1) $f(x,y,z) = xy^2 + z\ln(z)$

a) $\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = \left(y^2, 2xy, \ln(z)+1\right)$

$\nabla f(0,1,1) = (1,0,1)$

$\vec{PQ} = Q - P = (1,2,2) - (0,1,1) = (1,1,1)$

$\vec{u} = \frac{\vec{PQ}}{\|\vec{PQ}\|} = \frac{1}{\sqrt{3}}(1,1,1)$

$f'(P;\vec{u}) = \nabla f(P)\cdot\vec{u} = \frac{1}{\sqrt{3}}(1,0,1)\cdot(1,1,1) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

b) $\vec{r}(t) = \left(e^t - 1, \cos(t), 1+\operatorname{sen}(t)\right)$

$\vec{r}\,'(t) = \left(e^t, -\operatorname{sen}(t), \cos(t)\right)$

$g'(t) = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt} =$

$= \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)\cdot\left(x'(t), y'(t), z'(t)\right) =$

$= \nabla f(x,y,z)\cdot\vec{r}\,'(t)$

$\vec{r}\,'(0) = (1,0,1)$

$\vec{r}(0) = (0,1,1) = P$

$g'(0) = \nabla f(0,1,1)\cdot\vec{r}\,'(0) = (1,0,1)\cdot(1,0,1) = 2$

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2) $f(x,y,z) = 0$ em que $f(x,y,z) = 2z\cos(\pi x) - e^{y-1} - 1$

a)

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = \left(-2\pi z \operatorname{sen}(\pi x), -e^{y-1}, 2\cos(\pi x)\right)$$

$$\nabla f(1,1,-1) = (0,-1,-2)$$

O vector $\vec{u} = (0,-1,-2)$ é um vector normal à superfície no ponto $R = (1,1,-1)$.

b) Equação cartesiana do plano tangente à superfície em R é

$$(X - R) \cdot \vec{u} = 0 \Leftrightarrow (x-1, y-1, z+1) \cdot (0,-1,-2) = 0 \Leftrightarrow$$

$$\Leftrightarrow -y + 1 - 2z - 2 = 0 \Leftrightarrow y + 2z = -1$$

3) Linha: $\vec{r}(t) = \left(e^{-2t}, \operatorname{sen}(2t), 1 - t^3\right), t \in \mathbb{R}$

a) $\vec{r}\,'(t) = \left(-2e^{-2t}, 2\cos(2t), -3t^2\right)$

$S = (1,0,1) = \vec{r}(0)$

$\vec{r}\,'(0) = (-2, 2, 0)$

A equação vectorial da recta tangente à linha no ponto S é

$$\vec{x}(\theta) = S + \theta\, \vec{r}\,'(0) = (1,0,1) + \theta(-2,2,0), \quad \theta \in \mathbb{R}$$

b) $B(0) = T(0) \times N(0) = \dfrac{\vec{r}\,'(0) \times \vec{r}\,''(0)}{\| \vec{r}\,'(0) \times \vec{r}\,''(0) \|}$

$\vec{r}\,''(t) = \left(4e^{-2t}, -4\operatorname{sen}(2t), -6t\right)$

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$\vec{r}''(0) = (4,0,0)$

$$\vec{r}'(0) \times \vec{r}''(0) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 2 & 0 \\ 4 & 0 & 0 \end{vmatrix} = (0,0,-8)$$

$\| \vec{r}'(0) \times \vec{r}''(0) \| = 8$

$$B(0) = \frac{1}{8}(0,0,-8) = (0,0,-1) = -\vec{k}$$

4) $z = f(x,y)$, tal que $e^{yz} + x + z^2 = 2$

a) Derivando em ordem a x

$$y \frac{\partial z}{\partial x} e^{yz} + 1 + 2z \frac{\partial z}{\partial x} = 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{\partial z}{\partial x}\left[y e^{yz} + 2z \right] = -1 \Leftrightarrow \frac{\partial z}{\partial x} = \frac{-1}{y e^{yz} + 2z}$$

Derivando $\frac{\partial z}{\partial x}$ em ordem a x

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{-(-1)\frac{\partial}{\partial x}(y e^{yz} + 2z)}{(y e^{yz} + 2z)^2} \Leftrightarrow$$

$$\Leftrightarrow \frac{\partial^2 z}{\partial x^2} = \frac{y^2 \frac{\partial z}{\partial x} e^{yz} + 2\frac{\partial z}{\partial x}}{(y e^{yz} + 2z)^2} = \frac{(y^2 e^{yz} + 2)}{(y e^{yz} + 2z)^2} \frac{(-1)}{y e^{yz} + 2z} \Leftrightarrow$$

$$\Leftrightarrow \frac{\partial^2 z}{\partial x^2} = \frac{-y^2 e^{yz} - 2}{(y e^{yz} + 2z)^3}$$

b) Considerando $x = y = 0$ obtém-se para z

$$e^0 + z^2 = 2 \Leftrightarrow z^2 = 1 \Leftrightarrow z = -1 \vee z = 1$$

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$$\frac{\partial z}{\partial x}(0,0,-1) = \frac{-1}{-2} = \frac{1}{2}$$

$$\frac{\partial^2 z}{\partial x^2}(0,0,-1) = \frac{-2}{-8} = \frac{1}{4}$$

$$\frac{\partial z}{\partial x}(0,0,1) = -\frac{1}{2}$$

$$\frac{\partial^2 z}{\partial x^2}(0,0,1) = -\frac{1}{4}$$

5) Curva : $\vec{r}(s)$: $\|\vec{r}(s)\| = k$, $\forall_{s\in[0,a]} \wedge k>0$

$$\|\vec{r}(s)\| = k \Leftrightarrow \vec{r}(s)\cdot\vec{r}(s) = k^2$$

Derivando em ordem a s (comprimento de arco)

$$\vec{r}\,'(s)\cdot r(s) + \vec{r}(s)\cdot\vec{r}\,'(s) = 0 \Leftrightarrow 2\,\vec{r}\,'(s)\cdot\vec{r}(s) = 0 \Leftrightarrow$$

$$\Leftrightarrow \vec{r}\,'(s)\cdot\vec{r}(s) = \vec{r}(s)\cdot\vec{r}\,'(s) = 0$$

Derivando $\vec{r}(s)\cdot\vec{r}\,'(s) = 0$ em ordem a s

$$\vec{r}\,'(s)\cdot\vec{r}\,'(s) + \vec{r}(s)\cdot\vec{r}\,''(s) = 0 \Leftrightarrow \vec{r}(s)\cdot\vec{r}\,''(s) = -\|\vec{r}\,'(s)\|^2$$

Considerando a parametrização $\vec{r}(t)$, $t\in I$ para a mesma curva, verifica-se

$$\vec{r}\,'(t) = \frac{d\vec{r}}{ds}\frac{ds}{dt} \Leftrightarrow \frac{d\vec{r}}{ds} = \frac{\vec{r}\,'(t)}{s'(t)} = \frac{\vec{r}\,'(t)}{\|\vec{r}\,'(t)\|}$$

ou seja, $\frac{d\vec{r}}{ds} = \vec{r}\,'(s)$ é versor (norma unitária)

Conclui-se, então,

$$\vec{r}(s)\cdot\vec{r}\,''(s) = -1$$

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