone Triangle is \ representable \ by \ a \ Positive Semidefinite Cone Triangle \ and \ Exponential Cone \ constraints.$ 

Indeed,  $\log \det(X) = \log(\delta_1) + \cdots + \log(\delta_n)$  where  $\delta_1, ..., \delta_n$  are the eigenvalues of X.

Adapting the method from [1, p. 149], we see that  $t \leq u \log(\det(X/u))$  for u > 0 if and only if there exists a lower triangular matrix such that

$$\begin{pmatrix} X \\ \top & \text{Diag}() \end{pmatrix} \succeq 0$$
$$t \le u \log_{(11}/u) + u \log_{(22}/u) + \dots + u \log_{(nn}/u)$$

[1] Ben-Tal, Aharon, and Arkadi Nemirovski. Lectures on modern convex optimization: analysis, algorithms, and engineering applications. Society for Industrial and Applied Mathematics, 2001. "'

source

MathOptInterface.Bridges.Constraint.IndicatorActiveOnFalseBridge - Type.

IndicatorActiveOnFalseBridge{T}

The IndicatorActiveOnFalseBridge replaces an indicator constraint activated on 0 with a variable  $z_0$  with the constraint activated on 1, with a variable  $z_1$ . It stores the added variable and added constraints:

- $z_1 \in \mathbb{B}$  in zero\_one\_cons
- $z_0+z_1==1$  in 'indisjunction\_cons'
- The added ACTIVATE\_ON\_ONE indicator constraint in indicator\_cons\_index.

source

MathOptInterface.Bridges.Constraint.IndicatorSOS1Bridge - Type.

IndicatorSOS1Bridge{T,S<:MOI.AbstractScalarSet}</pre>

The IndicatorS0S1Bridge replaces an indicator constraint of the following form:  $z \in \mathbb{B}, z == 1 \implies f(x) \in S$  with a SOS1 constraint:  $z \in \mathbb{B}, slack$  free,  $f(x) + slack \in S, SOS1(slack, z)$ .

source

 ${\tt MathOptInterface.Bridges.Constraint.SemiToBinaryBridge-Type.}$ 

| SemiToBinaryBridge{T, S <: MOI.AbstractScalarSet}

The SemiToBinaryBridge replaces a Semicontinuous constraint:  $x \in$  Semicontinuous(l,u) is replaced by:  $z \in \{0,1\}$ ,  $x \le z \cdot u$ ,  $x \ge z \cdot l$ .

The SemiToBinaryBridge replaces a Semiinteger constraint:  $x \in Semiinteger(l,u)$  is replaced by:  $z \in \{0,1\}$ ,  $x \in \mathbb{Z}$ ,  $x \leq z \cdot u$ ,  $x \geq z \cdot l$ .

source

MathOptInterface.Bridges.Constraint.ZeroOneBridge - Type.

|ZeroOneBridge{T}

The ZeroOneBridge splits a MOI.VariableIndex-in-MOI.ZeroOne constraint into a MOI.VariableIndex-in-MOI.Integer constraint and a MOI.VariableIndex-in-MOI.Interval(0, 1) constraint.

## Variable bridges

MathOptInterface.Bridges.Variable.AbstractBridge - Type.

```
AbstractBridge
```

Subtype of MathOptInterface.Bridges.AbstractBridge for variable bridges.

source

MathOptInterface.Bridges.Variable.SingleBridgeOptimizer - Type.

```
SingleBridgeOptimizer{BT<:AbstractBridge, OT<:MOI.ModelLike} <:
AbstractBridgeOptimizer</pre>
```

The SingleBridgeOptimizer bridges any constrained variables supported by the bridge BT. This is in contrast with the MathOptInterface.Bridges.LazyBridgeOptimizer which only bridges the constrained variables that are unsupported by the internal model, even if they are supported by one of its bridges.

### Note

Two bridge optimizers using variable bridges cannot be used together as both of them assume that the underlying model only returns variable indices with nonnegative values.

source

MathOptInterface.Bridges.Variable.add all bridges - Function.

```
add all bridges(bridged model, :: Type{T}) where {T}
```

Add all bridges defined in the Bridges. Variable submodule to bridged\_model. The coefficient type used is T.

source

**Bridges implemented** MathOptInterface.Bridges.Variable.FlipSignBridge - Type.

```
FlipSignBridge{T, S1, S2}
```

Bridge constrained variables in S1 into constrained variables in S2 by multiplying the variables by -1 and taking the point reflection of the set across the origin. The flipped MOI.VectorOfVariables-in-S constraint is stored in the flipped\_constraint field by convention.

source

MathOptInterface.Bridges.Variable.ZerosBridge - Type.

```
| ZerosBridge{T} <: Bridges.Variable.AbstractBridge
```

Transforms constrained variables in MathOptInterface.Zeros to zeros, which ends up creating no variables in the underlying model.

The bridged variables are therefore similar to parameters with zero values. Parameters with non-zero value can be created with constrained variables in MOI. EqualTo by combining a VectorizeBridge and this bridge. The functions cannot be unbridged, given a function, we cannot determine, if the bridged variables were used.

The dual values cannot be determined by the bridge but they can be determined by the bridged optimizer using MathOptInterface.Utilities.get\_fallback if a CachingOptimizer is used (since ConstraintFunction cannot be got as functions cannot be unbridged).

```
{\tt MathOptInterface.Bridges.Variable.FreeBridge-Type.}
```

```
FreeBridge{T} <: Bridges.Variable.AbstractBridge</pre>
```

Transforms constrained variables in MOI. Reals to the difference of constrained variables in MOI. Nonnegatives.

source

MathOptInterface.Bridges.Variable.NonposToNonnegBridge - Type.

```
NonposToNonnegBridge{T} <:
FlipSignBridge{T, MOI.Nonpositives, MOI.Nonnegatives}
```

Transforms constrained variables in Nonpositives into constrained variables in Nonnegatives.

source

MathOptInterface.Bridges.Variable.VectorizeBridge - Type.

```
VectorizeBridge{T, S}
```

Transforms a constrained variable in scalar\_set\_type(S, T) where S <: VectorLinearSet into a constrained vector of one variable in S. For instance, VectorizeBridge{Float64, MOI.Nonnegatives} transforms a constrained variable in MOI.GreaterThan{Float64} into a constrained vector of one variable in MOI.Nonnegatives.

source

MathOptInterface.Bridges.Variable.SOCtoRSOCBridge - Type.

Same transformation as MOI.Bridges.Constraint.SOCtoRSOCBridge.

source

MathOptInterface.Bridges.Variable.RSOCtoSOCBridge - Type.

```
RSOCtoSOCBridge{T} <:

← Bridges.Variable.SetMapBridge{T,MOI.SecondOrderCone,MOI.RotatedSecondOrderCone}
```

Same transformation as MOI.Bridges.Constraint.RSOCtoSOCBridge.

source

MathOptInterface.Bridges.Variable.RSOCtoPSDBridge - Type.

```
RSOCtoPSDBridge{T} <: Bridges.Variable.AbstractBridge
```

Transforms constrained variables in MathOptInterface.RotatedSecondOrderCone to constrained variables in MathOptInterface.PositiveSemidefiniteConeTriangle.

## **Objective bridges**

MathOptInterface.Bridges.Objective.AbstractBridge - Type.

AbstractBridge

Subtype of MathOptInterface.Bridges.AbstractBridge for objective bridges.

source

 ${\tt MathOptInterface.Bridges.Objective.SingleBridgeOptimizer-Type.}$ 

```
| SingleBridgeOptimizer{BT<:AbstractBridge, OT<:MOI.ModelLike} <: AbstractBridgeOptimizer
```

The SingleBridgeOptimizer bridges any objective functions supported by the bridge BT. This is in contrast with the MathOptInterface.Bridges.LazyBridgeOptimizer which only bridges the objective functions that are unsupported by the internal model, even if they are supported by one of its bridges.

source

MathOptInterface.Bridges.Objective.add\_all\_bridges - Function.

```
add_all_bridges(bridged_model, ::Type{T}) where {T}
```

Add all bridges defined in the Bridges.Objective submodule to bridged\_model. The coefficient type used is T.

source

**Bridges implemented** MathOptInterface.Bridges.Objective.SlackBridge - Type.

```
SlackBridge{T, F, G}
```

The SlackBridge converts an objective function of type G into a MOI.VariableIndex objective by creating a slack variable and a F-in-MOI.LessThan constraint for minimization or F-in-MOI.LessThan constraint for maximization where F is MOI.Utilities.promote\_operation(-, T, G, MOI.VariableIndex}. Note that when using this bridge, changing the optimization sense is not supported. Set the sense to MOI.FEASIBILITY\_SENSE first to delete the bridge in order to change the sense, then re-add the objective.

source

MathOptInterface.Bridges.Objective.FunctionizeBridge - Type.

```
FunctionizeBridge{T}
```

The FunctionizeBridge converts a VariableIndex objective into a ScalarAffineFunction{T} objective.

# **Chapter 28**

# **FileFormats**

## 28.1 Overview

### The FileFormats submodule

The FileFormats module provides functionality for reading and writing MOI models using write\_to\_file and read\_from\_file.

### Supported file types

You must read and write files to a FileFormats. Model object. Specifc the file-type by passing a FileFormats. FileFormat enum. For example:

## The Conic Benchmark Format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_CBF)
A Conic Benchmark Format (CBF) model
```

## The LP file format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_LP)
A .LP-file model
```

# The MathOptFormat file format

```
| julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MOF)
| A MathOptFormat Model
```

### The MPS file format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MPS)
A Mathematical Programming System (MPS) model
```

# The NL file format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_NL)
An AMPL (.nl) model
```

## The SDPA file format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_SDPA)
A SemiDefinite Programming Algorithm Format (SDPA) model
```

### Write to file

To write a model src to a MathOptFormat file, use:

```
julia> src = MOI.Utilities.Model{Float64}()
MOIU.Model{Float64}
julia> MOI.add_variable(src)
MathOptInterface.VariableIndex(1)
julia> dest = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MOF)
A MathOptFormat Model
julia> MOI.copy_to(dest, src)
MathOptInterface.Utilities.IndexMap with 1 entry:
 VariableIndex(1) => VariableIndex(1)
julia> MOI.write_to_file(dest, "file.mof.json")
julia> print(read("file.mof.json", String))
 "name": "MathOptFormat Model",
 "version": {
   "major": 1,
   "minor": 0
 },
  "variables": [
   {
      "name": "x1"
   }
 ],
 "objective": {
   "sense": "feasibility"
  "constraints": []
```

## Read from file

To read a MathOptFormat file, use:

```
julia> dest = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MOF)
A MathOptFormat Model

julia> MOI.read_from_file(dest, "file.mof.json")

julia> MOI.get(dest, MOI.ListOfVariableIndices())
1-element Vector{MathOptInterface.VariableIndex}:
    MathOptInterface.VariableIndex(1)

julia> rm("file.mof.json") # Clean up after ourselves.
```

## **Detecting the filetype automatically**

Instead of the format keyword, you can also use the filename keyword argument to FileFormats. Model. This will attempt to automatically guess the format from the file extension. For example:

```
julia> src = MOI.Utilities.Model{Float64}()
MOIU.Model{Float64}
julia> dest = MOI.FileFormats.Model(filename = "file.cbf.gz")
A Conic Benchmark Format (CBF) model
julia> MOI.copy_to(dest, src)
MathOptInterface.Utilities.IndexMap()
julia> MOI.write_to_file(dest, "file.cbf.gz")
julia> src_2 = MOI.FileFormats.Model(filename = "file.cbf.gz")
A Conic Benchmark Format (CBF) model
julia> src = MOI.Utilities.Model{Float64}()
MOIU.Model{Float64}
julia> dest = MOI.FileFormats.Model(filename = "file.cbf.gz")
A Conic Benchmark Format (CBF) model
julia> MOI.copy_to(dest, src)
MathOptInterface.Utilities.IndexMap()
julia> MOI.write_to_file(dest, "file.cbf.gz")
julia> src_2 = MOI.FileFormats.Model(filename = "file.cbf.gz")
A Conic Benchmark Format (CBF) model
julia> MOI.read_from_file(src_2, "file.cbf.gz")
julia> rm("file.cbf.gz") # Clean up after ourselves.
```

Note how the compression format (GZip) is also automatically detected from the filename.

### **Unsupported constraints**

In some cases src may contain constraints that are not supported by the file format (e.g., the CBF format supports integer variables but not binary). If so, copy src to a bridged model using Bridges.full\_bridge\_optimizer:

```
src = MOI.Utilities.Model{Float64}()
x = MOI.add_variable(model)
MOI.add_constraint(model, x, MOI.ZeroOne())
dest = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_CBF)
bridged = MOI.Bridges.full_bridge_optimizer(dest, Float64)
MOI.copy_to(bridged, src)
MOI.write_to_file(dest, "my_model.cbf")
```

#### Note

Even after bridging, it may still not be possible to write the model to file because of unsupported constraints (e.g., PSD variables in the LP file format).

### Read and write to io

In addition to write\_to\_file and read\_from\_file, you can read and write directly from IO streams using Base.write and Base.read!:

```
julia> src = MOI.Utilities.Model{Float64}()
MOIU.Model{Float64}

julia> dest = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MPS)
A Mathematical Programming System (MPS) model

julia> MOI.copy_to(dest, src)
MathOptInterface.Utilities.IndexMap()

julia> io = IOBuffer();

julia> write(io, dest)

julia> seekstart(io);

julia> src_2 = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MPS)
A Mathematical Programming System (MPS) model

julia> read!(io, src_2);
```

## Validating MOF files

MathOptFormat files are governed by a schema. Use JSONSchema.jl to check if a .mof.json file satisfies the schema.

First, construct the schema object as follows:

```
julia> import JSON, JSONSchema

julia> schema = JSONSchema.Schema(JSON.parsefile(MOI.FileFormats.MOF.SCHEMA_PATH))
A JSONSchema
```

Then, check if a model file is valid using isvalid:

If we construct an invalid file, for example by mis-typing name as NaMe, the validation fails:

Use JSONSchema.validate to obtain more insight into why the validation failed:

```
julia> JSONSchema.validate(schema, bad_model)
Validation failed:
path:        [variables][1]
instance:        Dict{String, Any}("NaMe" => "x")
schema key:        required
schema value: Any["name"]
```

## 28.2 API Reference

## **File Formats**

Functions to help read and write MOI models to/from various file formats. See The FileFormats submodule for more details.

MathOptInterface.FileFormats.Model - Function.

```
Model(
    ;
    format::FileFormat = FORMAT_AUTOMATIC,
    filename::Union{Nothing, String} = nothing,
    kwargs...
)
```

Return model corresponding to the FileFormat format, or, if format == FORMAT\_AUTOMATIC, guess the format from filename.

The filename argument is only needed if format == FORMAT\_AUTOMATIC.

kwargs are passed to the underlying model constructor.

source

MathOptInterface.FileFormats.FileFormat - Type.

```
FileFormat
```

List of accepted export formats.

• FORMAT\_AUTOMATIC: try to detect the file format based on the file name

- FORMAT\_CBF: the Conic Benchmark format
- FORMAT\_LP: the LP file format
- FORMAT\_MOF: the MathOptFormat file format
- FORMAT\_MPS: the MPS file format
- FORMAT\_NL: the AMPL .nl file format
- FORMAT\_SDPA: the SemiDefinite Programming Algorithm format

# **Chapter 29**

# **Utilities**

## 29.1 Overview

### The Utilities submodule

The Utilities submodule provides a variety of functionality for managing MOI. ModelLike objects.

#### **Utilities.Model**

Utilities.Model provides an implementation of a ModelLike that efficiently supports all functions and sets defined within MOI. However, given the extensibility of MOI, this might not cover all use cases.

Create a model as follows:

```
julia> model = MOI.Utilities.Model{Float64}()
MOIU.Model{Float64}
```

## **Utilities.UniversalFallback**

Utilities.UniversalFallback is a layer that sits on top of any ModelLike and provides non-specialized (slower) fallbacks for constraints and attributes that the underlying ModelLike does not support.

For example, Utilities.Model doesn't support some variable attributes like VariablePrimalStart, so JuMP uses a combination of Universal fallback and Utilities.Model as a generic problem cache:

```
julia> model = MOI.Utilities.UniversalFallback(MOI.Utilities.Model{Float64}())
MOIU.UniversalFallback{MOIU.Model{Float64}}
fallback for MOIU.Model{Float64}
```

## Warning

Adding a UniversalFallback means that your model will now support all constraints, even if the inner-model does not! This can lead to unexpected behavior.

## Utilities.@model

For advanced use cases that need efficient support for functions and sets defined outside of MOI (but still known at compile time), we provide the Utilities.@model macro.

The @model macro takes a name (for a new type, which must not exist yet), eight tuples specifying the types of constraints that are supported, and then a Bool indicating the type is a subtype of MOI.AbstractOptimizer (if true) or MOI.ModelLike (if false).

The eight tuples are in the following order:

- 1. Un-typed scalar sets, e.g., Integer
- 2. Typed scalar sets, e.g., LessThan
- 3. Un-typed vector sets, e.g., Nonnegatives
- 4. Typed vector sets, e.g., PowerCone
- 5. Un-typed scalar functions, e.g., VariableIndex
- 6. Typed scalar functions, e.g., ScalarAffineFunction
- 7. Un-typed vector functions, e.g., VectorOfVariables
- 8. Typed vector functions, e.g., VectorAffineFunction

The tuples can contain more than one element. Typed-sets must be specified without their type parameter, i.e., MOI.LessThan, not MOI.LessThan{Float64}.

Here is an example:

```
julia> MOI.Utilities.@model(
                                       MyNewModel,
                                                                                                                                          # Un-typed scalar sets
# Typed scalar sets
# Un-typed vector sets
                                        (MOI.Integer,),
                                       (MOI.GreaterThan,),
                                       (MOI.Nonnegatives,),
                                       (MOI.PowerCone,),
                                                                                                                                                      # Typed vector sets
                                       (MOI.PowerCone,),  # Typed vector sets
(MOI.VariableIndex,),  # Un-typed scalar functions
                                       ({\tt MOI.ScalarAffineFunction,)}, \qquad {\tt\# Typed \ scalar \ functions}
                                       (MOI.VectorOfVariables,),
                                                                                                                                                     # Un-typed vector functions
                                       (MOI.VectorAffineFunction,), # Typed vector functions
                                                                                                                                                        # <:MOI.AbstractOptimizer?</pre>
                                       true.
\label{lem:mathOptInterface.Utilities.GenericOptimizer T, MathOptInterface.Utilities.Objective Container T, MathOptInterface.Utilities.Objective C, MathOptInterface
\quad \hookrightarrow \quad \mathsf{MathOptInterface}. \\ \mathsf{Utilities}. \\ \mathsf{VariablesContainer} \\ \mathsf{T} \}, \ \mathsf{MyNewModelFunctionConstraints} \\ \mathsf{T} \} \} \ \mathsf{where} \ \mathsf{T} 
julia> model = MyNewModel{Float64}()
MOIU.GenericOptimizer{Float64, MOIU.ObjectiveContainer{Float64}, MOIU.VariablesContainer{Float64},
```

### Warning

MyNewModel supports every VariableIndex-in-Set constraint, as well as VariableIndex, ScalarAffineFunction, and ScalarQuadraticFunction objective functions. Implement MOI.supports as needed to forbid constraint and objective function combinations.

As another example, PATHSolver, which only supports VectorAffineFunction-in-Complements defines its optimizer as:

However, PathOptimizer does not support some VariableIndex-in-Set constraints, so we must explicitly define:

Finally, PATH doesn't support an objective function, so we need to add:

```
julia> MOI.supports(::PathOptimizer, ::MOI.ObjectiveFunction) = false
```

### Warning

This macro creates a new type, so it must be called from the top-level of a module, e.g., it cannot be called from inside a function.

# **Utilities.CachingOptimizer**

A [Utilities.CachingOptimizer] is an MOI layer that abstracts the difference between solvers that support incremental modification (e.g., they support adding variables one-by-one), and solvers that require the entire problem in a single API call (e.g., they only accept the A, b and c matrices of a linear program).

It has two parts:

- 1. A cache, where the model can be built and modified incrementally
- 2. An optimizer, which is used to solve the problem

A Utilities.CachingOptimizer may be in one of three possible states:

- NO OPTIMIZER: The CachingOptimizer does not have any optimizer.
- EMPTY\_OPTIMIZER: The CachingOptimizer has an empty optimizer, and it is not synchronized with the cached model. Modifications are forwarded to the cache, but not to the optimizer.
- ATTACHED\_OPTIMIZER: The CachingOptimizer has an optimizer, and it is synchronized with the cached model. Modifications are forwarded to the optimizer. If the optimizer does not support modifications, and error will be thrown.

Use Utilities.attach optimizer to go from EMPTY OPTIMIZER to ATTACHED OPTIMIZER:

# Info

You must be in ATTACHED\_OPTIMIZER to use optimize!.

Use Utilities.reset\_optimizer to go from ATTACHED\_OPTIMIZER to EMPTY\_OPTIMIZER:

#### Info

Calling MOI.empty! (model) also resets the state to EMPTY\_OPTIMIZER. So after emptying a model, the modification will only be applied to the cache.

Use Utilities.drop\_optimizer to go from any state to NO\_OPTIMIZER:

Pass an empty optimizer to Utilities.reset\_optimizer to go from NO\_OPTIMIZER to EMPTY\_OPTIMIZER:

Deciding when to attach and reset the optimizer is tedious, and you will often write code like this:

```
try
    # modification
catch
    MOI.Utilities.reset_optimizer(model)
    # Re-try modification
end
```

To make this easier, Utilities.CachingOptimizer has two modes of operation:

- AUTOMATIC: The CachingOptimizer changes its state when necessary. Attempting to add a constraint or perform a modification not supported by the optimizer results in a drop to EMPTY\_OPTIMIZER mode.
- MANUAL: The user must change the state of the CachingOptimizer. Attempting to perform an operation in the incorrect state results in an error.

By default, AUTOMATIC mode is chosen. However, you can create a CachingOptimizer in MANUAL mode as follows:

### **Printing**

Use print to print the formulation of the model.

Use Utilities.latex\_formulation to display the model in LaTeX form:

```
julia> MOI.Utilities.latex_formulation(model)

$$ \begin{aligned}
\max\quad & x\_var \\
\text{Subject to}\\
 & \text{VariableIndex-in-ZeroOne} \\
 & x\_var \in \{0, 1\} \\
\end{aligned} $$
```

In IJulia, calling print or ending a cell with Utilities.latex\_formulation will render the model in LaTeX.

#### **Utilities.MatrixOfConstraints**

The constraints of Utilities. Model are stored as a vector of tuples of function and set in a Utilities. VectorOfConstraints. Other representations can be used by parametrizing the type Utilities. GenericModel (resp. Utilities. GenericOptimizer). For instance, if all non-VariableIndex constraints are affine, the coefficients of all the constraints can be stored in a single sparse matrix using Utilities. MatrixOfConstraints. The constraints storage can even be customized up to a point where it exactly matches the storage of the solver of interest, in which case copy\_to can be implemented for the solver by calling copy\_to to this custom model.

For instance, Clp defines the following model

```
MOI.Utilities.@product_of_scalar_sets(LP, MOI.EqualTo{T}, MOI.LessThan{T}, MOI.GreaterThan{T})
const Model = MOI.Utilities.GenericModel{
    Float64,
    MOI.Utilities.MatrixOfConstraints{
        Float64,
        MOI.Utilities.MutableSparseMatrixCSC{Float64,Cint,MOI.Utilities.ZeroBasedIndexing},
        MOI.Utilities.Hyperrectangle{Float64},
        LP{Float64},
    },
}
```

The copy\_to operation can now be implemented as follows (assuming that the Model definition above is in the Clp module so that it can be referred to as Model, to be distinguished with Utilities.Model):

```
function _copy_to(dest::Optimizer, src::Model)
   @assert MOI.is_empty(dest)
    A = src.constraints.coefficients
    row bounds = src.constraints.constants
    Clp_loadProblem(
        dest,
        A.n.
        A.m,
        A.colptr,
        A.rowval,
        A.nzval,
        src.lower_bound,
        src.upper_bound,
        # (...) objective vector (omitted),
        row_bounds.lower,
        row bounds.upper,
    # Set objective sense and constant (omitted)
    return
end
function MOI.copy_to(dest::Optimizer, src::Model)
    _copy_to(dest, src)
    return MOI.Utilities.identity_index_map(src)
end
function MOI.copy_to(
```

```
dest::Optimizer,
    src::MOI.Utilities.UniversalFallback{Model},
)
    # Copy attributes from `src` to `dest` and error in case any unsupported
    # constraints or attributes are set in `UniversalFallback`.
    return MOI.copy_to(dest, src.model)
end

function MOI.copy_to(
    dest::Optimizer,
    src::MOI.ModelLike,
)
    model = Model()
    index_map = MOI.copy_to(model, src)
    _copy_to(dest, model)
    return index_map
end
```

#### ModelFilter

Utilities provides Utilities.ModelFilter as a useful tool to copy a subset of a model. For example, given an infeasible model, we can copy the irreducible infeasible subsystem (for models implementing ConstraintConflictStatus) as follows:

```
my_filter(::Any) = true
function my_filter(ci::MOI.ConstraintIndex)
    status = MOI.get(dest, MOI.ConstraintConflictStatus(), ci)
    return status != MOI.NOT_IN_CONFLICT
end
filtered_src = MOI.Utilities.ModelFilter(my_filter, src)
index_map = MOI.copy_to(dest, filtered_src)
```

#### **Fallbacks**

The value of some attributes can be inferred from the value of other attributes.

For example, the value of ObjectiveValue can be computed using ObjectiveFunction and VariablePrimal.

When a solver gives direct access to an attribute, it is better to return this value. However, if this is not the case, Utilities.get\_fallback can be used instead. For example:

```
function MOI.get(model::Optimizer, attr::MOI.ObjectiveFunction)
    return MOI.Utilities.get_fallback(model, attr)
end
```

## **DoubleDicts**

When writing MOI interfaces, we often need to handle situations in which we map ConstraintIndexs to different values. For example, to a string for ConstraintName.

One option is to use a dictionary like Dict{MOI.ConstraintIndex,String}. However, this incurs a performance cost because the key is not a concrete type.

The DoubleDicts submodule helps this situation by providing two types main types Utilities.DoubleDicts.DoubleDict and Utilities.DoubleDicts.IndexDoubleDict. These types act like normal dictionaries, but internally they use more efficient dictionaries specialized to the type of the function-set pair.

The most common usage of a DoubleDict is in the index\_map returned by copy\_to. Performance can be improved, by using a function barrier. That is, instead of code like:

```
index_map = MOI.copy_to(dest, src)
for (F, S) in MOI.get(src, MOI.ListOfConstraintTypesPresent())
    for ci in MOI.get(src, MOI.ListOfConstraintIndices{F,S}())
        dest_ci = index_map[ci]
        # ...
    end
end
```

use instead:

```
function function_barrier(
    dest,
    src,
    index_map::MOI.Utilities.DoubleDicts.IndexDoubleDictInner{F,S},
) where {F,S}
    for ci in MOI.get(src, MOI.ListOfConstraintIndices{F,S}())
        dest_ci = index_map[ci]
        # ...
    end
    return
end

index_map = MOI.copy_to(dest, src)
for (F, S) in MOI.get(src, MOI.ListOfConstraintTypesPresent())
    function_barrier(dest, src, index_map[F, S])
end
```

# 29.2 API Reference

# **Utilities.Model**

MathOptInterface.Utilities.Model - Type.

An implementation of ModelLike that supports all functions and sets defined in MOI. It is parameterized by the coefficient type.

# **Examples**

```
model = Model{Float64}()
x = add_variable(model)
source
```

# **Utilities.UniversalFallback**

MathOptInterface.Utilities.UniversalFallback - Type.

```
UniversalFallback
```

The UniversalFallback can be applied on a MathOptInterface.ModelLike model to create the model UniversalFallback(model) supporting any constraint and attribute. This allows to have a specialized

implementation in model for performance critical constraints and attributes while still supporting other attributes with a small performance penalty. Note that model is unaware of constraints and attributes stored by UniversalFallback so this is not appropriate if model is an optimizer (for this reason, MathOptInterface.optimize! has not been implemented). In that case, optimizer bridges should be used instead.

source

#### Utilities.@model

MathOptInterface.Utilities.@model - Macro.

```
macro model(
    model_name,
    scalar_sets,
    typed_scalar_sets,
    vector_sets,
    typed_vector_sets,
    scalar_functions,
    typed_scalar_functions,
    vector_functions,
    typed_vector_functions,
    is_optimizer = false
)
```

Creates a type model\_name implementing the MOI model interface and containing scalar\_sets scalar sets typed\_scalar\_sets typed scalar sets, vector\_sets vector sets, typed\_vector\_sets typed vector sets, scalar\_functions scalar functions, typed\_scalar\_functions typed scalar functions, vector\_functions vector functions and typed\_vector\_functions typed vector functions. To give no set/function, write (), to give one set S, write (S,).

The function MathOptInterface.VariableIndex should not be given in scalar\_functions. The model supports MathOptInterface.VariableIndex-in-S constraints where S is MathOptInterface.EqualTo, MathOptInterface.Gm MathOptInterface.LessThan, MathOptInterface.Interval, MathOptInterface.Integer, MathOptInterface.ZeroOne, MathOptInterface.Semicontinuous or MathOptInterface.Semiinteger. The sets supported with the MathOptInterface.VariableIndex cannot be controlled from the macro, use the UniversalFallback to support more sets.

This macro creates a model specialized for specific types of constraint, by defining specialized structures and methods. To create a model that, in addition to be optimized for specific constraints, also support arbitrary constraints and attributes, use UniversalFallback.

otherwise, it is a GenericModel, which is a subtype of MathOptInterface.ModelLike.

If is\_optimizer = true, the resulting struct is a of GenericOptimizer, which is a subtype of MathOptInterface. AbstractOp

# **Examples**

The model describing an linear program would be:

```
@model(LPModel,
                                                                # Name of model
                                                                # untyped scalar sets
      (),
      (MOI.EqualTo, MOI.GreaterThan, MOI.LessThan, MOI.Interval), # typed scalar sets
      (MOI.Zeros, MOI.Nonnegatives, MOI.Nonpositives),
                                                               # untyped vector sets
      (),
                                                                # typed vector sets
                                                                # untyped scalar functions
      (MOI.ScalarAffineFunction,),
                                                                # typed scalar functions
                                                                # untyped vector functions
      (MOI. VectorOfVariables,),
                                                                # typed vector functions
      (MOI. VectorAffineFunction,),
```

```
false
)
```

Let MOI denote MathOptInterface, MOIU denote MOI.Utilities. The macro would create the following types with struct\_of\_constraint\_code:

```
struct LPModelScalarConstraints{T, C1, C2, C3, C4} <: MOIU.StructOfConstraints</pre>
    moi_equalto::C1
    moi_greaterthan::C2
    moi_lessthan::C3
    moi interval::C4
end
struct LPModelVectorConstraints{T, C1, C2, C3} <: MOIU.StructOfConstraints</pre>
    moi_zeros::C1
    moi_nonnegatives::C2
    moi_nonpositives::C3
struct LPModelFunctionConstraints{T} <: MOIU.StructOfConstraints</pre>
    moi_scalaraffinefunction::LPModelScalarConstraints{
        Τ.
        MOIU.VectorOfConstraints\{MOI.ScalarAffineFunction\{T\},\ MOI.EqualTo\{T\}\},
        MOIU.VectorOfConstraints{MOI.ScalarAffineFunction{T}, MOI.GreaterThan{T}},
        {\tt MOIU.VectorOfConstraints\{MOI.ScalarAffineFunction\{T\},\ MOI.LessThan\{T\}\},}
        MOIU.VectorOfConstraints{MOI.ScalarAffineFunction{T}, MOI.Interval{T}}
    }
    moi vectorofvariables::LPModelVectorConstraints{
        Τ,
        MOIU.VectorOfConstraints{MOI.VectorOfVariables, MOI.Zeros},
        MOIU.VectorOfConstraints{MOI.VectorOfVariables, MOI.Nonnegatives},
        MOIU.VectorOfConstraints{MOI.VectorOfVariables, MOI.Nonpositives}
    }
    moi_vectoraffinefunction::LPModelVectorConstraints{
        Τ.
        MOIU.VectorOfConstraints{MOI.VectorAffineFunction{T}, MOI.Zeros},
        MOIU.VectorOfConstraints{MOI.VectorAffineFunction{T}, MOI.Nonnegatives},
        MOIU.VectorOfConstraints{MOI.VectorAffineFunction{T}, MOI.Nonpositives}
    }
end
const LPModel{T} =
→ MOIU.GenericModel{T,MOIU.ObjectiveContainer{T},MOIU.VariablesContainer{T},LPModelFunctionConstraints{T}}
```

The type LPModel implements the MathOptInterface API except methods specific to optimizers like optimize! or get with VariablePrimal.

source

 ${\tt MathOptInterface.Utilities.GenericModel-Type.}$ 

```
mutable struct GenericModel{T,0,V,C} <: AbstractModelLike{T}</pre>
```

Implements a model supporting coefficients of type T and:

- An objective function stored in .objective::0
- Variables and VariableIndex constraints stored in .variable\_bounds::V
- F-in-S constraints (excluding VariableIndex constraints) stored in .constraints::C

All interactions should take place via the MOI interface, so the types 0, V, and C should implement the API as needed for their functionality.

source

MathOptInterface.Utilities.GenericOptimizer - Type.

```
mutable struct GenericOptimizer{T,0,V,C} <: AbstractOptimizer{T}</pre>
```

Implements a model supporting coefficients of type T and:

- An objective function stored in .objective::0
- Variables and VariableIndex constraints stored in .variable bounds::V
- F-in-S constraints (excluding VariableIndex constraints) stored in .constraints::C

All interactions should take place via the MOI interface, so the types 0, V, and C should implement the API as needed for their functionality.

source

.objective MathOptInterface.Utilities.ObjectiveContainer - Type.

```
ObjectiveContainer{T}
```

A helper struct to simplify the handling of objective functions in Utilities. Model.

source

.variables MathOptInterface.Utilities.VariablesContainer - Type.

```
struct VariablesContainer{T} <: AbstractVectorBounds
    set_mask::Vector{UInt16}
    lower::Vector{T}
    upper::Vector{T}
end</pre>
```

A struct for storing variables and VariableIndex-related constraints. Used in MOI.Utilities.Model by default.

source

MathOptInterface.Utilities.FreeVariables - Type.

```
mutable struct FreeVariables <: MOI.ModelLike
    n::Int64
    FreeVariables() = new(0)
end</pre>
```

A struct for storing free variables that can be used as the variables field of GenericModel or GenericModel. It represents a model that does not support any constraint nor objective function.

# **Example**

The following model type represents a conic model in geometric form. As opposed to VariablesContainer, FreeVariables does not support constraint bounds so they are bridged into an affine constraint in the MathOptInterface.Nonnegatives cone as expected for the geometric conic form.

```
julia> MOI.Utilities.@product_of_sets(
                   Cones,
                   MOI.Zeros,
                   MOI.Nonnegatives,
                   MOI.SecondOrderCone,
                   {\tt MOI.PositiveSemidefiniteConeTriangle,}
          );
          julia> const ConicModel{T} = MOI.Utilities.GenericOptimizer{
                   MOI.Utilities.ObjectiveContainer{T},
                   MOI.Utilities.FreeVariables.
                   MOI.Utilities.MatrixOfConstraints{
                             MOI.Utilities.MutableSparseMatrixCSC{
                                      Τ,
                                      MOI.Utilities.OneBasedIndexing,
                             },
                             Vector{T},
                             Cones\{T\},
                   },
          };
          julia> model = MOI.instantiate(ConicModel{Float64}, with_bridge_type=Float64);
          julia> x = MOI.add_variable(model)
          MathOptInterface.VariableIndex(1)
          julia> c = MOI.add constraint(model, x, MOI.GreaterThan(1.0))
          MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.GreaterThan{
                     Float64}}(1)
          julia> MOI.Bridges.is_bridged(model, c)
          true
          julia> bridge = MOI.Bridges.bridge(model, c)
         {\tt MathOptInterface.Bridges.Constraint.VectorizeBridge} \{Float 64, \ {\tt MathOptInterface.Bridges.Constraint.VectorizeBridge} \} \\
                     Vector Affine Function \{Float64\}, \ Math Opt Interface. Nonnegatives, \ Math Opt Interface. Variable Index \ Math Opt Interface \ Mat
                     }(MathOptInterface.ConstraintIndex{MathOptInterface.VectorAffineFunction{Float64},
                    MathOptInterface.Nonnegatives}(1), 1.0)
          julia> bridge.vector_constraint
          MathOptInterface.ConstraintIndex{MathOptInterface.VectorAffineFunction{Float64}, MathOptInterface
                      .Nonnegatives}(1)
          julia> MOI.Bridges.is_bridged(model, bridge.vector_constraint)
         false
        source
.constraints MathOptInterface.Utilities.VectorOfConstraints - Type.
          mutable struct VectorOfConstraints{
```

F<:MOI.AbstractFunction,
S<:MOI.AbstractSet,</pre>

```
} <: MOI.ModelLike
    constraints::CleverDicts.CleverDict{
        MOI.ConstraintIndex{F,S},
        Tuple{F,S},
        typeof(CleverDicts.key_to_index),
        typeof(CleverDicts.index_to_key),
    }
end</pre>
```

A struct storing F-in-S constraints as a mapping between the constraint indices to the corresponding tuple of function and set.

source

MathOptInterface.Utilities.StructOfConstraints - Type.

```
| abstract type StructOfConstraints <: MOI.ModelLike end
```

A struct storing a subfields other structs storing constraints of different types.

```
See Utilities.@struct_of_constraints_by_function_types and Utilities.@struct_of_constraints_by_set_types. source
```

MathOptInterface.Utilities.@struct\_of\_constraints\_by\_function\_types - Macro.

```
Utilities.@struct_of_constraints_by_function_types(name, func_types...)
```

Given a vector of n function types (F1, F2,..., Fn) in func\_types, defines a subtype of StructOfConstraints of name name and which type parameters {T, C1, C2, ..., Cn}. It contains n field where the ith field has type Ci and stores the constraints of function type Fi.

The expression Fi can also be a union in which case any constraint for which the function type is in the union is stored in the field with type Ci.

source

MathOptInterface.Utilities.@struct of constraints by set types - Macro.

```
Utilities.@struct_of_constraints_by_set_types(name, func_types...)
```

Given a vector of n set types (S1, S2,..., Sn) in func\_types, defines a subtype of StructOfConstraints of name name and which type parameters {T, C1, C2, ..., Cn}. It contains n field where the ith field has type Ci and stores the constraints of set type Si. The expression Si can also be a union in which case any constraint for which the set type is in the union is stored in the field with type Ci. This can be useful if Ci is a MatrixOfConstraints in order to concatenate the coefficients of constraints of several different set types in the same matrix.

source

MathOptInterface.Utilities.struct\_of\_constraint\_code - Function.

```
| struct_of_constraint_code(struct_name, types, field_types = nothing)
```

Given a vector of n Union{SymbolFun,\_UnionSymbolFS{SymbolFun}} or Union{SymbolSet,\_UnionSymbolFS{SymbolSet}} in types, defines a subtype of StructOfConstraints of name name and which type parameters {T, F1, F2, ..., Fn} if field\_types is nothing and a {T} otherwise. It contains n field where the ith field has type Ci if field\_types is nothing and type field\_types[i] otherwise. If types is vector of

Union{SymbolFun,\_UnionSymbolFs{SymbolFun}} (resp. Union{SymbolSet,\_UnionSymbolFs{SymbolSet}}) then the constraints of that function (resp. set) type are stored in the corresponding field.

This function is used by the macros <code>@model</code>, <code>@struct\_of\_constraints\_by\_function\_types</code> and <code>@struct\_of\_constraints\_by\_source</code>

# **Caching optimizer**

MathOptInterface.Utilities.CachingOptimizer - Type.

CachingOptimizer

CachingOptimizer is an intermediate layer that stores a cache of the model and links it with an optimizer. It supports incremental model construction and modification even when the optimizer doesn't.

#### Constructors

```
CachingOptimizer(cache::MOI.ModelLike, optimizer::AbstractOptimizer)
```

Creates a CachingOptimizer in AUTOMATIC mode, with the optimizer optimizer.

The type of the optimizer returned is CachingOptimizer{typeof(optimizer), typeof(cache)} so it does not support the function reset\_optimizer(::CachingOptimizer, new\_optimizer) if the type of new\_optimizer is different from the type of optimizer.

```
CachingOptimizer(cache::MOI.ModelLike, mode::CachingOptimizerMode)
```

Creates a CachingOptimizer in the NO\_OPTIMIZER state and mode mode.

The type of the optimizer returned is CachingOptimizer{MOI.AbstractOptimizer, typeof(cache)} so it does support the function reset\_optimizer(::CachingOptimizer, new\_optimizer) if the type of new\_optimizer is different from the type of optimizer.

# About the type

# States

A CachingOptimizer may be in one of three possible states (CachingOptimizerState):

- NO OPTIMIZER: The CachingOptimizer does not have any optimizer.
- EMPTY\_OPTIMIZER: The CachingOptimizer has an empty optimizer. The optimizer is not synchronized with the cached model.
- ATTACHED\_OPTIMIZER: The CachingOptimizer has an optimizer, and it is synchronized with the cached model.

#### Modes

A CachingOptimizer has two modes of operation (CachingOptimizerMode):

- MANUAL: The only methods that change the state of the CachingOptimizer are Utilities.reset\_optimizer, Utilities.drop\_optimizer, and Utilities.attach\_optimizer. Attempting to perform an operation in the incorrect state results in an error.
- AUTOMATIC: The CachingOptimizer changes its state when necessary. For example, optimize! will automatically call attach\_optimizer (an optimizer must have been previously set). Attempting to add a constraint or perform a modification not supported by the optimizer results in a drop to EMPTY\_OPTIMIZER mode.

```
source
```

MathOptInterface.Utilities.attach optimizer - Function.

```
attach optimizer(model::CachingOptimizer)
```

Attaches the optimizer to model, copying all model data into it. Can be called only from the EMPTY\_OPTIMIZER state. If the copy succeeds, the CachingOptimizer will be in state ATTACHED\_OPTIMIZER after the call, otherwise an error is thrown; see MathOptInterface.copy to for more details on which errors can be thrown.

source

MathOptInterface.Utilities.reset\_optimizer - Function.

```
reset_optimizer(m::CachingOptimizer, optimizer::MOI.AbstractOptimizer)
```

Sets or resets m to have the given empty optimizer optimizer.

Can be called from any state. An assertion error will be thrown if optimizer is not empty.

The CachingOptimizer m will be in state EMPTY\_OPTIMIZER after the call.

source

```
reset_optimizer(m::CachingOptimizer)
```

Detaches and empties the current optimizer. Can be called from ATTACHED\_OPTIMIZER or EMPTY\_OPTIMIZER state. The CachingOptimizer will be in state EMPTY\_OPTIMIZER after the call.

source

MathOptInterface.Utilities.drop\_optimizer - Function.

```
drop_optimizer(m::CachingOptimizer)
```

Drops the optimizer, if one is present. Can be called from any state. The CachingOptimizer will be in state NO\_OPTIMIZER after the call.

source

MathOptInterface.Utilities.state - Function.

```
| state(m::CachingOptimizer)::CachingOptimizerState
```

 $Returns\ the\ state\ of\ the\ CachingOptimizer\ m.\ See\ Utilities.\ CachingOptimizer.$ 

source

MathOptInterface.Utilities.mode - Function.

```
| mode(m::CachingOptimizer)::CachingOptimizerMode
```

Returns the operating mode of the CachingOptimizer m. See Utilities.CachingOptimizer.

source

# **Mock optimizer**

MathOptInterface.Utilities.MockOptimizer - Type.

```
MockOptimizer
```

MockOptimizer is a fake optimizer especially useful for testing. Its main feature is that it can store the values that should be returned for each attribute.

## **Printing**

MathOptInterface.Utilities.latex\_formulation - Function.

```
latex_formulation(model::MOI.ModelLike; kwargs...)
```

Wrap model in a type so that it can be pretty-printed as text/latex in a notebook like IJulia, or in Documenter.

To render the model, end the cell with latex\_formulation(model), or call display(latex\_formulation(model)) in to force the display of the model from inside a function.

Possible keyword arguments are:

- simplify\_coefficients: Simplify coefficients if possible by omitting them or removing trailing zeros.
- default name: The name given to variables with an empty name.
- print\_types : Print the MOI type of each function and set for clarity.

source

# Copy utilities

A layer to filter out various components of model.

| ModelFilter(filter::Function, model::MOI.ModelLike)

MathOptInterface.Utilities.ModelFilter - Type.

The filter function takes a single argument, which is eacy element from the list returned by the attributes below. It returns true if the element should be visible in the filtered model and false otherwise.

The components that are filtered are:

- Entire constraint types via:
  - MOI.ListOfConstraintTypesPresent
- · Individual constraints via:
  - MOI.ListOfConstraintIndices{F,S}
- Specific attributes via:
  - MOI.ListOfModelAttributesSet
  - MOI.ListOfConstraintAttributesSet
  - MOI.ListOfVariableAttributesSet

# Warning

The list of attributes filtered may change in a future release. You should write functions that are generic and not limited to the five types listed above. Thus, you should probably define a fallback filter(::Any) = true.

See below for examples of how this works.

#### Note

This layer has a limited scope. It is intended by be used in conjunction with MOI.copy\_to.

## Example: copy model excluding integer constraints

Use the do syntax to provide a single function.

```
filtered_src = MOI.Utilities.ModelFilter(src) do item
    return item != (MOI.VariableIndex, MOI.Integer)
end
MOI.copy_to(dest, filtered_src)
```

# **Example: copy model excluding names**

Use type dispatch to simplify the implementation:

```
my_filter(::Any) = true # Note the generic fallback!
my_filter(::MOI.VariableName) = false
my_filter(::MOI.ConstraintName) = false
filtered_src = MOI.Utilities.ModelFilter(my_filter, src)
MOI.copy_to(dest, filtered_src)
```

# Example: copy irreducible infeasible subsystem

```
my_filter(::Any) = true # Note the generic fallback!
function my_filter(ci::MOI.ConstraintIndex)
    status = MOI.get(dest, MOI.ConstraintConflictStatus(), ci)
    return status != MOI.NOT_IN_CONFLICT
end
filtered_src = MOI.Utilities.ModelFilter(my_filter, src)
MOI.copy_to(dest, filtered_src)
```

## **MatrixOfConstraints**

MathOptInterface.Utilities.MatrixOfConstraints - Type.

```
mutable struct MatrixOfConstraints{T,AT,BT,ST} <: MOI.ModelLike
    coefficients::AT
    constants::BT
    sets::ST
    caches::Vector{Any}
    are_indices_mapped::Vector{BitSet}
    final_touch::Bool
end</pre>
```

Represent ScalarAffineFunction and VectorAffinefunction constraints in a matrix form where the linear coefficients of the functions are stored in the coefficients field, the constants of the functions or sets are stored in the constants field. Additional information about the sets are stored in the sets field.

This model can only be used as the constraints field of a MOI.Utilities.AbstractModel.

When the constraints are added, they are stored in the caches field. They are only loaded in the coefficients and constants fields once MOI.Utilities.final\_touch is called. For this reason, MatrixOfConstraints should not be used by an incremental interface. Use MOI.copy\_to instead.

The constraints can be added in two different ways:

- 1. With add constraint, in which case a canonicalized copy of the function is stored in caches.
- 2. With pass\_nonvariable\_constraints, in which case the functions and sets are stored themselves in caches without mapping the variable indices. The corresponding index in caches is added in are\_indices\_mapped. This avoids doing a copy of the function in case the getter of CanonicalConstraintFunction does not make a copy for the source model, e.g., this is the case of VectorOfConstraints.

We illustrate this with an example. Suppose a model is copied from a src::MOI.Utilities.Model to a bridged model with a MatrixOfConstraints. For all the types that are not bridged, the constraints will be copied with pass\_nonvariable\_constraints. Hence the functions stored in caches are exactly the same as the ones stored in src. This is ok since this is only during the copy\_to operation during which src cannot be modified. On the other hand, for the types that are bridged, the functions added may contain duplicates even if the functions did not contain duplicates in src so duplicates are removed with MOI.Utilities.canonical.

#### Interface

The .coefficients::AT type must implement:

```
AT()
MOI.empty(::AT)!
MOI.Utilities.add_column
MOI.Utilities.set_number_of_rows
MOI.Utilities.allocate_terms
MOI.Utilities.load_terms
MOI.Utilities.final_touch
```

The .constants::BT type must implement:

• BT()

```
• Base.empty!(::BT)
      • Base.resize(::BT)
      • MOI.Utilities.load_constants
      • MOI.Utilities.function_constants
      • MOI.Utilities.set from constants
   The .sets::ST type must implement:
      • ST()
      • MOI.is empty(::ST)
      • MOI.empty(::ST)
      • MOI.dimension(::ST)
      • MOI.is_valid(::ST, ::MOI.ConstraintIndex)
      • MOI.get(::ST, ::MOI.ListOfConstraintTypesPresent)
      • MOI.get(::ST, ::MOI.NumberOfConstraints)
      • MOI.get(::ST, ::MOI.ListOfConstraintIndices)
      • MOI.Utilities.set_types
      • MOI.Utilities.set index
      • MOI.Utilities.add set
      • MOI.Utilities.rows
      • MOI.Utilities.final_touch
   source
.coefficients MathOptInterface.Utilities.add_column - Function.
   add_column(coefficients)::Nothing
   Tell coefficients to pre-allocate datastructures as needed to store one column.
   source
MathOptInterface.Utilities.allocate terms - Function.
   allocate_terms(coefficients, index_map, func)::Nothing
   Tell coefficients that the terms of the function func where the variable indices are mapped with index_map
   will be loaded with load terms.
   The function func must be canonicalized before calling allocate_terms. See is_canonical.
   source
MathOptInterface.Utilities.set_number_of_rows - Function.
   | set_number_of_rows(coefficients, n)::Nothing
   Tell coefficients to pre-allocate datastructures as needed to store n rows.
   source
```

```
MathOptInterface.Utilities.load_terms - Function.
   load_terms(coefficients, index_map, func, offset)::Nothing
   Loads the terms of func to coefficients, mapping the variable indices with index_map.
   The ith dimension of func is loaded at the (offset + i)th row of coefficients.
   The function must be allocated first with allocate terms.
   The function func must be canonicalized, see is canonical.
   source
MathOptInterface.Utilities.final_touch - Function.
   final_touch(coefficients)::Nothing
   Informs the coefficients that all functions have been added with load_terms. No more modification is
   allowed unless MOI.empty! is called.
   final_touch(sets)::Nothing
   Informs the sets that all functions have been added with add_set. No more modification is allowed unless
   MOI.empty! is called.
   source
MathOptInterface.Utilities.extract function - Function.
   extract_function(coefficients, row::Integer, constant::T) where {T}
   Return the MOI.ScalarAffineFunction{T} function corresponding to row row in coefficients.
    extract_function(
        coefficients,
        rows::UnitRange,
        constants::Vector{T},
    ) where{T}
   Return the MOI. VectorAffineFunction{T} function corresponding to rows rows in coefficients.
   source
MathOptInterface.Utilities.MutableSparseMatrixCSC - Type.
    mutable struct MutableSparseMatrixCSC{Tv,Ti<:Integer,I<:AbstractIndexing}</pre>
        indexing::I
        m::Int
        n::Int
        colptr::Vector{Ti}
        rowval::Vector{Ti}
```

Matrix type loading sparse matrices in the Compressed Sparse Column format. The indexing used is indexing, see AbstractIndexing. The other fields have the same meaning than for SparseArrays. SparseMatrixCSC except that the indexing is different unless indexing is OneBasedIndexing. In addition, nz\_added is used to cache the number of non-zero terms that have been added to each column due to the incremental nature of load\_terms.

The matrix is loaded in 5 steps:

nzval::Vector{Tv}
nz\_added::Vector{Ti}

end

- 1. MOI.empty! is called.
- 2. MOI.Utilities.add\_column and MOI.Utilities.allocate\_terms are called in any order.
- MOI.Utilities.set\_number\_of\_rows is called.
- 4. MOI.Utilities.load\_terms is called for each affine function.
- MOI.Utilities.final\_touch is called.

source

MathOptInterface.Utilities.AbstractIndexing - Type.

```
abstract type AbstractIndexing end
```

Indexing to be used for storing the row and column indices of MutableSparseMatrixCSC. See ZeroBasedIndexing and OneBasedIndexing.

source

MathOptInterface.Utilities.ZeroBasedIndexing - Type.

```
struct ZeroBasedIndexing <: AbstractIndexing end
```

Zero-based indexing: the ith row or column has index i - 1. This is useful when the vectors of row and column indices need to be communicated to a library using zero-based indexing such as C libraries.

source

MathOptInterface.Utilities.OneBasedIndexing - Type.

```
struct ZeroBasedIndexing <: AbstractIndexing end
```

One-based indexing: the ith row or column has index i. This enables an allocation-free conversion of MutableSparseMatrixCSC to SparseArrays.SparseMatrixCSC.

source

.constants MathOptInterface.Utilities.load\_constants - Function.

```
|load_constants(constants, offset, func_or_set)::Nothing
```

This function loads the constants of func\_or\_set in constants at an offset of offset. Where offset is the sum of the dimensions of the constraints already loaded. The storage should be preallocated with resize! before calling this function.

This function should be implemented to be usable as storage of constants for MatrixOfConstraints.

The constants are loaded in three steps:

- Base.empty! is called.
- 2. Base.resize! is called with the sum of the dimensions of all constraints.
- MOI.Utilities.load\_constants is called for each function for vector constraint or set for scalar constraint.

```
function_constants(constants, rows)
   This function returns the function constants that were loaded with load constants at the rows rows.
   This function should be implemented to be usable as storage of constants for MatrixOfConstraints.
    source
MathOptInterface.Utilities.set from constants - Function.
   set_from_constants(constants, S::Type, rows)::S
   This function returns an instance of the set S for which the constants where loaded with load constants
   at the rows rows.
   This function should be implemented to be usable as storage of constants for MatrixOfConstraints.
    source
MathOptInterface.Utilities.Hyperrectangle - Type.
    struct Hyperrectangle{T} <: AbstractVectorBounds</pre>
        lower::Vector{T}
        upper::Vector{T}
    end
   A struct for the .constants field in MatrixOfConstraints.
    source
.sets MathOptInterface.Utilities.set_index - Function.
   | set_index(sets, ::Type{S})::Union{Int,Nothing} where {S<:MOI.AbstractSet}
   Return an integer corresponding to the index of the set type in the list given by set types.
   If S is not part of the list, return nothing.
    source
MathOptInterface.Utilities.set_types - Function.
   set_types(sets)::Vector{Type}
    Return the list of the types of the sets allowed in sets.
    source
MathOptInterface.Utilities.add_set - Function.
   add_set(sets, i)::Int64
    Add a scalar set of type index i.
   add_set(sets, i, dim)::Int64
   Add a vector set of type index i and dimension dim.
   Both methods return a unique Int64 of the set that can be used to reference this set.
```

```
MathOptInterface.Utilities.rows - Function.
   rows(sets, ci::MOI.ConstraintIndex)::Union{Int,UnitRange{Int}}
   Return the rows in 1:MOI.dimension(sets) corresponding to the set of id ci.value.
   For scalar sets, this returns an Int. For vector sets, this returns an UnitRange{Int}.
   source
MathOptInterface.Utilities.num_rows - Function.
   num_rows(sets::OrderedProductOfSets, ::Type{S}) where {S}
   Return the number of rows corresponding to a set of type S. That is, it is the sum of the dimensions of the
   sets of type S.
   source
MathOptInterface.Utilities.set_with_dimension - Function.
   set_with_dimension(::Type{S}, dim) where {S<:MOI.AbstractVectorSet}</pre>
   Returns the instance of S of MathOptInterface.dimension dim. This needs to be implemented for sets of
   type S to be useable with MatrixOfConstraints.
   source
MathOptInterface.Utilities.ProductOfSets - Type.
   abstract type ProductOfSets{T} end
   Represents a cartesian product of sets of given types.
   source
MathOptInterface.Utilities.MixOfScalarSets - Type.
   abstract type MixOfScalarSets{T} <: ProductOfSets{T} end</pre>
   Product of scalar sets in the order the constraints are added, mixing the constraints of different types.
   Use @mix_of_scalar_sets to generate a new subtype.
   source
MathOptInterface.Utilities.@mix of scalar sets - Macro.
   @mix_of_scalar_sets(name, set_types...)
   Generate a new MixOfScalarSets subtype.
   Example
    @mix_of_scalar_sets(
        MixedIntegerLinearProgramSets,
        MOI.GreaterThan{T},
        MOI.LessThan{T},
        MOI.EqualTo{T},
        MOI.Integer,
```

```
source
```

MathOptInterface.Utilities.OrderedProductOfSets - Type.

```
abstract type OrderedProductOfSets{T} <: ProductOfSets{T} end
```

Product of sets in the order the constraints are added, grouping the constraints of the same types contiguously.

Use @product\_of\_sets to generate new subtypes.

source

MathOptInterface.Utilities.@product\_of\_sets - Macro.

```
@product_of_sets(name, set_types...)
```

Generate a new OrderedProductOfSets subtype.

#### **Example**

```
@product_of_sets(
    LinearOrthants,
    MOI.Zeros,
    MOI.Nonnegatives,
    MOI.Nonpositives,
    MOI.ZeroOne,
)
```

# Fallbacks

MathOptInterface.Utilities.get\_fallback - Function.

```
get_fallback(model::MOI.ModelLike, ::MOI.ObjectiveValue)
```

Compute the objective function value using the VariablePrimal results and the ObjectiveFunction value.

source

```
| get_fallback(model::MOI.ModelLike, ::MOI.DualObjectiveValue, T::Type)::T
```

Compute the dual objective value of type T using the ConstraintDual results and the ConstraintFunction and ConstraintSet values. Note that the nonlinear part of the model is ignored.

source

Compute the value of the function of the constraint of index constraint\_index using the VariablePrimal results and the ConstraintFunction values.

Compute the dual of the constraint of index ci using the ConstraintDual of other constraints and the ConstraintFunction values. Throws an error if some constraints are quadratic or if there is one another MOI.VariableIndex-in-S or MOI.VectorOfVariables-in-S constraint with one of the variables in the function of the constraint ci.

source

#### **Function utilities**

The following utilities are available for functions:

```
MathOptInterface.Utilities.eval_variables - Function.
```

```
eval_variables(varval::Function, f::AbstractFunction)
```

Returns the value of function f if each variable index vi is evaluated as varval(vi). Note that varval should return a number, see <a href="substitute\_variables">substitute\_variables</a> for a similar function where varval returns a function.

source

MathOptInterface.Utilities.map\_indices - Function.

```
|map_indices(index_map::Function, attr::MOI.AnyAttribute, x::X)::X where {X}
```

Substitute any MOI. VariableIndex (resp. MOI. ConstraintIndex) in x by the MOI. VariableIndex (resp. MOI. ConstraintIndex) of the same type given by index\_map(x).

# When to implement this method for new types $\boldsymbol{X}$

This function is used by implementations of MOI.copy\_to on constraint functions, attribute values and submittable values. If you define a new attribute whose values x::X contain variable or constraint indices, you must also implement this function.

source

```
map_indices(
    variable_map::AbstractDict{T,T},
    x::X,
)::X where {T<:MOI.Index,X}</pre>
```

Shortcut for map indices(vi -> variable map[vi], x).

source

MathOptInterface.Utilities.substitute\_variables - Function.

```
substitute_variables(variable_map::Function, x)
```

Substitute any MOI.VariableIndex in x by variable\_map(x). The variable\_map function returns either MOI.VariableIndex or MOI.ScalarAffineFunction, see eval\_variables for a similar function where variable\_map returns a number.

This function is used by bridge optimizers on constraint functions, attribute values and submittable values when at least one variable bridge is used hence it needs to be implemented for custom types that are meant to be used as attribute or submittable value.

WARNING: Don't use substitude\_variables(::Function, ...) because Julia will not specialize on this. Use instead substitude variables(::F, ...) where F<:FunctionF.

```
MathOptInterface.Utilities.filter_variables - Function.
```

```
filter_variables(keep::Function, f::AbstractFunction)
```

Return a new function f with the variable vi such that !keep(vi) removed.

WARNING: Don't define filter\_variables(::Function, ...) because Julia will not specialize on this. Define instead filter variables(::F, ...) where {F<:Function}.

source

MathOptInterface.Utilities.remove\_variable - Function.

```
remove_variable(f::AbstractFunction, vi::VariableIndex)
```

Return a new function f with the variable vi removed.

source

```
remove_variable(f::MOI.AbstractFunction, s::MOI.AbstractSet, vi::MOI.VariableIndex)
```

Return a tuple (g, t) representing the constraint f-in-s with the variable vi removed. That is, the terms containing the variable vi in the function f are removed and the dimension of the set s is updated if needed (e.g. when f is a VectorOfVariables with vi being one of the variables).

source

MathOptInterface.Utilities.all\_coefficients - Function.

```
all_coefficients(p::Function, f::MOI.AbstractFunction)
```

Determine whether predicate p returns true for all coefficients of f, returning false as soon as the first coefficient of f for which p returns false is encountered (short-circuiting). Similar to all.

source

MathOptInterface.Utilities.unsafe\_add - Function.

```
unsafe_add(t1::MOI.ScalarAffineTerm, t2::MOI.ScalarAffineTerm)
```

Sums the coefficients of t1 and t2 and returns an output MOI. Scalar Affine Term. It is unsafe because it uses the variable of t1 as the variable of the output without checking that it is equal to that of t2.

source

```
unsafe_add(t1::MOI.ScalarQuadraticTerm, t2::MOI.ScalarQuadraticTerm)
```

Sums the coefficients of t1 and t2 and returns an output MOI. ScalarQuadraticTerm. It is unsafe because it uses the variable's of t1 as the variable's of the output without checking that they are the same (up to permutation) to those of t2.

source

```
unsafe_add(t1::MOI.VectorAffineTerm, t2::MOI.VectorAffineTerm)
```

Sums the coefficients of t1 and t2 and returns an output MOI. VectorAffineTerm. It is unsafe because it uses the output\_index and variable of t1 as the output\_index and variable of the output term without checking that they are equal to those of t2.

```
MathOptInterface.Utilities.isapprox_zero - Function.
   isapprox_zero(f::MOI.AbstractFunction, tol)
   Return a Bool indicating whether the function f is approximately zero using tol as a tolerance.
   Important note
   This function assumes that f does not contain any duplicate terms, you might want to first call canonical
   if that is not guaranteed. For instance, given
   f = MOI.ScalarAffineFunction(MOI.ScalarAffineTerm.([1, -1], [x, x]), 0).
   then isapprox_zero(f) is false but isapprox_zero(MOIU.canonical(f)) is true.
   source
MathOptInterface.Utilities.modify function - Function.
   modify_function(f::AbstractFunction, change::AbstractFunctionModification)
   Return a new function f modified according to change.
   source
MathOptInterface.Utilities.zero with output dimension - Function.
   | zero_with_output_dimension(::Type{T}, output_dimension::Integer) where {T}
   Create an instance of type T with the output dimension output dimension.
   This is mostly useful in Bridges, when code needs to be agnostic to the type of vector-valued function that
   is passed in.
   source
The following functions can be used to canonicalize a function:
MathOptInterface.Utilities.is canonical - Function.
   is_canonical(f::Union{ScalarAffineFunction, VectorAffineFunction})
   Returns a Bool indicating whether the function is in canonical form. See canonical.
   source
   is_canonical(f::Union{ScalarQuadraticFunction, VectorQuadraticFunction})
   Returns a Bool indicating whether the function is in canonical form. See canonical.
   source
MathOptInterface.Utilities.canonical - Function.
    canonical(
        f::Union{
            ScalarAffineFunction,
            VectorAffineFunction,
            ScalarQuadraticFunction,
            VectorQuadraticFunction,
        },
```

)

Returns the function in a canonical form, i.e.

- A term appear only once.
- The coefficients are nonzero.
- The terms appear in increasing order of variable where there the order of the variables is the order of their value.
- For a AbstractVectorFunction, the terms are sorted in ascending order of output index.

The output of canonical can be assumed to be a copy of f, even for VectorOfVariables.

## **Examples**

```
If x (resp. y, z) is VariableIndex(1) (resp. 2, 3). The canonical representation of ScalarAffineFunction([y, x, z, x, z], [2, 1, 3, -2, -3], 5) is ScalarAffineFunction([x, y], [-1, 2], 5).
    source

MathOptInterface.Utilities.canonicalize! - Function.

| canonicalize!(f::Union{ScalarAffineFunction, VectorAffineFunction})

Convert a function to canonical form in-place, without allocating a copy to hold the result. See canonical.
    source
    | canonicalize!(f::Union{ScalarQuadraticFunction, VectorQuadraticFunction})
```

Convert a function to canonical form in-place, without allocating a copy to hold the result. See canonical.

source

The following functions can be used to manipulate functions with basic algebra:

MathOptInterface.Utilities.scalar\_type - Function.

```
| scalar_type(F::Type{<:MOI.AbstractVectorFunction})

Type of functions obtained by indexing objects obtained by calling eachscalar on functions of type F. source
```

```
MathOptInterface.Utilities.scalarize - Function.
| scalarize(func::MOI.VectorOfVariables, ignore_constants::Bool = false)
```

Returns a vector of scalar functions making up the vector function in the form of a Vector {MOI.SingleVariable}.

See also eachscalar.

```
source
| scalarize(func::MOI.VectorAffineFunction{T}, ignore_constants::Bool = false)
```

 $Returns\ a\ vector\ of\ scalar\ function\ s\ making\ up\ the\ vector\ function\ in\ the\ form\ of\ a\ Vector\ \{MOI.Scalar\ Affine\ Function\ \{T\}\}.$ 

```
See also eachscalar.
```

```
scalarize(func::MOI.VectorQuadraticFunction{T}, ignore_constants::Bool = false)
```

Returns a vector of scalar functions making up the vector function in the form of a Vector{MOI.ScalarQuadraticFunction{T} See also each scalar.

source

MathOptInterface.Utilities.eachscalar - Function.

```
eachscalar(f::MOI.AbstractVectorFunction)
```

Returns an iterator for the scalar components of the vector function.

See also scalarize.

source

```
eachscalar(f::MOI.AbstractVector)
```

Returns an iterator for the scalar components of the vector.

source

MathOptInterface.Utilities.promote\_operation - Function.

```
promote_operation(
    op::Function,
    ::Type{T},
    ArgsTypes::Type{<:Union{T, MOI.AbstractFunction}}...,
) where {T}</pre>
```

Returns the type of the MOI.AbstractFunction returned to the call operate(op, T, args...) where the types of the arguments args are ArgsTypes.

source

MathOptInterface.Utilities.operate - Function.

```
operate(
    op::Function,
    ::Type{T},
    args::Union{T,MOI.AbstractFunction}...,
)::MOI.AbstractFunction where {T}
```

Returns an MOI.AbstractFunction representing the function resulting from the operation op(args...) on functions of coefficient type T. No argument can be modified.

source

MathOptInterface.Utilities.operate! - Function.

```
operate!(
    op::Function,
    ::Type{T},
    args::Union{T, MOI.AbstractFunction}...,
)::MOI.AbstractFunction where {T}
```

Returns an MOI.AbstractFunction representing the function resulting from the operation op(args...) on functions of coefficient type T. The first argument can be modified. The return type is the same than the method operate(op, T, args...) without!

MathOptInterface.Utilities.operate\_output\_index! - Function.

```
operate_output_index!(
    op::Function,
    ::Type{T},
    output_index::Integer,
    func::MOI.AbstractVectorFunction
    args::Union{T, MOI.AbstractScalarFunction}...
)::MOI.AbstractFunction where {T}
```

Returns an MOI. AbstractVectorFunction where the function at output\_index is the result of the operation op applied to the function at output\_index of func and args. The functions at output index different to output\_index are the same as the functions at the same output index in func. The first argument can be modified.

source

MathOptInterface.Utilities.vectorize - Function.

```
vectorize(x::AbstractVector{MOI.VariableIndex})
```

Returns the vector of scalar affine functions in the form of a MOI.VectorAffineFunction{T}.

source

```
vectorize(funcs::AbstractVector{MOI.ScalarAffineFunction{T}}) where T
```

Returns the vector of scalar affine functions in the form of a MOI. VectorAffineFunction{T}.

source

```
vectorize(funcs::AbstractVector{MOI.ScalarQuadraticFunction{T}}) where T
```

Returns the vector of scalar quadratic functions in the form of a MOI. VectorQuadraticFunction  $\{T\}$ .

source

# **Constraint utilities**

The following utilities are available for moving the function constant to the set for scalar constraints:

```
{\tt MathOptInterface.Utilities.shift\_constant-Function}.
```

```
| shift_constant(set::MOI.AbstractScalarSet, offset)
```

Returns a new scalar set new\_set such that func-in-set is equivalent to func + offset-in-new\_set.

Only define this function if it makes sense to!

Use supports\_shift\_constant to check if the set supports shifting:

```
if supports_shift_constant(typeof(old_set))
    new_set = shift_constant(old_set, offset)
    f.constant = 0
    add_constraint(model, f, new_set)
else
    add_constraint(model, f, old_set)
end
```

Adds the scalar constraint obtained by moving the constant term in func to the set in model. If allow\_modify\_function is true then the function func can be modified.

source

)

MathOptInterface.Utilities.normalize\_constant - Function.

```
normalize_constant(
    func::MOI.AbstractScalarFunction,
    set::MOI.AbstractScalarSet;
    allow_modify_function::Bool = false,
)
```

Return the func-in-set constraint in normalized form. That is, if func is MOI.ScalarQuadraticFunction or MOI.ScalarAffineFunction, the constant is moved to the set. If allow\_modify\_function is true then the function func can be modified.

source

The following utility identifies those constraints imposing bounds on a given variable, and returns those bound values:

 ${\tt MathOptInterface.Utilities.get\_bounds-Function}.$ 

```
get_bounds(model::MOI.ModelLike, ::Type{T}, x::MOI.VariableIndex)
```

Return a tuple (lb, ub) of type  $Tuple\{T, T\}$ , where lb and ub are lower and upper bounds, respectively, imposed on x in model.

source

The following utilities are useful when working with symmetric matrix cones.

```
MathOptInterface.Utilities.is_diagonal_vectorized_index - Function.
```

```
is_diagonal_vectorized_index(index::Base.Integer)
   Return whether index is the index of a diagonal element in a MOI. AbstractSymmetricMatrixSetTriangle
   set.
   source
MathOptInterface.Utilities.side dimension for vectorized dimension - Function.
   | side_dimension_for_vectorized_dimension(n::Integer)
   Return the dimension d such that MOI.dimension (MOI.PositiveSemidefiniteConeTriangle(d)) is n.
   source
DoubleDicts
MathOptInterface.Utilities.DoubleDicts.DoubleDict - Type.
   | DoubleDict{V}
   An optimized dictionary to map MOI. ConstraintIndex to values of type V.
   Works as a AbstractDict{MOI.ConstraintIndex,V} with minimal differences.
   If V is also a MOI. ConstraintIndex, use IndexDoubleDict.
   Note that MOI. ConstraintIndex is not a concrete type, opposed to MOI. ConstraintIndex {MOI. VariableIndex,
   MOI.Integers}, which is a concrete type.
   When looping through multiple keys of the same Function-in-Set type, use
   inner = dict[F, S]
   to return a type-stable DoubleDictInner.
   source
MathOptInterface.Utilities.DoubleDicts.DoubleDictInner - Type.
   | DoubleDictInner{F,S,V}
   A type stable inner dictionary of DoubleDict.
   source
MathOptInterface.Utilities.DoubleDicts.IndexDoubleDict - Type.
   IndexDoubleDict
   A specialized version of [DoubleDict] in which the values are of type MOI.ConstraintIndex
   When looping through multiple keys of the same Function-in-Set type, use
   inner = dict[F, S]
   to return a type-stable IndexDoubleDictInner.
   source
```

 ${\tt MathOptInterface.Utilities.DoubleDicts.IndexDoubleDictInner-Type.}$ 

| IndexDoubleDictInner{F,S}

A type stable inner dictionary of  ${\tt IndexDoubleDict}.$ 

# **Chapter 30**

# **Test**

# 30.1 Overview

## The Test submodule

The Test submodule provides tools to help solvers implement unit tests in order to ensure they implement the MathOptInterface API correctly, and to check for solver-correctness.

We use a centralized repository of tests, so that if we find a bug in one solver, instead of adding a test to that particular repository, we add it here so that all solvers can benefit.

# How to test a solver

The skeleton below can be used for the wrapper test file of a solver named FooBar.

```
module TestFooBar
import FooBar
using MathOptInterface
using Test
const MOI = MathOptInterface
const OPTIMIZER = MOI.instantiate(
   MOI.OptimizerWithAttributes(FooBar.Optimizer, MOI.Silent() => true),
const BRIDGED = MOI.instantiate(
   MOI.OptimizerWithAttributes(FooBar.Optimizer, MOI.Silent() => true),
   with bridge type = Float64,
# See the docstring of MOI.Test.Config for other arguments.
const CONFIG = MOI.Test.Config(
   # Modify tolerances as necessary.
   atol = 1e-6,
   rtol = 1e-6,
   # Use MOI.LOCALLY_SOLVED for local solvers.
   optimal_status = MOI.OPTIMAL,
   # Pass attributes or MOI functions to `exclude` to skip tests that
```

```
# rely on this functionality.
    exclude = Any[MOI.VariableName, MOI.delete],
0.00
    runtests()
This function runs all functions in the this Module starting with `test_`.
function runtests()
    for name in names(@__MODULE__; all = true)
        if startswith("$(name)", "test_")
            @testset "$(name)" begin
                getfield(@__MODULE__, name)()
            end
        end
   end
end
    test_runtests()
This function runs all the tests in MathOptInterface.Test.
Pass arguments to `exclude` to skip tests for functionality that is not
implemented or that your solver doesn't support.
function test_runtests()
   MOI.Test.runtests(
        BRIDGED,
        CONFIG,
        exclude = [
            "test_attribute_NumberOfThreads",
            "test_quadratic_",
        # This argument is useful to prevent tests from failing on future
        # releases of MOI that add new tests. Don't let this number get too far
        # behind the current MOI release though! You should periodically check
        # for new tests in order to fix bugs and implement new features.
        exclude_tests_after = v"0.10.5",
    )
    return
end
    test_SolverName()
You can also write new tests for solver-specific functionality. Write each new
test as a function with a name beginning with `test_`.
function test_SolverName()
   @test MOI.get(FooBar.Optimizer(), MOI.SolverName()) == "FooBar"
    return
end
end # module TestFooBar
```

```
# This line at tne end of the file runs all the tests!
TestFooBar.runtests()
```

Then modify your runtests.jl file to include the MOI\_wrapper.jl file:

#### Info

The optimizer BRIDGED constructed with instantiate automatically bridges constraints that are not supported by OPTIMIZER using the bridges listed in Bridges. It is recommended for an implementation of MOI to only support constraints that are natively supported by the solver and let bridges transform the constraint to the appropriate form. For this reason it is expected that tests may not pass if OPTIMIZER is used instead of BRIDGED.

# How to debug a failing test

When writing a solver, it's likely that you will initially fail many tests! Some failures will be bugs, but other failures you may choose to exclude.

There are two ways to exclude tests:

• Exclude tests whose names contain a string using:

```
MOI.Test.runtests(
    model,
    config;
    exclude = String["test_to_exclude", "test_conic_"],
)
```

This will exclude tests whose name contains either of the two strings provided.

• Exclude tests which rely on specific functionality using:

```
MOI.Test.Config(exclude = Any[MOI.VariableName, MOI.optimize!])
```

This will exclude tests which use the MOI.VariableName attribute, or which call MOI.optimize!.

Each test that fails can be independently called as:

```
model = FooBar.Optimizer()
config = MOI.Test.Config()
MOI.empty!(model)
MOI.Test.test_category_name_that_failed(model, config)
```

You can look-up the source code of the test that failed by searching for it in the src/Test/test\_category.jl file.

## Tip

Each test function also has a docstring that explains what the test is for. Use? MOI.Test.test\_category\_name\_that\_fail from the REPL to read it.

#### How to add a test

To detect bugs in solvers, we add new tests to MOI.Test.

As an example, ECOS errored calling optimize! twice in a row. (See ECOS.jl PR #72.) We could add a test to ECOS.jl, but that would only stop us from re-introducing the bug to ECOS.jl in the future, but it would not catch other solvers in the ecosystem with the same bug! Instead, if we add a test to MOI.Test, then all solvers will also check that they handle a double optimize call!

For this test, we care about correctness, rather than performance. therefore, we don't expect solvers to efficiently decide that they have already solved the problem, only that calling optimize! twice doesn't throw an error or give the wrong answer.

#### Step 1

Install the MathOptInterface julia package in dev mode (ref):

```
julia> ]
(@v1.6) pkg> dev MathOptInterface
```

## Step 2

From here on, proceed with making the following changes in the ~/.julia/dev/MathOptInterface folder (or equivalent dev path on your machine).

# Step 3

Since the double-optimize error involves solving an optimization problem, add a new test to src/Test/UnitTest-s/solve.jl:

```
test unit optimize! twice(model::MOI.ModelLike, config::Config)
Test that calling `MOI.optimize!` twice does not error.
This problem was first detected in ECOS.jl PR#72:
https://github.com/jump-dev/ECOS.jl/pull/72
function test_unit_optimize!_twice(
   model::MOI.ModelLike,
   config::Config{T},
) where {T}
   # Use the `@requires` macro to check conditions that the test function
    # requires in order to run. Models failing this `@requires` check will
    # silently skip the test.
   @requires MOI.supports_constraint(
        model.
        MOI.VariableIndex,
        MOI.GreaterThan{Float64},
   @requires _supports(config, MOI.optimize!)
    # If needed, you can test that the model is empty at the start of the test.
```

```
# You can assume that this will be the case for tests run via `runtests`.
   # User's calling tests individually need to call `MOI.empty!` themselves.
   @test MOI.is_empty(model)
   # Create a simple model. Try to make this as simple as possible so that the
   # majority of solvers can run the test.
   x = MOI.add_variable(model)
   MOI.add_constraint(model, x, MOI.GreaterThan(one(T)))
   MOI.set(model, MOI.ObjectiveSense(), MOI.MIN_SENSE)
        model,
        MOI.ObjectiveFunction{MOI.VariableIndex}(),
    )
   # The main component of the test: does calling `optimize!` twice error?
   MOI.optimize!(model)
   MOI.optimize!(model)
   # Check we have a solution.
   @test MOI.get(model, MOI.TerminationStatus()) == MOI.OPTIMAL
    # There is a three-argument version of `Base.isapprox` for checking
   # approximate equality based on the tolerances defined in `config`:
   @test isapprox(MOI.get(model, MOI.VariablePrimal(), x), one(T), config)
    # For code-style, these tests should always `return` `nothing`.
end
```

#### Info

Make sure the function is agnoistic to the number type T! Don't assume it is a Float64 capable solver!

We also need to write a test for the test. Place this function immediately below the test you just wrote in the same file:

```
function setup_test(
    ::typeof(test_unit_optimize!_twice),
    model::MOI.Utilities.MockOptimizer,
    ::Config,
)

MOI.Utilities.set_mock_optimize!(
    model,
    (mock::MOI.Utilities.MockOptimizer) -> MOIU.mock_optimize!(
    mock,
    MOI.OPTIMAL,
    (MOI.FEASIBLE_POINT, [1.0]),
    ),
    )
    return
end
```

Finally, you also need to implement Test.version\_added. If we added this test when the latest released version of MOI was v0.10.5, define:

```
version_added(::typeof(test_unit_optimize!_twice)) = v"0.10.6"
```

## Step 6

Commit the changes to git from ~/.julia/dev/MathOptInterface and submit the PR for review.

## Tip

If you need help writing a test, open an issue on GitHub, or ask the Developer Chatroom

## 30.2 API Reference

## The Test submodule

Functions to help test implementations of MOI. See The Test submodule for more details.

MathOptInterface.Test.Config - Type.

```
Config(
    ::Type{T} = Float64;
    atol::Real = Base.rtoldefault(T),
    rtol::Real = Base.rtoldefault(T),
    optimal_status::MOI.TerminationStatusCode = MOI.OPTIMAL,
    infeasible_status::MOI.TerminationStatusCode = MOI.INFEASIBLE,
    exclude::Vector{Any} = Any[],
) where {T}
```

Return an object that is used to configure various tests.

# **Configuration arguments**

- atol::Real = Base.rtoldefault(T): Control the absolute tolerance used when comparing solutions
- rtol::Real = Base.rtoldefault(T): Control the relative tolerance used when comparing solutions.
- optimal\_status = MOI.OPTIMAL: Set to MOI.LOCALLY\_SOLVED if the solver cannot prove global optimality.
- infeasible\_status = MOI.INFEASIBLE: Set to MOI.LOCALLY\_INFEASIBLE if the solver cannot prove global infeasibility.
- exclude = Vector{Any}: Pass attributes or functions to exclude to skip parts of tests that require certain functionality. Common arguments include:
  - MOI.delete to skip deletion-related tests
  - MOI.optimize! to skip optimize-related tests
  - MOI.ConstraintDual to skip dual-related tests
  - MOI. VariableName to skip setting variable names
  - MOI.ConstraintName to skip setting constraint names

## **Examples**

For a nonlinear solver that finds local optima and does not support finding dual variables or constraint names:

```
Config(
    Float64;
    optimal_status = MOI.LOCALLY_SOLVED,
```

source

MathOptInterface.Test.runtests - Function.

```
runtests(
   model::MOI.ModelLike,
   config::Config;
   include::Vector{String} = String[],
   exclude::Vector{String} = String[],
   warn_unsupported::Bool = false,
   exclude_tests_after::VersionNumber = v"999.0.0",
)
```

Run all tests in MathOptInterface. Test on model.

## **Configuration arguments**

- config is a Test.Config object that can be used to modify the behavior of tests.
- If include is not empty, only run tests that contain an element from include in their name.
- If exclude is not empty, skip tests that contain an element from exclude in their name.
- exclude takes priority over include.
- If warn\_unsupported is false, runtests will silently skip tests that fail with UnsupportedConstraint or UnsupportedAttribute. When warn\_unsupported is true, a warning will be printed. For most cases the default behavior (false) is what you want, since these tests likely test functionality that is not supported by model. However, it can be useful to run warn\_unsupported = true to check you are not skipping tests due to a missing supports\_constraint method or equivalent.
- exclude\_tests\_after is a version number that excludes any tests to MOI added after that version number. This is useful for solvers who can declare a fixed set of tests, and not cause their tests to break if a new patch of MOI is released with a new test.

See also: setup test.

# **Example**

```
config = MathOptInterface.Test.Config()
MathOptInterface.Test.runtests(
    model,
    config;
    include = ["test_linear_"],
    exclude = ["VariablePrimalStart"],
    warn_unsupported = true,
    exclude_tests_after = v"0.10.5",
)
```

MathOptInterface.Test.setup\_test - Function.

```
setup_test(::typeof(f), model::MOI.ModelLike, config::Config)
```

Overload this method to modify model before running the test function f on model with config. You can also modify the fields in config (e.g., to loosen the default tolerances).

This function should either return nothing, or return a function which, when called with zero arguments, undoes the setup to return the model to its previous state. You do not need to undo any modifications to config.

This function is most useful when writing new tests of the tests for MOI, but it can also be used to set test-specific tolerances, etc.

See also: runtests

## **Example**

```
function MOI.Test.setup_test(
    ::typeof(MOI.Test.test_linear_VariablePrimalStart_partial),
    mock::MOIU.MockOptimizer,
    ::MOI.Test.Config,
)

MOIU.set_mock_optimize!(
    mock,
    (mock::MOIU.MockOptimizer) -> MOIU.mock_optimize!(mock, [1.0, 0.0]),
)
mock.eval_variable_constraint_dual = false

function reset_function()
    mock.eval_variable_constraint_dual = true
    return
end
return reset_function
end
```

 ${\tt MathOptInterface.Test.version\_added-Function}.$ 

```
version_added(::typeof(function_name))
```

Returns the version of MOI in which the test function\_name was added.

This method should be implemented for all new tests.

See the exclude tests after keyword of runtests for more details.

source

MathOptInterface.Test.@requires - Macro.

```
@requires(x)
```

Check that the condition x is true. Otherwise, throw an RequirementUnmet error to indicate that the model does not support something required by the test function.

# **Examples**

```
@requires MOI.supports(model, MOI.Silent())
@test MOI.get(model, MOI.Silent())
source
MathOptInterface.Test.RequirementUnmet - Type.
```

```
RequirementUnmet(msg::String) <: Exception
```

An error for throwing in tests to indicate that the model does not support some requirement expected by the test function.