

# QUBO.jl



## A tale of implementation and benchmarking of a Quantum Optimization Ecosystem in Julia

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ISMP 2024 □ July 22, 2024 □ Montreal, Canada



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PUC-Rio, PSR



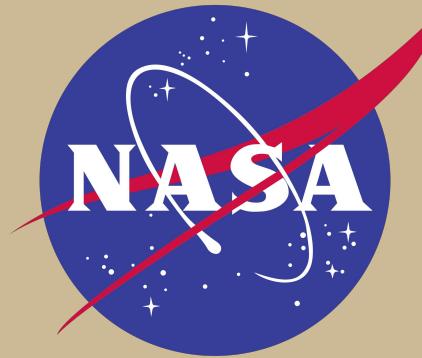
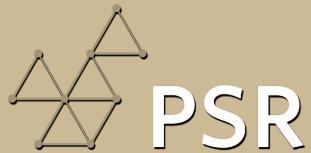
Joaquim Diás Caroia  
PSR



Nelson Maculan  
UFRJ



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USRA  
NASA Quantum AI Lab



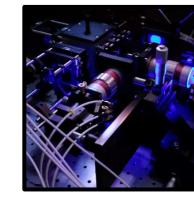
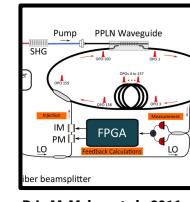
# **QUBO: Quadratic Unconstrained Binary Optimization**

$$\begin{aligned} \min \quad & \mathbf{x}' \mathbf{Q} \mathbf{x} + \boldsymbol{\ell}' \mathbf{x} + c && \text{OBJECTIVE FUNCTION} \\ \text{s.t.} \quad & \mathbf{x} \in \{0, 1\}^n && \text{BINARY VARIABLES} \end{aligned}$$

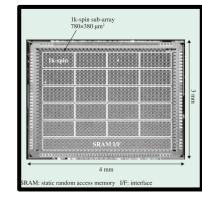
# QUBO: Quadratic Unconstrained Binary Optimization

$$\begin{aligned} \min \quad & \mathbf{x}' \mathbf{Q} \mathbf{x} + \ell' \mathbf{x} + c \\ \text{s.t.} \quad & \mathbf{x} \in \{0, 1\}^n \end{aligned}$$

OBJECTIVE FUNCTION  
BINARY VARIABLES



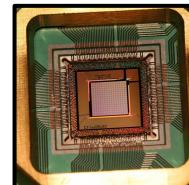
Microsoft Research



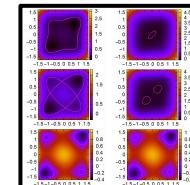
Hitachi, Yamaoka et al.

Quantum Annealing,  
Variational Quantum Eigensolver,  
Alternating Optimization Ansatz,  
Ising Machine, Analog  
Simulated Bifurcation  
Annealing...

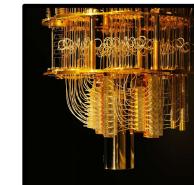
Digital Annealing,  
Quantum Coherent  
Ansatz, Iterative  
Machine, CMOS



D-Wave



Toshiba, Goto et al., 2019



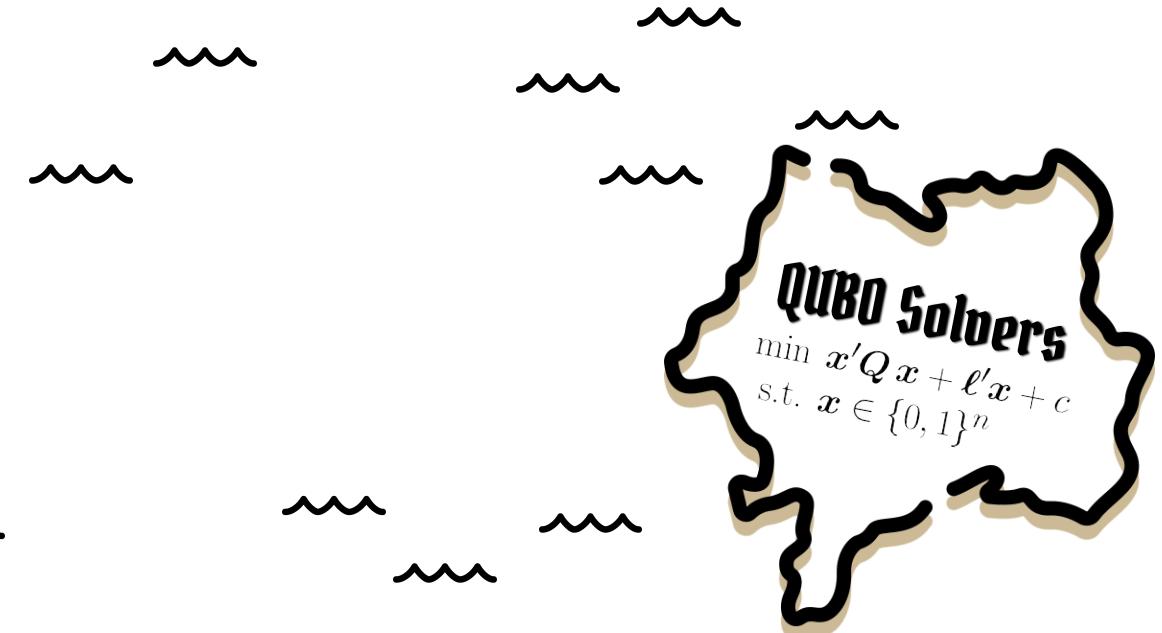
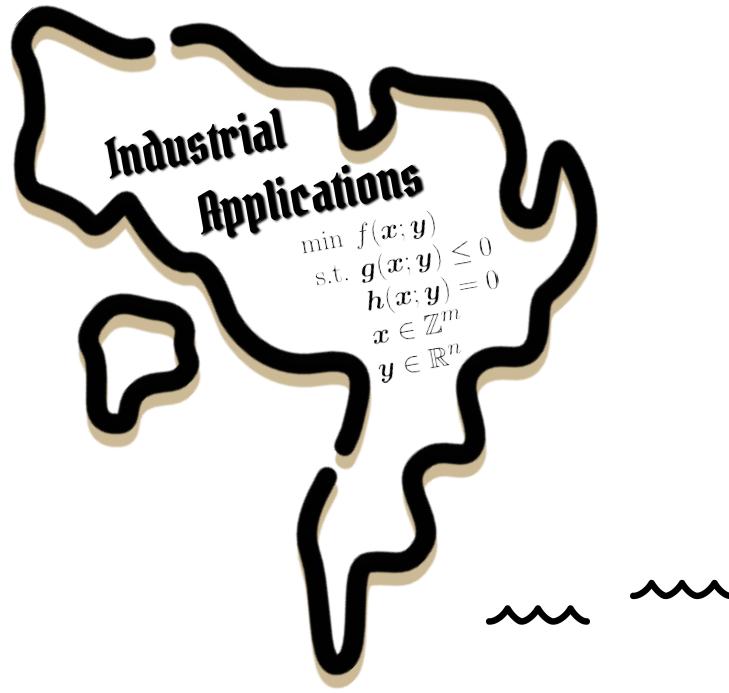
IBM



Fujitsu

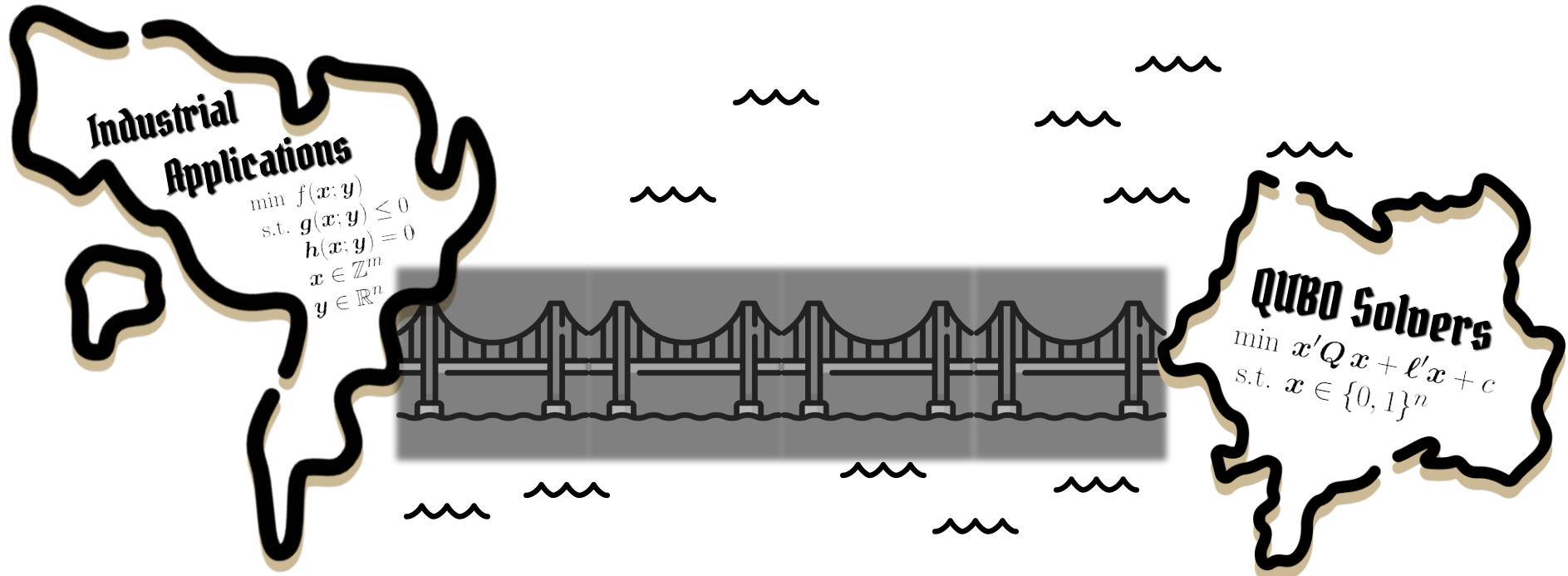


# Our Goal





# Our Goal





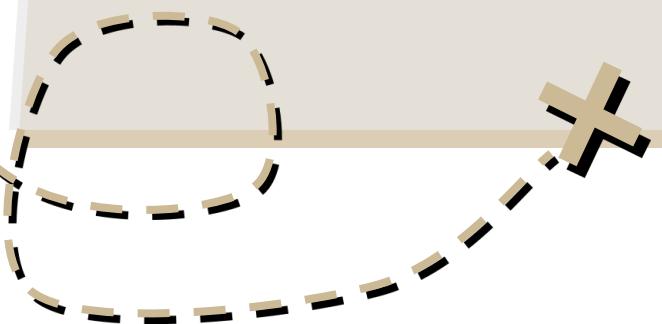
# Tasks



# Tasks

## *Access Solvers*

- Common software Interface





# Tasks

## *Access Solvers*

- Common software Interface

## *Reformulate Models*

- Mathematical Programming Compiler

# Solution Overview

JuMP Model  
(MINLP)

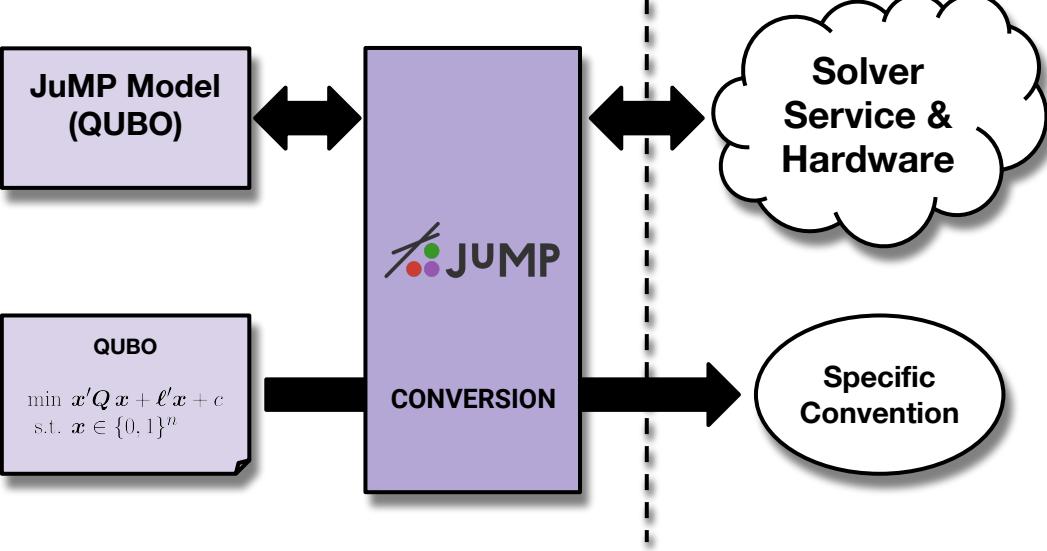
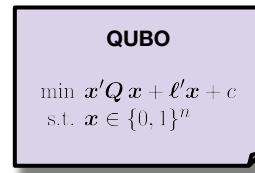
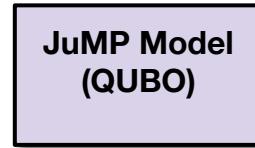
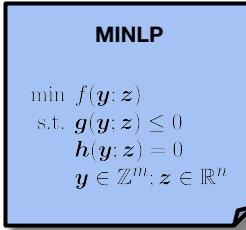
MINLP

$$\begin{aligned} & \min f(\mathbf{y}; \mathbf{z}) \\ \text{s.t. } & g(\mathbf{y}; \mathbf{z}) \leq 0 \\ & h(\mathbf{y}; \mathbf{z}) = 0 \\ & \mathbf{y} \in \mathbb{Z}^m, \mathbf{z} \in \mathbb{R}^n \end{aligned}$$

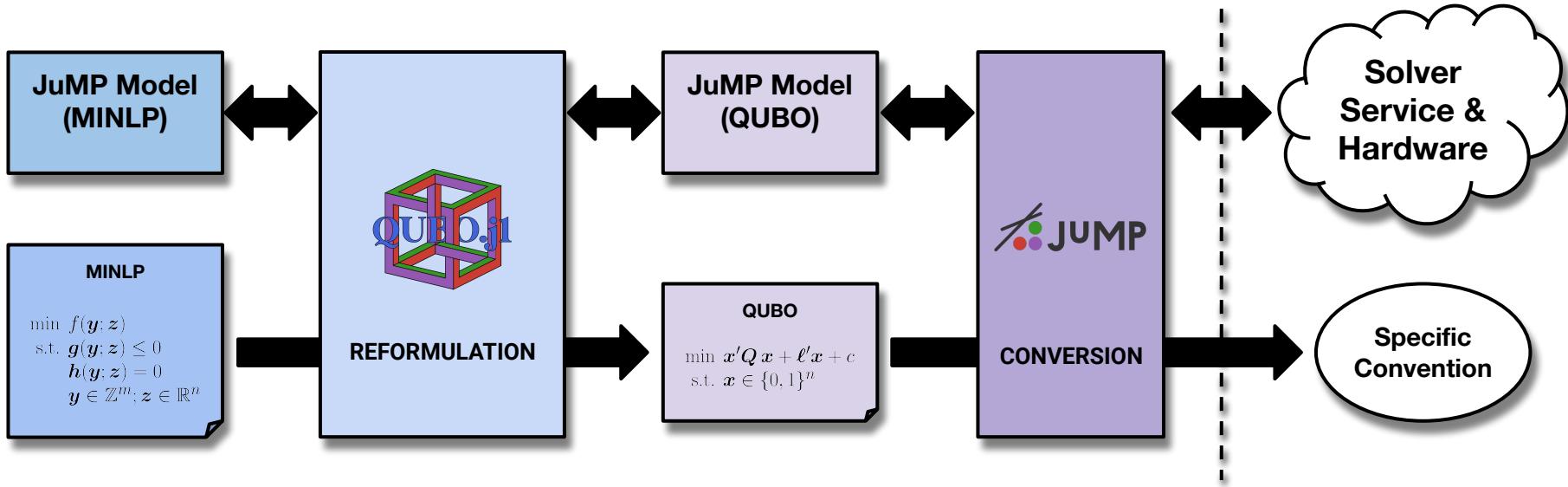
Solver  
Service &  
Hardware

Specific  
Convention

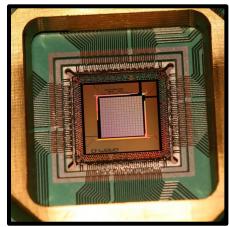
# Solution Overview



# Solution Overview



# Integrating an heterogeneous Solver Landscape



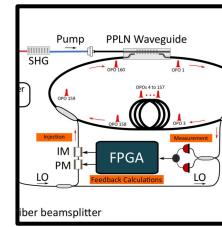
D-Wave



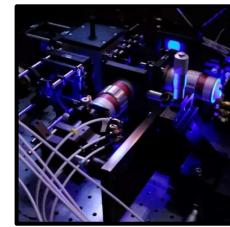
Fujitsu



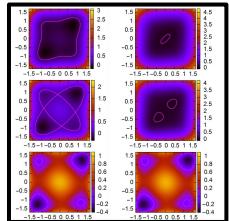
IBM



P. L. McMahon et al., 2016

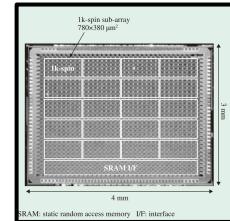


Microsoft Research



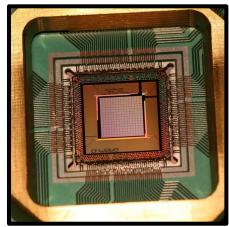
Toshiba, Goto et al., 2019

**Quantum Annealing, Digital Annealing, Variational Quantum Eigensolver, Quantum Alternating Optimization Ansatz, Coherent Ising Machine, Analog Iterative Machine, Simulated Bifurcation Machine, CMOS Annealing...**



Hitachi, Yamaoka et al.

# Integrating an heterogeneous Solver Landscape



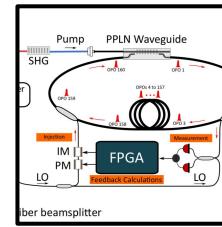
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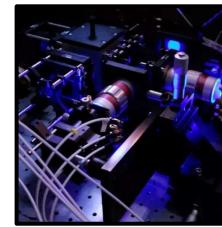
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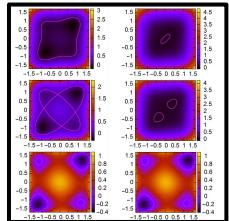
IBM



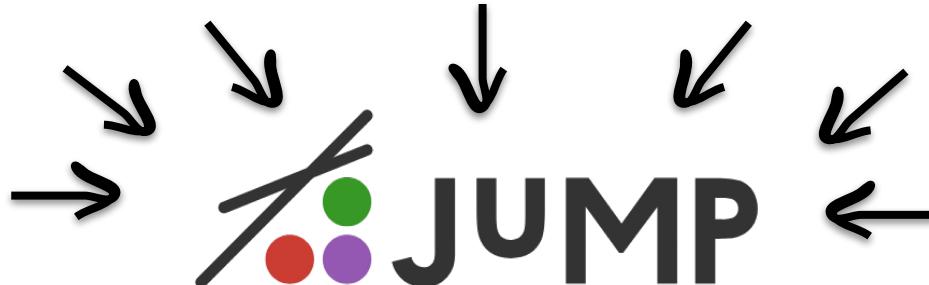
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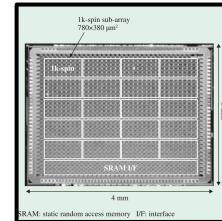
Microsoft Research



Toshiba, Goto et al., 2019



Julia Mathematical Programming



Hitachi, Yamaoka et al

# A Common Solver Interface



```
using JuMP
using QiskitOpt # IBM Qiskit Optimization

model = Model(QiskitOpt.QAOA.Optimizer)

@variable(model, x[1:n], Bin)
@objective(
    model,
    Min,
    x' * Q * x + ℓ' * x + c
)

optimize!(model)

@show objective_value(model)
@show value.(x)
```

```
using JuMP
using DWave # DWave Quantum Annealing

model = Model(DWave.Optimizer)

@variable(model, x[1:n], Bin)
@objective(
    model,
    Min,
    x' * Q * x + ℓ' * x + c
)

optimize!(model)

@show objective_value(model)
@show value.(x)
```

```
using JuMP
using PySA # NASA Parallel Tempering

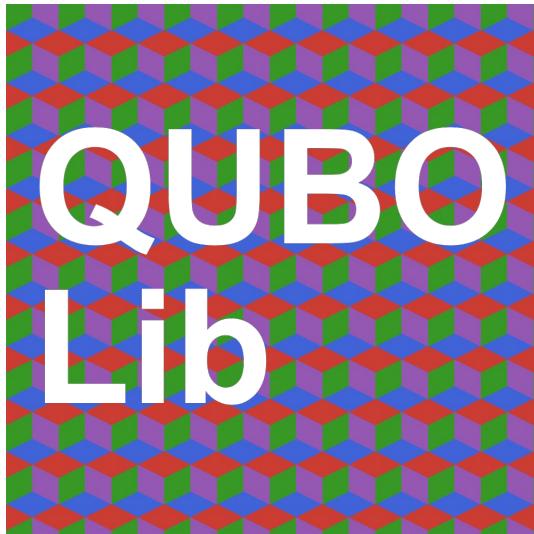
model = Model(PySA.Optimizer)

@variable(model, x[1:n], Bin)
@objective(
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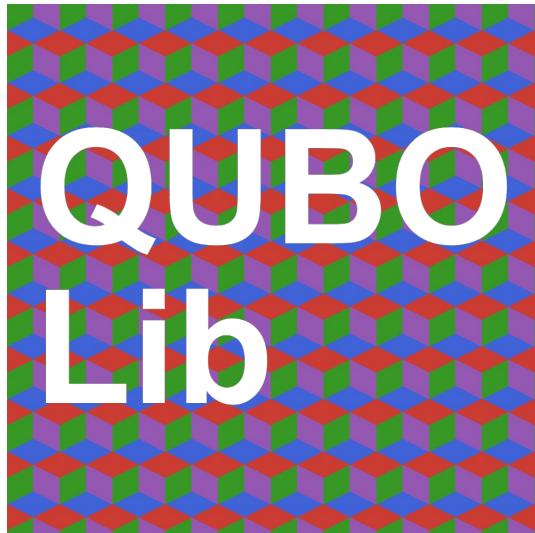
optimize!(model)

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```

# *Testing and Benchmarking Solvers*



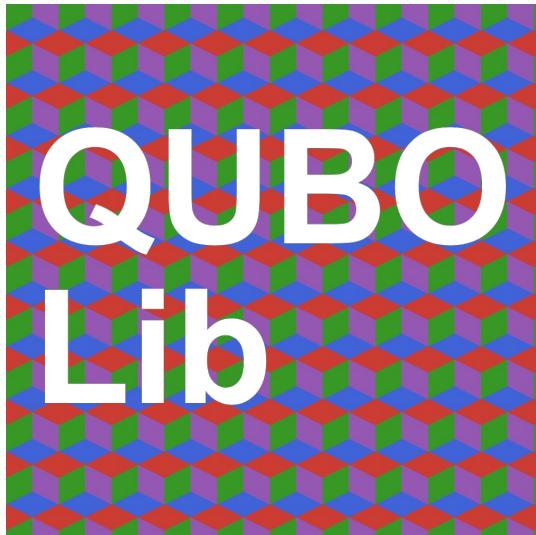
# *Testing and Benchmarking Solvers*



| Sources Summary          |           |            |
|--------------------------|-----------|------------|
| Collection               | Instances | Size Range |
| arXiv:2103.008464 (3R3X) | 2300      | 16 - 4096  |
| arXiv:1903.100928 (3R3X) | 3200      | 16 - 4096  |
| arXiv:1903.100928 (5R5X) | 307       | 24 - 24576 |
| qplib*                   | 23        | 120 - 1225 |

\*QPLIB: A Library of Quadratic Programming Instances, Mathematical Programming Computation, 2018

# *Testing and Benchmarking Solvers*



```
QUBOLib.load_index() do index
    db = QUBOLib.database(index)
    df = DBInterface.execute(
        db,
        """
        SELECT instance FROM Instances
        WHERE dimension < 100 AND quadratic_density < 0.5;
        """,
        ) |> DataFrame

    codes = collect(Int, df[!, :instance])

    @info "Running DWave Neural"
    QUBOLib.run!(
        index, DWave.Neal.Optimizer, codes; solver = "dwave-neal"
    )

    @info "Running DWave (Quantum)"
    QUBOLib.run!(
        index, DWave.Optimizer, codes; solver = "dwave"
    )
end
```



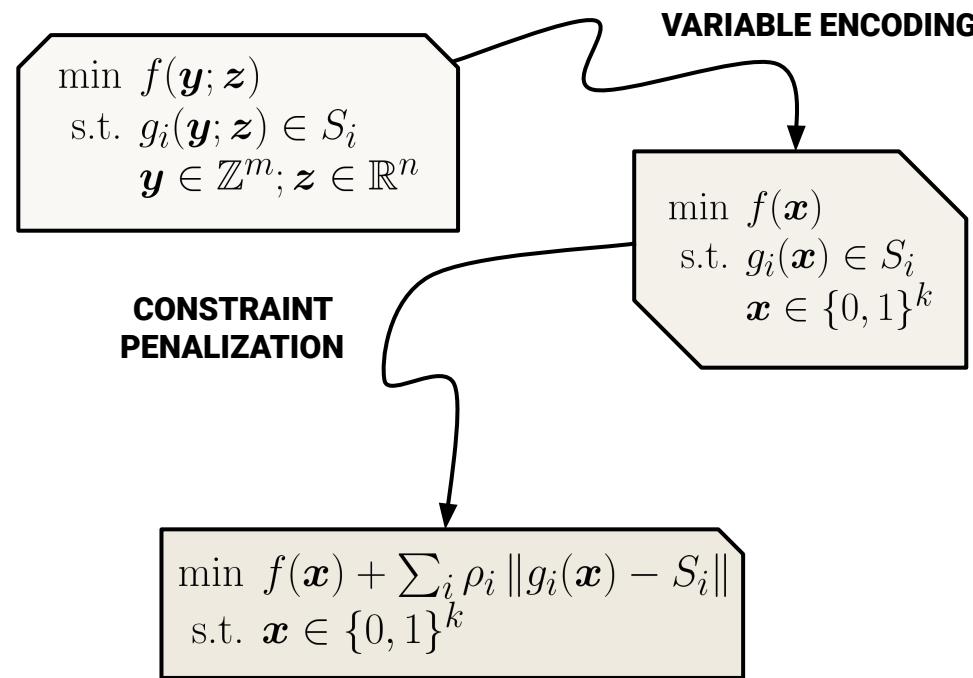
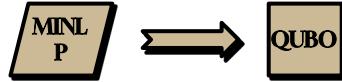
$$\begin{aligned} \min \quad & f(\mathbf{y}; \mathbf{z}) \\ \text{s.t. } & g_i(\mathbf{y}; \mathbf{z}) \in S_i \\ & \mathbf{y} \in \mathbb{Z}^m; \mathbf{z} \in \mathbb{R}^n \end{aligned}$$

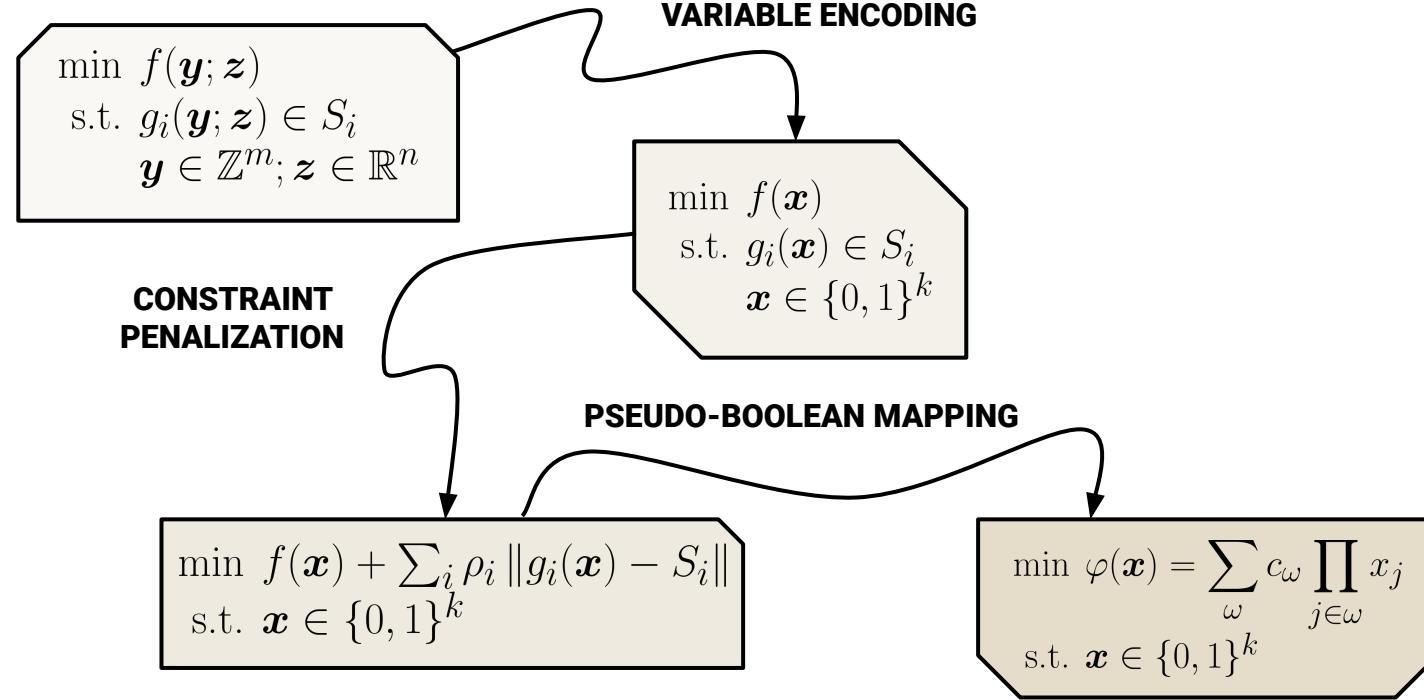


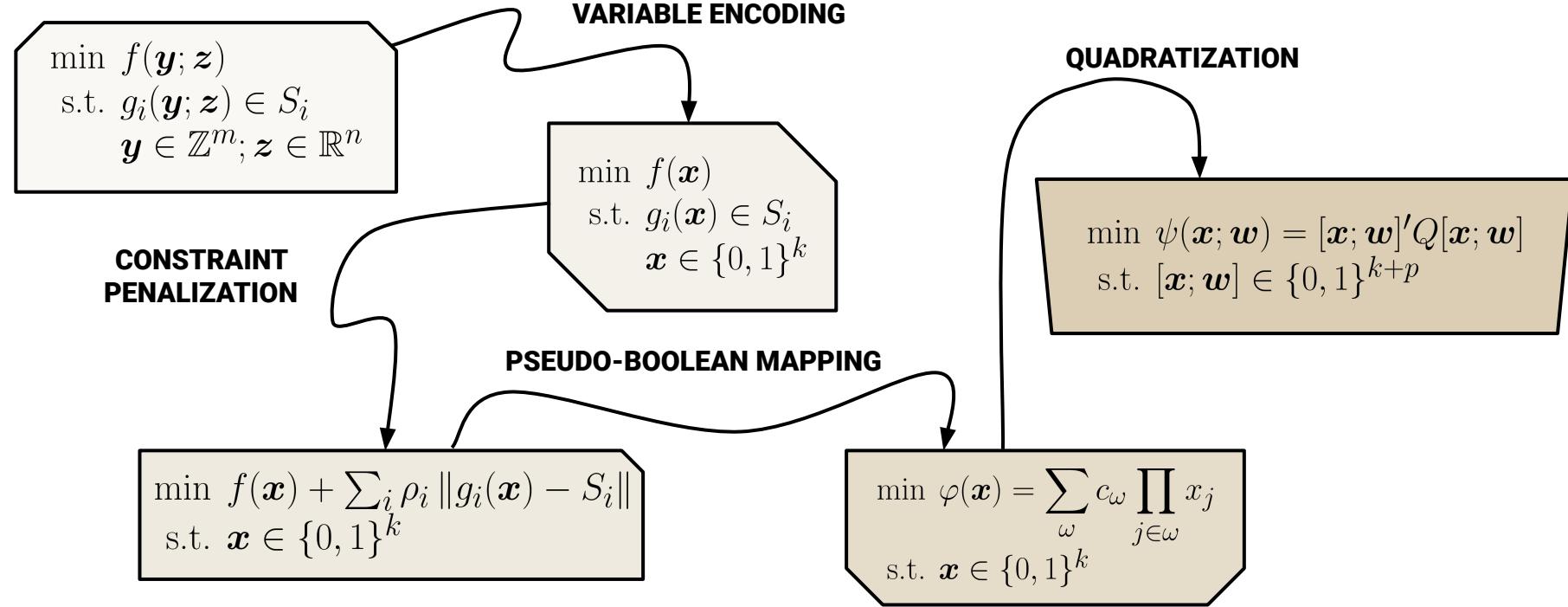
$$\begin{aligned} & \min f(\mathbf{y}; \mathbf{z}) \\ \text{s.t. } & g_i(\mathbf{y}; \mathbf{z}) \in S_i \\ & \mathbf{y} \in \mathbb{Z}^m; \mathbf{z} \in \mathbb{R}^n \end{aligned}$$

### VARIABLE ENCODING

$$\begin{aligned} & \min f(\mathbf{x}) \\ \text{s.t. } & g_i(\mathbf{x}) \in S_i \\ & \mathbf{x} \in \{0, 1\}^k \end{aligned}$$







# A Compiler for Mathematical Programming

C, C++, Julia, Rust...

AMPL, JuMP, Pyomo...

# A Compiler for Mathematical Programming

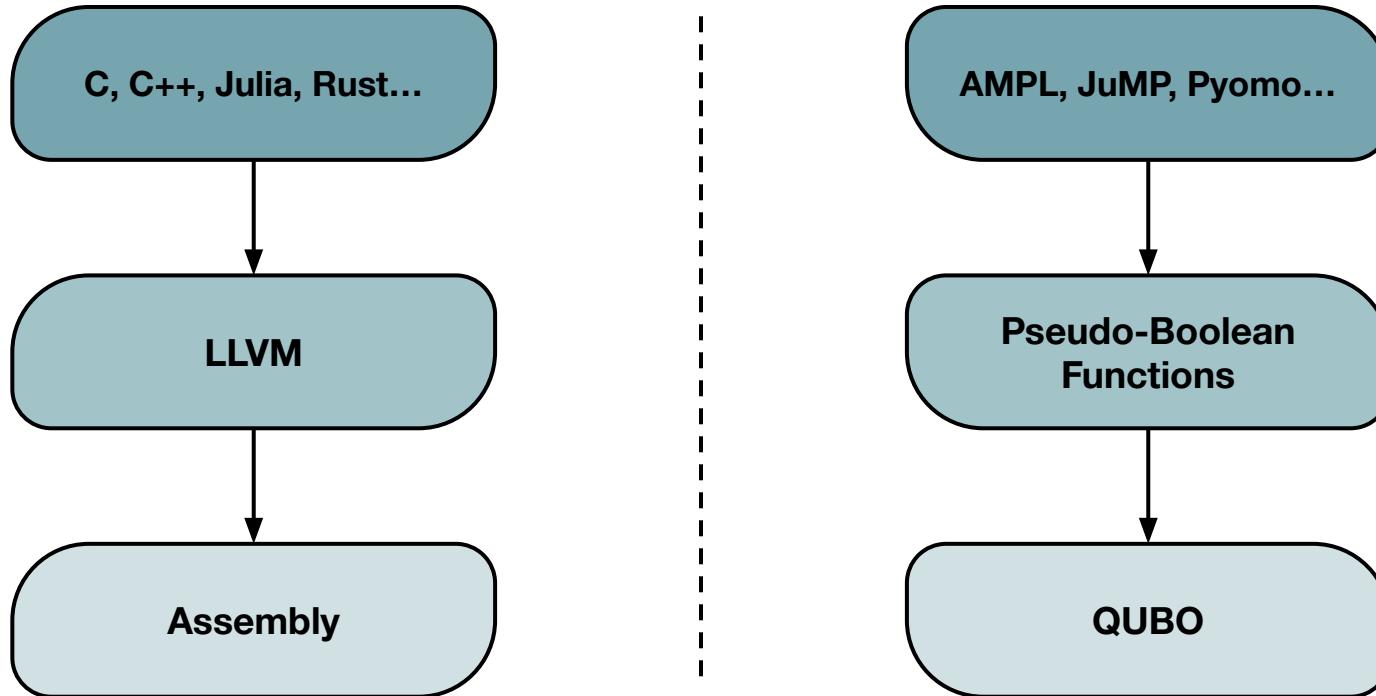
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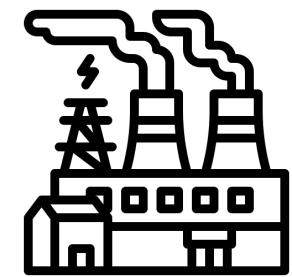
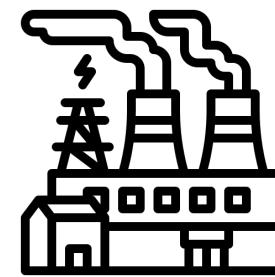
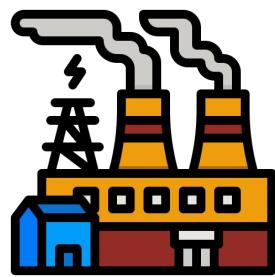
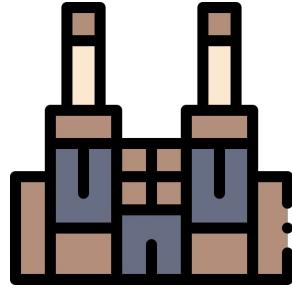
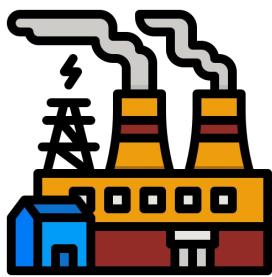
Assembly

QUBO

# A Compiler for Mathematical Programming



# Example: Generation capacity expansion



# Example: Generation capacity expansion

$$\min_{\mathbf{g}, \mathbf{u}, \mathbf{x}} \sum_t \mathbf{c}' \mathbf{g}^{(t)} + \mathbf{i}' \mathbf{x}$$

$$\text{s.a. } \sum_j g_j^{(t)} = d^{(t)} \quad \forall t$$

$$g_j^{(t)} \leq u_j^{(t)} G_j^{\text{(max)}} \quad \forall j, t$$

$$u_j^{(t)} \leq x_j \quad \forall j, t$$

$$g_j^{(t)} \in [0, G_j^{\text{(max)}}] \quad \forall j, t$$

$$u_j^{(t)} \in \{0, 1\} \quad \forall j, t$$

$$x_j \in \{0, 1\} \quad \forall j$$



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BALANCE

$$g_j^{(t)} \leq u_j^{(t)} G_j^{\max} \quad \forall j, t$$

$$u_j^{(t)} \leq x_j \quad \forall j, t$$

$$g_j^{(t)} \in [0, G_j^{\max}]$$

OPERATION

$$u_j^{(t)} \in \{0, 1\} \quad \forall j, t$$

$$x_j \in \{0, 1\} \quad \forall j$$



# Example: Generation capacity expansion

$$\min_{\mathbf{g}, \mathbf{u}, \mathbf{x}} \sum_t \mathbf{c}' \mathbf{g}^{(t)} + \mathbf{i}' \mathbf{x}$$

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**UNIT COMMITMENT**

$$u_j^{(t)} \leq x_j \quad \forall j, t$$

$$g_j^{(t)} \in [0, G_j^{(\max)}] \quad \forall j, t$$

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**UNIT COMMITMENT**

$$x_j \in \{0, 1\} \quad \forall j$$



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$$x_j \in \{0, 1\} \quad \forall j$$



# Example: Generation capacity expansion

$$\begin{array}{ll}\text{min}_{\mathbf{g}, \mathbf{u}, \mathbf{x}} & \text{OPERATION} \quad \text{INVESTMENT} \\ \sum_t \mathbf{c}' \mathbf{g}^{(t)} + \mathbf{i}' \mathbf{x} & \\ \text{s.a. } \sum_j g_j^{(t)} = d^{(t)} & \forall t \quad \text{BALANCE} \\ g_j^{(t)} \leq u_j^{(t)} G_j^{(\max)} & \forall j, t \quad \text{UNIT COMMITMENT} \\ u_j^{(t)} \leq x_j & \forall j, t \quad \text{INVESTMENT} \\ g_j^{(t)} \in [0, G_j^{(\max)}] & \forall j, t \quad \text{OPERATION} \\ u_j^{(t)} \in \{0, 1\} & \forall j, t \quad \text{UNIT COMMITMENT} \\ x_j \in \{0, 1\} & \forall j \quad \text{INVESTMENT}\end{array}$$



# Example: Generation capacity expansion

| OPERATION   | INVESTMENT |
|---|------------|
| $\min_{\mathbf{g}, \mathbf{u}, \mathbf{x}} \sum_t \mathbf{c}' \mathbf{g}^{(t)} + \mathbf{i}' \mathbf{x}$                                    |            |
| s.a. $\sum_j g_j^{(t)} = d^{(t)} \quad \forall t$ <span style="font-size: 10px; margin-left: 100px;"><b>BALANCE</b></span>                  |            |
| $g_j^{(t)} \leq u_j^{(t)} G_j^{(\max)} \quad \forall j, t$ <span style="font-size: 10px; margin-left: 100px;"><b>UNIT COMMITMENT</b></span> |            |
| $u_j^{(t)} \leq x_j \quad \forall j, t$   |            |
| $g_j^{(t)} \in [0, G_j^{(\max)}] \quad \forall j, t$ <span style="font-size: 10px; margin-left: 100px;"><b>OPERATION</b></span>             |            |
| $u_j^{(t)} \in \{0, 1\} \quad \forall j, t$ <span style="font-size: 10px; margin-left: 100px;"><b>UNIT COMMITMENT</b></span>                |            |
| $x_j \in \{0, 1\} \quad \forall j$ <span style="font-size: 10px; margin-left: 100px;"><b>INVESTMENT</b></span>                              |            |



**REFORMULATION**

$\rho_{balance} \left( \sum_t \left( \sum_j g_j^{(t)} - d^{(t)} \right)^2 \right)$

$\rho_{invest} \left( \sum_{j,t} \left( g_j^{(t)} + s_{UB} - x_j G_j^{(\max)} \right)^2 \right)$

$$g_j^{(t)} = \alpha \sum_k 2^k y_{k,j}^{(t)} \text{ s.t. } y_{k,j}^{(t)} \in \{0, 1\}$$

# Example: Generation capacity expansion

| OPERATION  | INVESTMENT             |
|--|------------------------|
| $\min_{\mathbf{g}, \mathbf{u}, \mathbf{x}} \sum_t \mathbf{c}' \mathbf{g}^{(t)} + \mathbf{i}' \mathbf{x}$ |                        |
|  |                        |
| s.a. $\sum_j g_j^{(t)} = d^{(t)} \quad \forall t$  | <b>BALANCE</b>         |
|  |                        |
| $g_j^{(t)} \leq u_j^{(t)} G_j^{(\max)} \quad \forall j, t$   | <b>UNIT COMMITMENT</b> |
|  |                        |
| $u_j^{(t)} \leq x_j \quad \forall j, t$  | <b>INVESTMENT</b>      |
|  |                        |
| $g_j^{(t)} \in [0, G_j^{(\max)}] \quad \forall j, t$   | <b>OPERATION</b>       |
|  |                        |
| $u_j^{(t)} \in \{0, 1\} \quad \forall j, t$  | <b>UNIT COMMITMENT</b> |
|  |                        |
| $x_j \in \{0, 1\} \quad \forall j$   | <b>INVESTMENT</b>      |
|  |                        |



```

1  using JuMP
2  using PySA
3
4  model = Model(PySA.Optimizer)
5
6  @variable(model, 0 ≤ g[1:T,j=1:n] ≤ Gmax[j])
7  @variable(model, u[1:T,1:n], Bin)
8  @variable(model, x[1:n], Bin)
9
10 @objective(model, Min, sum(c'g[t,:] for t=1:T) + i'x)
11
12 @constraint(model, [t=1:T], sum(g[t,:]) = d[t])
13 @constraint(model, [t=1:T,j=1:n], g[t,j] ≤ u[t,j] * Gmax[j])
14 @constraint(model, [t=1:T,j=1:n], u[t,j] ≤ x[j])
15
16 optimize!(model)
17
18 @show objective_value(model)
19 @show value.(x)

```

snappyf.com

# Example: Generation capacity expansion

| OPERATION  | INVESTMENT             |
|--|------------------------|
| $\min_{\mathbf{g}, \mathbf{u}, \mathbf{x}} \sum_t \mathbf{c}' \mathbf{g}^{(t)} + \mathbf{i}' \mathbf{x}$ |                        |
|  | <b>BALANCE</b>         |
| s.a. $\sum_j g_j^{(t)} = d^{(t)} \quad \forall t$  |                        |
|  | <b>UNIT COMMITMENT</b> |
| $g_j^{(t)} \leq u_j^{(t)} G_j^{(\max)} \quad \forall j, t$   |                        |
|  | <b>INVESTMENT</b>      |
| $u_j^{(t)} \leq x_j \quad \forall j, t$  |                        |
|  | <b>OPERATION</b>       |
| $g_j^{(t)} \in [0, G_j^{(\max)}] \quad \forall j, t$   |                        |
|  | <b>UNIT COMMITMENT</b> |
| $u_j^{(t)} \in \{0, 1\} \quad \forall j, t$  |                        |
|  | <b>INVESTMENT</b>      |
| $x_j \in \{0, 1\} \quad \forall j$   |                        |



```

1  using JuMP
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6  @variable(model, 0 ≤ g[1:T,j=1:n] ≤ Gmax[j])
7  @variable(model, u[1:T,1:n], Bin)
8  @variable(model, x[1:n], Bin)
9
10 @objective(model, Min, sum(c'g[t,:] for t=1:T) + i'x)
11
12 @constraint(model, [t=1:T], sum(g[t,:]) = d[t])
13 @constraint(model, [t=1:T,j=1:n], g[t,j] ≤ u[t,j] * Gmax[j])
14 @constraint(model, [t=1:T,j=1:n], u[t,j] ≤ x[j])
15
16 optimize!(model)
17
18 @show objective_value(model)
19 @show value.(x)

```

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# Example: Generation capacity expansion

| OPERATION  | INVESTMENT             |
|--|------------------------|
| $\min_{\mathbf{g}, \mathbf{u}, \mathbf{x}} \sum_t \mathbf{c}' \mathbf{g}^{(t)} + \mathbf{i}' \mathbf{x}$ |                        |
|  | <b>BALANCE</b>         |
| s.a. $\sum_j g_j^{(t)} = d^{(t)} \quad \forall t$  |                        |
|  | <b>UNIT COMMITMENT</b> |
| $g_j^{(t)} \leq u_j^{(t)} G_j^{(\max)} \quad \forall j, t$   |                        |
|  | <b>INVESTMENT</b>      |
| $u_j^{(t)} \leq x_j \quad \forall j, t$  |                        |
|  | <b>OPERATION</b>       |
| $g_j^{(t)} \in [0, G_j^{(\max)}] \quad \forall j, t$   |                        |
|  | <b>UNIT COMMITMENT</b> |
| $u_j^{(t)} \in \{0, 1\} \quad \forall j, t$  |                        |
|  | <b>INVESTMENT</b>      |
| $x_j \in \{0, 1\} \quad \forall j$   |                        |



```

1  using JuMP
2  using PySA
3
4  model = Model(PySA.Optimizer)

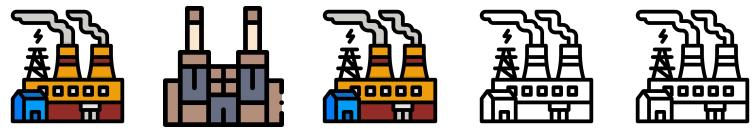
6  @variable(model, 0 ≤ g[1:T,j=1:n] ≤ Gmax[j])
7  @variable(model, u[1:T,1:n], Bin)
8  @variable(model, x[1:n], Bin)
9
10 @objective(model, Min, sum(c'g[t,:] for t=1:T) + i'x)
11
12 @constraint(model, [t=1:T], sum(g[t,:]) = d[t])
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14 @constraint(model, [t=1:T,j=1:n], u[t,j] ≤ x[j])
15
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18 @show objective_value(model)
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```

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# Example: Generation capacity expansion

| OPERATION  | INVESTMENT                                   |
|--|--|
| $\min_{\mathbf{g}, \mathbf{u}, \mathbf{x}} \sum_t \mathbf{c}' \mathbf{g}^{(t)} + \mathbf{i}' \mathbf{x}$ |  |
|  | s.a.   |
|  | $\sum_j g_j^{(t)} = d^{(t)} \quad \forall t$ |
|  | BALANCE                                      |
| $g_j^{(t)} \leq u_j^{(t)} G_j^{(\max)} \quad \forall j, t$   | UNIT COMMITMENT                              |
| $u_j^{(t)} \leq x_j \quad \forall j, t$  | INVESTMENT                                   |
| $g_j^{(t)} \in [0, G_j^{(\max)}] \quad \forall j, t$   | OPERATION                                    |
| $u_j^{(t)} \in \{0, 1\} \quad \forall j, t$  | UNIT COMMITMENT                              |
| $x_j \in \{0, 1\} \quad \forall j$   | INVESTMENT                                   |



```

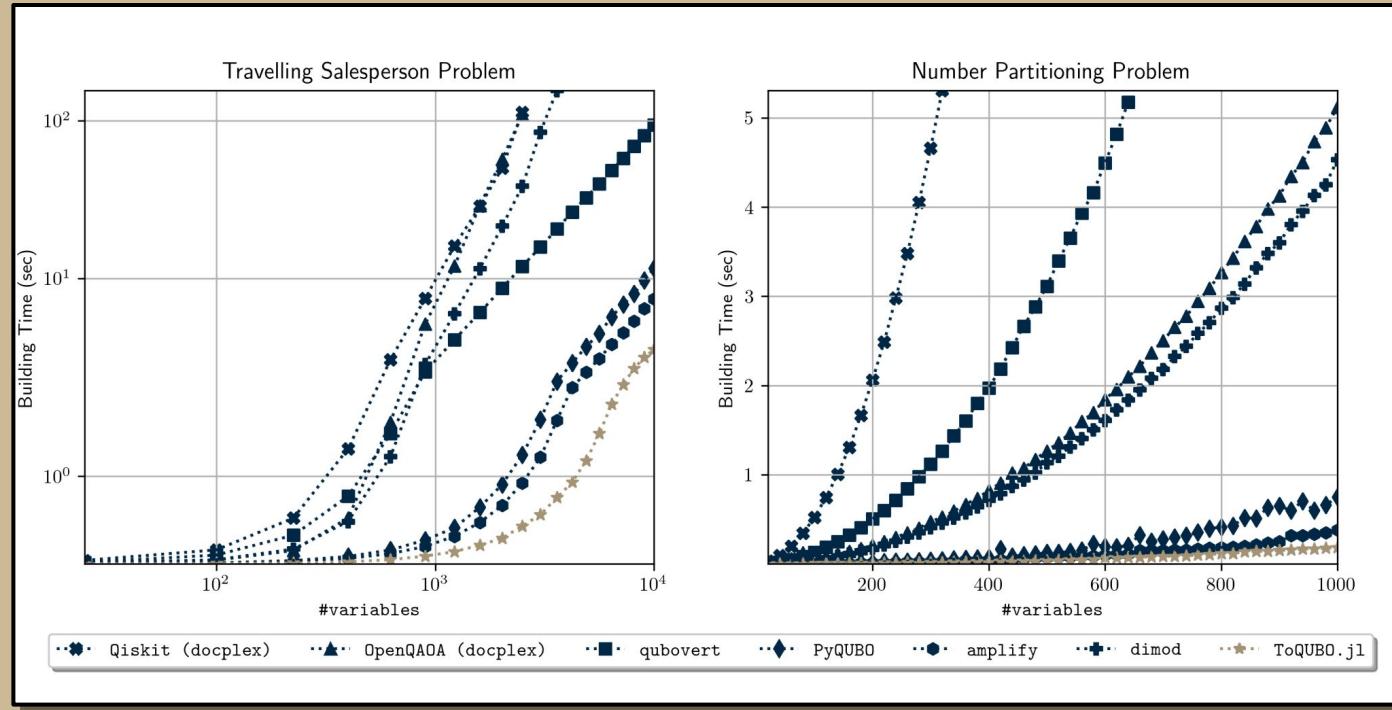
1 using JuMP, QUBO
2 using PySA
3
4 model = Model() → ToQUBO.Optimizer(PySA.Optimizer))

6 @variable(model, 0 ≤ g[1:T,j=1:n] ≤ Gmax[j])
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```

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# Reformulation Performance



# Qualitative Analysis

**CITATION ALERT**

Towards an Automatic Framework for Solving Optimization Problems with Quantum Computers, arXiv:2406.12840, 2024

TABLE I: Comparing the support provided by proposed framework and existing libraries and framework in each step of quantum optimization.

✓ indicates that the corresponding action is performed automatically.

✗ signifies that a proper function is available for implementing the step.

✗ indicates that the method is not fully supported.

+ denotes that logarithmic encoding is also compatible with bases different from two.

\* signifies that the encoding techniques can be exploited only for constraints translation.

† indicates that the polynomial reduction is implemented by exploiting the corresponding qubovert function.

| Supports for each step |                        | Existing Libraries |               |            |             |               |               | Existing Frameworks |              | Proposed Framework |
|------------------------|------------------------|--------------------|---------------|------------|-------------|---------------|---------------|---------------------|--------------|--------------------|
|                        |                        | pyqubo [31]        | qubovert [32] | dimod [33] | Qiskit [34] | fixstars [35] | openQAOA [36] | AutoQUBO [37]       | QUBO.jl [38] |                    |
| Integer Encoding       | Floating Encoding      | ✗                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✗                   | ✓            | ✓                  |
|                        | Logarithmic [39]       | ✓                  | ✓             | ✓          | ✓           | ✓*            | ✗             | ✗                   | ✓            | ✓+                 |
|                        | Unitary [39]           | ✓                  | ✓             | ✗          | ✗           | ✓*            | ✗             | ✗                   | ✓            | ✓                  |
|                        | Dictionary [39]        | ✓                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✗                   | ✓            | ✓                  |
|                        | Domain-Wall [40]       | ✓                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✗                   | ✓            | ✓                  |
|                        | Bounded-Coeff [41]     | ✗                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✗                   | ✓            | ✓                  |
| Penalty Functions      | Arithmetic [42]        | ✗                  | ✗             | ✗          | ✗           | ✓*            | ✗             | ✗                   | ✓            | ✓                  |
|                        | Equality [22] [21]     | ✗                  | ✓             | ✓          | ✓           | ✓             | ✓             | ✗                   | ✓            | ✓                  |
|                        | Inequality [22]        | ✗                  | ✓             | ✓          | ✓           | ✓             | ✓             | ✗                   | ✓            | ✓                  |
|                        | Boolean [22]           | ✓                  | ✓             | ✓          | ✗           | ✓             | ✗             | ✗                   | ✗            | ✓                  |
|                        | UB positive [43]       | ✗                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✗                   | ✗            | ✓                  |
|                        | MQC [43]               | ✗                  | ✗             | ✓          | ✗           | ✗             | ✗             | ✗                   | ✗            | ✓                  |
| Penalty Weight         | VLM [44]               | ✗                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✓                   | ✓            | ✓                  |
|                        | MOMC [43]              | ✗                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✗                   | ✗            | ✓                  |
|                        | MOC [43]               | ✗                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✗                   | ✗            | ✓                  |
|                        | UB Naive [45], [46]    | ✗                  | ✗             | ✗          | ✓           | ✗             | ✗             | ✓                   | ✓            | ✓                  |
|                        | UB posiform [45], [46] | ✗                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✓                   | ✗            | ✓                  |
|                        | Polynomial Reduction   | ✓                  | ✓             | ✓          | ✗           | ✓             | ✗             | ✓                   | ✓            | ✓†                 |
| Solvers                | Dwave QA               | ✓                  | ✗             | ✓          | ✗           | ✓             | ✗             | ✓                   | ✓            | ✓                  |
|                        | QAOA                   | ✗                  | ✗             | ✗          | ✓           | ✗             | ✓             | ✗                   | ✓            | ✓                  |
|                        | VQE                    | ✗                  | ✗             | ✗          | ✓           | ✗             | ✓             | ✗                   | ✓            | ✓                  |
|                        | GAS                    | ✗                  | ✗             | ✗          | ✓           | ✗             | ✗             | ✗                   | ✗            | ✓                  |
|                        | SA                     | ✓                  | ✓             | ✓          | ✓           | ✓             | ✓             | ✓                   | ✓            | ✓                  |
| Solution Decoding      |                        | ✓                  | ✓             | ✓          | ✓           | ✓             | ✓             | ✓                   | ✓            | ✓                  |
| Check Constraints      |                        | ✓                  | ✓             | ✓          | ✗           | ✓             | ✗             | ✓                   | ✓            | ✓                  |
| Penalty Update         | Sequential [47]        | ✗                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✗                   | ✗            | ✓                  |
|                        | Scaled [47]            | ✗                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✗                   | ✗            | ✓                  |
|                        | Binary search [47]     | ✗                  | ✗             | ✗          | ✗           | ✗             | ✗             | ✗                   | ✗            | ✓                  |



# Conclusions & Future Work

## MAIN TAKEAWAYS

- ❑ *QUBO.jl gives OR practitioners a smooth experience for accessing advanced hardware*
- ❑ *Provides tools & infrastructure for benchmarking novel optimization technologies*



# Conclusions & Future Work

## MAIN TAKEAWAYS

- ❑ *QUBO.jl gives OR practitioners a smooth experience for accessing advanced hardware*
- ❑ *Provides tools & infrastructure for benchmarking novel optimization technologies*

## NEXT STEPS

- ❑ *Set up benchmarking service and use results to guide reformulation*
- ❑ *Expand Reformulation Library and Solver support*
- ❑ *Investigate architecture-oriented compilation*



# Thanks!

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