Curso MIEM / MIEGI / MIEIC Data / 04 / 21

Disciplina Análise Matemática II / Complementos Matemática Ano 1º Semestre 2º

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Espaço reservado para o avaliador

MIEM / MIEGI - Capítulo 1 - Exercício 28 a)

MIEIC - Ficha 1 - Exercício 38 a)

Curva $y = e^{-x}$, $x \in \mathbb{R}$

Parametrização : $\vec{r}(t) = (t, e^{-t})$, $t \in \mathbb{R}$

$\vec{r}\,'(t) = (1, -e^{-t})$, $t \in \mathbb{R}$

$\|\vec{r}\,'(t)\| = \left[1 + e^{-2t}\right]^{1/2}$, $t \in \mathbb{R}$

$$\vec{T}(t) = \frac{1}{\left[1+e^{-2t}\right]^{1/2}} (1, -e^{-t}) \quad , \quad t \in \mathbb{R}$$

$$\vec{T}\,'(t) = -\frac{1}{2}(-2)\,e^{-2t}\left[1+e^{-2t}\right]^{-3/2}(1, -e^{-t}) + \frac{1}{\left[1+e^{-2t}\right]^{1/2}}(0, e^{-t}) =$$

$$= \frac{e^{-2t}}{\left[1+e^{-2t}\right]^{3/2}}(1, -e^{-t}) + \frac{e^{-t}}{\left[1+e^{-2t}\right]^{1/2}}(0, 1) =$$

$$= \frac{e^{-2t}}{\left[1+e^{-2t}\right]^{3/2}}(1, -e^{-t}) + \frac{e^{-t}(1+e^{-2t})}{\left[1+e^{-2t}\right]^{3/2}}(0, 1) =$$

$$= \frac{1}{\left[1+e^{-2t}\right]^{3/2}}\left(e^{-2t}, \cancel{-e^{-3t}} + e^{-t} + \cancel{e^{-3t}}\right) =$$

$$= \frac{e^{-t}}{\left[1+e^{-2t}\right]^{3/2}}(e^{-t}, 1) \quad , \quad t \in \mathbb{R}$$

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$$\|\vec{T}'(t)\| = \frac{e^{-t}}{[1+e^{-2t}]^{3/2}} \, [1+e^{-2t}]^{1/2} =$$

$$= \frac{e^{-t}}{1+e^{-2t}}$$

Curvatura :

$$k(t) = \frac{\|\vec{T}'(t)\|}{\|\vec{r}'(t)\|} = \frac{e^{-t}}{[1+e^{-2t}]^{3/2}} \quad , \ t \in \mathbb{R} \quad e \quad k(t) > 0$$

$$k'(t) = \frac{-e^{-t}[1+e^{-2t}]^{3/2} - e^{-t}(3/2)(-2e^{-2t})[1+e^{-2t}]^{1/2}}{[1+e^{-2t}]^{3}} =$$

$$= \frac{-e^{-t}(1+e^{-2t}) + 3e^{-3t}}{[1+e^{-2t}]^{5/2}} = \frac{-e^{-t} + 2e^{-3t}}{[1+e^{-2t}]^{5/2}}$$

$$k'(t) = 0 \iff -e^{-t} + 2e^{-3t} = 0 \iff$$

$$\iff 2e^{-3t} = e^{-t} \iff \ln(2e^{-3t}) = \ln(e^{-t}) \iff$$

$$\iff \ln(2) - 3t = -t \iff t = \frac{1}{2}\ln(2) \iff$$

$$\iff t = \ln(\sqrt{2})$$

$k'(t) = 0$ no ponto $Q = \left(\ln(\sqrt{2}), e^{-\ln(\sqrt{2})}\right) =$

$$= \left(\ln(\sqrt{2}), e^{\ln(1/\sqrt{2})}\right) =$$

$$= \left(\ln(\sqrt{2}), \frac{1}{\sqrt{2}}\right)$$

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O valor da curvatura no ponto Q é

$$\kappa(\ln(\sqrt{2})) = \frac{e^{-\ln(\sqrt{2})}}{\left[1+e^{-2\ln(\sqrt{2})}\right]^{3/2}} = \frac{1/\sqrt{2}}{\left[1+1/2\right]^{3/2}} =$$

$$= \frac{1/\cancel{\sqrt{2}}}{\frac{3}{2}\,\frac{\sqrt{3}}{\cancel{\sqrt{2}}}} = \frac{2}{3\sqrt{3}} = \frac{2\sqrt{3}}{9}$$

e corresponde ao valor máximo da curvatura em qualquer ponto da curva.