① $C: \begin{cases} x^2+y^2+z^2=2 \\ z=1 \end{cases}$ $\quad \begin{cases} x^2+y^2=1 \\ z=1 \end{cases}$

$\bar{r}(\theta) = (\cos\theta, \operatorname{sen}\theta, 1)$, $\theta \in [0, 2\pi]$

$\bar{r}'(\theta) = (-\operatorname{sen}\theta, \cos\theta, 0)$

$\bar{f}(x,y,z) = (P,Q,R) = (-xz, y, y)$

$\bar{f}[\bar{r}(\theta)] = (-\cos\theta, \operatorname{sen}\theta, \operatorname{sen}\theta)$

$\bar{f}[\bar{r}(\theta)] \cdot \bar{r}'(\theta) = \operatorname{sen}\theta\cos\theta + \operatorname{sen}\theta\cos\theta = 2\operatorname{sen}\theta\cos\theta$

$$\int_C -P\,dx + Q\,dy + R\,dz = \int_0^{2\pi} 2\operatorname{sen}\theta\cos\theta\,d\theta = 0$$

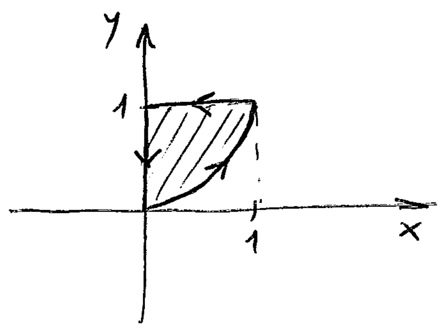
---

② $\bar{f}(x,y) = (P,Q) = (2y^3 + \beta y x^2 + 2, \alpha x y^2 + x^3 + 1)$

$C: y=1, y=x^3, 0 \le x \le 1$ (directo)

a) $\alpha = \beta = 0$

$\bar{f}(x,y) = (2y^3+2, x^3+1) = (P,Q)$



$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 3x^2 - 6y^2$$

$$\oint_C \bar{f}\cdot d\bar{r} = 3\int_0^1\int_{x^3}^1 x^2 - 2y^2\,dy\,dx =$$

$$= 3\int_0^1 \left[x^2y - \tfrac{2}{3}y^3\right]_{x^3}^1 dx = 3\int_0^1 x^2 - \tfrac{2}{3} - x^5 + \tfrac{2}{3}x^9\,dx =$$

$$= 3\left[\frac{x^3}{3} - \frac{2}{3}x - \frac{x^6}{6} + \frac{1}{15}x^{10}\right]_0^1 =$$

$$= 3\left(\frac{1}{3} - \frac{2}{3} - \frac{1}{6} + \frac{1}{15}\right) = -1 - \frac{1}{2} + \frac{1}{5} = -\frac{3}{2} + \frac{1}{5} = -\frac{13}{10}$$

b) $$\frac{\partial P}{\partial y} = 6y^2 + \beta x^2 \qquad \frac{\partial Q}{\partial x} = \alpha y^2 + 3x^2$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \iff \alpha = 6 \wedge \beta = 3$$

c) $$\frac{\partial \varphi}{\partial x} = P = 2y^3 + 3yx^2 + 2 \Rightarrow \varphi(x,y) = 2xy^3 + yx^3 + 2x + \phi_1(y) + k_1$$

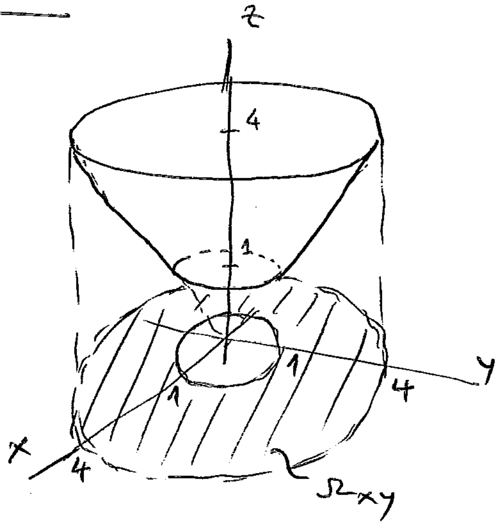
$$\frac{\partial \varphi}{\partial y} = Q = 6xy^2 + x^3 + 1 \Rightarrow \varphi(x,y) = 2xy^3 + yx^3 + y + \phi_2(x) + k_2$$

$$\varphi(x,y) = 2xy^3 + yx^3 + 2x + y + k$$

$$\int_C \bar{f} \cdot d\bar{r} = \varphi(1,1) - \varphi(0,0) = 2 + 1 + 2 + 1 + k - k = 6$$

---

(3) $$z = \sqrt{x^2+y^2}, \quad 1 \le z \le 4$$



$$\Omega_{xy} : 1 \le x^2+y^2 \le 4^2$$

$$\bar{r}(x,y) = \left(x, y, \sqrt{x^2+y^2}\right)$$

$$\frac{\partial \bar{r}}{\partial x} = \left(1, 0, \frac{x}{\sqrt{x^2+y^2}}\right)$$

$$\frac{\partial \bar{r}}{\partial y} = \left(0, 1, \frac{y}{\sqrt{x^2+y^2}}\right)$$

$$\bar{N}(x,y) = \left(\frac{-x}{\sqrt{x^2+y^2}}, \frac{-y}{\sqrt{x^2+y^2}}, 1\right)$$

$$\| N(x,y) \| = \sqrt{\frac{x^2+y^2}{x^2+y^2} + 1} = \sqrt{2}$$

$$A(S) = \iint_\Omega \| N(x,y) \| \, dx\, dy = \sqrt{2} \iint_\Omega dx\, dy = \sqrt{2}\, A(\Omega) = 15\sqrt{2}\,\pi$$

④ $S = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2 \leq 1 \wedge z = xy\}$

$\bar{f}(x,y,z) = (P,Q,R) = (y,x,z)$

a) $\bar{r}(x,y) = (x,y,xy) \longrightarrow R_1: 0 \leq x^2+y^2 \leq 1$

$\bar{r}(u,v) = (u\cos v,\ u \operatorname{sen} v,\ u^2 \operatorname{sen} v \cos v)$ (30)

$(u,v) \in R \qquad R: 0 \leq v \leq 2\pi \wedge 0 \leq u \leq 1$

$\frac{\partial \bar{r}}{\partial x} = (1,0,y)$

$\frac{\partial \bar{r}}{\partial u} = (\cos v,\ \operatorname{sen} v,\ 2u \operatorname{sen} v \cos v)$

$\frac{\partial r}{\partial y} = (0,1,x)$

$\frac{\partial \bar{r}}{\partial v} = (-u\operatorname{sen} v,\ u\cos v,\ u^2\cos^2 v - u^2 \operatorname{sen}^2 v) =$

$= u(-\operatorname{sen} v,\ \cos v,\ u\cos^2 v - u \operatorname{sen}^2 v)$

$\bar{N}(x,y) = (-y,-x,1)$

$\bar{N}(u,v) = u(u \operatorname{sen} v \cos^2 v - u \operatorname{sen}^3 v - 2u \operatorname{sen} v \cos^2 v,\ -2u \operatorname{sen}^2 v \cos v - u\cos^3 v +$
$+ u \cos v \operatorname{sen}^2 v,\ 1) =$

$= u(-u\operatorname{sen}^3 v - u \operatorname{sen} v \cos^2 v,\ -u\cos^3 v - u\cos v \operatorname{sen}^2 v,\ 1)$

$= u(-u\operatorname{sen} v(\operatorname{sen}^2 v + \cos^2 v),\ -u\cos v(\cos^2 v + \operatorname{sen}^2 v),\ 1) =$ (40)

$= u(-u\operatorname{sen} v,\ -u\cos v,\ 1)$

$\|\bar{N}(x,y)\| = \sqrt{1+(x^2+y^2)}$

$\|\bar{N}(u,v)\| = u\sqrt{u^2+1}$

$\bar{n}(u,v) = \frac{\bar{N}(u,v)}{\|\bar{N}(u,v)\|} = \frac{1}{\sqrt{u^2+1}}(-u\operatorname{sen} v, -u\cos v, 1)$

$\bar{n}(x,y) = \frac{1}{\sqrt{1+(x^2+y^2)}}(-y,-x,1)$ (30)

b) $\bar{f}[\bar{r}(u,v)] = (u\operatorname{sen} v,\ u\cos v,\ u^2 \operatorname{sen} v \cos v)$

$\bar{f}[\bar{r}(u,v)] \cdot \bar{N}(u,v) = u\left[-u^2 \operatorname{sen}^2 v \cancel{+} u^2 \cos^2 v + u^2 \operatorname{sen} v \cos v\right] =$

$= u^3(-1 + \operatorname{sen} v \cos v)$

$\bar{f}[\bar{r}(x,y)] = (y,x,xy)$

$\iint_S (\bar{f}\cdot\bar{n})\,dS = \int_0^{2\pi}\int_0^1 u^3(-1+\operatorname{sen} v\cos v)\,du\,dv =$

$[\bar{f}[\bar{r}(x,y)] \cdot \bar{N}(x,y) = -y^2 + x^2 + xy$

$= \frac{1}{4}\int_0^{2\pi} -1 + \operatorname{sen} v \cos v\,dv = -\frac{\pi}{2}$

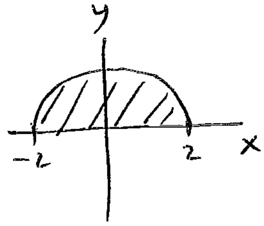
$\iint_{R_1} \cancel{xy - x^2 - y^2}\, dR_1 = \int_0^{2\pi}\int_0^1 \cancel{r^3 \operatorname{sen}\theta\cos\theta - r^3}\,dr\,d\theta = 2\pi\left[\frac{r^2}{2}+\frac{r^4}{4}\right]_0^1 = -\frac{\pi}{2}$

$xy - x^2 - y^2$ 75 (60)

$r^3 \operatorname{sen}\theta\cos\theta - r^3$ 25

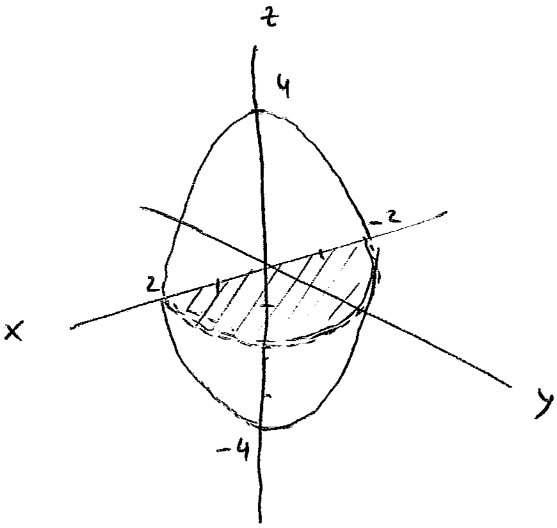
(5) $$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{x^2+y^2-4}^{4-x^2-y^2} dz\,dy\,dx$$

a)



$z = x^2+y^2-4$

$z = 4-(x^2+y^2)$



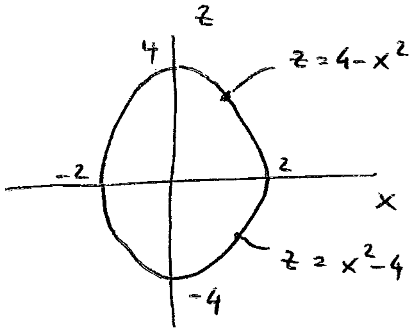
b) $0 \le \theta \le \pi \ \wedge \ 0 \le r \le 2$

$r^2-4 \le z \le 4-r^2$

$$\int_0^{\pi}\int_0^{2}\int_{r^2-4}^{4-r^2} r\,dz\,dr\,d\theta = \pi\int_0^2 r\left[4-r^2-r^2+4\right]dr =$$

$$= \pi\, 2\int_0^2 4r-r^3\,dr = 2\pi\left[2r^2-\frac{1}{4}r^4\right]_0^2 = 8\pi$$

c) $\Omega_{xz}: \ -2 \le x \le 2 \ \wedge \ x^2-4 \le z \le 4-x^2$



$\Omega_1: \ -2 \le x \le 2 \ \wedge \ 0 \le z \le 4-x^2$

$0 \le y \le \sqrt{4-x^2-z}$

$\Omega_2: \ -2 \le x \le 2 \ \wedge \ x^2-4 \le z \le 0$

$0 \le y \le \sqrt{4+z-x^2}$

$$\int_{-2}^{2}\int_{0}^{4-x^2}\int_{0}^{\sqrt{4-x^2-z}} dy\,dz\,dx + \int_{-2}^{2}\int_{x^2-4}^{0}\int_{0}^{\sqrt{4+z-x^2}} dy\,dz\,dx$$

$4-x^2-y^2 = z$

$y^2 = 4-x^2-z$

$y = \pm\sqrt{4-x^2-z}$

$x^2+y^2-4 = z$

$y^2 = 4+z-x^2$

$y = \pm\sqrt{4+z-x^2}$